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Fuzzy observer-controller design in finite frequency domain: application to wind turbine

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Abstract: This work presents the generalized Kalman Yakubovich-Popuv (gKYP) combined with the Takagi Sugeno (T-S) fuzzy model to design a fuzzy robust state feedback controller and a fuzzy robust observer-based in finite frequency (FF) domain. T-S fuzzy model is well known for its efficiency to control complex nonlinear systems. However, for wind generator system, the unknown parts are large and produce disturbances parameters. In order to attenuate, the level of the disturbances parameters observer based is utilized to estimate the unknown parts of the wind system. The control design method is based on Lyapunov function, the generalized gKYP with projection lemma, a PDC (Parallel Distributed Compensation) structure and the finite frequency (FF) technique. The proposed approach is formulated linear matrix inequalities (LMIs) to prove the asymptotic stability in (FF) domain. Finally, an example of wind turbine is showing the validity of the proposed new approach.

Keywords: State feedback Controller; Observer-based controller; Takagi Sugeno fuzzy model; Finite frequency (FF); Linear matrix inequality (LMI); Wind system.

Introduction

Wind energy technology has evolved rapidly over the last decades and is nowadays becomes the most competitive of the new power generation sources in the world. Therefore, wind energy has shown the fastest rate of growth of any form of electricity generation with its development and security of supply. Its exploitation has been one of the most dynamically growing for the last years. By the end of 2010, wind turbines were generating 2.5% of the world electricity consumption, and the global wind installed capacity exceeded 197 GW [21][17]. The control plays a very important role to make wind technology more profitable and reliable. The first objective of control of wind turbine is to extract as much energy from the wind as possible. The issue of the wind turbine control is the uncontrollable and stochastic nature of the principal component, to deal with this problem, robust controller conditions are designed to cope with nonlinear dynamics through the use of linearization techniques, as well as other methods, have been widely applied to the design of wind control systems. Nonlinearities and system uncertainties are the most important difficulties in designing controllers that ensure stability and good performances. Recently, fuzzy control has attracted growing, attention, essentially because it can give an effective solution to the control of complex nonlinear systems [22]. In this study we are interested in the T-S fuzzy model to represent a class of nonlinear systems of the wind turbine, these fuzzy systems are described by a family of fuzzy IF-THEN rules which makes the control algorithm easily understood [1][2].

In this work, we are focused in control of wind system formed by a turbine, a driving shaft and a double fed induction machine. The control plays a very important role to make wind technology more profitable and reliable the primary objective of control of wind turbine is to extract as much energy from the wind as possible [13]. It is well known that the celebrated gKYP lemma effectively builds a bridge between the frequency-domain approach and time-domain approach [23]. However, when the external disturbance belongs to a certain frequency range which is known beforehand, it is not favorable to control the system in the full frequency domain, because this may introduce some conservatism and poor performances. Recently, the control synthesis in a finite frequency domain has been addressed, and there have appeared many results in this domain [15] [16].

Hence, this paper presents a novel approach based on Finite Frequency T-S fuzzy model to deal with problem of control for nonlinear systems. In this approach, observer-based is used to estimate the unknown parts of the wind system, LMIs design combined with gKYP lemma are defined and employed to find feedback gains and observer gains of a fuzzy controller. Next, the proposed feedback control is established by the FF approach to command the wind turbine system in a specific band of frequency. These conditions are formulated as LMIs. The proposed design approach is practically useful because LMIs can be solved very efficiently using the convex optimization techniques.

The paper is arranged as follows: Section (II) introduce the T-S fuzzy model with disturbances parameters and gives the proposed control law. In section (III) a sufficient condition based on the finite frequency technique, the gKYP lemma, LMIs conditions and observer-based controller are developed in order to command a wind turbine system and guarantee the performances. Section (IV) simulation results are given to highlight the effectiveness of the design procedure of the observer and of the controller wind turbine. Section (V) concludes this paper.

System description and problem formulation

In this section, we present a TS fuzzy model with a disturbances parameter and an observer-based controller applied to ensure the stability of the whole system.

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T-S fuzzy model

T–S fuzzy model has nowadays become a popular since it showed its efficiency to control a complex nonlinear system and has been used for nonlinear systems. The fuzzy system is described by fuzzy If–Then rule, which represented local linear input–output relations of nonlinear system [1]. The (TS) fuzzy model used in this paper is presented as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \lambda_i(z(t))[A_i x(t) + B_i u(t) + B_{2i}\delta(t)] \\ y(t) = \sum_{i=1}^{r} \lambda_i(z(t))[C_i x(t)] \end{cases} \tag{1}$$

Given a pair of $(x(t), u(t))$, the final outputs of the fuzzy systems are inferred as follows

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} \mu_i(z(t))[A_i x(t) + B_i u(t) + B_{2i}\delta(t)]}{\sum_{i=1}^{r} \mu_i(z(t))} \tag{2}$$

