ORIGINAL ARTICLE

A new diagnosis approach of hybrid systems through Observers and hybrid automata

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Abstract: This paper gives a new approach of fault detection and isolation (FDI) of hybrid dynamical systems based on hybrid observers and hybrid automata; a methodology for the design of dynamical observers has been proposed. The hybrid observer composed of two blocks: a location observer that identifies the current mode and a continuous observer that detects faults. Although this approach is interesting, it is still unable to detect instantly the change of the continuous state; as a result and in case of a fault, the system cannot identify correctly the defected mode. In this paper, a new version is proposed to improve this approach and reach good diagnosis results.

Keywords: Hybrid observers; Diagnosis; FDI; Hybrid dynamical systems; Hybrid automata.

Introduction

Nowadays, companies are increasingly subject to market competition. In order to ensure their future, they have to face different socio-economic problems, which leads to an important complexity in their production systems; thus, several researches have been done in order to understand, analyze and control the behavior of complex systems like: Aeroelastic systems[1], ultrasonic machining systems [2], atomic microscope [3], etc.

The mastery of a process is inseparable from its supervision. This supervision allows businesses to ensure and maintain the safety of their equipment; in this sense, several studies have been made on different systems such as: ball bearings [4], Cutting tools [5,6],etc. However, they remain insufficient when compared to the immense number of current industrial systems, that is why we decided to work on the diagnosis of hybrid dynamic systems that constitutes a large part of the complex industrial systems.

Today, most systems are hybrid dynamical ones; systems that exhibit complex hybrid behavior, composed of continuous and discrete states. According to the classic view, the behavior of hybrid dynamical systems is described by piecewise continuous series; each continuous evolution corresponds to a mode. When a controlled or spontaneous discrete transition occurs, the system jumps from a mode to another, which means the discrete state changes. The mode transitions that govern the continuous behavior of the system result from an external control signals that change directly the discrete state of the system [7]; for example, an operator turns off or on a valve; as a result, the signal switches abruptly from low to high, or an internal continuous variable that exceeds their predefined thresholds; for example, the level of liquid in a tank reaches a defined threshold and leads to opening a valve. Thus, the real challenge in hybrid dynamical systems diagnosis is to analyze the system behavior and consider, in the same time, the interaction between the continuous and the discrete dynamics.

The authors investigated for years the use of a hybrid formalism to solve diagnosis problems; the model-based fault detection technique has attracted much attention, and the main idea of this technique is to construct a residue that indicates the presence of a fault. The classical approaches based on the generation of residues are: identification algorithms [8], parity-space approaches [9] and observers-based methods [10-12].Among all these approaches, observer-based technique is regarded as one of the most popular and robust methods.

For this reason, we have developed a new approach based on observers, usually used in the field of continuous systems, combined with automata well-known in the field of discrete event systems [13] .For a physical system, an observer detects and locates faults using the residues by comparing the actual output of the reel system with the output of the model. Hybrid automata used to represent the dynamic evolution of the system and their propagation failures.

In hybrid systems, we use two blocks of observers: location observers that identify the active mode and continuous observers that detect and locate faults. The generation of residues, which is the engine of this method, has a delay, which makes our system unable to detect the current mode instantly. Most of the studies done have worked on the design of the observer by developing the mathematical part more than the detection method in order to improve the precision of observers[14-17].This theory is stagnated today and reaches a high level of maturity. For this reason, we have decided to work on the detection algorithm by integrating the time as a primordial axis, which has generated good results and opened new perspectives to resolve the problem of diagnosis on both sides: method and design.

In order to present our methodology, this paper is organized as follows: section II, gives a general idea on hybrid dynamical systems, Observers and Hybrid Automata, which allow detection and fault localization; in Section III our diagnosis approach will be presented; an academic example in section IV is used to illustrate this method; finally, a conclusion is presented with some perspectives.

Diagnosis of hybrid systems through Observers and Hybrid automata

Hybrid systems

Hybrid systems are dynamical systems that involve the interaction of two types of dynamics: discrete jumps and continuous flows.

Continuous state is the state that takes values in Euclidean space $\mathbb{R}^n$ for n $\geq$1. We use x$\in$ $\mathbb{R}^n$ to denote the state of a continuous dynamical system. This type of systems has been studied widely in mathematics engineering, physics, biology [18,19].

Discrete state is the state that takes values in a countable or finite set {q1, q2 ...}. We use q to denote the state of a discrete system. For example, a digital sensor that produces a binary output in the form of a logic “1” or a logic “0” and q$\in$ (“ON” or “OFF”).This type of systems has also been studied for many years. There are many interesting books available [20,21] .

The study of hybrid systems is relatively recent. Some books and articles have appeared on the subject [22-24]. In order to model the behavior of hybrid dynamical systems, we need a mathematical model that combines the dynamics of the continuous part with the dynamics of the discrete part; these mathematical models consist of differential equations on the one hand [25], and automata or other discrete-event models on the other hand.

