ORIGINAL ARTICLE

Optimal design of a fuzzy controller based on the imperialist competitive algorithm for fourth-order nonlinear systems

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Abstract: Nowadays, control and optimization of systems are two important and noticeable topics in all fields of science and engineering. One of the control methods, widely used because of its simplicity and accuracy, is the fuzzy control, which is based on the fuzzy logic theory. In the uncertainty conditions, this theory is able to formulate many inaccurate and vague implications, variables and systems in mathematical language, and provides requirements for reasoning, inference, control and decision making. This paper studies the stabilization of a Translation Oscillations with a Rotational Actuator (TORA) nonlinear fourth-order system by using a new fuzzy control technique which utilizes the small number of membership functions and rules. Besides, to determine the proper values for the controller parameters, the Imperialist Competitive Algorithm (ICA) is employed. Unlike the other optimization algorithms, ICA is not originated from evolutionary behavior of nature, but it is inspired by a human natural phenomenon called imperialism. In the optimization procedure, the objective function is defined as the sum of integrals of the rotor angle and the cart position. Finally, the comparison between the results obtained by the proposed strategy and some other researchers' methods is provided, which shows the superiority of this work.

Keywords: Fuzzy Control; Optimal Control; Imperialist Competitive Algorithm; Fourth-order Nonlinear System; TORA.

Introduction

Dealing with the destructive oscillations of rotary systems is an important topic in the field of industrial systems. These oscillations can appear via the rotor imbalance and cause harmful horizontal forces in rotor supports. One of the nonlinear, under-actuated and time-variant systems presented to study this issue is the TORA system [1-3].

On the other hand, development of intelligent control techniques has demonstrated their advantages over the classical control methods [4]. The reason lies in the fact that they are needless to know an accurate model of the systems, and are not sensitive to the precision of dynamic models, although they give fast dynamic responses. One of the drawbacks of the classical controllers is their inadaptability to nonlinear systems especially for higher order ones.

The fuzzy theory constitutes an intelligent control method which was initially introduced in 1965 by Zadeh in an article named "fuzzy sets" [5]. In his opinion, the classical control theory is too much concentrated on the accuracy and that is why it cannot work properly with the complicated systems.

Today, fuzzy systems are applied in a wide spectrum of sciences and technologies. Their remarkable applications during the past decades are: image segmentation [6]; vehicles active suspension systems [7]; fuzzy control of cement kilns [8]; control systems [9, 10]; human trafficking [11]; manufacturing processes [12] etc.

In fact, the most significant uses of the fuzzy systems are addressing the difficulties of control systems that successfully portray the efficiency and reliability of the fuzzy theory. For instant, Amirkhanzadeh et al. investigated a mixed-signal current-mode fuzzy logic controller in [13]. Chen et al. introduced a combined fuzzy-based power control with window-based transmission rate management in multimedia cellular systems in [14]. Jesus and Barbosa implemented smith-fuzzy fractional control of systems with time delay in [15]. Kumar and Kumar employed a hybridized ABC-GA for optimization of fractional order fuzzy pre-compensated FOPID control for a 2-DOF robot manipulator in [16]. Márquez-Vera et al. investigated the stable fuzzy control based observer via linear matrix inequalities in a fermentation process in [17]. Sheng and Zhang applied fuzzy adaptive hybrid impedance control for mirror milling system in [18]. Shi and Li implemented adaptive fuzzy control for feedback linearizable multi-input multi-output nonlinear systems with prescribed performance in [19]. Feng and Wu introduced a robust adaptive fuzzy controller for a class of nonlinear coupled beam systems with boundary uncertainties [20].

On the other hand, determining the proper values of the control parameters plays a vital role for achieving the optimal behavior of the designed controller. A suitable and usual way to select these parameters is employing the evolutionary algorithms. For example, in [21], a multi-objective genetic algorithm was utilized for optimum design of fuzzy-PID controllers for fourth order nonlinear systems. In [22], the particle swarm optimization was used for the proportional–derivative control to analyze nonlinear behavior in the atomic force microscope. In [23], the genetic algorithm was applied for optimal design of adaptive robust PID control subject to fuzzy rules and sliding modes for multi-input multi-output uncertain chaotic systems. In [24], a new multi-objective particle swarm optimization was proposed for optimal fuzzy inverse dynamics control of a parallelogram mechanism. In [25], the multi-objective particle swarm optimization was implemented for guarantee of stability of nonlinear systems using optimal fuzzy controllers.

