A One-parameter Approximation for Soil Hydraulic Functions

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Abstract: Modeling of unsaturated porous flow requires the knowledge of soil hydraulic functions. Such functions often involve multiple soil parameters that need to be estimated for different soil types. This study explores the use of a single-parameter hydraulic functions based on the linear-type $K \cdot \eta - \psi$ relationship applied in stochastic subsurface hydrology [1-3]. We first estimate the parameters of above linear-type $K \cdot \eta - \psi$ relationship through widely used soil hydraulic functions. Then we explore the possibilities and implications of expressing those parameters with a single soil parameter and few constants. Our results suggest that those constants are not very sensitive to the variation of soil parameters and a single-parameter soil hydraulic functions yield reasonably accurate representation. Preliminary validation of the proposed model using observed data yields promising results.

Keywords: soil hydraulic functions; soil properties; soil moisture; soil parameters.

1. Introduction:

The soil hydraulic functions, i.e. K- θ -h relationship, is of fundamental importance for modeling unsaturated porous flow, where θ is the volumetric water content, h is the hydraulic suction and K is the unsaturated hydraulic conductivity. A significant knowledge base exists, in the soil physics and groundwater literature that focuses on the development and refinement of methods to describe such K- θ -hrelationship [4-8]. Most of them, however, are expressed in a highly nonlinear fashion and consequently analytical result is extremely difficult to obtain when such relationships are used with other modeling approaches.

As an alternative, a linear approximation of ψ - θ relationship [3], $\theta = \theta_s - C\psi$, and a quasilinear approximation of ψ -*K* relationship

[9], $\ln(K) = \ln(K_{c}) - \psi/B$, have been applied [1-3]' where ψ is the absolute hydraulic suction defined as the difference between hydraulic suction and bubbling pressure ($\psi = h$ h_b), θ_s is the saturated soil moisture, C is the specific capacity of soil moisture, B is the thickness of capillary fringe and K_s is the saturated hydraulic conductivity. Such approximations of soil hydraulic functions provide a dramatic simplification for various modeling applications. Although the gains in simplification are achieved at the cost of accuracy, the reality is that even the most complicated soil hydraulic functions cannot precisely address the behavior of soil-moisture flow because so much is unknown and uncertain in the unsaturated porous media. Pullan [10] summarizes the state of art "we should be surprised and delighted if our prediction

Accepted for Publication: Feb. 26, 2003

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hold to 10%". Given that some simplification of soil hydraulic functions are unavoidable if analytical solution of soil-water model is desired, it appears that the above linear-type relationship may provide sufficient accuracy for various applications.

A key objective of this study is to present a further simplification of the linear-type soil hydraulic functions such that the proposed representation requires the calculation of only one parameter. Because the information of soil properties is integrated into a single parameter, the one-parameter model could be of great value for investigating the effect of soil properties for various soil-water applications. The proposed one-parameter model is validated by comparing the results with those from widely used van Genuchten's [11] soil hydraulic functions and observed data.

2. Relationship between soil physical parameters

We start from the aforementioned lineartype hydraulic functions with soil moisture converted into soil wetness, $\eta = (\theta - \theta_r)/(\theta_s - \theta_r)$, where θ_r is the residual soil moisture:

$$\eta = 1 - C_s \psi \tag{1}$$

$$\psi = -B\ln(K/K_s) \tag{2}$$

where $C_s = C/(\theta_s - \theta_r)$ is the specific capacity of soil wetness. By combining Eqs. (1) and (2) to eliminate ψ , the following *K*- η relationship is obtained:

$$\eta = 1 + A \ln \left(K / K_s \right) \tag{3}$$

where A, a dimensionless parameter, is defined as the product of B and C_s . Eq. (3) shows a power-law relationship between soil wetness and hydraulic conductivity. Similar power-law functions have also been reported [9, 12]. Note that Eqs. (1~3) involve four soil parameters (A, B, C_s and K_s). Usually, these parameters are considered independent of unsaturated soil hydraulic properties (ψ , K and θ) and assumed to vary only as a function of soil type. Mantoglou and Gelhar [3] argue that such constant assumption is valid for an intermediate range of ψ values provided local hysteresis is relatively small. We will show later that while constant assumption of parameters B and C_s is only valid for certain range of soil moisture, parameter A does not suffer from such range restrictions.