Observers

Observers are widely used in literature for diagnosis of linear and non-linear systems. Several studies have been proposed in the linear part: [26,27] and the non-linear part: [10,11] .

Observers are auxiliary systems that allow reconstructing the internal state of the reel system, from the measured inputs and outputs. The detection of defects is based on the evaluation of the residues that are calculated by taking the difference between measured and observed outputs. This evaluation consists in defining a threshold to detect the presence of changes. In the presence of a defect, the value of residues must be different to zero and equal to zero when the system is safe; these residues have to be robust and sensitive to disturbances and uncertainties in order to avoid false alarms.

Hybrid Automata

A hybrid automata is an extended finite state automata with continuous variables. The discrete states of the hybrid system are represented by edges of the automata, and the continuous states are represented by differential equations [28,29].

A hybrid automata is formally described by the tuple H= (Q, X, f, Init, Inv, J)

• Q={qi ; i= 1,2 ..,n} is a finite set of the discrete modes. It represents the discrete state of H.

• X= {xi; i= 1, 2 .., n} is a finite set of continuous states. It represents the continuous variables of H.

• f:Q×X→ $\mathbb{R}^n$ is a function called the vector field. It represents the continuous flow in each mode.

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• Init⊂Q×$\mathbb{R}^n$ represents the initial state of H, which is a pair (q,x) composed of the mode q and the continuous variable x.
• Inv⊂Q×$\mathbb{R}^n$ is the invariant condition. As long as the system is in the mode q the state belongs to Inv.
• J:Q×$\mathbb{R}^n$ →P(Q×$\mathbb{R}^n$) is the jump condition. It defines the conditions of the system to jump from a mode to another and the values affected to the continuous state after jump.

Detection and localization concept

The observer-based method consists of two parts: a location observer that identifies the current location and a continuous observer that estimates the continuous state. The hybrid observer identifies the current location of the plant after a number of steps and converges exponentially to the continuous state [12,14].

The residuals generation consists in comparing measurement outputs from the system to their estimation outputs from observer.

Consider a nonlinear dynamical system that has m outputs and functions in n modes.

Mode identification

We associate to each mode an observer. As shown in Figure 1, each observer receives all the inputs outputs of the system; reconstructed outputs by each observer $\breve{y}$ (t) are compared, at any moment, to the outputs measured $y(t)$ to generate residual vector $r_i(t)$ represented by the equation below:

$$r_i(t) = y(t) - \breve{y}(t) \tag{1}$$

Each component of the residual vector $r_i(t)$ converges to zero only when the system evolves in mode i; otherwise, it departs significantly from zero.

The residue $r_i(t)$ will then be evaluated to generate the modes signature.

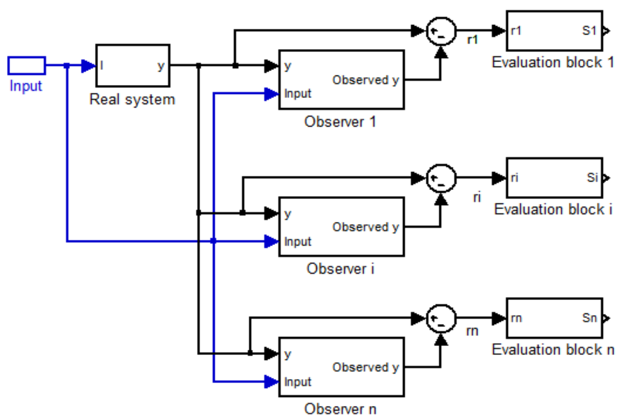


Figure 1. Observers for mode detection.

The based residual evaluation consists in comparing each element calculated to a threshold Mi, which is defined according to the disturbances modeling errors and noise measurements; we then use the decision logic next :
If |ri(t)|> Mi then Si =0
Else Si=1

The identification of the current mode consists in making the correspondence between the signature obtained and the signature shown in the theoretical table of signatures presented below:

Table 1. Theoretical table of modes signatures

Signature	***Mode 1***	***Mode i***	***Mode n***	***Unknown mode***
S1	1	0	0	0
Si	0	1	0	0
Sn	0	0	1	0

Fault detection and isolation

If the mode has been identified using the first module, we use M observers corresponding to the identified mode i as shown in Figure 2, and each one of these observers is sensitive to a single output.