The present paper has tried to control a fourth-order nonlinear system by a fuzzy control method. Further, to reach the best possible control response, the parameters of the introduced fuzzy system have been optimized by means of the ICA [26]. The fuzzy controller parameters are supposed to be the design variables, which their optimum values would be obtained regarding the considered objective function as the absolute values of the state vector error.

Dynamical Equations of the TORA System

Since TORA is a dynamical high-order nonlinear under-actuated system, it would be hard to design a proper controller for it with high accuracy and stability. As shown in Figure 1, the system is fixed at one side through a linear spring with stiffness k, while an eccentric rotor revolves inside the cart. The dynamical equations of the system could be obtained by Lagrange's approach and written as follows.

$$(M+m)\ddot{x}_c + m\,r(\ddot{\theta}\cos\cos\theta - \dot{\theta}^2 \\ \sin\sin\theta) + kx_c = F \qquad (1)$$

$$(I+mr^2)\dot{\theta}^2 + m\ddot{x}_c r\cos\cos\theta = T \qquad (2)$$

where M denotes the summation of cart and rotor masses, m is the rotor mass, x_c refers to the position of the cart, F is the control force, θ represents the rotation angle of the eccentric mass, k denotes the stiffness of the spring, I is the inertia moment of the eccentric mass, T refers to the control torque, and r is the rotor radius. IF the normalized variables are considered as follows.

$$x_d = x_c\sqrt{\frac{M+m}{I+mr^2}},\ F_d = \frac{F}{k}\sqrt{\frac{M+m}{I+mr^2}},$$

$$u = T\frac{M+m}{k(I+mr^2)},\quad \varepsilon = \frac{mr}{\sqrt{(I+mr^2)(M+m)}}$$

After normalizing the previous equations, they would be articulated as [2]:

$$\ddot{x}_d + x_d = \varepsilon(\dot{\theta}^2\sin\sin\theta - \ddot{\theta}\cos\cos\theta) + F_d \qquad (3)$$

$$\ddot{\theta} = u - \varepsilon\ddot{x}_d\cos\cos\theta \qquad (4)$$

where 0< $\varepsilon < 1$, F_d represents the actuation effect, and u is the control input. Herein, the ε value is taken as 0.1. If the state variables are defined as below:

$$x_1 = x_d + \varepsilon\sin\sin\theta \qquad (5)$$
$$x_2 = \dot{x}_d + \varepsilon\dot{\theta}\cos\cos\theta \qquad (6)$$
$$x_3 = \theta \qquad (7)$$
$$x_4 = \dot{\theta} \qquad (8)$$

then, the dynamical equations of the system in the state-space from would be mentioned as follows.

$$\dot{x}_1 = x_2 \qquad (9)$$
$$\dot{x}_2 = -x_1 + \varepsilon\sin\sin x_3 + F_d \qquad (10)$$
$$\dot{x}_3 = x_4 \qquad (11)$$

$$\dot{x}_4 = -\frac{\varepsilon\cos\cos x_3}{1-\varepsilon^2\cos^2 x_3}(x_1 - \varepsilon(1+x_4^2) \\ \sin\sin x_3 - F_d) + \frac{1}{1-\varepsilon^2\cos^2 x_3}u \qquad (12)$$

Fuzzy Controller Design

The fuzzy controller is a system stabilization approach based on the fuzzy logic that its simple articulation would be as: "calculation by words rather than numbers" or "control by statements instead of equations". A fuzzy system includes some rules called the "rule base" expressed as fuzzy IF-THEN statements.

Further, it contains an inference engine to combine the fuzzy rules and map the input fuzzy sets to the output fuzzy sets. Moreover, a fuzzifier is utilized to convert a real value point to a fuzzy set as the input of the inference engine, and a defuzzifier is applied to map the output of the inference engine as a fuzzy set to a crisp point. A compact formulation of this kind of the single input-single output fuzzy systems can be expressed as follows [27].

