International Journal of Applied Science and Engineering
2003. 1, 2: 114-122

# The Periodic Preventive Maintenance Policy for Deteriorating Systems by Using Improvement Factor Model

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**Abstract:** Many researchers have shown that the imperfect preventive maintenance (PM) can reduce the wear out and aging effects of deteriorating systems (or machines) to a certain level between the conditions of as good as new and of as bad as old. The concept of the improvement factor is used to measure the extent of the restoration for a deteriorating system in this paper. The proposed improvement factor is considered as a variable depending upon the system's age (or the operating time), the number of PM performed in the specified finite time span, and the cost ratio of each PM to the replacement. By applying Lie and Chun's model, the proposed improvement factor model consists of three different functions to measure various age restoration situations.

By minimizing the expected cost rate per unit time over a finite time span, an optimal preventive maintenance policy for a deteriorating system is proposed in this paper. It is assumed that the periodic PM is performed for the deteriorating system with a minimal repair at each failure.

In this paper, it is considered that a deteriorating system undergoes *N* times of periodic PM with a minimal repair at each failure during the specified finite time span (*T*) and is replaced at *T*. The expression to compute the expected cost rate per unit time is derived and the optimal number of PM is also obtained for the Weibull failure case.

**Keywords:** imperfect maintenance; preventive maintenance; reliability; improvement factor.

## 1. Introduction

It has been shown that the imperfect preventive maintenance can rejuvenate deteriorating systems (or machines) and reduce the failure rate [1-15]. Nakagawa [11] has proposed that the effective age of a system is reduced by a certain units of time after each imperfect PM being performed. Canfield [1] has presented the effect of the imperfect PM on the hazard function of which the hazard rate at age $t$ is restored to the hazard rate at a younger age, while the hazard level remains unchanged. Similar to Nakagawa's concept [11], Malik [8] as well as Lie and Chun [7] has proposed the improvement factor ($k$) to investigate the age restoration of a system. In other words, the imperfect PM can reduce a system's age from $t$ to $t/k$ and is resulting in restoring the system's reliability to $R(t/k)$ from $R(t)$. The restoration effect is affected by several factors, such as, equipment age, time interval of the periodic PM, and cost of PM.

Malik [8] has proposed the concept of improvement factor, $k$, to measure the restoration of age and failure rate of a system after

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performing maintenance. It is equivalent to the minimal repair if $k = 1$ and is equivalent to the perfect maintenance if $k = \infty$, is performed. Malik [8] has further proposed that a system can be restored to the age $t/k$ after being maintained at age $t$. In turn, the reliability and failure rate of a system can be restored to the level at age $t/k$. Lie and Chun [7] have proposed that the improvement factor is affected by the maintenance cost and the system's age.

Canfield [1] has found that operational stress of a system (or equipment) will be induced during the operating time period and will result in deteriorating condition which causes the failure rate to be increased over time. Hence, Canfield [1] has suggested that PM be performed to reduce the operational stress. Canfield [1] has also presented that the failure rate can be restored to the level at $t-\tau$, where $\tau$ can be treated as an improvement factor and $0 \leqq \tau \leqq T_p$, $T_p$ is the time interval between each maintenance. Chan and Shaw [2] have also proposed that the failure rate of the system can be reduced by performing PM. The study has shown that the reduction of failure rate is determined by the system's age and the number of PM being performed.

Park et al [12] have proposed that the operational stress can be reduced by performing PM, in turn, to slow down the deteriorating rate of the equipment. Park et al [12] have presented that the optimal maintenance intervals and the optimal number of maintenance can be obtained by minimizing the expected cost per unit time for an infinite time span. The study has been widely used for surveying the optimal maintenance policy.

The improvement factor model has been proposed to measure the restoration effect of the PM by several researchers [1,7,8,9]. However, most of above studies assume that the improvement factors be constants. Although Lie and Chun [7] consider the improvement factors as variables, yet, some parameters are not well defined. Hence, the purpose of this paper is to propose a robust model considering the improvement factor as a variable affected by system's age, cost, and numbers of maintenance performed.