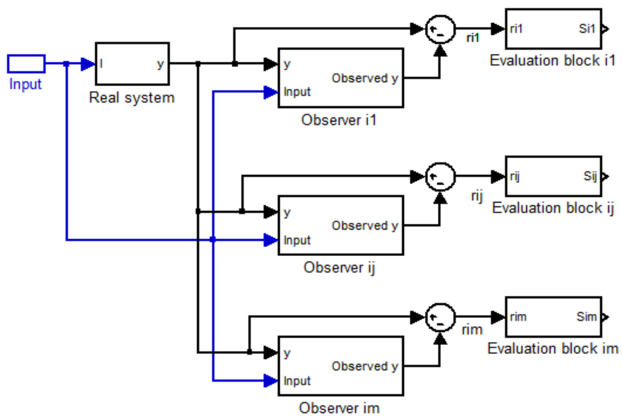


Figure 2. Observers for faults detection.

The residue rij will then be evaluated to generate the sensors signature. The based residual evaluation consists in comparing each element calculated to a threshold Mij.
If |rij(t)|> Mij then Si =1, which means the output j is faulty.
Else Sij=0

The identification of the defected output makes the correspondence between the signature obtained and the signature shown in the theoretical table of signatures below:

Table 2. Theoretical table of faults signatures

Signature	***C1***	***Cj***	***CM***

Si1	1	0	0
Sij	0	1	0
SiM	0	0	1

Limits and improvements

When the system jumps from a mode to another, observers make a delay τ to respond as shown in Figure 3; therefore, the modes signature generated equals to zero, and the first module is unable to detect the current mode.

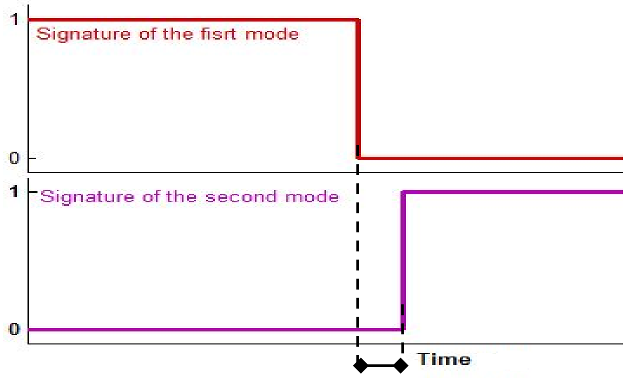


Figure 3. The unknown range.

It is the major disadvantage of this method; to settle this issue and make the approach of observers more robust, we have integrated a new procedure based on time.

We will analyze the different cases that we can have using only the first method as shown in Figure 4 and demonstrate how our approach is able to settle these problems using the time constraints; then, we will propose a synthesis in the end.

The previously and the posteriorly detected modes are respectively the mode before and after the range τ.

Hybrid dynamical systems are largely applied in the industrial automation area, and they operate periodically in a cyclical manner. We focus on this feature to build our detection and isolation concept.

First of all, we calculate the safety time of each mode, we model the system by hybrid automata, then we add the time constraints [T1, Ti,Tn] as shown in Figure 5 .

We study the case when the modes signature= [0,…,0]

If the defaults signature = [0,…,0]   the system is safe.

Else the system is faulty and we can have two cases:

-the previously and the posteriorly detected modes are the same; in this case, the defected mode is the same one detected before and after the default.

-the previously and the posteriorly detected modes are different that leads to two cases:

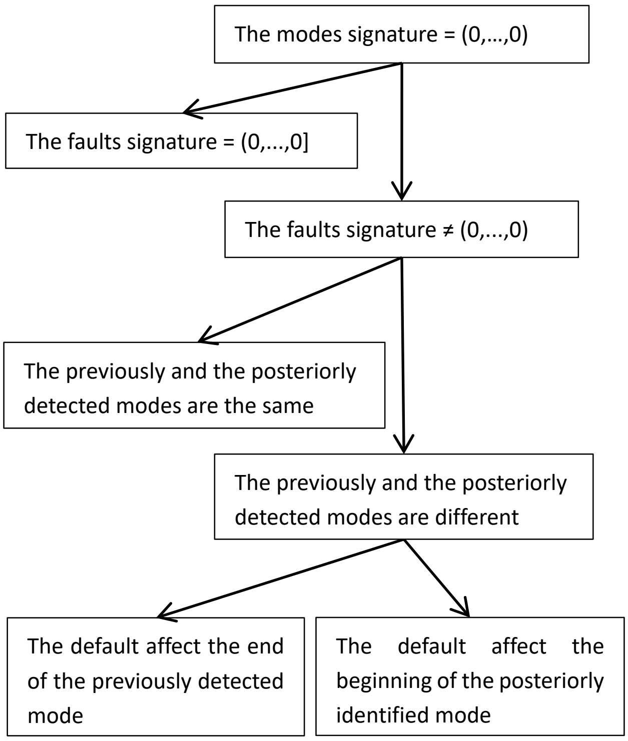


Figure 4. Case analysis.