Where: $z_1(t) = [z_1(t), z_2(t), \ldots\ldots, z_p(t)]$

$$\mu_i = \prod_{i=1}^{n} M_{ij} z(t) \quad , \quad \lambda_i(z(t)) = \frac{\mu_i(z(t))}{\sum_{i=1}^{r} \mu_i(z(t))}$$

Where M_{ij} a Fuzzy set is, $x(t) \in R^n$ is the system state vector, $u(t) \in R^m$ is the control input vector, $\delta(t) \in R^p$ is the disturbance input vector, $y(t) \in R^p$ is the system output, A_i, B_i, C_i, B_{2i} are known constant matrix that describe the nominal system, $z_1(t) = [z_1(t), z_2(t), \ldots\ldots, z_p(t)]$ are the premise variables.

Observer-based controller

In the present work, our objective is the control of wind turbine system in finite frequency domain to deal with this issue, the control law is based on the classical structure of a PDC command, and the fuzzy control is designed as:

$$u(t) = -\sum_{j=1}^{r} \lambda_j(z(t)) k_j \hat{x}(t) \tag{3}$$

Where k_j are gain matrices with appropriate dimension $j = 1, 2, \ldots, r$

In this case in order to reduce the disturbances parameters caused by the wind speed, observer-based is used to estimate the unknown parts of the wind system. Therefore, the following T-S observer is considered to deal with the state estimation of T-S nonlinear system:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(t))h_j(z(t))[A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))] \\ \hat{y}(t) = \sum_{i=1}^{r} \lambda_i(z(t))[C_i\hat{x}(t)] \end{cases} \tag{4}$$

Where $L_j(j=1,\dots,r)$ the observers are gains to be determined and $\hat{x}(t)$ is the state estimation.

Let us consider the estimation error :

$$\dot{e}_x(t) = \dot{x}(t) - \dot{\hat{x}}(t) \tag{5}$$

By substituting (1), (3) and (4) in (5), we get the following augmented system:

$$\dot{\tilde{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(z(t))\lambda_j(z(t))[\bar{A}\tilde{x}(t) + \bar{B}\delta(t)]$$

$$(1 \le i \prec j \le r) \tag{6}$$

Where $\dot{\tilde{x}}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix}$, $\tilde{x}(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}$,

$$\bar{A} = \begin{bmatrix} A_i - B_i k_j & B_i k_j \\ 0 & A_i - L_j C_i \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_{2i} \\ B_{2i} \end{bmatrix}$$

Stabilization analysis of observer-based controller in finite frequency domain

The objective is to design a T–S fuzzy model controller, based on the finite frequency technique, observer-based and the gKYP lemma, which stabilizes the augmented system (6) and achieves the performances. The finite frequency control problem is to design a controller such that two conditions are satisfied [10]:

- Closed-loop system (4) is asymptotically stable
- the following finite frequency index holds:

$$\int_{\omega_1 \le \omega \le \omega_2} \chi(\omega)^T \chi(\omega) d\omega \le \gamma^2 \int_{\omega_1 \le \omega \le \omega_2} w(\omega)^T w(\omega) d\omega \tag{7}$$

Where $\omega \in [\omega_1, \omega_2]$, ω_1, ω_2 represent the upper and lower bounds of the concerned frequency, γ>0 is a prescribed scalar.
The interval frequency given by:

$$\begin{cases} \omega \in R, & |\omega| \le \omega_1, & \omega_1 \ge 0 & (LF) \\ \omega \in R, & \omega_1 \le \omega \le \omega_2, & 0 \le \omega_1 \le \omega_2 & (MF) \\ \omega \in R, & |\omega| \ge \omega_h, & \omega_h \ge 0, & (HF) \end{cases} \tag{8}$$

Where LF, MF and HF stand for low-, middle-, and high-frequency ranges, respectively.
To facilitate the presentation, we introduce the essential lemmas, which we will be used in the proof of our results.

Lemma1. (Generalized KYP Lemma [10]) The Kalman Yakubovich-Pupov (gKYP) lemma is a useful tool in control theory, since it relates frequency-domain inequalities (FDIs) to linear matrix inequalities (LMIs). Consider the linear system $(\bar{A}, \bar{B}, C)$, Let $\gamma > 0$ be a given scalar.