## 2. Preventive maintenance model with improvement factors

The improvement factor model proposed by Lie and Chun's concept [7] is used in this paper and is expressed as below and shown in Figure 1.

$$\frac{1}{k} = (\frac{C_{pm} - C_{pr}}{C_{pr}})^m, \ m = 2,4,6,8,10,... \tag{1}$$

$$\frac{1}{k} = -(\frac{C_{pm}}{C_{pr}}) + 1 \tag{2}$$

$$\frac{1}{k} = (\frac{C_{pm}}{C_{pr}})^m + 1, \ m = 2,4,6,8,10,... \tag{3}$$

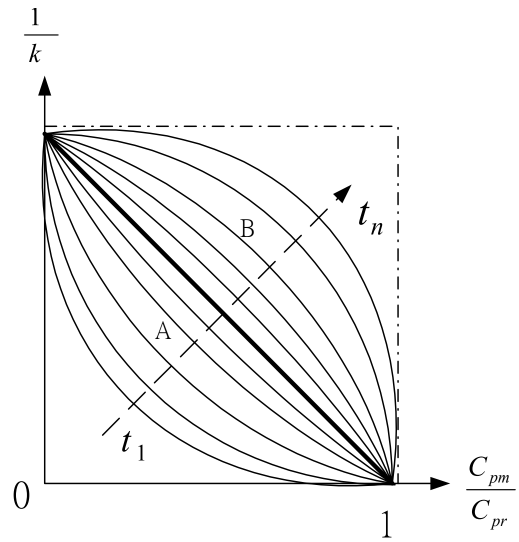


**Figure 1.** The relationship between the improvement factor and the PM cost ratio

It is noticed that, for the case of $1 \leqq k \leqq \infty$, the improvement factor is approaching 1 when $C_{pm}$ is approaching zero; whereas the improvement factor is approaching $\infty$ when $C_{pm}$ is approaching $C_{pr}$. Unfortunately, $m$ is not clearly defined in the above Equations.

Based on the concepts of Lie and Chun's improvement factor, a new improvement factor model with explicit definition of $m$ is proposed in this paper. The assumptions made for the proposed improvement factor model are:

1. Minimal repair is performed when failure occurs and PMs are performed within a finite time span.
2. PM will be performed on the equipment for a total number of $N$ times within the finite time span $[0,T]$. The equipment will be replaced at $N$+1 times on which $T$ is reached.
3. The equipment is treated as a system or a single unit.
4. The system (equipment) will be deteriorating over time and leading to the increase of the failure rate and decrease of the reliability.
5. The improvement factor of each PM is a variable which is affected by the system's age ($t$), the total number of PM performed over the specified finite time span ($N$), and the cost ratio of each PM to the replacement ($C_{pm}/C_{pr}$). It is also assumed that the system's age ($t$) is measured by the number of PM performed ($i$) at time $t$.
6. The cost ratio of each PM to the replacement ($C_{pm}/C_{pr}$) is a constant.

By applying the 5$^{\text{th}}$ assumption, the $m$ in Eqs. (1) and (3) of Lie and Chun's improvement factor can be redefined as $m_i$ as follows.

$$m_i = \begin{cases} N-i, & \text{for Eq.(1)} \\ i, & \text{for Eq.(3)} \end{cases} \tag{4}$$

Hence, based on Eqs. (1) to (4), the proposed improvement factor model after performing the $i^{\text{th}}$ PM is presented below.

$$\frac{1}{b_{1i}} = \left( \frac{C_{pr} - a \cdot C_{pm}}{C_{pr}} \right)^{N-i} \tag{5}$$

$$\frac{1}{b_{2i}} = -\left( \frac{a \cdot C_{pm}}{C_{pr}} \right) + 1 \tag{6}$$

$$\frac{1}{b_{3i}} = -\left( \frac{a \cdot C_{pm}}{C_{pr}} \right)^{i} + 1 \tag{7}$$

where $a$ is an adjustment parameter for the improvement factor and can be determined by the historical data or by experience. From Eqs. (5) to (7), it can be seen that the above improvement factor model includes three types of restoration effect, respectively.

Type #1: The restoration effect is more sensitive to the smaller amount of $C_{pm}/C_{pr}$ ratio and is less sensitive to the larger amount of $C_{pm}/C_{pr}$ ratio (as shown in Figure 2(a)).