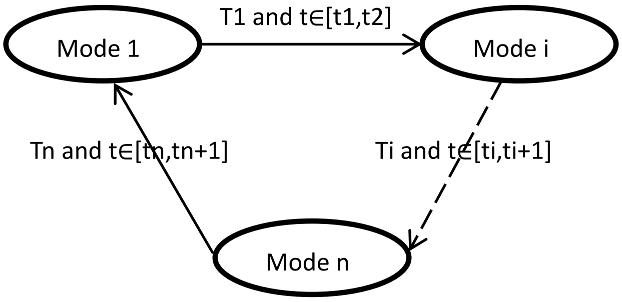


Figure 5. Integration of time constraints.

The fault affects the end of the previously detected mode

If the fault affects the end of a mode automatically, it influences its normal time; we give an example to explain that.

We suppose, in the example shown in Figure 6, that the system functions in the first mode, and jumps to another mode when the output y = 3, which means the normal time of the mode is 3 seconds.

We add a default d=1 at the range of time [2.5,3] ,and we note that the injection of the default sensor has changed the time of the mode to 2.5 seconds.

The constraints of times will stop the propagation of the default to not affect the others modes, so we can identify clearly the defected mode, which is the mode where the system was stopped.

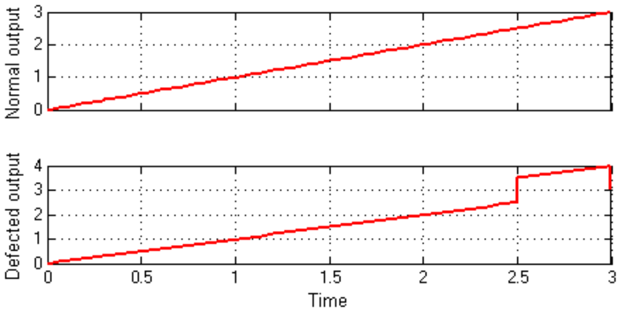


Figure 6. The impact of the fault on the mode time.

The fault affects the beginning of a mode

Thanks to constraints of time, our system cannot jump to another mode if only the normal conditions have been realized so as not to disturb all the function of the system; so, the system behaves normally and the defected mode is the last mode detected before the occurrence of the default.

In both cases, if we have a default in the unknown range the defected mode is the first mode detected after the occurrence of the default.

Thanks to this approach, we can detect and locate all types of sensors defaults at any time.

We will apply this method to an academic example to test the effectiveness and the robustness of our approach.

APPLICATION EXAMPLE

Description of the system

We consider as example a three tanks hydraulic system represented in Figure 7; the water arrives at a first tank T1, of section S1= 2 m^2, with a volume flow q1 and a height $l1$. At the exit of this tank a valve V1, of hydraulic resistance R1 =20 N.s.m^{-5}, lets the fluid passes to a second tank T2, of section S2= 1.5 m^2 and height $l2$. At the exit of this tank a valve V2, of hydraulic resistance R2=20 N.s.m^{-5}, lets the fluid passes in a third tank T3, of section S3= 1 m^2 and height $l3$.Thus, the outlet flow of this tank is authorized by a valve V3, of hydraulic resistance R3=20 N.s.m^{-5}.

In order to facilitate the study, the two valves V1 and V2 are left opened, and only the valve V3 is acted on.

We can have three states of the valve V3:

Closed: mode 1

Half-Opened: mode 2

Opened: mode 3

The objective is to maintain the liquid level in the two tanks T1 and T3 on a well-defined level as:

$$\begin{cases} l3 \le 1 \\ l1 \ge \ 0.9 \end{cases} \tag{2}$$

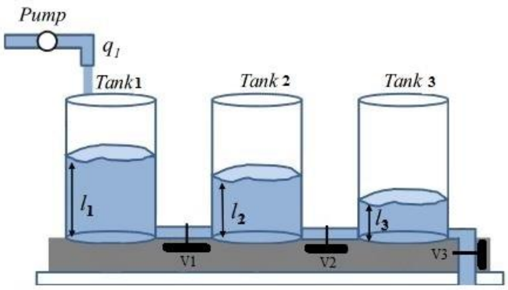


Figure 7. Three tanks hydraulic system.

State equation

The volumetric flow rate Q is the volume of fluid, which passes per unit time; it is defined by:

$$Q = \frac{dV}{dt} \tag{3}$$

The volume variation dV of a tank is equal to the difference of inputs and outputs flows rates as shown below:

$$Q = \sum input \ \ flow - \sum output \ \ flow \tag{4}$$

The volume of fluid inside a tank is the surface of the tank multiplied by the level of the liquid and written as:

$$V = S * Level \tag{5}$$

The state equation of the three tanks system is then obtained:

$$\begin{cases} \frac{dl1}{dt} = \frac{q1}{s1} + \frac{g}{R1*S1}(l2 - l1) \\ \frac{dl2}{dt} = \frac{g}{R1*S2}(l1 - l2) + \frac{g}{R2\ *S2}(l3 - l2) \\ \frac{dl3}{dt} = \frac{g}{R2*S3}(l2 - l3) - \frac{g}{R3*S3} l3 \end{cases} \tag{6}$$