Given a symmetric matrix, the following statements are equivalent.
There exist symmetric matrices $P = P^T, Q = Q^T \succ 0$ such that:

$$\begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix}^T \Xi \begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \prec 0 \tag{9}$$

- **Low-frequency range** $|\omega| \le \omega_1$

$$\Xi = \begin{bmatrix} -Q & P \\ P & \omega_1^2 Q \end{bmatrix} \tag{10}$$

- **Middle-frequency range** $\omega_1 \le \omega \le \omega_2$

$$\omega_c = \frac{\omega_1 + \omega_2}{2}$$

$$\Xi = \begin{bmatrix} -Q & P + j\omega_c Q \\ P - j\omega_c Q & -\omega_1\omega_2 Q \end{bmatrix} \tag{11}$$

- **High-frequency range** $|\omega| \ge \omega_h$

$$\Xi = \begin{bmatrix} Q & P \\ P & -\omega_h^2 Q \end{bmatrix} \quad (12)$$

Lemma 2. (Projection Lemma [12]). Given a symmetric matrix $\Omega \in \Re^{m\times m}$ and two matrices Γ, Π of column dimension m, there exists a matrix F such that the following LMI holds:

$$\Omega + \Gamma F \Pi + \Pi^T F \Gamma^T < 0 \quad (13)$$

If and only if the following projection inequalities with respect to F are satisfied

$$\Gamma^{\perp} \Omega \Gamma^{\perp T} < 0 \quad (14)$$

$$\Pi^{\perp} \Omega \Pi^{\perp T} < 0 \quad (15)$$

Remark 1 The objective is to ensure the stability of closed-loop system and to make the performance of the closed-loop system from entire domain to frequency domain. Lemma 1and Lemma2 give us an approach to obtain the LMI condition which guarantees the asymptotic stability of the closed loop system.

Using lemma 1 and 2, this problem can be formulated in terms of convex optimization, which can guarantee the asymptotic stability and the FF performance of the whole system (6) as shown in the next theorem.

Theorem 1. For γ>0 system (6) is asymptotically stable if there exist hermitian matrices $\tilde{P} = \begin{bmatrix} \tilde{P}_1 & \tilde{P}_2 \\ * & \tilde{P}_3 \end{bmatrix}$, $\tilde{Q} = \begin{bmatrix} \tilde{Q}_1 & \tilde{Q}_2 \\ * & \tilde{Q}_3 \end{bmatrix} \succ 0$ a symmetric matrix $\tilde{Y} = \begin{bmatrix} \tilde{Y}_1 & \tilde{Y}_2 \\ * & \tilde{Y}_3 \end{bmatrix} \succ 0$, matrices X_j, Z_j and $F = \begin{bmatrix} F_1 & F_1 \\ F_1 & F_1 \end{bmatrix}$ such that the following LMIs conditions holds in (16) for any $1 \le i \prec j \le r$:

$$\begin{cases} \Phi \prec 0 \\ \Psi \prec 0 \end{cases} \quad (16)$$

Where

$$\Phi = \begin{bmatrix} -\bar{F}_1^T - \bar{F}_1 & -\bar{F}_1^T - \bar{F}_1 & \Phi_{13} & \Phi_{14} \\ * & -\bar{F}_1^T - \bar{F} & \Phi_{23} & \Phi_{24} \\ * & * & \Phi_{33} & \Phi_{34} \\ * & * & * & \Phi_{44} \end{bmatrix} \prec 0 \quad (17)$$

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \bar{F}_1 B_{2i} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \bar{F}_1 B_{2i} \\ * & * & \Psi_{33} & \Psi_{34} & 0 \\ * & * & * & \Psi_{44} & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} \prec 0 \quad (18)$$

Where $\bar{F}_1 = F_1^{-1}$

Where for (17) we have:

$$\begin{aligned}
\Phi_{13} &= \tilde{Y}_1 + A_i \bar{F}_1^T - B_i X_j - \bar{F}_1 \\
\Phi_{14} &= \tilde{Y}_2 + B_i X_j + A_i \bar{F}_1^T - C_i Z_j - \bar{F}_1 \\
\Phi_{23} &= \tilde{Y}_2 + A_i \bar{F}_1^T - B_i X_j - \bar{F}_1 \\
\Phi_{24} &= \tilde{Y}_3 + B_i X_j + A_i \bar{F}_1^T - C_i Z_j - \bar{F}_1 \\
\Phi_{33} &= A_i \bar{F}_1^T - B_i X_j + \bar{F}_1 A_i - X_j^T B_i^T \\
\Phi_{34} &= B_i X_j + X_j^T B_i^T + A_i^T \bar{F}_1^T + \bar{F}_1 A_i - C_i Z_j - Z_j^T C_i^T \\
\Phi_{44} &= B_i X_j + X_j^T B_i^T + A_i^T \bar{F}_1^T + \bar{F}_1 A_i - C_i Z_j - Z_j^T C_i^T
\end{aligned}$$

For (18) the expressions of Ψ given as follows:

- **Low-frequency (LF) domain** $|\omega| \le \omega_1$

$$\begin{aligned}
\Psi_{11} &= -\tilde{Q}_1 - \bar{F}_1^T - \bar{F}_1 \\
\Psi_{12} &= -\tilde{Q}_2 - \bar{F}_1^T - \bar{F}_1 \\
\Psi_{13} &= \tilde{P}_1 + A_i \bar{F}_1^T - B_i X_j - \bar{F}_1 \\
\Psi_{14} &= \tilde{P}_2 + A_i \bar{F}_1^T - C_i Z_j - \bar{F}_1 + B_i X_j \\
\Psi_{22} &= -\tilde{Q}_3 - \bar{F}_1^T - \bar{F}_1 \\
\Psi_{23} &= \tilde{P}_2 + A_i \bar{F}_1^T - B_i X_j - \bar{F}_1 \\
\Psi_{24} &= \tilde{P}_3 + A_i \bar{F}_1^T - C_i Z_j - \bar{F}_1 + B_i X_j \\
\Psi_{33} &= \omega_l^2 Q_1 + A_i \bar{F}_1^T - B_i X_j + \bar{F}_1 A_i^T - X_j^T B_i^T \\
\Psi_{34} &= \omega_l^2 \tilde{Q}_2 + B_i X_j + A_i \bar{F}_1^T - C_i Z_j + X_j^T B_i^T + \bar{F}_1 A_i^T - Z_j^T C_i^T \\
\Psi_{44} &= \omega_l^2 \tilde{Q}_3 + B_i X_j + A_i \bar{F}_1^T - C_i Z_j + X_j^T B_i^T + \bar{F}_1 A_i^T - Z_j^T C_i^T
\end{aligned}$$

- **Middle-frequency (MF) domain** $\omega_1 \le \omega \le \omega_2$

$$\Psi_{11} = -\tilde{Q}_1 - \bar{F}_1^T - \bar{F}_1$$
$$\Psi_{12} = -\tilde{Q}_2 - \bar{F}_1^T - \bar{F}_1$$
$$\Psi_{13} = \tilde{P}_1 + j\omega_c Q_1 + A_i \bar{F}_1^T - B_i X_j - \bar{F}_1$$
$$\Psi_{14} = \tilde{P}_2 + j\omega_c Q_2 + A_i \bar{F}_1^T - C_i Z_j - \bar{F}_1 + B_i X_j$$
$$\Psi_{22} = -\tilde{Q}_3 - \bar{F}_1^T - \bar{F}_1$$
$$\Psi_{23} = \tilde{P}_2 + j\omega_c Q_2 + A_i \bar{F}_1^T - B_i X_j - \bar{F}_1$$
$$\Psi_{24} = \tilde{P}_3 + j\omega_c Q_3 + A_i \bar{F}_1^T - C_i Z_j - \bar{F}_1 + B_i X_j$$
$$\Psi_{33} = -\omega_1\omega_2 \tilde{Q}_1 + A_i \bar{F}_1^T - B_i X_j + \bar{F}_1 A_i^T - X_j^T B_i^T$$
$$\Psi_{34} = -\omega_1\omega_2 \tilde{Q}_2 + B_i X_j + A_i \bar{F}_1^T - C_i Z_j + X_j^T B_i^T + \bar{F}_1 A_i^T - Z_j^T C_i^T$$
$$\Psi_{44} = -\omega_1\omega_2 \tilde{Q}_3 + B_i X_j + A_i \bar{F}_1^T - C_i Z_j + X_j^T B_i^T + \bar{F}_1 A_i^T - Z_j^T C_i^T$$
$$\omega_c = \frac{\omega_1 + \omega_2}{2}$$

- **High-frequency (HF) domain** $|\omega| \geq \omega_h$

$$\Psi_{11} = -\tilde{Q}_1 - \bar{F}_1^T - \bar{F}_1$$
$$\Psi_{12} = -\tilde{Q}_2 - \bar{F}_1^T - \bar{F}_1$$
$$\Psi_{13} = \tilde{P}_1 + A_i \bar{F}_1^T - B_i X_j - \bar{F}_1$$
$$\Psi_{14} = \tilde{P}_2 + A_i \bar{F}_1^T - C_i Z_j - \bar{F}_1 + B_i X_j$$
$$\Psi_{22} = -\tilde{Q}_3 - \bar{F}_1^T - \bar{F}_1$$
$$\Psi_{23} = \tilde{P}_2 + A_i \bar{F}_1^T - B_i X_j - \bar{F}_1$$
$$\Psi_{24} = \tilde{P}_3 + A_i \bar{F}_1^T - C_i Z_j - \bar{F}_1 + B_i X_j$$
$$\Psi_{33} = -\omega_h^2 Q_1 + A_i \bar{F}_1^T - B_i X_j + \bar{F}_1 A_i^T - X_j^T B_i^T$$
$$\Psi_{34} = -\omega_h^2 \tilde{Q}_2 + B_i X_j + A_i \bar{F}_1^T - C_i Z_j + X_j^T B_i^T + \bar{F}_1 A_i^T - Z_j^T C_i^T$$
$$\Psi_{44} = -\omega_h^2 \tilde{Q}_3 + B_i X_j + A_i \bar{F}_1^T - C_i Z_j + X_j^T B_i^T + \bar{F}_1 A_i^T - Z_j^T C_i^T$$

Remark1. Wind energy conversion systems are physical systems that can be affected by external disturbances in very specific frequency ranges, so controlling these systems in a full band of frequencies may introduce some restriction in the frequency range and the performance of the control system. However, the lemma Kalman Yakubovich Popove (KYP) treats the properties over the entire frequency domain. To study this problem in a specific frequency band, the researchers developed a new version of the generalized Kalman-Yakubovich-Popove (KYP) lemma. The synthesis of the control law in the finite frequency domain has received a lot of attention given its efficiency in terms of stability and control.