Type #2: The relationship between the restoration effect and the $C_{pm}/C_{pr}$ ratio is linear (as shown in Figure 2(b)).

Type #3: The restoration effect is more sensitive to the larger amount of $C_{pm}/C_{pr}$ ratio and is less sensitive to the smaller amount of $C_{pm}/C_{pr}$ ratio (as shown in Figure 2(c)).

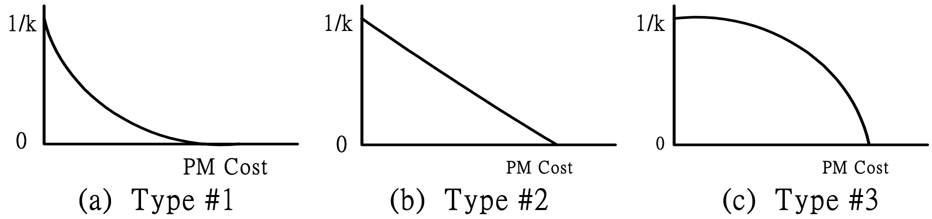


**Figure 2.** Three different types of restoration effect for the proposed improvement factor model

The system can be restored to a younger age, called effective age, after each PM. Let $w_i^-$ and $w_i^+$ represent the effective age before and after the $i^{th}$ PM, respectively, as expressed below.

$$w_i^- = w_{i-1}^+ + \frac{T}{N+1} \tag{8}$$

$$w_i^+ = w_{i-1}^+ + \frac{T/(N+1)}{b_{hi}}, \quad h=1,2,3;\ i=1,2,\ldots,N \tag{9}$$

By substituting the random variable $t$ of the reliability function by $w_i^-$ and $w_i^+$, the reliability function, $R_i(t)$, for the effective age can be obtained and shown in the following Equations.

$$R_i^-(t) = R_i^-(w_i^-) = R_i^-(w_{i-1}^+ + \frac{T}{N+1}) \tag{10}$$

$$R_i^+(t) = R_i^+(w_i^+) = R_i^+(w_{i-1}^+ + \frac{T/(N+1)}{b_{hi}}) \tag{11}$$

It is then reasonable to use the total number, $N$, as a decision variable for determining maintenance policy so that an optimal $N^*$ can be found by minimizing the cost objective function. Hence, the cost objective function of the improvement factor model is proposed below.

$$TC = C_{pm} N + C_{mr} \left\{ \int_0^{W_1^-} \lambda(t)dt + \int_{W_1^+}^{W_2^-} \lambda(t)dt + \ldots + \int_{W_N^+}^{W_N^+ + \frac{T}{N+1}} \lambda(t)dt \right\} \tag{12}$$

where $C_{mr}$ is the minimal repair cost per failure, $\lambda(t)$ is the hazard rate function. The optimal solution $N^*$ can be obtained by applying numerical method to minimize Eq. (12).

## 3. Results and discussion

To verify the model, an example is given for this study. Table 1 shows the parameters used for the case of periodic PM with the Weibull failure distribution. It should be noted that the value of parameter $a$ of the Eq. (5) is smaller than that of Eq. (7). It is because Eq. (5) represents the restoration of Type #1 improvement factor; whereas Eq. (7) is categorized for the case of Type #3 improvement factor.

Figures 3 and 4 present the sensitivity of effective age, $w$, to the PM cost, $C_{pm}$, and to the total number of PM, $N$, respectively. It can be verified that either larger $C_{pm}$ or larger $N$ has a better effective age.

The relationship between total cost and the total number of PM in the finite time span for all three types of improvement factors are presented in Figure 5. It can be seen that only Type #1 improvement factor can generate the optimal solution for the $N$ in this example. Figure 6 shows the sensitivity of each type of improvement factor to the system's age. It is noted that the effect of Type #1 improvement factor is highly sensitive when the system is at an older age; whereas the effect of Type #3 improvement factor is highly sensitive when the system is at a younger age. On the other hand, the restoration effect of Type #2 improvement factor is not affected by the system's age.