We can represent our system under the state space equations bellow:

$$\begin{cases} \dot{X} = A.X + B.u \\ \ \ Y = C.X \end{cases} \tag{7}$$

The system matrix is:

$$A = \begin{pmatrix} -\frac{g}{R1*S1} & \frac{g}{R1*S1} & 0 \\ \frac{g}{R1*S2} & -\frac{g}{R1*S2} - \frac{g}{R2*S2} & \frac{g}{R2*S2} \\ 0 & \frac{g}{R2*S3} & -\frac{g}{R2*S3} - \frac{g}{R3*S3} \end{pmatrix} \tag{8}$$

The control matrix is:

$$B = \begin{pmatrix} \frac{q1}{S1} \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

The output matrix is:

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{10}$$

Observer-based fault diagnosis method

The simplest kind of observers that we can use to applicate our method is the Luenberger one, represented by the mathematical model:

$$\begin{cases} \dot{\breve{X}} = A.\breve{X} + B.u + L(Y - \breve{Y}) \\ \breve{Y} = C.\breve{X} \end{cases} \tag{11}$$

L is the gain of the observer, and the pair [A, C] is observable

Diagnosis method based on hybrid Automata

The dynamic model represented by the hybrid automata contains all the normal states of the system, which allows us to follow its temporal evolution. Therefore, thanks to the integration of time in transitions, we are able to locate a fault by quantifying the time spent in the mode. All the times concerning the faults detection and localization have been found mathematically or from simulations.

Figure 8 shows the model of our system represented by hybrid automata. To jump from a mode to another, the system must satisfy the conditions on the continuous state and the external conditions represented by the value of the variable m, which gives the opportunity to choose the accurate mode.

Simulation results

The simulation of the dynamic behavior has been performed by the software Matlab .The normal evolution of the discrete and the continuous states are presented in figure 9. The switching between modes occurs when the conditions are realized. The Simulation time is fixed at 35s to show all the different modes of the system.

In order to test the effectiveness of our proposed diagnosis approach, we will inject different sensors faults in different times and we will present the responses of the system.

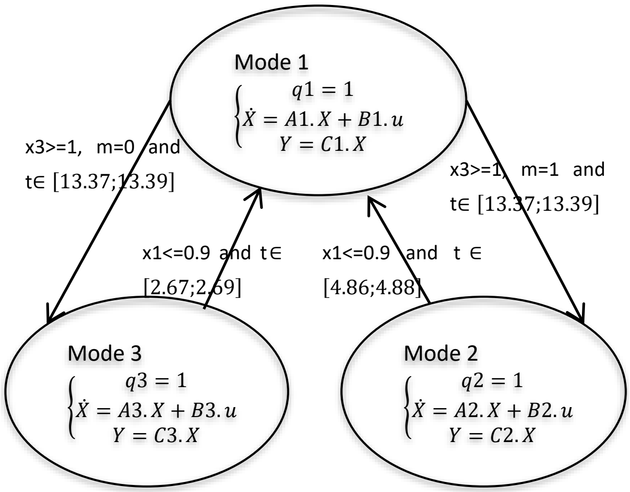


Figure 8. Hybrid automata for the system.

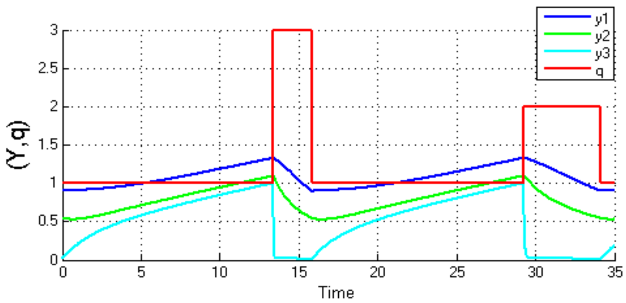


Figure 9. The discrete and the continuous states of the system.

First case

We inject a fault on the level sensor L2 at the instant t = 30s.Our aim is to detect the active mode and locate the defected output.

Figure 10 shows the response of the location observers represented by residues (r1, r2, r3) and their sensitivity to detect the different modes of the system.

We note that at the instant 30s, our observers detect an unknown mode (the mode signature (r1, r2, r3) = (0, 0, 0)) and return to the second mode.

The previously and the posteriorly detected modes surrounding the unknown range are the same, so the unknown mode is the mode 2.

The other block of continuous observers used to locate the defected sensor in the second mode.

Figure 11 shows the response of the residues (r21, r22, r23); at the time 30s, we notice that the residue r2 exceeds their respective thresholds, which means our system has a problem in the output y2 which corresponds to the sensor L2.

