International Journal of Applied Science and Engineering
2003. 1, 2: 172-176

Short Communication

Determining the Optimum Process Mean for the Larger-The-Better Weibull Quality Characteristic

Chung-Ho Chen

Department of Industrial Management,
Southern Taiwan University of Technology,
Tainan 710, Taiwan, R.O.C.

Abstract: In Cho and Leonard's piecewise linear loss function for the larger-the-better quality characteristic, the quality loss is positively and linearly proportional to the deviation from the lower specification limit (LSL) when the quality characteristic is less than the LSL. The quality loss is equal to zero when the quality characteristic is greater than or equal to the LSL. However, the quality loss should be equal to zero when the value of quality characteristic approaches infinity for the larger-the-better characteristic. In this paper, we modify Cho and Leonard's piecewise linear loss function for measuring the product quality and determine the optimum process mean. Assuming that the quality characteristic follows a Weibull distribution and is larger-the-better, the application of determining the optimum process mean for the modified Cho and Leonard's model with piecewise linear loss function is discussed.

Keywords: piecewise linear loss function; process mean; process standard deviation; Weibull distribution.

1. Introduction

In the modern method of quality improvement, the process characteristic is controlled in the target value with the smallest variability. Three types of process characteristic are usually considered: the nominal-is-best, the smaller-the-better, and the larger-the-better types. For the nominal-is-best characteristic, the product has the two-sided specification limits, i.e., the lower specification limit (LSL) and the upper specification limit (USL), and the constant target value. For the smaller-the-better characteristic, the product has a one-sided upper specification limit, USL, and the target value is equal to zero. For the larger-the-better characteristic, the product has a one-sided lower specification limit, LSL, and the target value approaches infinity.

The selection of optimum process mean will directly affect the process fraction defective, scrap/rework cost, and the loss to the customer. Cho and Leonard [2] presented the case that the piecewise linear loss of product is roughly proportional to the deviation of the quality characteristic from its specification limits. The linear loss function is usually applied in the filling/canning problem for determining the optimum manufacturing target, for example, Calsson [1], Golhar and Pollock [3], Misiorek and Barnett [5], and Lee et al. [4] etc.

For the nominal-the-best quality characteristic, the normal distribution is usually used for describing the behavior of product characteristic. In the real world, however, many quality characteristics often show various types of skewed distributions. Hence, we need

Corresponding e-mail: chench@mail.stut.edu.tw
Accepted for Publication: July 22, 2003

a practical distribution for fitting the real situations. The Weibull distribution has been successfully applied in describing the lifetime of certain electronic systems. This makes the Weibull distribution a much better candidate for fitting the larger-the-better quality characteristics. In Cho and Leonard's piecewise linear loss function [2] for the larger-the-better quality characteristic, the quality loss is positively and linearly proportional to the deviation from the LSL when the quality characteristic is less than the LSL. The quality loss is equal to zero when the quality characteristic is greater than or equal to the LSL. However, the quality loss should be equal to zero when the value of quality characteristic approaches infinity for the larger-the-better characteristic. Hence, it is not reasonable that the quality loss at the LSL is equal to zero as shown in Cho and Leonard's piecewise linear loss function [2]. In this paper, we modify Cho and Leonard's piecewise linear loss function [2] for measuring the product quality and determine the optimum process mean for the larger-the-better Weibull quality characteristic. Finally, the application of determining the optimum process mean for the modified Cho and Leonard's model with piecewise linear loss function is discussed.

2. Piecewise linear loss function

Cho and Leonard [2] presented the piecewise linear loss function for the larger-the-better characteristic as follows:

$$L(x) = \begin{cases} D_L(T_L - x) & \text{if } x < T_L \\ 0 & \text{if } x \geq T_L \end{cases} \quad (1)$$

where T_L is the lower specification limit; D_L is the quality loss coefficient when the quality characteristic is less than the T_L. Figure 1 shows the Cho and Leonard's piecewise linear loss function [2] for the larger-the-better quality characteristic.