The sensitivity of the adjustable parameter, $a$, in the improvement factor model has been tested for all three types. Figure 7 illustrates the effect of Type#1 improvement factor that the larger the $a$, the better the restoration. Same results can also be found for the other two types of improvement factor.

Furthermore, the effects of both PM cost ($C_{pm}$) and the number of PM ($N$) on the reliability are presented in Figures 8 and 9, respectively. It can be verified that either larger $C_{pm}$ or larger $N$ has a better reliability condition.

In the analysis of the relationship between the $C_{pm}$ and the improvement factor, Table 2 shows the results of the case that $C_{pm}$ = 8000 and $C_{pm}/C_{mr}$ =0.0133 for Type #1 improvement factor. Table 2 lists the given number of $C_{pm}$ and $C_{pm}/C_{pr}$ and the values of $N^*$, $T_p$, and $TC(N^*)$ which are obtained by using the numerical analysis method. For Type #1 improvement factor and the case of $C_{pm}$ = 8000 and $C_{pm}/C_{mr}$ =0.0133, Figure 10 shows the relationship between $N$ and $TC$ with various ratios of $C_{pm}/C_{pr}$. It also shows that an optimal $N^*$ is found in this example if $C_{pm}/C_{pr} >$ 0.0001. However, $N^* = 0$ when $C_{pm}/C_{pr} \leq$ 0.0001; in other words, the optimal maintenance policy is the repair maintenance after failure occurs.

**Table 1.** The parameters for the case of periodic PM with the Weibull failure distribution

| Minimum acceptable reliability | Maintenance cost | | | Adjustable parameters |
|---|---|---|---|---|
| $R_T$ | $C_{pm}$ | $C_{pr}$ | $C_{mr}$ | $a$ |
| 0.45 at $T$=43800 hrs | $8000 | $10^7 ($C_{pm}/C_{pr}$= 8x10^-4) | $6x10^-5 ($C_{pm}/C_{mr}$ =0.0133) | 25 for $b_1$ 1200 for $b_2$, $b_3$ |