Proof. First, we consider the Middle-frequency case, on other hand, the proof of low and high frequency cases is similar to the middle frequency case.

In this part we select for variables P, Q, Y the following structures

$$P = \begin{bmatrix} P_1 & P_2 \\ * & P_3 \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \succ 0 \quad Y = \begin{bmatrix} Y_1 & Y_2 \\ * & Y_3 \end{bmatrix} \succ 0 \tag{19}$$

Then the slack variable

$$F = \begin{bmatrix} F_1 & F_1 \\ F_1 & F_1 \end{bmatrix} \tag{20}$$

$$\begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \\ 0 & I \end{bmatrix}^T \Xi \begin{bmatrix} \bar{A} & \bar{B} \\ I & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix} \prec 0 \tag{21}$$

- **Middle-frequency (MF) domain** $\omega_1 \leq \omega \leq \omega_2$

$$\omega_c = \frac{\omega_1 + \omega_2}{2}$$

$$\Xi = \begin{bmatrix} -Q_1 & -Q_2 & P_1 + j\omega_c Q_1 & P_2 + j\omega_c Q_2 & 0 & 0 \\ * & -Q_3 & P_2 + j\omega_c Q_2 & P_3 + j\omega_c Q_3 & 0 & 0 \\ * & * & -\omega_1\omega_2 Q_1 & -\omega_1\omega_2 Q_2 & 0 & 0 \\ * & * & * & -\omega_1\omega_2 Q_3 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \end{bmatrix} \tag{22}$$

In this study we consider the middle-frequency case, by applying the projection lemma 2 we obtain the inequality

$$\Omega + \Gamma F \Pi + \Pi^T F \Gamma^T < 0 \tag{23}$$

Where

$$\Gamma = \begin{bmatrix} I \\ I \\ 0 \end{bmatrix} \quad \Pi = \begin{bmatrix} -I & \bar{A} & \bar{B} \end{bmatrix}$$

$$\Omega = \begin{bmatrix} -Q & P + j\omega_c Q & 0 \\ P - j\omega_c Q & -\omega_1\omega_2 Q & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\begin{bmatrix} -Q & P + j\omega_c Q & 0 \\ P - j\omega_c Q & -\omega_1\omega_2 Q & 0 \\ 0 & 0 & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} I \\ I \\ 0 \end{bmatrix} F \begin{bmatrix} -I & \bar{A} & \bar{B} \end{bmatrix} + \begin{bmatrix} -I & \bar{A} & \bar{B} \end{bmatrix}^T F^T \begin{bmatrix} I \\ I \\ 0 \end{bmatrix}^T \prec 0 \tag{24}$$

After calculating (24) and replacing $P, Q, F, \bar{A}$ and $\bar{B}$ by their expressions we get:

$$\zeta=\begin{bmatrix}-Q_1-F_1-F_1^T & -Q_2-F_1-F_1^T & \zeta_{13} & \zeta_{14} & F_1B_{2i}\\ * & -Q_3-F_1-F_1^T & \zeta_{23} & \zeta_{24} & F_1B_{2i}\\ * & * & \zeta_{33} & \zeta_{34} & 0\\ * & * & * & \zeta_{44} & 0\\ * & * & * & * & -\gamma^2 I\end{bmatrix}\prec 0 \qquad (25)$$

$$\zeta_{13}=P_1+j\omega_c Q_1+F_1(A_i-B_ik_j)-F_1^T$$
$$\zeta_{14}=P_2+j\omega_c Q_2+F_1B_ik_j+F_1(A_i-L_jC_i)-F_1^T$$
$$\zeta_{23}=P_2+j\omega_c Q_2+F_1(A_i-B_ik_j)-F_1^T$$
$$\zeta_{24}=P_3+j\omega_c Q_2+F_1B_ik_j+F_1(A_i-L_jC_i)-F_1^T$$
$$\zeta_{33}=-\omega_1\omega_2 Q_1+F_1(A_i-B_ik_j)+(A_i-B_ik_j)^T F_1^T$$
$$\zeta_{34}=-\omega_1\omega_2 Q_2+F_1B_ik_j+F_1(A_i-L_jC_i)+k_j^TB_i^TF_1^T+(A_i-L_jC_i)^T F_1^T$$
$$\zeta_{44}=-\omega_1\omega_2 Q_3+F_1B_ik_j+F_1(A_i-L_jC_i)+k_j^TB_i^TF_1^T+(A_i-L_jC_i)^T F_1^T$$

