

# A New Approach to Compose Load Devices in Electric Power Systems

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**Abstract:** This paper presents a new aggregate model for voltage and frequency dependent load components in power system studies. The model is built using a modular approach that allows the easy integration of dissimilar load data specification formats, such as exponential forms and polynomial representations. As opposed to conventional load models in which the exponents of the exponential or polynomial representations are constant, the proposed model dynamically adjusts these exponents. The composite load components are represented by network equivalents connected to the transmission system maintaining the topological structure of the original power network. The proposed aggregate load model includes more information on the nature of the load devices than conventional models and is thus valid for a wider range of voltage and frequency variations. This paper presents the basic equations of the new proposed aggregate load model, as well as comparison with conventional models.

**Keywords:** aggregate load modeling; power system modeling; voltage and frequency dependence of power system loads.

## 1. Introduction

Electrical load components have strong effects on the behavior of electric power systems and should be modeled accurately in power flow and stability studies [1-3]. The voltage and frequency dependence of the loads influences system stability parameters such as maximum power transfer and fault clearing time. Accurate load modeling can avoid costly system design miscalculations and erroneous system operation based on imprecise operational limits [1].

Millions of different load components are consuming energy in an electric power system at a given moment, resulting in a large and complicated network. In this scenario, even if

each load had a rather simple mathematical representation, it would be practically impossible to model each individual load component in the network separately due to the heavy computational burden. In reality load devices may differ considerably from each other in their operational characteristics and require widely different mathematical models. In order to overcome the dilemma of having a global load model when the individual load components may have widely different behavior, load aggregation must be done very carefully.

The measurement-based method and the component-based approach are currently the most commonly used techniques to aggregate load devices in power system studies [1]. The

first technique is based on taking measurements of real and reactive power under changing voltage and frequency. The parameters of the aggregate load model are then derived from the measured data. The component-based approach, on the other hand, is based on the analytical development of an aggregate model from the models and parameters of the individual load components. The advantage of the measurement-based approach is that it is based on actual (not assumed) behavior of the loads. However, the data obtained at one bus in the system may not be applicable to other buses due to dissimilar load compositions at the buses. To account for load composition changes requires continuous measurements. Determination of wide range of voltage and frequency is also not practical. The component-based approach, however, does not require particular measurements but only the load composition in terms of typical loads. The technique makes use of the known load data in different formats and applies the available load component representations. This paper focuses on the component-based approach of load aggregation.

## 2. Brief description of load models

There exist some conventional static load models used in electric power system studies. A brief description of different load representations can assist one to appreciate the importance of developing accurate aggregate load models in power system modeling.

### 2.1. Conventional aggregate load models

A static load model represents the active power and the reactive power of a load at any instant as functions of the voltage magnitude and of the frequency [4-5]. To model voltage and frequency dependence, the real power and the reactive power of the load are expressed as exponential functions or polynomials of these quantities [6-8].

Static load models can be used to approximate the behavior of combined loads in power system studies. Methods for aggregating loads have been introduced in the past. Particularly, several aggregate load models are to be briefly evaluated [9-11]. The behavior of a power system depends not only on the aggregate loads but also on the transmission network impedances which separate the loads. Both the loads and the network impedances influence the system voltages and the power flow. To include the effects of the network impedances in an aggregate load model, it has been proposed that the load is in series with an equivalent circuit which has an inductor in series with the parallel combination of an inductor and a capacitor [9]. This model, however, does not maintain the original topological network structure and is not accurate enough. In the model of [10], both the network and the load components are combined into a single load component. Again, this model cannot take into account the physical separation between the loads. In addition, the model in [10] treats the exponents of the exponential functions of the real power and the reactive power of the loads as constant. This is true only under the rated operating condition. To facilitate the work of aggregating static loads, a simplified load aggregation technique (SLA) has also been proposed [11]. In this model, the aggregate static load representation has the same form as the individual load components. That is, the real power and the reactive power of the loads are the products of a voltage exponential function and a linear frequency function. The exponents of the exponential function of the aggregate load are the weighted average value of the exponents of the exponential functions of the individual loads. The model, however, does not include the effects of network impedances and treats the exponents of the exponential functions as constant, resulting in an inaccurate aggregate load representation.