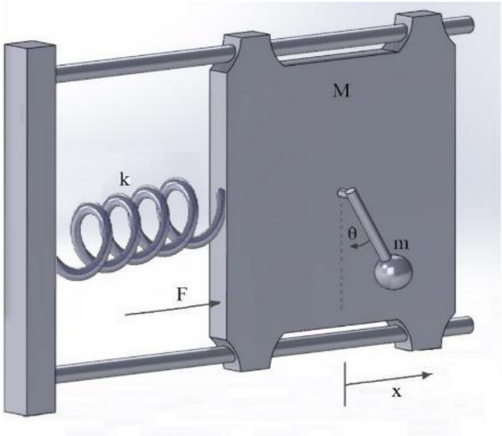

Figure 1. Configuration of the TORA system.

$$y = \frac{\sum_{i=1}^{M} \mu_i(x)\bar{y}^i}{\sum_{i=1}^{M} \mu_i(x)} \qquad \textbf{(13)}$$

where $\mu_i(x)$ is the membership function of the rule ith, M is the number of the rules, x is the input and $\underline{y}$ is the output of each rule.

In the present work, the fuzzy rule based consists three rules with one input and one output. The product inference engine, the singleton fuzzifier and the center average defuzzifier are implemented for the designed fuzzy system. In the proposed control strategy, the control effort u for the fourth-order TORA system is defined as follows.

$$u = \sum_{i=1}^{4} C_i E_i \qquad \textbf{(14)}$$

where i is the number of the state variable. C_i would be defined as: $C_i = \alpha_i + \beta_i F_i$, where α_i and β_i are two constant values that will be determined by the ICA. A block diagram of the proposed fuzzy controller is illustrated in Figure 2. F_i and E_i are the fuzzy variables obtained through Equation (14). Table 1 and Figure 3 respectively express the fuzzy rules and membership functions for fuzzy variable E_i. Furthermore, the associated fuzzy rules and membership functions of the fuzzy variable F_i are presented in Table 2 and Figure 4, respectively. Besides, xn_i is the normalized value of the state variable x_i $(i = 1,2,3,4)$ which can be calculated via Equation (15).

$$\boldsymbol{xn_i = \frac{x_i}{n_i}} \qquad \textbf{(15)}$$

Table 1:. Fuzzy rules corresponding to the fuzzy variable E_i.

$\boldsymbol{xn_i}$ (i=1,2,3,4)	$\boldsymbol{E_i}$ (i=1,2,3,4)
N	1.0
Z	0.0
P	-1.0

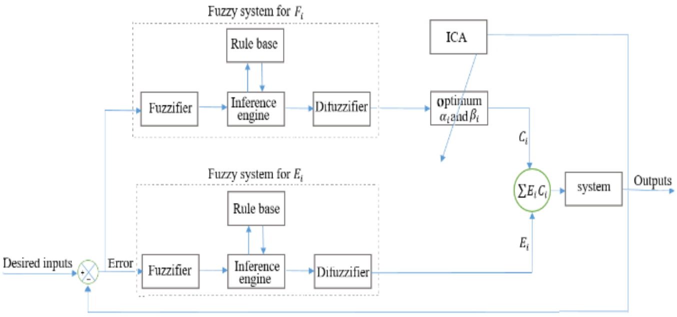

Figure 2. A block diagram of the proposed fuzzy controller.

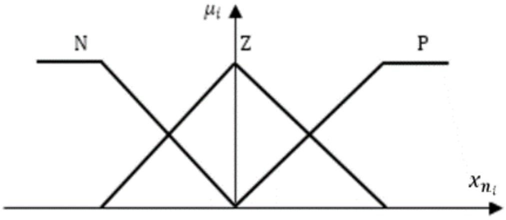

Figure 3. Membership functions corresponding to the fuzzy variable E_i.

Table 2. Fuzzy rules corresponding to the fuzzy variable F_i.

$\boldsymbol{\lvert xn_i \rvert = (i = 1,2,3,4)}$	$\boldsymbol{F_i = (i = 1,2,3,4)}$
S	0.0
M	0.5
B	1.0

Table 3. The normalization values of the fuzzy controller states.