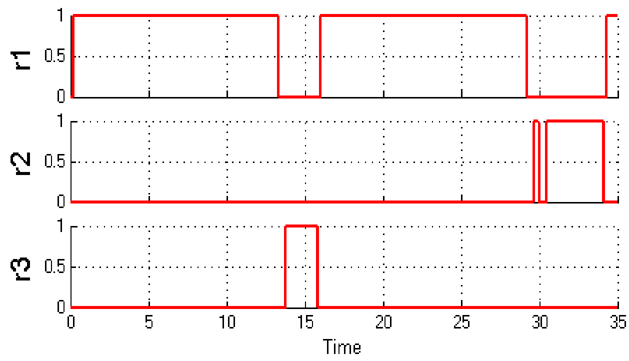
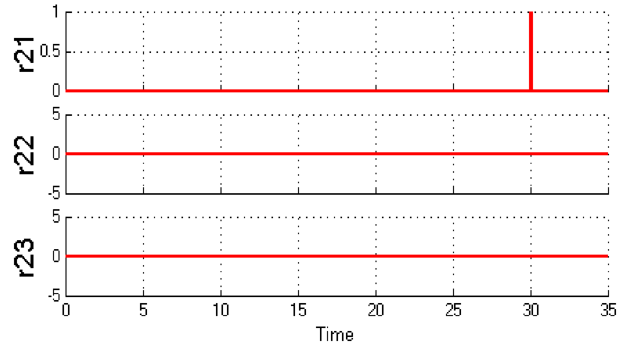


Figure 10. Response of the residues (r1, r2, r3).

Figure 11. Response of the residues (r21, r22, r23)



Second case

In this case, we inject a fault at the extremities of a mode, which impacts its normal duration time, as a solution the constraints of time, will stop the propagation of the fault so as not to disturb other modes ,and help to detect the defected mode.

We inject a fault on the level sensor L1 at the instant t = 15.5s.Figure 12 shows the response of the residues (r1, r2, r3) and their sensitivity to detect the different modes of the system.

At the time 15.5, our observers detect an unknown mode, and our system stays in the same mode, which is the third mode.

The previously and the posteriorly detected modes surrounding the unknown range are the same, so the unknown mode is the third mode.

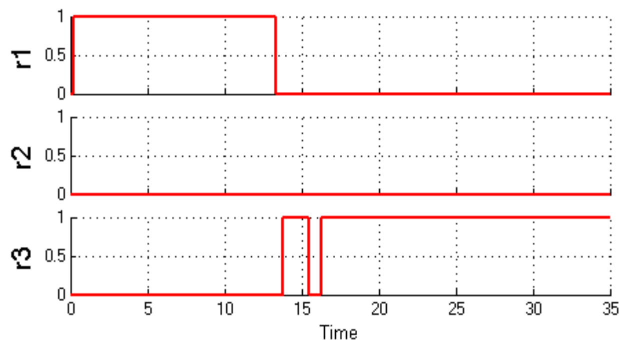


Figure 12. Response of the residues (r1, r2, r3).

The other set of observers gives more information to determine exactly the output affected by the fault. Figure 13 shows the response of the residues (r31, r32, r33); we can note that we have a default in the output 1.

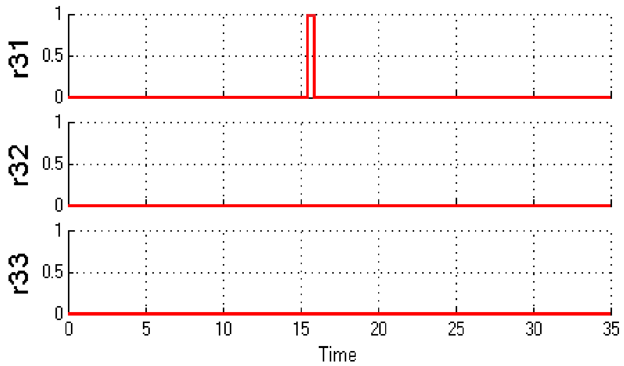


Figure 13. Response of the residues (r31, r32, r33).

Third case

There is always a delay to detect a mode using only observers; our system is unable to detect the active mode in a period of time; Consequently, we can't define the defected mode.

To show the effectiveness of our approach, we inject a fault in the beginning of a mode when our observers cannot detect the active mode,

In this range of time τ and at t=15.8s, we insert a default; we can note in the figure 14 that the set of locations observers cannot detect the active mode in certain range of time. Therefore, we initiate the other set of observers to know if there is a default in this range or not.

