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Analyzing Fuzzy System Reliability Using Vague Set Theory

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Abstract: Reliability modeling is the most important discipline of reliable engineering. A new method for analyzing fuzzy system reliability using vague set theory is demonstrated, where the reliabilities of the components of a system are represented by vague sets defined in the universe of discourse [0, 1]. The proposed method can model and analyze the fuzzy system reliability in a more flexible and more intelligent manner.

Keywords: fuzzy system reliability; false-membership function; truth-membership function; vague sets.

1. Introduction

In [18], Kaufmann et al. pointed out that reliability modeling is the most important one of the disciplines of reliable engineering. The conventional reliability of a system behavior is fully characterized in the context of probability measures. However, because of the inaccuracy and uncertainties of data, the estimation of precise values of probability becomes very difficult in many systems [11]. In recent years, some researchers focused on using fuzzy set theory [23] for fuzzy system reliability analysis [2-5], [7], [8], [11], [12], [15], [19], [20]. In [3], Cai et al. presented the following two assumptions:

(1) Fuzzy-state assumption: At any time, the system may be either in the fuzzy success state or the fuzzy failure state.
(2) Possibility assumption: The system behavior can be fully characterized by possibility measures.

In [5], Cai presented an introduction to system failure engineering and its use of fuzzy methodology. In [7], we presented a method for fuzzy system reliability analysis using fuzzy number arithmetic operations. In [11], we presented a method for fuzzy system reliability analysis based on fuzzy time series and the α–cuts operations of fuzzy numbers. In [13], Cheng et al. presented a method for fuzzy system reliability analysis by interval of confidence. In [19], Mon et al. presented a method for fuzzy system reliability analysis for components with different membership functions via non-linear programming techniques. In [20], Singer presented a fuzzy set approach for fault tree and reliability analysis. In [21], Suresh et al. presented a comparative study of probabilistic and fuzzy methodologies for uncertainty analysis using fault trees. In [22], Utkin et al. presented a system of functional equations for fuzzy reliability analysis of various systems in the possibility context.

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In this paper, we present a new method for analyzing fuzzy system reliability using vague sets [10], [16], where the reliabilities of the components of a system are represented byvague sets defined in the universe of discourse [0, 1]. The grade of membership of an element x in a vague set is represented by a vague value $[t_x, 1 - f_x]$ in [0, 1], where t_x indicates the degree of truth, f_x indicates the degree of false, $1 - t_x - f_x$ indicates the unknown part, $0 \leq t_x \leq 1 - f_x \leq 1$, and $t_x + f_x \leq 1$. The notion of vague sets is similar to that of intuitionistic fuzzy sets [1], and both of them are generalizations of the notion of fuzzy sets [23]. The proposed method can model and analyze fuzzy system reliability in a more flexible and more intelligent manner. It can provide us with a more flexible and more intelligent way for analyzing fuzzy system reliability.

This paper is organized as follows. In Section 2, we briefly review some definitions and arithmetic operations of vague sets from [10] and [16]. In Section 3, we present a new method for analyzing fuzzy system reliability based on vague set theory. The conclusions are discussed in Section 4.