## 2.2 Proposed load model

To closely represent the behavior of a group of different load components, a new aggregate load model has been developed. The proposed aggregate load model accurately represents both voltage and frequency dependence of load components, includes the effects of the network impedances, and considers the adjustment of the exponential approximating functions. When the voltage and the frequency change, the exponents of the approximating functions are adjusted to minimize their deviation from their correct values. The new model is valid for wider ranges of voltage and frequency and with better accuracy than the conventional models. Due to the flexibility of its formulation, the model can easily accommodate different types of available modeling data, such as exponential forms and polynomial representations. The network impedances are aggregated as equivalent connecting impedances. The combination of the aggregate loads and the aggregate connecting impedances closely match the topology of the original network.

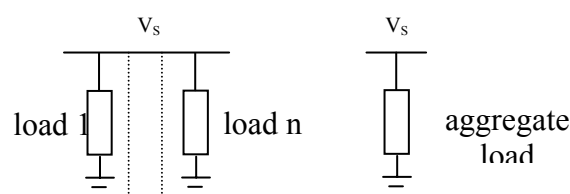
## 3. Aggregation of loads at a same bus

In power networks many different load devices can be connected to a common bus. Figure 1 shows a group of electrical load components connected to a same bus in a simple bus network. Traditionally, the available data of a load device are its power factor and the exponents of the exponential functions of its real power and reactive power. On the per unit bases of voltage and frequency, the real power and the reactive power of the  $i$ -th load in the group can be represented by the following algebraic equations:

$$P_i = P_{oi} V_i^{p_{vi}} \omega^{p_{\omega i}} \quad (1)$$

$$Q_i = Q_{oi} V_i^{q_{vi}} \omega^{q_{\omega i}} \quad (2)$$

where  $P_{oi}$  and  $Q_{oi}$  are the real power and the reactive power of the load component in the initial state, respectively;  $V_i$  and  $\omega$  are the per unit voltage amplitude and frequency, respectively;  $p_{vi}$ ,  $p_{\omega i}$ ,  $q_{vi}$ , and  $q_{\omega i}$  are the exponents of the exponential functions of the real power and the reactive power of the load component.



**Figure 1.** Loads at a same bus and their aggregate model

## 3.1. Adjustment scheme

In Eqs. (1) and (2) the exponents of the exponential functions of the real power and the reactive power are normally obtained at or near the rated operating condition. However, for each individual load component, these exponents actually depend on voltage and frequency. In order for the aggregate load model to be valid under other than the rated condition, these exponents should be adjusted. Since the adjustment of the exponent  $p_{vi}$  is similar to the adjustment of the exponents  $p_{\omega i}$ ,  $q_{vi}$ , and  $q_{\omega i}$ , only the modification of  $p_{vi}$  is considered here.

At the rated frequency (1.0 p.u.), taking the derivative with respect to voltage on both sides of Eq. (1) produces

$$\frac{\partial P_i}{\partial V_i} = p_{vi} P_{oi} V_i^{p_{vi}-1} \quad (3)$$

In the voltage range from (1 - MΔV) to 1 p.u. with M being an integer, representing the right side of Eq. (3) by a Taylor's series and neglecting the second and higher order terms give

$$\begin{aligned}
 \left(\frac{\partial P_i}{P_{oi} \partial V}\right)_{V=1} &\approx p_{vi} \\
 \left(\frac{\partial P_i}{P_{oi} \partial V}\right)_{V=1-\Delta V} &\approx p_{vi} - (p_{vi}^2 - p_{vi})\Delta V \\
 &\dots\dots\dots \\
 \left(\frac{\partial P_i}{P_{oi} \partial V}\right)_{V=1-M\Delta V} &\approx p_{vi} - (p_{vi}^2 - p_{vi})(M\Delta V)
 \end{aligned} \tag{4}$$

From Eq. (4) one can see that the exponent  $p_{vi}$  is the normalized rate of power change with respect to voltage. Accordingly, in the voltage range from  $(1 - M\Delta V)$  to 1 p.u. with  $M$  being an integer, the value of  $p_{vi}$  can be approximated by averaging the rates of power changes in Eq. (4) as

$$p_{vi-ad} \approx \frac{\sum_{j=1}^M \left(\frac{1}{P_{oi}}\right) \left[\left(\frac{\partial P_i}{\partial V}\right)_{V_j}\right]^2}{\sum_{k=1}^M \left(\frac{\partial P_i}{\partial V}\right)_{V_k}} \tag{5}$$