Experimental data from published literature show that the parameters $(K_s, B \text{ and } C_s)$ have a monotonic relationship with soil types. For example, the estimated data from Le et al. [13] and Carsel and Parrish [14] show that the K_s values monotonically decrease from coarse-texture soil to fine-texture soil. Also the results from Philip [15] and Morrison and Szecsody [16] show that the B values monotonically increase from coarse-texture soil to fine-texture soil. In addition, the soil-water characteristic curves for different soil types suggest that coarse-texture soil tends to have steeper slope than fine-texture soil (for a review see Leong and Rahardjo [17]). We note here that the slopes are related to C_s values. We will show later that parameter A also displays such a monotonic relationship. To summarize, the above discussion suggests that the soil parameters are correlated with each other. Therefore, a key motivation of this study is to explore the possibilities and implications of expressing these four parameters with a single parameter by exploring their internal consistency and correlation.

3. Model development

This section describes a framework to represent $K \cdot \eta - \psi$ relationship with a oneparameter model. It is a two-step approach. In the first step, we obtain the values of the relevant parameters (*A*, *B*, *C*_s and *K*_s) for different soil textures, where the parameters *A*, *B* and *C*_s are estimated by comparing the lineartype hydraulic functions with other commonly used soil hydraulic functions, for example van Genuchten's [11] soil hydraulic functions, and the K_s is obtained from observed data. In the second step, we analyze the relationship between these parameters and explore the possibility that these parameters can be expressed as a single parameter.

Step 1: Parameter estimation

The first step is to estimate the parameters *A*, *B* and *C*_s for different soil textures. To do so, we rewrite Eqs. (1) and (3) as linear forms so that the soil deficit φ , defined as $\varphi = 1 - \eta$, is the independent variable:

$$\psi = \frac{1}{C_s} \varphi \tag{4}$$

$$\ln(K_r) = -\frac{1}{A}\varphi \tag{5}$$

where $K_r = K/K_s$, is the relative hydraulic conductivity. Parameters A, B and C_s are estimated by using other more accurate and widely used $K \cdot \eta - \psi$ expressions to approximate the above linear relationship. Here we use van Genuchten's [11] soil hydraulic functions. By rewriting van Genuchten's $K \cdot \eta - \psi$ expressions in a form that φ is the independent variable, the following expression is obtained:

$$\psi = a^{-1} \left[\left(1 - \varphi \right)^{-\frac{1}{m}} - 1 \right]^{\frac{1}{m}}$$
(6)

$$\ln(K_r) = \ln\left\{ \left(1 - \varphi\right)^{1/2} \left\{ 1 - \left(1 - \left(1 - \varphi\right)^{1/m}\right)^m \right\}^2 \right\}$$
(7)

where parameters *n* and *a* are constants primarily depending on soil types and m=1-1/n. Values of *n* and *a* for twelve different soil textures are as in Carsel and Parrish [14].

A comparison between Eqs. (4) and (6) shows that the linear fit of the ψ - φ relationship using Eq. (6) can be applied to estimate parameter $C_{\rm s}$, where the slope of the approximated linear line is equivalent to $1/C_s$. Similarly, a comparison between Eq. (5) and Eq. (6) show that the linear fit of the $\ln(K/K_s)-\phi$ relationship using Eq. (7) can be applied to estimate parameter A, where the slope of the approximated linear fit is equivalent to -1/A. Finally parameter B is derived as $B = A/C_s$. range of absolute hydraulic suction (ψ) and soil wetness (η). As shown in Figure 1, such a range varies with texture type. Finer-texture soil, such as silt clay, covers a wide range of absolute hydraulic suction ($\psi < 20$ m), but it includes only a small range of moisture deficit ($\varphi = 0.1 \sim 0.2$, equivalent to soil wetness $\eta =$ 0.8~0.9). On the other hand, coarser-texture soil, such as sand, covers a wide range of moisture deficit ($\varphi = 0.1 \sim 0.7$, equivalent to soil wetness $\eta = 0.3 \sim 0.9$), but it includes only a small range of absolute hydraulic suction (ψ < 0.12 m).

Figure 1 shows ψ - φ relationship and its linear fit using Eq. (6) for different soil texture. As mentioned above, the linear ψ - ϕ expression is a mathematical simplification and thus such relationship is only valid for limited Figure 2 shows the $ln(K_r)-\varphi$ relationship and its linear fit using Eq. (7) for 12 different soil texture. It can be seen that the linear approximation is valid for a wider range of moisture deficit ($\varphi = 0 \sim 0.7$), which is equivalent to soil wetness ($\eta = 0.3 \sim 1$). In other words, parameter A can be assumed to be constant for a given soil and its value would primarily depend on soil types. Table 1 lists the estimated values of parameters A, B and C_s and their valid range of η and ψ as well as observed K_s values from Carsel and Parrish [14]. We note that these four parameters show monotonic relationships with soil types.