2. Basic concepts of vague sets

In 1965, Zadeh proposed the theory of fuzzy sets [23]. Roughly speaking, a fuzzy set is a class with fuzzy boundaries. Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$. The grade of membership of an element u_i in a fuzzy set is represented by a real value between zero and one, where $u_i \in U$. In [16], Gau et al. pointed out that this single value combines the evidence for $u_i \in U$ and the evidence against $u_i \in U$. It does not indicate the evidence for $u_i \in U$ and the evidence against $u_i \in U$, respectively, and it does not indicate how much there is of each. Furthermore, Gau et al. also pointed out that the single value tells us nothing about its accuracy. Thus, in [16], Gau et al. presented the concepts of vague sets. In [10], we presented the arithmetic operations between vague sets. Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$, with a generic element of U denoted by u_i. A vague set $\tilde{A}$ in the universe of discourse U is characterized by a truth-membership function $t_{\tilde{A}}$, $t_{\tilde{A}}: U \rightarrow [0, 1]$, and a false-membership function $f_{\tilde{A}}$, $f_{\tilde{A}}: U \rightarrow [0, 1]$, where $t_{\tilde{A}}(u_i)$ is a lower bound of the grade of membership of u_i derived from the evidence for u_i, $f_{\tilde{A}}(u_i)$ is a lower bound on the negation of u_i derived from the evidence against u_i, and $t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$. The grade of membership of u_i in the vague set $\tilde{A}$ is bounded by a subinterval $[t_{\tilde{A}}(u_i), 1- f_{\tilde{A}}(u_i)]$ of [0, 1]. The vague value $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ indicates that the exact grade of membership $\mu_{\tilde{A}}(u_i)$ of u_i is bounded by $t_{\tilde{A}}(u_i) \leq \mu_{\tilde{A}}(u_i) \leq 1 - f_{\tilde{A}}(u_i)$, where $t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$. For example, a vague set $\tilde{A}$ in the universe of discourse U is shown in Figure 1.

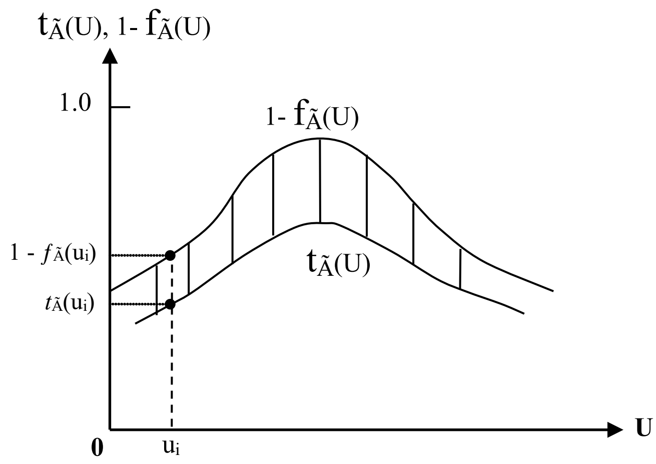

Figure 1. A vague set.

When the universe of discourse U is a finite set, a vague set $\tilde{A}$ of the universe of discourse U can be represented as

$$\tilde{A} = \sum_{i=1}^{n} [t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)] / u_i. \quad (1)$$

When the universe of discourse U is an infinite set, a vague set $\tilde{A}$ of the universe of discourse can be represented as

$$\tilde{A} = \int_U [t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)] / u_i, \quad u_i \in U. \tag{2}$$

Definition 2.1: Let $\tilde{A}$ be a vague set of the universe of discourse U with the truth-membership function $t_{\tilde{A}}$ and the false-membership function $f_{\tilde{A}}$, respectively. The vague set $\tilde{A}$ is convex if and only if for all u_1, u_2 in U,

$$t_{\tilde{A}}(\lambda u_1 + (1-\lambda)u_2) \geq \text{Min}(t_{\tilde{A}}(u_1), t_{\tilde{A}}(u_2)), \tag{3}$$

$$1 - f_{\tilde{A}}(\lambda u_1 + (1-\lambda) u_2) \geq \text{Min}(1 - f_{\tilde{A}}(u_1), 1 - f_{\tilde{A}}(u_2)), \tag{4}$$

where $\lambda \in [0, 1]$.

Definition 2.2: A vague set $\tilde{A}$ of the universe of discourse U is called a normal vague set if $\exists$ $u_i \in U$, such that $1- f_{\tilde{A}}(u_i) = 1$. That is, $f_{\tilde{A}}(u_i) = 0$.

Definition 2.3: A vague number is a vague subset in the universe of discourse U that is both convex and normal.

In the following, we introduce some arithmetic operations of triangular vague sets [10].