In order to solve the nonlinear problem in the controller and observer terms, we perform a congruence transformation by multiplying both sides of (25) by the full rank matrix $\mathrm{diag}\{F_1^{-1}\quad F_1^{-1}\quad F_1^{-1}\quad F_1^{-1}\quad I\}$ and its transpose from the left and right, and after some easy manipulation we get the inequality (18). With $\bar{F}_1=F_1^{-1}$

We considered the next variable change:

$$L_jC_i=C_iQ_j$$
$$L_jC_iF_1^{-T}=C_iQ_jF_1^{-T}=C_iZ_j$$
$$Z_j=Q_jF_1^{-T}$$

Remark 2. Note that the linear matrix inequality (25) has complex variables. According to [10], the LMI in complex variables can be transformed to an LMI of larger dimension in real variables. This means that inequality $N_1+jN_2\prec 0$ is equivalent to $\begin{bmatrix}N_1 & N_2\\ -N_2 & N_1\end{bmatrix}\prec 0$, which implies the LMI in (25) can be addressed.

Let us consider a candidate of Lyapunov function, $\bar{A}$ is stable if and only if there exists $Y=Y^T\succ 0$ such that:

$$\bar{A}^TY+Y\bar{A}\prec 0 \qquad (26)$$

Which is rewritten in the form

$$\begin{bmatrix}\bar{A}\\ I\end{bmatrix}^T\begin{bmatrix}0 & Y\\ Y & 0\end{bmatrix}\begin{bmatrix}\bar{A}\\ I\end{bmatrix}\prec 0 \qquad (27)$$

By Lemma 2, we get

$$\begin{bmatrix}0 & Y\\ Y & 0\end{bmatrix}+\begin{bmatrix}F\\ F\end{bmatrix}\begin{bmatrix}-I & \bar{A}\end{bmatrix}+\begin{bmatrix}-I & \bar{A}\end{bmatrix}^T\begin{bmatrix}F\\ F\end{bmatrix}^T\prec 0 \qquad (28)$$

Where $Y=\begin{bmatrix}Y_1 & Y_2\\ * & Y_3\end{bmatrix}\succ 0$ $F=\begin{bmatrix}F_1 & F_1\\ F_1 & F_1\end{bmatrix}$,

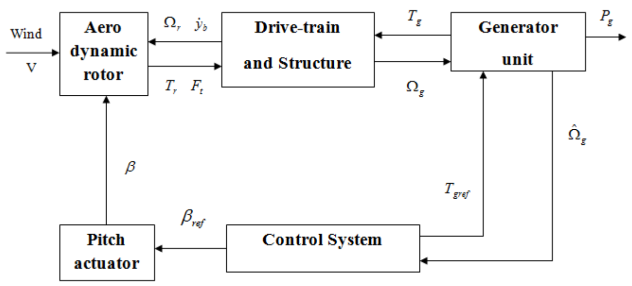
$$\bar{A}=\begin{bmatrix}A_i-B_ik_j & B_ik_j\\ 0 & A_i-L_jC_i\end{bmatrix}$$

In order to stabilize the closed loop system the projection Lemma is applied, after some easy manipulations we get the LMI (18).

Illustrative example

In this section, we will apply the above approach to design a finite frequency observer-based controller on the wind turbine system described as follows:

Figure 1. Structure of the considered Wind Energy Conversion System (WECS) Controller.



The wind energy conversion system (WECS) model is described by combining a model of the mechanical structure of a wind turbine and nonlinear model. As it is shown in Figure.1, the system is formed by the rotor, the mechanical structure and by a generator unit. The control system acts on generator in order to command a wind turbine system [4].

The mathematics model of the wind turbine is clearly described in [3], which is represented by the following state representation:

$$\begin{cases}\dot{x}(t)=A(z)x(t)+Bu(t)+B_2V(t)\\ y(t)=Cx(t)\end{cases} \qquad (33)$$

Where:

$$A(z)=\begin{bmatrix}0 & 1 & -1 & 0\\ -\dfrac{K_s}{J_r} & -\dfrac{B_s}{J_r} & \dfrac{B_s}{J_r} & \dfrac{T_{r\beta}(z_0)}{J_r}\\ \dfrac{K_s}{J_g} & \dfrac{B_s}{J_g} & -\dfrac{(B_s+B_g)}{J_g} & 0\\ 0 & 0 & 0 & -\dfrac{1}{\tau}\end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{B_g}{J_g} \\ \frac{1}{\tau} & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ \frac{T_{rv}(z_0)}{J_r} \\ 0 \\ 0 \end{bmatrix}$$

$$C = [0 \quad 0 \quad 1 \quad 0]$$

$$z = [V, \beta]^T, x(t) = [\theta_s \quad \Omega_r \quad \Omega_g \quad \beta]^T \quad u = [\beta_d, \Omega_z]^T$$

Where θ_s denotes the torsion angle, Ω_r is angular velocity of rotor, Ω_g is angular velocity of generator, k_s is the stiffness of the transmission, B_s is the damping of the transmission, J_r and J_g are the inertia of the rotor and the generator, respectively. T_r is the aerodynamic torque V is the wind speed β and β_d are the actual and desired pitch angles, respectively.