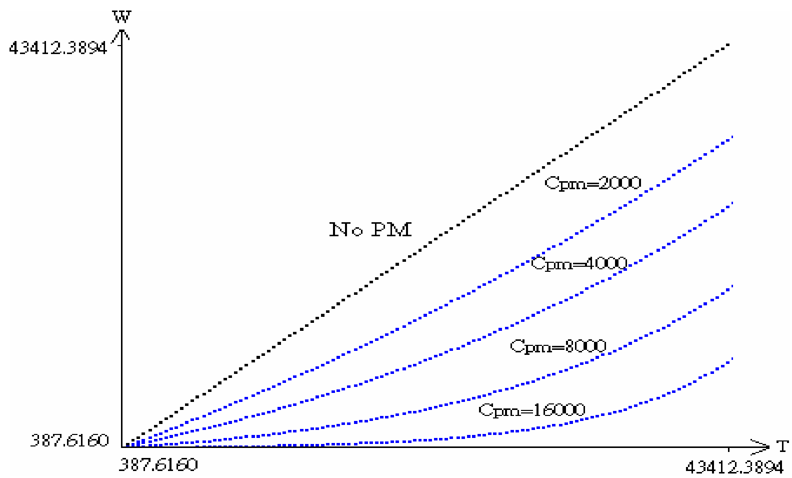


**Figure 3.** The sensitivity of the effective age to the PM cost

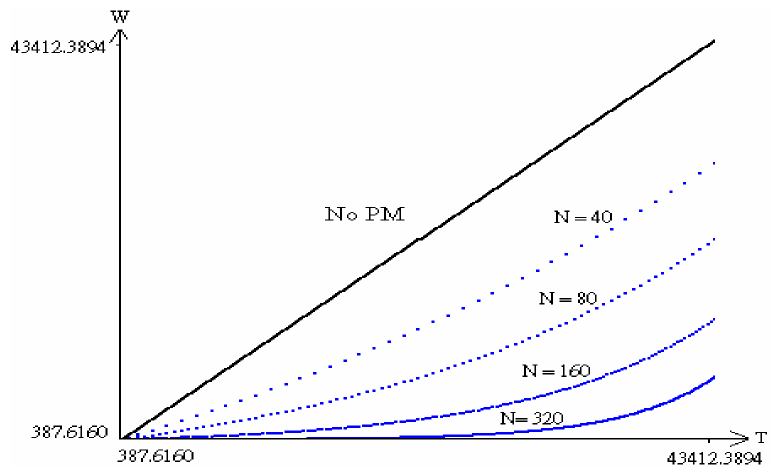


**Figure 4.** The sensitivity of the effective age to the number of PM

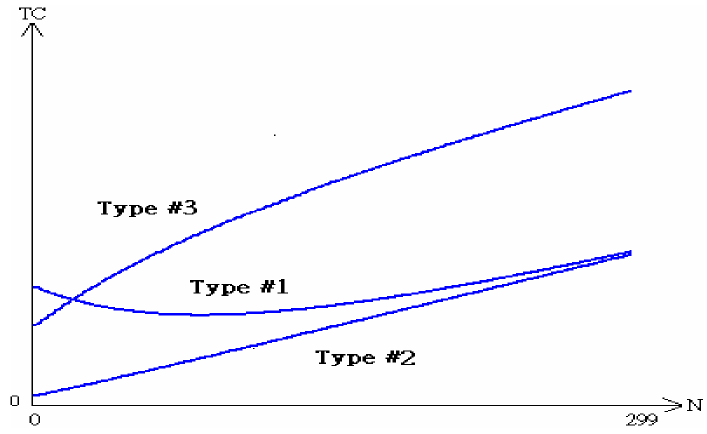


**Figure 5.** The relationship between total cost and the total number of PM for the three types of improvement factors

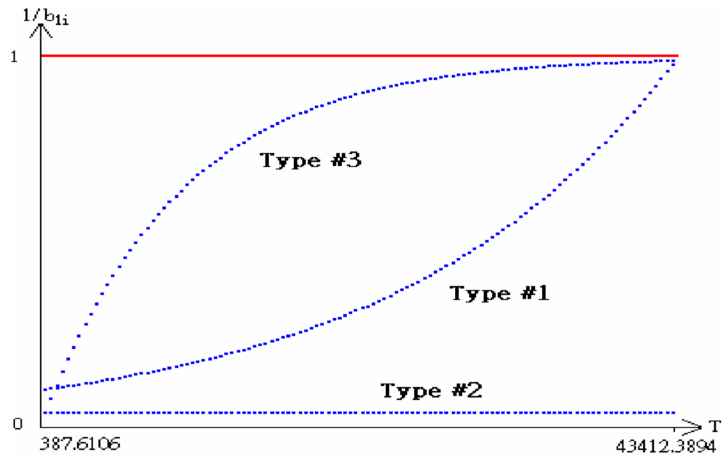


**Figure 6.** The sensitivity of each type of improvement factor to the system's age

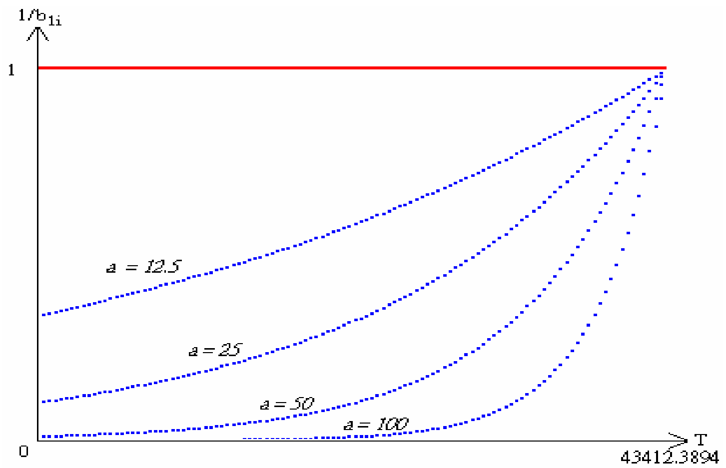


**Figure 7.** The sensitivity of the adjustable parameter for Type #1 improvement factor

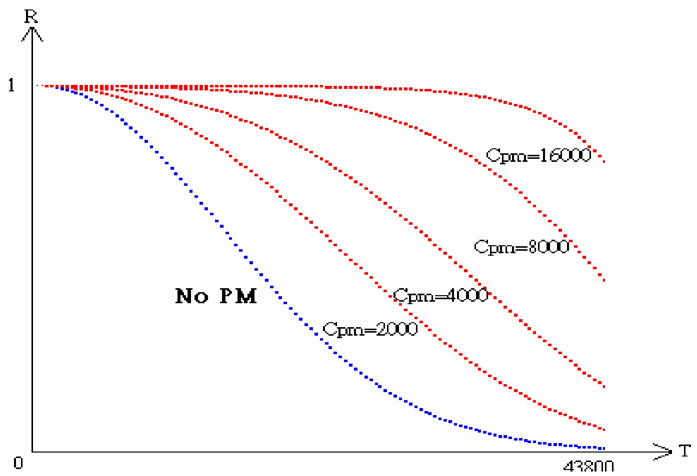