Let us consider the triangular vague set $\tilde{A}$ shown in Figure 2, where the triangular vague set $\tilde{A}$ can be parameterized by a tuple <[(a, b, c); μ_1], [(a, b, c); μ_2]>. For convenience, the tuple <[(a, b, c); μ_1], [(a, b, c); μ_2]> can also be abbreviated into <[(a, b, c); μ_1; μ_2]>, where $0 \leq \mu_1 \leq \mu_2 \leq 1$.

Some arithmetic operations between triangular vague sets are as follows:

Case 1: Consider the triangular vague sets $\tilde{A}$ and $\tilde{B}$ shown in Figure 3, where

$$\tilde{A} = \langle[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle = \langle[(a_1, b_1, c_1); \mu_1; \mu_2]\rangle,$$

$$\tilde{B} = \langle[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]\rangle = \langle[(a_2, b_2, c_2); \mu_1; \mu_2]\rangle,$$

and $0 \leq \mu_1 \leq \mu_2 \leq 1$. The arithmetic operations between the triangular vague sets $\tilde{A}$ and $\tilde{B}$ are defined as follows:

$$\begin{aligned}\tilde{A} \oplus \tilde{B} &= \langle[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \oplus \langle[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]\rangle \\ &= \langle[(a_1 + a_2, b_1 + b_2, c_1 + c_2); \mu_1], [(a_1 + a_2, b_1 + b_2, c_1 + c_2); \mu_2]\rangle \\ &= \langle[(a_1 + a_2, b_1 + b_2, c_1 + c_2); \mu_1; \mu_2]\rangle,\end{aligned} \tag{5}$$

$$\begin{aligned}\tilde{B} \ominus \tilde{A} &= \langle[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]\rangle \ominus \langle[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \\ &= \langle[(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mu_1], [(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mu_2]\rangle \\ &= \langle[(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mu_1; \mu_2]\rangle,\end{aligned} \tag{6}$$

$$\begin{aligned}\tilde{A} \otimes \tilde{B} &= \langle[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \otimes \langle[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]\rangle \\ &= \langle[(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mu_1], [(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mu_2]\rangle \\ &= \langle[(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mu_1; \mu_2]\rangle,\end{aligned} \tag{7}$$

$$\begin{aligned}\tilde{B} \oslash \tilde{A} &= \langle[(a_2, b_2, c_2); \mu_1], [(a_2, b_2, c_2); \mu_2]\rangle \oslash \langle[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]\rangle \\ &= \langle[(a_2/c_1, b_2/b_1, c_2/a_1); \mu_1], [(a_2/c_1, b_2/b_1, c_2/a_1); \mu_2]\rangle \\ &= \langle[(a_2/c_1, b_2/b_1, c_2/a_1); \mu_1; \mu_2]\rangle.\end{aligned} \tag{8}$$

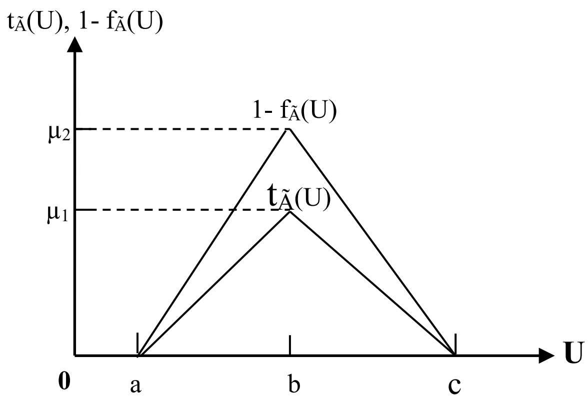

Figure 2. A triangular vague set.