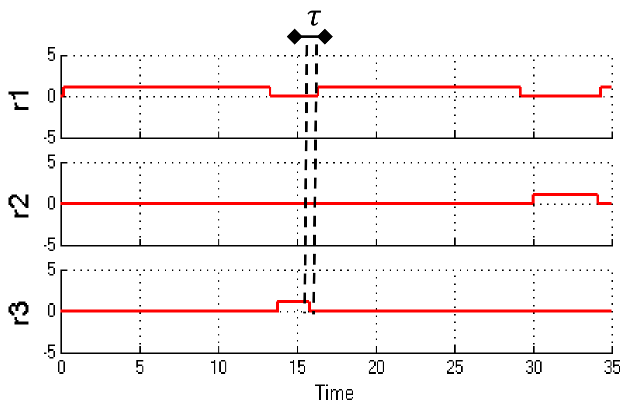


Figure 14. Response of the residues (r1, r2, r3).

With continuous observers, we can detect the defected output. Figure 15 shows the response of the residues (r11, r12, r13) ;we notice that we have a default in the first output, and the detected mode after this default is the mode 1; so, we have a default in the mode 1 in the first output at t=15.8 .

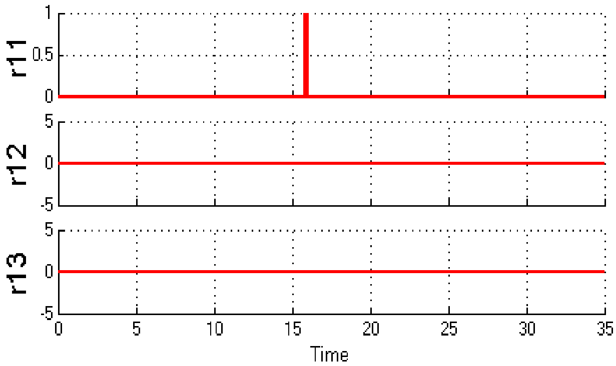


Figure 15. Response of the residues (r11, r12, r13).

Conclusion

Our contribution concerns the diagnosis of hybrid systems based on Observer and Hybrid Automata; in the first place, we generate residues from observers (modes and faults observers), which must be zero in the normal behavior of the system and sensitive to any fault. Then, we analyze these residues, using the constraints of time to reinforce the precision of detection and isolation .The diagnosis method presented in this paper detects and locates each sensor defect at any time, and it can be applied easily on many hybrid dynamical systems.

A perspective of this work is to generalize this approach to include the diagnosis problem of actuators faults.

References

[1] C.-C. Wang, C.-L. Chen, and H.-T. Yau, “Bifurcation and Chaotic Analysis of Aeroelastic Systems,” *J. Comput. Nonlinear Dyn.*, vol. 9, no. 2, pp. 021004, avr. 2014.
doi: 10.1115/1.4025124

[2] C.-T. Hsieh, H.-T. Yau, and C.-C. Wang, “Control circuit design and chaos analysis in an ultrasonic machining system,” *Eng. Comput.*, vol. 34, no. 7, pp. 2189-2211, oct. 2017.
doi: 10.1108/EC-02-2017-0044

[3] C.-T. Hsieh, H.-T. Yau, C.-C. Wang, and Y.-S. Hsieh, “Particle swarm optimization used with proportional–derivative control to analyze nonlinear behavior in the atomic force microscope,” *Adv. Mech. Eng.*, vol. 8, no. 9, pp. 1-10, 2016.
doi: 10.1177/1687814016667271

[4] C.-J. Lin, W.-L. Chu, C.-C. Wang, C.-K. Chen, and I.-T. Chen, “Diagnosis of ball-bearing faults using support vector machine based on the artificial fish-swarm algorithm,” *J. Low Freq. Noise Vib. Act. Control*, pp. 1-14, juill. 2019.
doi: 10.1177/1461348419861822

[5] H.-T. Yau, C.-C. Wang, J.-Y. Chang, and X.-Y. Su, “A Study on the Application of Synchronized Chaotic Systems of Different Fractional Orders for Cutting Tool Wear Diagnosis and Identification,” *IEEE Access*, vol. 7, pp. 15903-15911, 2019.
doi: 10.1109/ACCESS.2019.2894815

[6] B.-L. Jian, C.-C. Wang, J.-Y. Chang, X.-Y. Su, and H.-T. Yau, “Machine Tool Chatter Identification Based on Dynamic Errors of Different Self-Synchronized Chaotic Systems of Various Fractional Orders,” *IEEE Access*, vol. 7, pp. 67278-67286, 2019.
doi: 10.1109/ACCESS.2019.2917094

[7] T. E. Mezyani, “Méthodologie de surveillance des systèmes dynamiques hybrides,” PhD Thesis, Lille 1, 2005.