**Figure 8.** The effect of PM cost to the system's reliability for Type #1 improvement factor

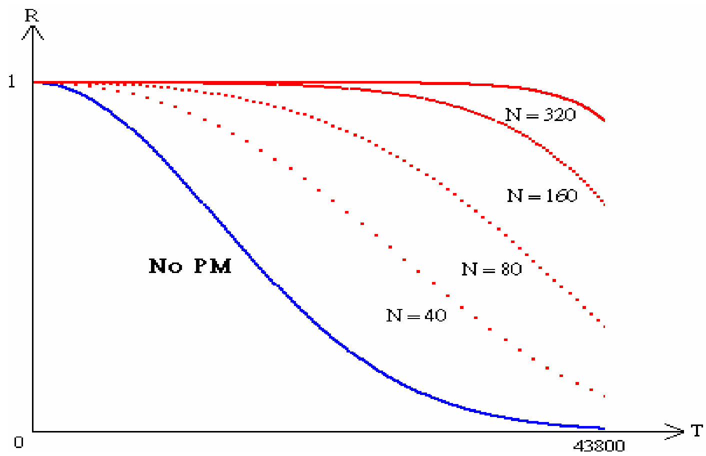


**Figure 9.** The effect of the number of PM to the system's reliability for Type #1 improvement factor

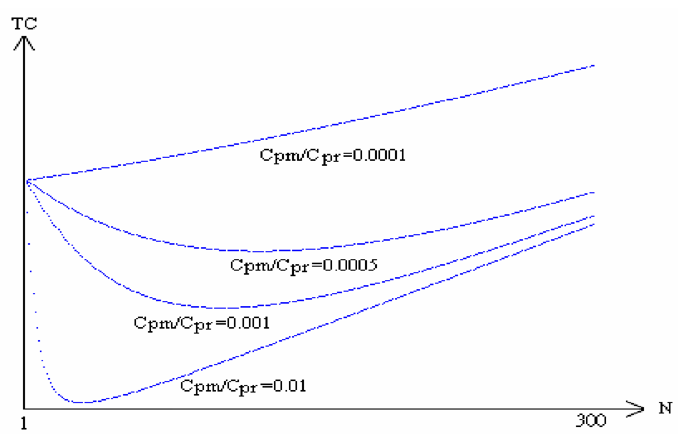


**Figure 10.** The relationship between *N* and *TC* when the ratio of $C_{pm}/C_{pr}$ increases for Type #1 improvement factor

**Table 2.** The relationship between the $C_{pm}$ and the optimal results of Type #1 improvement factor for the case that $C_{pm}$ = 8000 and $C_{pm}/C_{mr}$ =0.0133