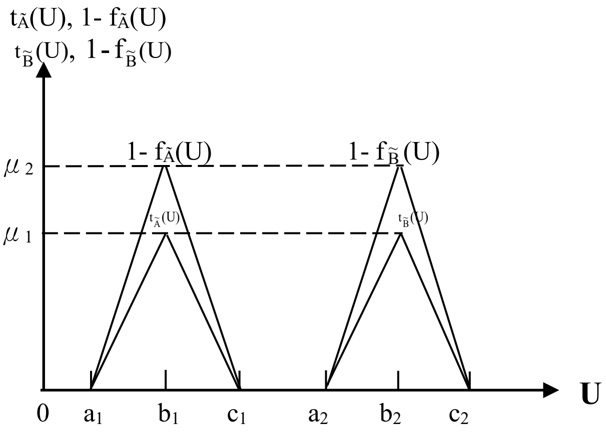

Figure 3. Triangular vague sets $\tilde{A}$ and $\widetilde{B}$ (Case 1).

Case 2: Consider the triangular vague sets $\tilde{A}$ and $\widetilde{B}$ shown in Figure 4, where

$$\widetilde{A} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]>,$$

$$\widetilde{B} = <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]>,$$

and $0 \le \mu_3 \le \mu_1 \le \mu_4 \le \mu_2 \le 1$. The arithmetic operations between the triangular vague sets $\tilde{A}$ and $\widetilde{B}$ are defined as follows:

$$\tilde{A} \oplus \widetilde{B} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]> \oplus <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]>$$
$$= <[(a_1 + a_2, b_1 + b_2, c_1 + c_2); \mathrm{Min}(\mu_1, \mu_3)], [(a_1 + a_2, b_1 + b_2, c_1 + c_2); \mathrm{Min}(\mu_2, \mu_4)]>, \quad (9)$$

$$\widetilde{B} \ominus \tilde{A} = <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]> \ominus <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]>$$
$$= <[(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mathrm{Min}(\mu_1, \mu_3)], [(a_2 - c_1, b_2 - b_1, c_2 - a_1); \mathrm{Min}(\mu_2, \mu_4)]>, \quad (10)$$

$$\tilde{A} \otimes \widetilde{B} = <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]> \otimes <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]>$$
$$= <[(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mathrm{Min}(\mu_1, \mu_3)], [(a_1 \times a_2, b_1 \times b_2, c_1 \times c_2); \mathrm{Min}(\mu_2, \mu_4)]>, \quad (11)$$

$$\widetilde{B} \oslash \tilde{A} = <[(a_2, b_2, c_2); \mu_3], [(a_2, b_2, c_2); \mu_4]> \oslash <[(a_1, b_1, c_1); \mu_1], [(a_1, b_1, c_1); \mu_2]>$$
$$= <[(a_2/c_1, b_2/b_1, c_2/a_1); \mathrm{Min}(\mu_1, \mu_3)], [(a_2/c_1, b_2/b_1, c_2/a_1); \mathrm{Min}(\mu_2, \mu_4)]>. \quad (12)$$

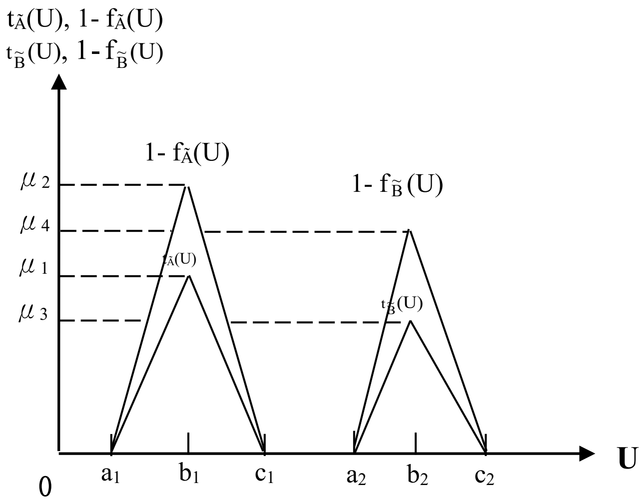

Figure 4. Triangular vague sets $\tilde{A}$ and $\widetilde{B}$ (Case 2).