Treating the terms on the right side of Eq. (5) as the sum of an arithmetic series and simplifying the respective arithmetic series produces [12]

$$p_{vi-ad} \approx f_1(M\Delta V) f_2(M\Delta V) \tag{6}$$

where

$$\begin{aligned}
 f_1(M\Delta V) &= \frac{p_{vi}}{1 + \frac{M\Delta V}{2} - \frac{M\Delta V p_{vi}}{2}}; \\
 f_{2a}(M\Delta V) &= 1 + M\Delta V(1 - p_{vi}); \\
 f_{2b}(M\Delta V) &= \frac{M(2M + 1)(\Delta V)^2(1 - p_{vi})^2}{6};
 \end{aligned}$$

$$f_2(M\Delta V) = f_{2a}(M\Delta V) + f_{2b}(M\Delta V).$$

A similar procedure can be applied to obtain  $p_{vi-ad}$  in the voltage range from 1 to  $(1 + M\Delta V)$  p.u.. Since in the calculation of  $p_{vi-ad}$  the constant exponent  $p_{vi}$  is used to compute the exponents, errors may have been introduced. Further adjustment is needed to reduce the deviation of  $p_{vi-ad}$  from the correct value. Let the power function with the exponent  $p_{vi-ad}$  and the correct exponent  $(p_{vi-ad} + \Delta p_{vi-ad})$  be

$$P_{i-ad} = P_{oi} V^{p_{vi-ad}} \tag{7}$$

$$P_{i-cor} = P_{oi} V^{p_{vi-ad} + \Delta p_{vi-ad}} \tag{8}$$

From Eqs. (7) and (8), in the voltage range from  $(1 - N\Delta V)$  to 1 p.u. with  $N$  being an integer, the difference between  $\partial P_{i-ad}/\partial V$  and  $\partial P_{i-cor}/\partial V$  becomes

$$\Delta \left(\frac{\partial P_{i-ad}}{\partial V}\right)_{V=1-N\Delta V} = P'_{i-cor} - P'_{i-ad} \tag{9}$$

where

$$\begin{aligned}
 P'_{i-cor} &= \left(\frac{\partial P_{i-cor}}{\partial V}\right)_{V=1-N\Delta V}; \\
 P'_{i-ad} &= \left(\frac{\partial P_{i-ad}}{\partial V}\right)_{V=1-N\Delta V}.
 \end{aligned}$$

From Eqs. (5) and (9), treating the terms on the right side of Eq. (5) as the sum of an arithmetic series and simplifying the respect

tive arithmetic series gives the modified value of  $p_{vi-ad}$  as [12]

$$P_{vi-cor} \approx a \left( 1 + \frac{\frac{1}{2}(N\Delta V)b}{a + \frac{1}{2}(N\Delta V)b} \right) \quad (10)$$

where

$$a = 2 p_{vi-ad} - p_{vi};$$

$$b = -3 p_{vi-ad}^2 + 2 p_{vi} p_{vi-ad} + 2 p_{vi-ad} - p_{vi}.$$

### 3.2. Aggregation of loads

The real power of the aggregate load must equal the total real power of the individual loads. Similarly, its reactive power must also equal the total reactive power of the individual loads. Thus we have the following relations:

$$P_o V^{p_v} \omega^{p_\omega} = \sum_{i=1}^n P_{oi} V^{p_{vi-cor}} \omega^{p_{oi-cor}} \quad (11)$$

$$Q_o V^{q_v} \omega^{q_\omega} = \sum_{i=1}^n Q_{oi} V^{q_{vi-cor}} \omega^{q_{oi-cor}} \quad (12)$$

where  $P_o$  and  $Q_o$ , are the real power and the reactive power of the aggregate load in the initial state, respectively;  $p_v$ ,  $p_\omega$ ,  $q_v$ , and  $q_\omega$  are the exponents of the exponential functions of the real power and the reactive power of the aggregate load, respectively;  $P_{oi}$  and  $Q_{oi}$ , are the real power and the reactive power of the  $i$ -th load in the initial state, respectively;  $p_{vi-cor}$ ,  $p_{oi-cor}$ ,  $q_{vi-cor}$ , and  $q_{oi-cor}$  are the modified exponents of the exponential functions of the real power and the reactive power of the  $i$ -th load, respectively;  $n$  is the number of loads.