In order to obtain the best possible performance from this highly nonlinear system, the following subsection gives a T-S fuzzy representation.

The variables in the wind turbine are assumed varying in the operating range: $V_1 \le V \le V_2, \beta_1 \le \beta \le \beta_2$ Consequently the nonlinear system (1) can be represented by the following four IF-THEN rules [3].

If β is F_1^1 and V is F_2^1 then $\begin{cases} \dot{x} = A_1 x + B_1 u + B_{21}\delta \\ y = Cx \end{cases}$

If β is F_1^1 and V is F_2^2 then $\begin{cases} \dot{x} = A_2 x + B_2 u + B_{22}\delta \\ y = Cx \end{cases}$

If β is F_1^2 and V is F_2^1 then $\begin{cases} \dot{x} = A_3 x + B_3 u + B_{23}\delta \\ y = Cx \end{cases}$

If β is F_1^2 and V is F_2^2 then $\begin{cases} \dot{x} = A_4 x + B_4 u + B_{24}\delta \\ y = Cx \end{cases}$

Then the equivalent T-S fuzzy model is equivalent to equation (1).

Where the membership functions, the disturbances level, the parameters of systems are given by:

$$\delta(t) = V(t)$$

$$\lambda_1(z) = F_1^1(\beta)F_2^1(V), \lambda_2(z) = F_1^1(\beta)F_2^2(V)$$
$$\lambda_3(z) = F_1^2(\beta)F_2^1(V), \lambda_4(z) = F_1^2(\beta)F_2^2(V)$$
$$F_1^1(\beta) = \frac{\beta - \beta_1}{\beta_2 - \beta_1}, F_1^2(\beta) = \frac{-\beta + \beta_1}{\beta_2 - \beta_1}$$
$$F_2^1(V) = \frac{V - V_1}{V_2 - V_1}, F_2^2(V) = \frac{-V + V_2}{V_2 - V_1}$$

$$A_1 = \begin{bmatrix} 0 & 1 & -1 & 0 \\ -\frac{K_s}{J_r} & -\frac{B_s}{J_r} & \frac{B_s}{J_r} & \frac{T_{r\beta 1}}{J_r} \\ \frac{K_s}{J_g} & \frac{B_s}{J_g} & -\frac{(B_s + B_g)}{J_g} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & -1 & 0 \\ -\frac{K_s}{J_r} & -\frac{B_s}{J_r} & \frac{B_s}{J_r} & \frac{T_{r\beta 2}}{J_r} \\ \frac{K_s}{J_g} & \frac{B_s}{J_g} & -\frac{(B_s + B_g)}{J_g} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau} \end{bmatrix}$$

$$A_2 = A_1, A_4 = A_3$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{B_g}{J_g} \\ \frac{1}{\tau} & 0 \end{bmatrix} \quad B_{21} = B_{23} = \begin{bmatrix} 0 \\ \frac{T_{rv1}}{J_r} \\ 0 \\ 0 \end{bmatrix} \quad B_{22} = B_{24} = \begin{bmatrix} 0 \\ \frac{T_{rv2}}{J_r} \\ 0 \\ 0 \end{bmatrix}$$

$$B = B_1 = B_2 = B_3 = B_4$$
$$C = [0 \quad 0 \quad 1 \quad 0]$$
$$C = C_1 = C_2 = C_3 = C_4$$

Numerical value:

$$K_s = 1.566 * 10^6 \text{ N/m}, \tau = 100\text{ms}$$
$$B_s = 3029.5\text{Nms/rad}, B_g = 15.993\text{Nms/rad}$$
$$J_r = 830000\text{Kg.m}^2, J_g = 5.9\text{Kg.m}^2$$

In the high wind speed, the considered operated range of pitch angle is $-2\text{deg} \le \beta \le 24\text{deg}$, the range of wind speed is $17\text{m/s} \le V \le 31\text{m/s}$ and then

$$T_{r\beta 1} = 723980, \quad T_{r\beta 2} = 376070$$
$$T_{rv1} = 106440, \quad T_{rv2} = 85370$$