3. Analyzing fuzzy system reliability based on vague sets

In this section, we present a new method for analyzing fuzzy system reliability based on vague set theory, where the reliabilities of the components of a system are represented by vague sets defined in the universe of discourse [0, 1].

Consider a serial system shown in Figure 5, where the reliability $\widetilde{R}_i$ of component P_i is represented by a vague set $<[(a_i, b_i, c_i); \mu_{i1}; \mu_{i2}]>$, where $0 \le \mu_{i1} \le \mu_{i2} \le 1$, and $1 \le i \le n$. Then, the reliability $\widetilde{R}$ of the serial system shown in Figure 5 can be evaluated as follows:

$$\widetilde{R} = \widetilde{R}_1 \otimes \widetilde{R}_2 \otimes \ldots \otimes \widetilde{R}_n$$

$$= <[(a_1, b_1, c_1); \mu_{11}; \mu_{12}]> \otimes <[(a_2, b_2, c_2); \mu_{21}; \mu_{22}]> \otimes \ldots \otimes <[(a_n, b_n, c_n); \mu_{n1}; \mu_{n2}]>$$

$$= <[(\prod_{i=1}^{n} a_i, \prod_{i=1}^{n} b_i, \prod_{i=1}^{n} c_i); \mathrm{Min}(\mu_{11}, \mu_{21}, \ldots, \mu_{n1}); \mathrm{Min}(\mu_{12}, \mu_{22}, \ldots, \mu_{n2})]>. \quad (13)$$

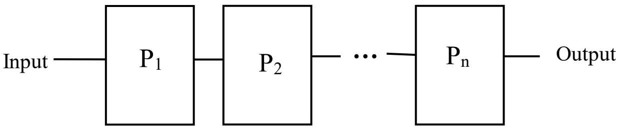

Figure 5. Configuration of a serial system.

Furthermore, consider the parallel system shown in Figure 6, where the reliability $\tilde{R}_i$ of component P_i is represented by a vague set $<[(a_i, b_i, c_i); \mu_{i1}; \mu_{i2}]>$, where $0 \le \mu_{i1} \le \mu_{i2} \le 1$, and $1 \le i \le n$. Then, the reliability $\tilde{R}$ of the parallel system shown in Figure 6 can be evaluated as follows:

$$\tilde{R} = 1 \ominus \prod_{i=1}^{n} (1 \ominus \tilde{R}_i)$$

$$= 1 \ominus (1 \ominus <[(a_1, b_1, c_1); \mu_{11}; \mu_{12}]>) \otimes (1 \ominus <[(a_2, b_2, c_2); \mu_{21}; \mu_{22}]>) \otimes \ldots \otimes (1 \ominus <[(a_n, b_n, c_n); \mu_{n1}; \mu_{n2}]>)$$

$$= 1 \ominus <[(1 - c_1, 1 - b_1, 1 - a_1); \mu_{11}; \mu_{12}]> \otimes <[(1 - c_2, 1 - b_2, 1 - a_2); \mu_{21}; \mu_{22}]> \otimes \ldots \otimes <[(1 - c_n, 1 - b_n, 1 - a_n); \mu_{n1}; \mu_{n2}]>$$

$$= 1 \ominus <[(\prod_{i=1}^{n}(1 - c_i), \prod_{i=1}^{n}(1 - b_i), \prod_{i=1}^{n}(1 - a_i)); \mathrm{Min}(\mu_{11}; \mu_{21}, \ldots, \mu_{n1}); \mathrm{Min}(\mu_{12}; \mu_{22}, \ldots, \mu_{n2})]>$$

$$= <[(1 - \prod_{i=1}^{n}(1 - a_i), 1 - \prod_{i=1}^{n}(1 - b_i), 1 - \prod_{i=1}^{n}(1 - c_i)); \mathrm{Min}(\mu_{11}; \mu_{21}, \ldots, \mu_{n1}); \mathrm{Min}(\mu_{12}; \mu_{22}, \ldots, \mu_{n2})]>. \quad (14)$$