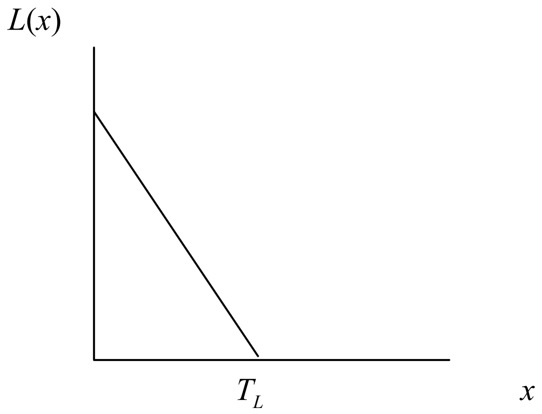

Figure 1. Cho and Leonard's piecewise linear loss function

In fact, the cost of exceeding the LSL should be considered in measuring the quality loss of product for the larger-the-better characteristic. Assuming that the quality loss at the T_L is A (where $A > 0$), the quality loss is positively and linearly proportional to the deviation from the LSL for $x < T_L$, and the quality loss is negatively and linearly proportional to the deviation from the LSL for $x > T_L$, the modified Cho and Leonard's piecewise linear loss function [2] can be defined as

$$L(x) = \begin{cases} A + D_L(T_L - x) & \text{if } x < T_L \\ A - D_U(x - T_L) & \text{if } x \geq T_L \end{cases} \quad (2)$$

where D_U is the quality loss coefficient when the quality characteristic is more than the T_L.

Figure 2 shows the modified Cho and Leonard's piecewise linear loss function [2] for the larger-the-better quality characteristic.

3. The cost model with piecewise linear loss function

Assume that the quality characteristic follows the Weibull distribution. The probability density function of X is defined as

$$f(x)=\frac{\beta}{\theta}(\frac{x-v}{\theta})^{\beta-1}e^{-(\frac{x-v}{\theta})^{\beta}}, x-v\geq 0 \quad (3)$$

where β is the scale parameter, $\beta>0$; θ is the scale parameter, $\theta>0$; v is the location parameter.

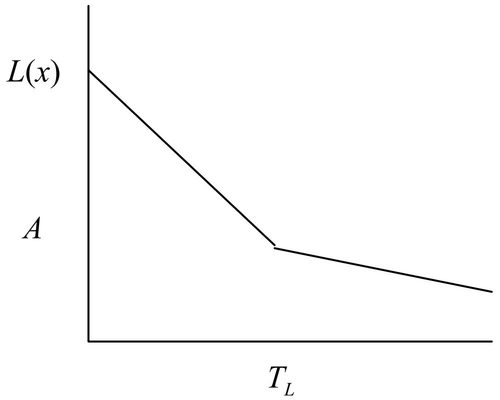

Figure 2. Modified Cho and Leonard'spiecewise linear loss function

The cumulative distribution function, the expected value, and the variance of the Weibull distribution, respectively, are

$$F(y)=P(X\leq y)=1-e^{-(\frac{y-v}{\theta})^{\beta}} \quad (4)$$

$$\mu=\theta\Gamma(1+\frac{1}{\beta})+v \quad (5)$$

$$\sigma^2=\theta^2[\Gamma(1+\frac{2}{\beta})-\Gamma^2(1+\frac{1}{\beta})] \quad (6)$$

where $\Gamma(.)$ is the gamma function.

3.1. Cho and Leonard's model with piecewise linear loss function

Using Eq. (1), the expected total loss per item in Cho and Leonard's model, denoted by TC_1, with piecewise linear loss function for the larger-the-better Weibull quality characteristic is to

Minimize

$$\begin{aligned}TC_1&=\int_0^{T_L}D_L(T_L-x)f(x)dx\\&=\int_0^{T_L}D_LT_Lf(x)dx-\int_0^{T_L}D_Lxf(x)dx\\&=D_LT_L[1-e^{-(\frac{T_L-v}{\theta})^{\beta}}]\text{-}D_L\{\text{-}T_Le^{-(\frac{T_L-v}{\theta})^{\beta}}\\&\quad+\frac{\theta}{\beta}\Gamma(\frac{1}{\beta})\mathrm{I}[\frac{1}{\beta},(\frac{T_L-v}{\theta})^{\beta}]\}\\&=D_LT_L-D_L\frac{\theta}{\beta}\Gamma(\frac{1}{\beta})\cdot\\&\quad\mathrm{I}[\frac{1}{\beta},(\frac{T_L-v}{\theta})^{\beta}]\end{aligned} \quad (7)$$

subject to $0\leq v\leq T_L$ (8)

where

$$I[a,y]=\frac{1}{\Gamma(a)}\int_0^y u^{a-1}e^{-u}du \quad (9)$$

For Eqs. (7) and (8), the parameters θ,β,D_L, and T_L are given and the parameter v is unknown. Hence, the Weibull quality characteristic has a known variance σ^2 and an unknown mean μ. $I[a, y]$ is the incomplete gamma function with $a > 0$ and $y > 0$. Since $\lim_{y\to\infty}I[a,y]=1$, the values of the incomplete gamma function are always assumed to be between zero and one. As the location parameter v (the process mean μ) increases, the $I[\frac{1}{\beta},(\frac{T_L-v}{\theta})^{\beta}]$ decreases. Hence, the minimum of TC_1 occurs when v = 0.