The simulation is performed with Matlab by using the Toolbox LMI. The resolution of LMIs given in (17) and provide the following gains. Feedback and observer gain matrices obtained by the following values:

$$k_1 = \begin{bmatrix} 0.0007 & -0.0003 & 0.1156 & -0.0022 \\ -181.4361 & 19.5755 & 0.2852 & -2.4789 \end{bmatrix}$$

$$k_2 = \begin{bmatrix} 0.0007 & -0.0004 & 0.1314 & -0.0043 \\ -181.6694 & 19.6006 & 0.3340 & -2.4821 \end{bmatrix}$$

$$k_3 = \begin{bmatrix} 0.0007 & -0.0002 & 0.1161 & 0.0006 \\ -181.3347 & 19.5646 & 0.2856 & -2.4776 \end{bmatrix}$$

$$k_3 = \begin{bmatrix} 0.0007 & -0.0003 & 0.1159 & -0.0026 \\ -181.2910 & 19.5599 & 0.2543 & -2.4769 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.0052 \\ 0.0001 \\ -0.2067 \\ -0.0806 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.0004 \\ 0.0000 \\ -0.2209 \\ -0.0620 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 0.00514 \\ 0.0001 \\ -0.2087 \\ -0.07020 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 0.0055 \\ 0.0001 \\ -0.2044 \\ -0.0896 \end{bmatrix}$$

and $\gamma = 0.110$

Comparing the results of [13] and [25] with the results in this paper, the obtained results show that the approach proposed has a good performances and minimizing variations in the speed of the generator and the control forces .

Figure 2 and figure 3 represent the time response of the generator speed with PDC command and a finite frequency Takagi Sugeno fuzzy with observer-based controller respectively. We can observe that the proposed condition with a finite frequency technique yields a good performances and the system is more stable with the approach proposed.

Figure 2. Generator rotated speed (rd/s) with H∞ PDC controller

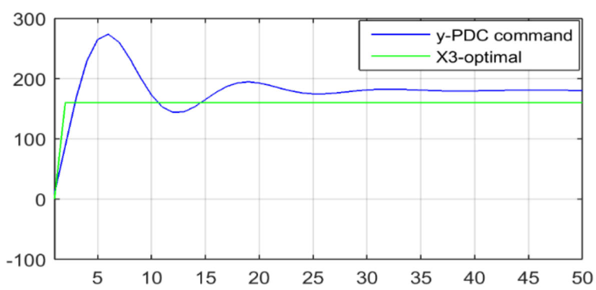


Figure 3. Generator rotated speed (rd/s) with FFTSF observer-based controller

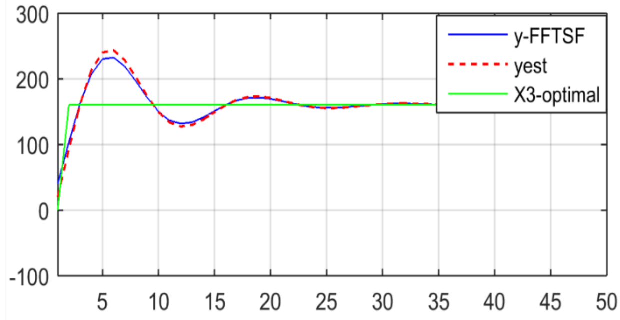


Figure 4. Membership function

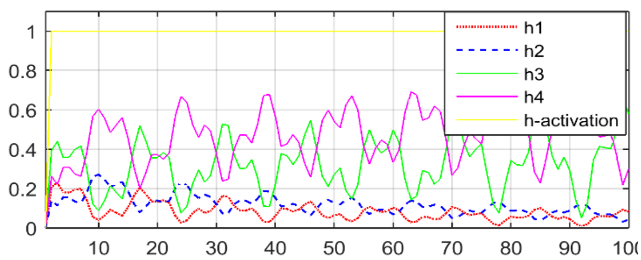
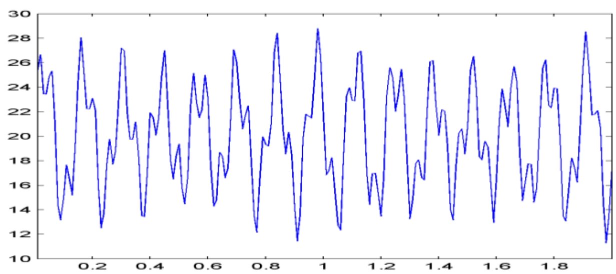


Figure 5. The profile of wind input speed of 17~31 m/s.



The wind speed input profile (17m/s < V < 31m/s) is given in figure (5) and the membership function given in figure (4).

Conclusion

This paper has investigated the problem of control in finite frequency domain for wind turbine system. We are based on the observer-based controller to estimate the unknown parts of the wind turbine. By the Generalized KYP lemma and Lyapunov function a sufficient stability conditions proposed to deal with problem of control in specific domain. The effectiveness of the control technique compared with a predictive controller has been realized .The Simulations results show that the control law attenuates the effect of wind and the approach proposed controller has a good performances.

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