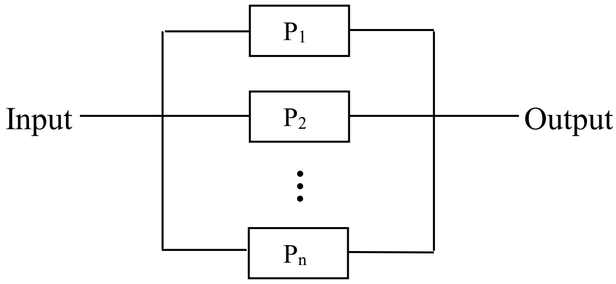

Figure 6. Configuration of a parallel system.

In the following, we use an example to illustrate the fuzzy system reliability analysis process of the proposed method.

Example 3.1: Consider the system shown in Figure 7, where the reliabilities of the components P_1, P_2, P_3 and P_4 are $\tilde{R}_1$, $\tilde{R}_2$, $\tilde{R}_3$ and $\tilde{R}_4$, respectively, where

$$\tilde{R}_1 = <[(a_1, b_1, c_1); \mu_{11}; \mu_{12}]>,$$
$$\tilde{R}_2 = <[(a_2, b_2, c_2); \mu_{21}; \mu_{22}]>,$$
$$\tilde{R}_3 = <[(a_3, b_3, c_3); \mu_{31}; \mu_{32}]>,$$
$$\tilde{R}_4 = <[(a_4, b_4, c_4); \mu_{41}; \mu_{42}]>,$$

$0 \le \mu_{i1} \le \mu_{i2} \le 1$, and $1 \le i \le 4$. Based on the previous discussion, we can see that the reliability $\tilde{R}$ of the system shown in Figure 7 can be evaluated as follows:

$$\tilde{R} = [1 \ominus (1 \ominus \tilde{R}_1) \otimes (1 \ominus \tilde{R}_2)] \otimes [1 \ominus (1 \ominus \tilde{R}_3) \otimes (1 \ominus \tilde{R}_4)]$$

$$= [1 \ominus (1 \ominus <[(a_1, b_1, c_1); \mu_{11}; \mu_{12}]>) \otimes (1 \ominus <[(a_2, b_2, c_2); \mu_{21}; \mu_{22}]>) \otimes [1 \ominus (1 \ominus <[(a_3, b_3, c_3); \mu_{31}; \mu_{32}]>) \otimes (1 \ominus <[(a_4, b_4, c_4); \mu_{41}; \mu_{42}]>)]$$

$$= [1 \ominus <[(1 - c_1, 1 - b_1, 1 - a_1); \mu_{11}; \mu_{12}]> \otimes <[(1 - c_2, 1 - b_2, 1 - a_2); \mu_{21};$$

$$\mu_{22}]>] \otimes [1 \ominus <[(1 - c_3, 1 - b_3, 1 - a_3); \mu_{31}; \mu_{32}]> \otimes <[(1 - c_4, 1 - b_4, 1 - a_4); \mu_{41}; \mu_{42}]>]$$

$$= [1 \ominus <[((1 - c_1)(1 - c_2), (1 - b_1)(1 - b_2), (1 - a_1)(1 - a_2)); \mathrm{Min}(\mu_{11}; \mu_{21}); \mathrm{Min}(\mu_{12}, \mu_{22})]>] \otimes [1 \ominus <[((1 - c_3)(1 - c_4), (1 - b_3)(1 - b_4), (1 - a_3)(1 - a_4)); \mathrm{Min}(\mu_{31}; \mu_{41}); \mathrm{Min}(\mu_{32}, \mu_{42})]>]$$