3.2. Modified Cho and Leonard's model with piecewise linear loss function

Using Eq. (2), the expected total loss per item of modified Cho and Leonard's piecewise linear loss function, denoted by TC_2, for the larger-the-better Weibull quality characteristic is to

Minimize

$$TC_2 = \int_{T_L}^{\infty}[A - D_U(x - T_L)]f(x)dx + \int_{0}^{T_L}[A + D_L(T_L - x)]f(x)dx$$

$$= A + \int_{0}^{T_L} D_L T_L f(x)dx - \int_{0}^{T_L} D_L xf(x)dx + \int_{T_L}^{\infty} D_U T_L f(x)dx - \int_{T_L}^{\infty} D_U xf(x)dx$$

$$= A + D_L T_L[1 - e^{-(\frac{T_L - v}{\theta})^{\beta}}] + D_U T_L e^{-(\frac{T_L - v}{\theta})^{\beta}} - D_L\{-T_L e^{-(\frac{T_L - v}{\theta})^{\beta}} + \frac{\theta}{\beta}\Gamma(\frac{1}{\beta})\cdot \mathrm{I}[\frac{1}{\beta}, (\frac{T_L - v}{\theta})^{\beta}]\} - D_U\{T_L e^{-(\frac{T_L - v}{\theta})^{\beta}} + \frac{\theta}{\beta}\Gamma(\frac{1}{\beta})[1 - \mathrm{I}[\frac{1}{\beta}, (\frac{T_L - v}{\theta})^{\beta}]]\}$$

$$= A + D_L T_L + (D_U - D_L)\frac{\theta}{\beta}\Gamma(\frac{1}{\beta})\cdot \mathrm{I}[\frac{1}{\beta}, (\frac{T_L - v}{\theta})^{\beta}] - D_U\frac{\theta}{\beta}\Gamma(\frac{1}{\beta}) \qquad (10)$$

subject to

$$0 \le v \le T_L \qquad (11)$$

For Eqs. (10) and (11), the parameters A, θ, β, D_L, D_U, and T_L are given and the parameter v is unknown. Hence, the Weibull quality characteristic has a known variance σ^2 and an unknown mean μ. $I[a, y]$ is the incomplete gamma function with $a > 0$ and $y > 0$. Since $\lim_{y\to\infty} I[a, y] = 1$, the values of the incomplete gamma function are always assumed to be between zero and one. As the location parameter v increases, the $I[\frac{1}{\beta}, (\frac{T_L - v}{\theta})^{\beta}]$ decreases. If $D_U - D_L$ is positive, then the minimum of TC_2 occurs when v = T_L. It means that the piecewise linear loss within specification decreases faster than that of out-of-specification. If $D_U - D_L$ is negative, then the minimum of TC_2 occurs when v = 0. It means that the piecewise linear out-of-specification loss decreases faster than that of within specification.

4. Concluding remarks

For the larger-the-better quality characteristic, one often hopes to have a large process mean. However, the manufacturing cost increases when the process mean increases, thus, it is not possible to increase the value of a larger-the-better quality characteristic without any bound. The modified Cho and Leonard's model [2] with piecewise linear loss function proposed in this article is similar to Carlsson's model [1] with expected net income per unit. The manufacturing cost has been considered in the modified Cho and Leonard's model [2]. From the above discussion, the optimum location parameter v should be zero for Cho and Leonard's model [2] and the optimum location parameter v should be zero or LSL for modified Cho and Leonard's model [2]. Further study can be extended to determining the optimum process mean for the smaller-the-better quality characteristic using the other distributions.

References

[1] Carlsson, O. 1984. Determining the most profitable process level for a production process under different sales conditions.

Journal of Quality Technology, 16: 44-49.

[2] Cho, B. R. and Leonard, M. S. 1997. Identification and extensions of quasi-convex quality loss functions. *International Journal of Reliability, Quality and Safety Engineering*, 4: 191-204.

[3] Golhar, D. Y. and Pollock, S. M. 1998. Determination of the optimal process mean and the upper limit of the canning problem. *Journal of Quality Technology*, 20: 188-192.

[4] Lee, M. K., Hong, S. H., and Elsayed, A. 2001. The optimum target value under single and two-stage screenings. *Journal of Quality Technology*, 33: 506-514.

[5] Misiorek, V. I. and Barrnett, N. S. 2000. Mean selection for filling processes under weights and measures requirements. *Journal of Quality Technology*, 32: 111-121.

[6] Taguchi, G. 1986. "*Introduction to Quality Engineering*". Asian Productivity Organization, Tokyo.