Step 2: Derivation of a one-parameter model

Here we examine the use of parameter A as the single parameter for the linear-type soil hydraulic functions. Other parameters (K_s , B and C_s) will be expressed as a function of A. A key advantage of using parameter A as a representative parameter to describe soil hydraulic functions is that parameter A is a dimensionless quantity with low sensitivity to variations in other parameters.

Figure 3 shows the scatter plot of B vs A, C_s vs A and K_s vs A. It can be seen that the relationship between parameters B and A resembles a hyperbolic curve and the relationship between parameters C_s and A as well as that between parameters K_s and A resemble parabolic curves. Therefore, based on the scat er plot we approximate the B-A and K_s -A re-



Figure 1. The ψ-φ relationship using van Genuchten's hydraulic function (dotted line) and its linear fit (solid line) for 12 different soil textures: (a) sand, (b) loamy sand, (c) sandy loam (d) sandy clay loam, (e) sandy loam, (f) silt loam



Figure 1. (*Continued*) (g) sandy clay, (h) silt, (i) clay loam, (j) silty clay loam, (k) agriculture clay, (l) silty clay

lationship as

$$B(A) = c_1 A^{-c_2}$$
, (m) (8)

and
$$K_s(A) = c_3 A^{c_4}$$
, (m/hr) (9)

respectively, where the coefficients c_1 through c_4 are constants that are independent of soil types. These constants are estimated once and kept fixed. The C_s -A relationship is approximated through $C_s = A/B$ as

$$C_s(A) = \frac{1}{c_1} A^{1+c_2}, (m^{-1})$$
 (10)

An optimization method is applied to estimate the constants c_1 through c_4 by solving the following two objective functions:

$$\underset{c_1,c_2}{\text{minimize}} f_{RMSE}(B) \times f_{RMSE}(C_s)$$
(11)

$$\underset{c_3,c_4}{\text{minimize}} f_{RMSE}(K_s)$$
(12)

The function $f_{RMSE}(x)$, where the variable x represents the parameters B, C_s or K_s , is the root mean square error (RMSE) between the estimated (listed in Table 1) and modeled (using Eqs. (8), (9) and (10) for parameters B, K_s and C_s , respectively) x values for 12 different

soil textures. For example, the function f_{RMSE} (*B*) is defined as the following:

$$f_{RMSE}(B) = \sqrt{\sum_{i=1}^{N} (B_i - \hat{B}_i)^2} / N$$
 (13)

where N = 12 (12 textural types), B_i is the estimated *B* value for soil texture $i, \hat{B}_i = c_1 A_i^{c_2}$,

and A_i is the *A* value for soil texture *i*. The optimization method is carried out with "FindMinimum" function of *MATHEMATICA* (version 3.0, Wolfram Research inc.). For details about this function, we refer to Stephen Wolfram [18]. The optimal values of c_1 through c_4 and the RMSEs between estimated and modeled (K_s , *B* and C_s) are listed



Figure 2. The ln(*K_l*)-φ relationship using van Genuchten's hydraulic function (dotted line) and its linear fit (solid line) for 12 different soil textures: (a) sand, (b) loamy sand, (c) sandy loam (d) sandy clay loam, (e) sandy loam, (f) silt loam



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in Table 2. It can be seen that the RMSEs are extremely small. Consequently, by combing Eqs. (1), (2) and (3) with Eqs. (8), (9) and (10), the K- η - ψ relationship can be expressed

as single-parameter A with overall constants c_1 through c_4 :

$$(1-\eta) = C_s(A)\psi \tag{14}$$

Texture	A	B	C_{s} (m ⁻¹)	K_s	Valid ranges of <i>B</i> and <i>C</i> _s	
		(m)		(m/hr)		
					ψ(m)	η
Sand	0.141	0.021	6.625	0.2970	0.03~0.12	0.1~0.7
Loamy sand	0.127	0.027	4.763	0.1459	0.04~0.14	0.1~0.6
Sandy loam	0.109	0.058	1.878	0.0442	0.04~0.31	0.1~0.6
Sandy clay loam	0.079	0.085	0.929	0.0131	0.07~0.39	0.1~0.4
Loam	0.086	0.124	0.697	0.0104	0.12~0.55	0.1~0.4
Silt loam	0.072	0.290	0.249	0.0045	0.18~1.39	0.1~0.4
Silt	0.068	0.402	0.169	0.0025	0.21~1.99	0.1~0.4
Clay loam	0.061	0.416	0.146	0.0026	0.12~2.18	0.1~0.4
Sandy clay	0.050	0.297	0.167	0.0015	0.20~1.39	0.1~0.3
Silty clay loam	0.050	0.801	0.062	0.0007	0.53~3.76	0.1~0.3
Agricultural clay	0.024	2.440	0.010	0.0020	2.18~12.51	0.1~0.2
Silty clay	0.024	3.904	0.006	0.0002	3.49~20.02	0.1~0.2