$$= <[(1 - (1 - a_1)(1 - a_2), 1 - (1 - b_1)(1 - b_2), 1 - (1 - c_1)(1 - c_2)); \mathrm{Min}(\mu_{11}; \mu_{21}); \mathrm{Min}(\mu_{12}, \mu_{22})]>] \otimes <[(1 - (1 - a_3)(1 - a_4), 1 - (1 - b_3)(1 - b_4), 1 - (1 - c_3)(1 - c_4)); \mathrm{Min}(\mu_{31}; \mu_{41}); \mathrm{Min}(\mu_{32}, \mu_{42})]>$$

$$= <[(a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2, c_1 + c_2 - c_1c_2); \mathrm{Min}(\mu_{11}; \mu_{21}); \mathrm{Min}(\mu_{12}, \mu_{22})]> \otimes <[(a_3 + a_4 - a_3a_4, b_3 + b_4 - b_3b_4, c_3 + c_4 - c_3c_4); \mathrm{Min}(\mu_{31}; \mu_{41}); \mathrm{Min}(\mu_{32}, \mu_{42})]>$$

$$= <[((a_1 + a_2 - a_1a_2)(a_3 + a_4 - a_3a_4), (b_1 + b_2 - b_1b_2)(b_3 + b_4 - b_3b_4), (c_1 + c_2 - c_1c_2)(c_3 + c_4 - c_3c_4)), \mathrm{Min}(\mu_{11}, \mu_{21}, \mu_{31}, \mu_{41}); \mathrm{Min}(\mu_{12}, \mu_{22}, \mu_{32}, \mu_{42})]>$$

$$= <[(a_1a_3 + a_1a_4 - a_1a_3a_4 + a_2a_3 + a_2a_4 - a_2a_3a_4 - a_1a_2a_3 - a_1a_2a_4 + a_1a_2a_3a_4, b_1b_3 + b_1b_4 - b_1b_3b_4 + b_2b_3 + b_2b_4 - b_2b_3b_4 - b_1b_2b_3 - b_1b_2b_4 + b_1b_2b_3b_4, c_1c_3 + c_1c_4 - c_1c_3c_4 + c_2c_3 + c_2c_4 - c_2c_3c_4 - c_1c_2c_3 - c_1c_2c_4 + c_1c_2c_3c_4); \mathrm{Min}(\mu_{11}, \mu_{21}, \mu_{31}, \mu_{41}); \mathrm{Min}(\mu_{12}, \mu_{22}, \mu_{32}, \mu_{42})]>. \quad (15)$$

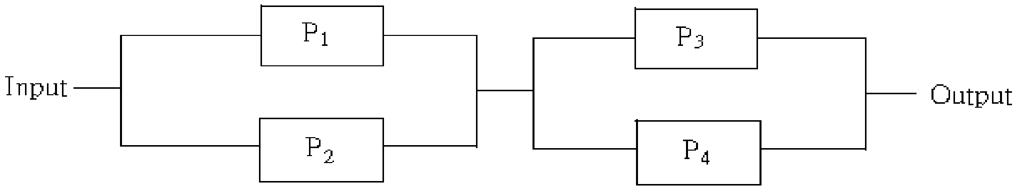

Figure 7. A system with four components P1, P2, P3, P4.

4. Conclusions

We have presented a new method for analyzing fuzzy system reliability using vague set theory, where the components of a system are represented by vague sets defined in the universe of discourse [0, 1]. The proposed method can model and analyze fuzzy system reliability in a more flexible and more intelligent manner. It can provide us with a more flexible and more intelligent way for fuzzy system reliability analysis.

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References

[1] Atanassov, K. 1986. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20: 87-96.

[2] Cai, K. Y., Wen, C. Y., and Zhang, M. L. 1991. Fuzzy variables as a basis for a theory of fuzzy reliability in possibility context. *Fuzzy Sets and Systems*, 42: 145-172.