$\boldsymbol{x_4}$	$\boldsymbol{x_3}$	$\boldsymbol{x_2}$	$\boldsymbol{x_1}$	$\boldsymbol{x_i}$
1.0	1.0	1.5	1.5	n_i

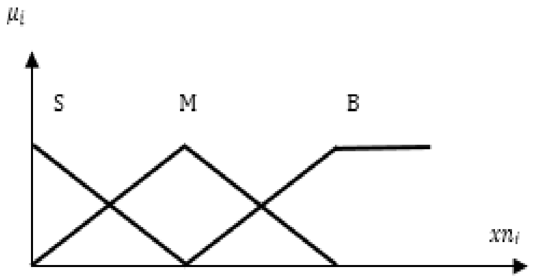

Figure 4. Membership functions corresponding to the fuzzy variable F_i.

Imperialist Competitive Algorithm

Most of the optimization algorithms have been established with respect to the evolutionary development of the natural phenomena. For instance, the genetic algorithm [28], the ant colony algorithm [29], the particle swarm algorithm [30] and the bee colony algorithm [31] are all inspired by nature.

In 2007, Atashpaz and Lucas originally presented an optimization algorithm derived from a human evolutionary behavior, i.e. the imperialism [26]. They called it as the Imperialist Competitive Algorithm (ICA), and simulated every action in the imperial path with a step in the optimization process. These actions can be articulated as: (1) formation of empires and colonies, (2) the assimilation policy, (3) the revolution, (4) the exchange of the colonial and imperial positions, (5) the imperialistic competition and (6) the elimination of weak empires.

In this paper, the objective function applied for the optimization process is considered as the sum of the areas under the displacement and angle graphs, illustrated in quation (16) that should be minimized. The fuzzy controller parameters α_i and B_i (i=1,2,3,4) are supposed to be the design variables, which their optimum values would be obtained regarding the considered objective function and by using the imperialist competitive optimization algorithm.

$$f = \int |x| dt + \int |\theta| dt \qquad \textbf{(16)}$$

Computer Simulation and Results Analysis

The values of the objective functions and design variables obtained via the optimization process are stated into Table 4. Figure 5 shows the displacement of the TORA system for initial conditions: $x(0) = 1$ and $\theta(0) = \dot{\theta}(0) = \dot{x}(0) = 0$. Figure 5 (a) is related to the fuzzy sliding mode control introduced in [2] and the fuzzy controller proposed in [3], while Figure 5 (b) corresponds to the optimal fuzzy control of this study. Comparison of these two figures clearly illustrates that the system vibration range is about -0.9^{m} to 0.7^{m} for Reference [2] and -0.8^{m} to 0.8^{m} for Reference [3], while it is between -0.2^{m} and 0.2^{m} for this work. In addition, the settling time is almost 20^{s} for Reference [2] and 60^{s} for Reference [3], where as it is less than 15^{s} for this research.

Figure 6 shows the rotation angle of the eccentric rotor for initial conditions: $x(0) = 1$ and $\theta(0) = \dot{\theta}(0) = \dot{x}(0) = 0$. Figure 6 (a) is related to the fuzzy sliding mode control approach introduced in Reference [2] and the fuzzy control method presented in Reference [3], while Figure 6 (b) corresponds to the optimal fuzzy control proposed by this study. It can be observed from this figure that the vibration range of the eccentric rotor lies between -1.5^{rad} to 1.6^{rad} for the fuzzy sliding mode and -3.5^{rad} to 3^{rad} for the fuzzy control approach, while it is around -0.03^{rad} to 0.07^{rad} in Figure 6 (b). Moreover, the settling time for the mentioned methods are respectively 55^{s} and 65^{s}, where as it is about 15^{s} for the proposed scheme. Thus, the superiority of the proposed method is completely clear.

Table 4. Design variables and objective function obtained through ICA.