Table 1. Relevant parameters for the linear-type hydraulic function

$$\ln(K) = -\frac{\psi}{B(A)} + \ln K_s(A)$$
(15)

$$\ln(K) = -\frac{(1-\eta)}{A} + \ln K_s(A)$$
(16)

where the functions B(A), $K_s(A)$ and $C_s(A)$ are as in Eqs. (8), (9) and (10), respectively.

4. Validation

The proposed model is based on the lineartype approximation of ψ - θ relationship by Mantoglou and Gelhar [3] and the ψ -K relationship by Gardner [9]. Implications of such approximation have been addressed in Section 2 and the feasibility of Gardner's [9] ψ -K relationship has also been reviewed by Pullan [10]. Our validation experiments will focus on two issues. First, we will compare and contrast model predicted K- η - ψ relationships to those with two observed soils, Hygiene Sandstone and Beit Netofa Clay. The observed soil hydraulic and physical properties for Hygiene Sandstone are taken from Brooks and Corey [19] and those for Beit Netofa Clay are taken from Rawitz [20]. These two soils were also compared by van Genuchten [11] with his proposed model. Second, we will focus on the appropriateness of the main assumption of the proposed model. A key assumption is that the values of constants c_1 through c_4 (as given in Table 2) are not expected to vary significantly for different regions or soil types. In the second experiment, we will evaluate the sensitivity of constants c_1 through c_4 by considering the variances of parameters $(A, B, C_s \text{ and } K_s)$.

4.1. Comparison with experimental data

For practical applications, one can determine parameter A with a single experiment of

Model	Constants	RMSEs
$B = c_1 A^{-c_2}$, (m)	$c_1 = 8.94 \times 10^{-5} \text{ (m)}$ $c_2 = 2.8$	9.27×10 ⁻² (m)
$C_s = \frac{1}{c_1} A^{1+c_2}, (m^{-1})$		5.92×10 ⁻² (m ⁻¹)
$K_s = c_3 A^{c_4}$, (m/hr)	$c_3 = 2.16 \times 10^5$ (m/hr) $c_4 = 6.9$	7.96×10 ⁻⁴ (m/hr)

Table 2. Constants for the one-parameter model and the RMSEs for B-A, $C_{s}-A$ and $K_{s}-A$ relationships

soil water characteristic curve (the $\psi-\eta$ relationship, Eq. (14)). The resulting *A* values are then applied to either the *K*- ψ relationship (Eq. (15)) or the *K*- η relationship (Eq. (16)) to estimate unsaturated hydraulic conductivity. It is important to note that the constants c_1 through c_4 are kept fixed (as given in Table 2) irrespective of soil types and geographical region.

Predictions obtained for Hygiene Sandston are shown in Figure 4. Figure 4a shows the linear fit and the observed data for the $\psi - \eta$ relationship. Using Eq. (14) and the linear fit of $\psi - \eta$ relationship (Figure 4a), the value of parameter A is estimated as A = 0.126. Figure 4b compares the observed and modeled $K-\psi$ curves. It can be seen that the modeled $K - \psi$ curves show close agreement with observed data. The observed and modeled $K-\eta$ curves are shown in Figure 4c. The high conductivity value is slightly overestimated. This overestimation can be traced back to the inability of the (Eq. (3)) in that the observed $K-\eta$ relationship does not follow the exponential approximation for high conductivity values. Results obtained from Beit Netofa Clay are shown in Figure 5. Figure 5a shows the linear fit and the observed data for the $\psi - \eta$ relationship. Again, we use this linear fit to estimate parameter A as A = 0.034. Because the observed data for K- η relationship is not available for Beit Netofa Clay, we only show the



Figure 4. Observed (dot points) and modeled curves (solid lines) of (a) $\psi - \eta$ relationship, (b) $K - \psi$ relationship and (c) $K - \eta$ relationship for Hygiene Sandstone

K- ψ curves here. The observed and modeled *K*- ψ curves (Figure 5b) show overestimation bias by the model. Predictions for Beit Netofa Clay appear to be less accurate compare to the Hygiene Sandstone. We note here that similar results were also reported by van Genuchten [11] for those two soils. Nevertheless, given the one-parameter representation of soil hydraulic functions, it appears that for both of these soils the general shapes of our model predicted curves have reasonably good agreement with the observed data.