[3] Cai, K. Y., Wen, C. Y., and Zhang, M. L. 1991. Posbist reliability behavior of typical systems with two types of failure. *Fuzzy Sets and Systems*, 43: 17-32.

[4] Cai, K. Y., Wen, C. Y., and Zhang, M. L. 1991. Fuzzy reliability modeling of gracefully degradable computing sys-

tems. *Reliability Engineering and System Safety*, 33: 141-157.

[5] Cai, K. Y. 1996. System failure engineering and fuzzy methodology: An introductory overview. *Fuzzy Sets and Systems*, 83: 113-133.

[6] Chen, S. M. 1988. A new approach to handling fuzzy decisionmaking problems. *IEEE Transactions on Systems, Man, and Cybernetics*, 18: 1012-1016.

[7] Chen, S. M. 1994. Fuzzy system reliability analysis using fuzzy number arithmetic operations. *Fuzzy Sets and Systems*, 64: 31-38.

[8] Chen, S. M. and Jong, W. T. 1996. Analyzing fuzzy system reliability using interval of confidence. *International Jounal of Information Management and Engineering*, 2: 16-23.

[9] Chen, S. M. 1996. New methods for subjective mental workload assessment and fuzzy risk analysis. *Cybernetics and Systems: An International Journal*, 27: 449-472.

[10] Chen, S. M. 1995. Arithmetic operations between vague sets. *Proceedings of the International Joint Conference of CFSA/IFIS/SOFT'95 on Fuzzy Theory and Applications*, Taipei, Taiwan, Republic of China: 206-211.

[11] Chen, S. M. 1996. New method for fuzzy system reliability analysis. *Cybernetics and Systems: An International Journal*, 27: 385-401.

[12] Chen, S. M. 1997. Fuzzy system reliability analysis based on vague set theory. *Proceedings of the 1997 IEEE International Conference on Systems, Man, and Cybernetics*, Orlando, Florida, U. S. A.: 1650-1655.

[13] Chen, S. M. 1997. Similarity measures between vague sets and between elements. *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 27: 153-158.

[14] Chen, S. M. and Shiau, Y. S. 1988. Vague reasoning and knowledge representation using extended fuzzy Petri nets. *Journal of Information Science and Engineering*, 14: 391-408.

[15] Cheng, C. H. and Mon, D. L. 1993. Fuzzy system reliability analysis by interval of confidence. *Fuzzy Sets and Systems*, 56: 29-35.

[16] Gau, W. L. and. Buehrer, D. J. 1993. Vague sets. *IEEE Transactions on Systems, Man, and Cybernetics*, 23: 610-614.

[17] Kandel, A. 1986. "*Fuzzy Mathematical Techniques with Applications*". Addison-Wesley, Massachusetts, U. S. A.

[18] Kanfmann, A. and Gupla, M. M. 1988. "*Fuzzy Mathematical Models in Engineering and Management Science*". North-Holland, Amsterdam.

[19] Mon, D. L. and Cheng, C. H. 1994. Fuzzy system reliability analysis for components with different membership functions. *Fuzzy Sets and Systems*, 64: 147-157.

[20] Singer, D. 1990. A fuzzy set approach to fault tree and reliability analysis. *Fuzzy Sets and Systems*, 34: 145-155.

[21] Suresh, P. V., Babar, A. K., and Raj, V. V. 1996. Uncertainty in fault tree analysis: A fuzzy approach. *Fuzzy Sets and Systems*, 83: 135-141.

[22] Utkin, L. V. and Gurov, S. V. 1996. A general formal approach for fuzzy reliability analysis in the possibility context. *Fuzzy Sets and Systems*, 83: 203-213.

[23] Zadeh, L. A. 1965. Fuzzy sets. *Information and Control*, 8: 338-353.

[24] Zimmermann, H. J. 1991. "*Fuzzy Set Theory and Its Applications*". Kluwer Academic Publishers, Boston, U. S. A.