Design variables	w_1	13.1083
	w_2	2.47187
	w_3	1.095925
	w_4	0.117261
	B_1	-0.936004
	B_2	2.39576
	B_3	-0.489798
	B_4	1.73808
Objective function	$\int \lvert x \rvert \, dt$	1.7857
	$\int \lvert \theta \rvert \, dt$	2.0158

Conclusion

In this research, a very simple fuzzy controller has been introduced to stabilize a TORA under-actuated nonlinear fourth-order system. The proposed controller has utilized two fuzzy variables calculated via the singleton fuzzifier, the product inference engine and the center average defuzzifier. In order to find the optimum values for the controller gains, the imperialist competitive optimization algorithm has been applied with an objective function as summation of integral absolute values of the position and angle. The obtained simulation results compared with those of reported in literature have illustrated the effectiveness of the introduced scenario.

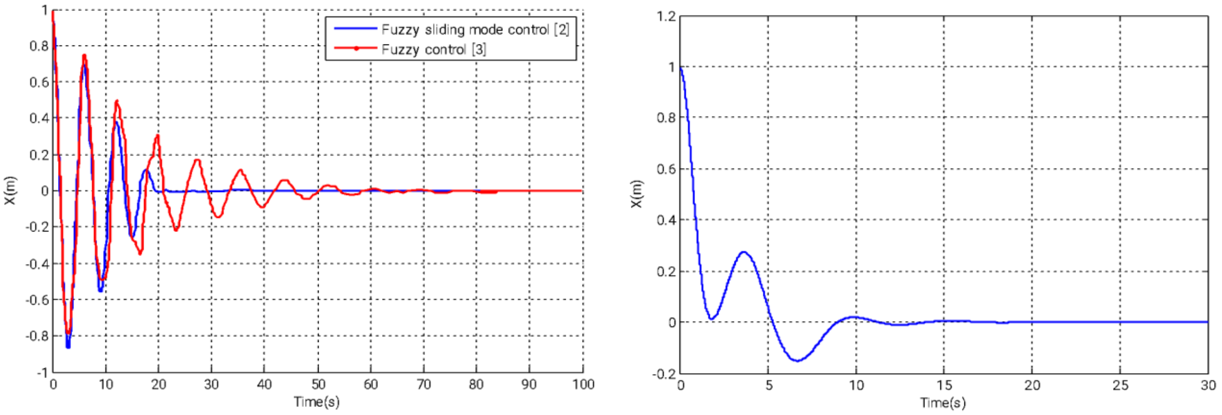

Figure 5. The system's displacement graph: (Left) References [2] and [3]; (Right) proposed fuzzy optimization control.

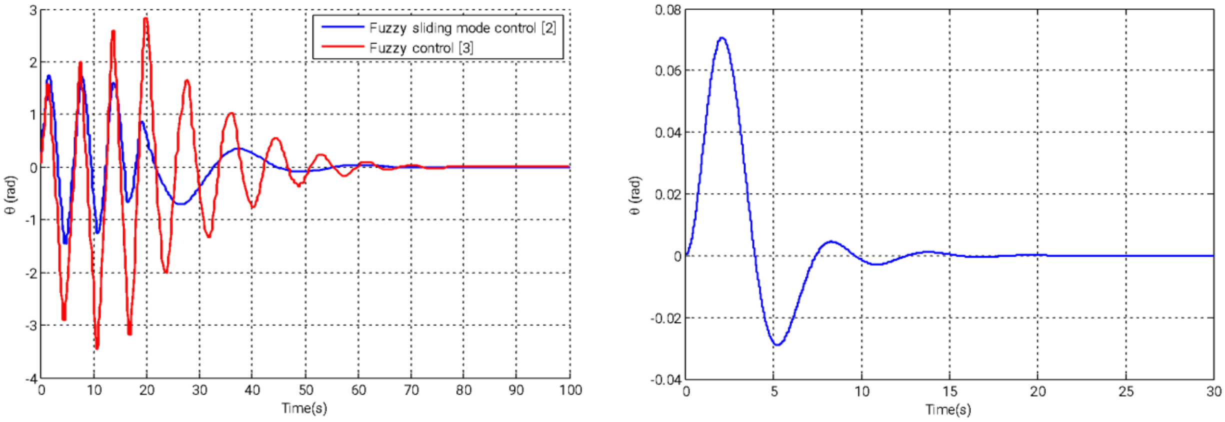

Figure 6. The rotor's rotation angle graph: (Left) References [2] and [3]; (Right) proposed fuzzy optimization control

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