4.2. Sensitivity of the constants c_1 through c_4

Here we consider the variances of parameters (A, B, C_s and K_s) to show that the constants c_1 through c_4 are not very sensitive to variations in the parameters. Consider these four parameters with small perturbations, the upper values denoted as (A^+ , B^+ , C_s^+ and K_s^+) and the lower values denoted as (A^- , B^- , C_s^- and K_s) are estimated based on the ten percent variances of van Genuchten's [11] parameters n, a and K_s . Details for estimating the upper and lower values are shown in Table 3. Figure 6a, 6b and 6c show the scatter plot of estimated B-A, C_s -A and K_s -A relationship, respectively, including the upper and lower values, together with their modeled relationship using Eqs. (8), (9) and (10) with constants c_1 through c_4 obtained from Table 2. The RMSEs between the modeled and estimated values for mean-value relationship, upper-value relationship and lower-value relationship are listed in Table 4. Based on the modeled and estimated results in Figure 6 and the RMSE listed in Table 4, it is apparent that the one-parameter relationships defined in Eq. Eqs. (8)~(10) with overall constants c_1 through c_4 listed in Table 2 are reasonably accurate to account for variances in parameters $(A, B, C_s \text{ and } K_s)$.



Figure 5. Observed (dot points) and modeled curves (solid lines) of (a) $\psi - \eta$ relationship and (b) *K*- ψ relationship for Beit Netofa Clay



Figure 6. The modeled and estimated relationship of soil parameters, including upper- and lower- value variation. Figure 6a, 6b and 6c are the modeled (solid line) *B-A* curve, *C_s-A* curve and *K_s-A* curve, respectively, together with their estimated values (dotted point)

Estima-	Based	van Genuchten's parameters	
tion	on		
<u>Upper value</u>	es		
A^+	Eq. (7)	$a = \overline{a} + 0.1\sigma_a, n = \overline{n} + 0.1\sigma_n$	
B^+	Eq. (6)	$a = \overline{a} + 0.1\sigma_a, n = \overline{n} + 0.1\sigma_n$	
$C_{ m s}^{+}$	Eq. (6)	$n = \overline{n} + 0.1\sigma_n$	
$K_{ m s}^{+}$	-	$K_s = \overline{K}_s + 0.1\sigma_{Ks}$	
Lower values			
A^{-}	Eq. (7)	$a = \overline{a} - 0.1\sigma_a, n = \overline{n} - 0.1\sigma_n$	
B^{-}	Eq. (6)	$a = \overline{a} - 0.1\sigma_a, n = \overline{n} - 0.1\sigma_n$	
$C_{\rm s}^{-}$	Eq. (6)	$n=\overline{n}-0.1\sigma_n$	
$K_{\rm s}^{-}$	-	$K_s = \overline{K}_s - 0.1\sigma_{Ks}$	

Table 3. Estimation of upper and lower values of parameters (A, B, C_s and K_s), where \bar{x} and σ_x are themean and standard variation of x, where x represents a, n or K_s

Table 4. The RMSEs for B - A, $C_{s} - A$ and $K_{s} - A$ relationships with variations

Relationship	RMSEs		
Mean-value relationship			
B-A relationship	9.27×10 ⁻² (m)		
C_s -A relationship	$5.92 \times 10^{-2} (m^{-1})$		
K_sA relationship	7.96×10 ⁻⁴ (m/hr)		
Upper-value relationship			
B^+ - A^+ relationship	8.28×10 ⁻² (m)		
C_s^+ - A^+ relationship	$6.50 \times 10^{-2} (m^{-1})$		
K_s^+ - A^+ relationship	1.01×10 ⁻³ (m/hr)		
Lower-value relationship			
$B^{-}A^{-}$ relationship	1.22×10 ⁻¹ (m)		
C_s - A relationship	5.60×10 ⁻² (m ⁻¹)		
$K_s^{-}-A^{-}$ relationship	8.04×10 ⁻⁴ (m/hr)		

5. Conclusions

A single-parameter hydraulic functions are developed based on the linear-type $K - \eta - \psi$ relationship used in stochastic subsurface hydrology [1-3]. This one-parameter model could be of great value for investigating the effect of soil texture on various soil-water applications. Considering the large degree of uncertainties for various unsaturated parameters, our results suggest that that this approach would provide a reasonable approximation of reality. Comparison of the one-parameter representations of hydraulic functions with observed data for two soils shows promising results.

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