| $C_{pm}/C_{pr}$ | $N^*$ | $T_p$ | $TC(N^*)$ | $R_T$ |
|---|---|---|---|---|
| 0.0001 | 0 | | 288.49 | 0.0083 |
| 0.0002 | 50 | 858.82 | 285.00 | 0.0234 |
| 0.0003 | 107 | 405.56 | 258.07 | 0.1040 |
| 0.0004 | 121 | 359.02 | 231.56 | 0.2001 |
| 0.0005 | 123 | 353.23 | 209.79 | 0.2879 |
| 0.0006 | 120 | 361.98 | 192.14 | 0.3585 |
| 0.0007 | 117 | 374.36 | 177.63 | 0.4179 |
| 0.0008 | 112 | 387.61 | 165.53 | 0.4690 |
| 0.0009 | 108 | 405.55 | 155.27 | 0.5071 |
| 0.001 | 103 | 421.15 | 146.47 | 0.5427 |
| 0.002 | 75 | 576.32 | 97.91 | 0.7270 |
| 0.003 | 60 | 718.03 | 76.92 | 0.7983 |
| 0.004 | 51 | 858.82 | 65.00 | 0.8330 |
| 0.005 | 44 | 973.33 | 57.28 | 0.8600 |
| 0.01 | 29 | 1460.0 | 40.49 | 0.9183 |

## 4. Conclusions

With the consideration of the imperfect preventive maintenance, the improvement factor model is proposed to investigate the restoration effect after performing PM on the deteriorating system or equipment. The improvement factor model considers system's age, the number of PM being performed, and the cost of PM as the factors affecting the restoration effect. By using the numerical analysis method, the total cost, the optimum number of PM and the PM time interval can be obtained by minimizing the total cost objective function. For the preventive maintenance, the analysis results have shown that the proposed improvement factor model provides a variety of choices to evaluate the restoration effect of a deteriorating system.


## Acknowledgements

This research has been supported by the National Science Council under the project number NSC90-2218-E-324-009.


## References


[ 1] Canfield, R. V. 1986. Cost optimization of periodic preventive maintenance. *IEEE Transactions on Reliability*, R-35, 1: 78-81.

[ 2] Chaudhuri, D. and Sahu, K. C. 1977. Preventive maintenance intervals for optimal reliability of deteriorating system. *IEEE Transactions on Reliability*, R-26, 5: 371-372.

[ 3] Chan, J. K. and Shaw, L. 1993. Modeling repairable systems with failure rates that depend on age and maintenance. *IEEE Transactions on Reliability*, 42, 4: 566-571.

[ 4] Chan, P. D. W. and Downs, T. 1978. Two criteria for preventive maintenance. *IEEE Transactions on Reliability*, R-27, 4: 272-273.

[ 5] Gu, H. Y. 1993. Studies on optimum preventive maintenance policies for general repair result. *Reliability Engineering and System Safety*, 41: 197-201.

[ 6] Jayabalan, V. and Chaudhuri, D. 1992. Optimal maintenance-replacement policy under imperfect maintenance. *Reliability Engineering and System Safety*, 36: 165-169.

[ 7] Lie, C. H. and Chun, Y. H. 1986. An algorithm for preventive maintenance policy. *IEEE Transactions on Reliability*, R-35, 1: 71-75.

[ 8] Malik, M. A. K. 1979. Reliable preventive maintenance scheduling. *AIIE Transactions* 11, 3: 221-228.

[ 9] Martorell, S., Sanchez, A., and Serradell, V. 1999. Age-dependent reliability model considering effects of maintenance and working conditions. *Reliability Engineering and System Safety*, 64: 19-31.

[10] Nakagawa, T. 1979. Optimal policies when preventive maintenance is imperfect. *IEEE Transactions on Reliability*, R-28, 4: 331-332.

[11] Nakagawa, T. 1980. Mean time to failure with preventive maintenance. *IEEE Transactions on Reliability*, R-29, 4: 341.

[12] Park, D. H., Hung, G. M., and Yum, J. K. 2000. Cost minimization for periodic maintenance policy of a system subject to slow degradation. *Reliability Engineering and System Safety*, 68: 105-112.

[13] Pham, H. and Wang, H. 1996. Imperfect maintenance. *European Journal of Operational Research*, 94: 425-438.

[14] Wang, H. and Pham, H. 1996. Optimal age-dependent preventive maintenance policies with imperfect maintenance. *International Journal of Reliability, Quality and Safety Engineering*, 3, 2: 119-135.

[15] Whitaker, L. R. and Samaniego, F. J. 1989. Estimating the reliability of systems subject to imperfect repair, *Journal of the American Statistical Association, Theory and Methods*, 84, 405: 301-309.