[8] R. Isermann, “Estimation of physical parameters for dynamic processes with application to an industrial robot,” in proceeding of *6th Mediterranean Electrotechnical Conference*, 1991, pp. 12-17 vol.1.
doi: 10.1109/MELCON.1991.161769

[9] R. J. Patton and J. Chen, “A Review of Parity Space Approaches to Fault Diagnosis,” *IFAC Proc. Vol.*, vol. 24, no. 6, pp. 65-81, sep. 1991.
doi: 10.1016/S1474-6670(17)51124-6

[10] P. M. Frank, “Advanced Fault Detection and Isolation Schemes Using Nonlinear and Robust Observers,” *IFAC Proc. Vol.*, vol. 20, no. 5, Part 3, pp. 63-68, juill. 1987.
doi: 10.1016/S1474-6670(17)55353-7

[11] H. Hammouri, M. Kinnaert, and E. H. El Yaagoubi, “Observer-based approach to fault detection and isolation for nonlinear systems,” *IEEE Trans. Autom. Control*, vol. 44, no. 10, pp. 1879-1884, oct. 1999.
doi: 10.1109/9.793728

[12] A. Takrouni-Hedfi, “Surveillance par observateur des systèmes dynamique hybrides,” PhD Thesis, Lille 1, 2013.

[13] Z. Kohavi, and N. K. Jha, *Switching and finite automata theory*. Cambridge University Press, 2009.
doi: 10.1017/CBO9780511816239

[14] A. Balluchi, L. Benvenuti, M. D. Di Benedetto, and A. L. Sangiovanni-Vincentelli, “Design of Observers for Hybrid Systems,” in *Hybrid Systems: Computation and Control*, Berlin, Heidelberg, 2002, pp. 76-89.
doi: 10.1007/3-540-45873-5_9

[15] Z.-H. Zhang, S. Li, H. Yan, and Q.-Y. Fan, “Sliding mode switching observer-based actuator fault detection and isolation for a class of uncertain systems,” *Nonlinear Anal. Hybrid Syst.*, vol. 33, pp. 322-335, août 2019.
doi: 10.1016/j.nahs.2019.04.001

[16] D. Du, V. Cocquempot, and B. Jiang, “Robust fault estimation observer design for switched systems with unknown input,” *Appl. Math. Comput.*, vol. 348, pp. 70-83, mai 2019.
doi: 10.1016/j.amc.2018.11.034

[17] T. Zhan, S. Ma, X. Liu, and H. Chen, “Impulsive observer design for a class of switched nonlinear

systems with unknown inputs," *J. Frankl. Inst.*, vol. 356, no. 12, pp. 6757‑6777, août 2019.
doi: 10.1016/j.jfranklin.2019.05.039

[18] M. Vidyasagar, *Nonlinear Systems Analysis*. Society for Industrial and Applied Mathematics, 2002.
doi: 10.1137/1.9780898719185

[19] S. Wiggins, *Introduction to applied nonlinear dynamical systems and chaos*, vol. 2. Springer Science & Business Media, 2003.
doi: 10.1007/b97481

[20] C. G. Cassandras and S. Lafortune, *Introduction to discrete event systems*. Springer Science & Business Media, 2009.
doi: 10.1007/978-0-387-68612-7

[21] B. Caillaud, P. Darondeau, L. Lavagno, and X. Xie, *Synthesis and control of discrete event systems*. Springer Science & Business Media, 2013.
doi: 10.1007/978-1-4757-6656-1

[22] A. S. Matveev and A. V. Savkin, *Qualitative theory of hybrid dynamical systems*. Springer Science & Business Media, 2012.
doi: 10.1007/978-1-4612-1364-2

[23] A. J. Van Der Schaft and J. M. Schumacher, *An introduction to hybrid dynamical systems*, vol. 251. Springer London, 2000.
doi: 10.1007/BFb0109998

[24] Q. Gaudel, "Approche intégrée de diagnostic et de pronostic pour la gestion de santé des systèmes hybrides sous incertitude," PhD Thesis, Toulouse, INSA, 2016.

[25] S. Sastry, *Nonlinear systems: analysis, stability, and control*, vol. 10. Springer Science & Business Media, 2013.
doi: 10.1007/978-1-4757-3108-8

[26] J.-F. Magni and P. Mouyon, "On residual generation by observer and parity space approaches," *IEEE Trans. Autom. Control*, vol. 39, no. 2, pp. 441‑447, févr. 1994.
doi: 10.1109/9.272354

[27] M. Staroswiecki, V. Cocquempot, and J. P. Cassar, "Observer based and parity space approaches for failure detection and identification," in *IMACS-IFAC international symposium, Lille, France*, 1991, vol. 25, pp. 536–541.

[28] S. Engell, "Modelling and analysis of hybrid systems," *Math. Comput. Simul.*, vol. 46, no. 5, pp. 445‑464, juin 1998.
doi: 10.1016/S0378-4754(98)00076-7

[29] J. Lygeros, K. H. Johansson, S. Sastry, and M. Egerstedt, "On the existence of executions of hybrid automata," in Proceedings of *38th IEEE Conference on Decision and Control (Cat. No.99CH36304)*, 1999, vol. 3, pp. 2249‑2254 vol.3.
doi: 10.1109/CDC.1999.831255