Again, only  $p_v$  is derived here. At the rated frequency (1.0 p.u.), take the derivative with respect to voltage on both side of Eq. (11). Then in the voltage range from  $(1 - N\Delta V)$  to 1 p.u. with  $N$  being an integer, treating the re

sulting terms on the right side of Eq. (11) as the sum of an arithmetic series and simplifying the respective arithmetic series give [12]

$$p_v^2 - \frac{1 + N\Delta V}{N\Delta V} p_v + f_L(N\Delta V) \approx 0 \quad (13)$$

where

$$f_{L1}(N\Delta V) \approx \sum_{i=1}^n \left( \frac{P_{oi}}{P_o} \right) \left( -A_i^2 + A_i + B_i \right);$$

$$f_{L2}(N\Delta V) \approx \frac{1}{N\Delta V} \sum_{i=1}^n \left( \frac{P_{oi}}{P_o} \right) A_i;$$

$$f_L(N\Delta V) \approx f_{L1}(N\Delta V) + f_{L2}(N\Delta V);$$

$$A_i = 2 p_{vi-cor} - p_{vi};$$

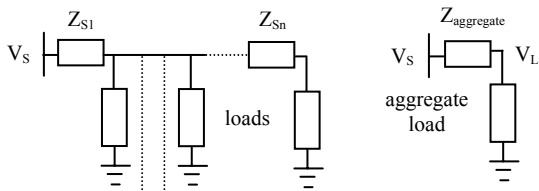
$$B_i = -3 p_{vi-cor}^2 + (2 p_{vi} + 1) p_{vi-cor} - p_{vi};$$

$$B_i = \frac{\frac{1}{2} A_i b_i}{A_i + \frac{1}{2} b_i (N\Delta V)}.$$

Finally, within the voltage range,  $p_v$  can then be obtained by solving Eq. (13). Again, a similar procedure can be used to derive  $p_v$  in the voltage range from 1 to  $(1 + N\Delta V)$  p.u..

### 4. Aggregation of loads at different buses

When electrical load devices are connected to different buses in a power system, the terminal voltages of the loads are different from their rated values due to the voltage drops in the network impedances, as shown in Figure 2.



**Figure 2.** A simple network and its aggregate Model

**4.1. Aggregation of network impedances**

The admittance of the *i*-th load is given by

$$Y_{Li} = G_{Li} + j B_{Li} \tag{14}$$

where

$$G_{Li} = P_{oi} V_i^{p_{vi-cor}-2} \omega^{p_{oi-cor}} ;$$

$$B_{Li} = -Q_{oi} V_i^{q_{vi-cor}-2} \omega^{q_{oi-cor}} .$$

The nodal matrix equation of the overall system can be constructed from the admittances of both the loads and the network as

$$[Y_{system}] [V_{bus}] = [I_{bus}] \tag{15}$$

where  $[Y_{system}]$  is the system admittance matrix;  $[V_{bus}]$  is the bus voltage vector;  $[I_{bus}]$  is the current vector.

Since the load admittances are functions of the terminal voltages, Eq. (15) is non-linear. However, the bus voltages can be obtained by solving the system matrix equation with the Newton Raphson's iterative method [2, 13]. After the bus voltages are calculated, the aggregate network admittance can be computed as

$$\bar{Y}_{aggregate} = \frac{\left| \sum_{i=1}^n \bar{I}_i \right|^2}{\sum_{j=1}^k \left| \bar{V}_{Y_j} \right|^2 (\bar{Y}_j)^*} \tag{16}$$

where  $V_{Y_j}$  is the amplitude of the voltage across the *j*-th network impedance  $1/Y_j$ ;  $I_i$  is the amplitude of the current through the *i*-th load; *n*

is the number of the considered loads; *k* is the number of the network impedances.

The terminal voltage of the aggregate load is then given by

$$\bar{V}_L = \bar{V}_s - \frac{\sum_{i=1}^n \bar{I}_i}{\bar{Y}_{aggregate}} \tag{17}$$

where  $V_s$  is the amplitude of the voltage at the bus point *s*.

**4.2. Aggregation of loads**

Again, the real power of the aggregate load must equal the total real power of the individual loads and its reactive power must also equal the total reactive power of the individual loads. Thus we have, at the rated frequency:

$$P_o V_L^{P_v} = \sum_{i=1}^n P_{oi} V_i^{p_{vi-cor}} \tag{18}$$

$$Q_o V_L^{Q_v} = \sum_{i=1}^n Q_{oi} V_i^{q_{vi-cor}} \tag{19}$$

where  $P_o$  and  $Q_o$  are the real power and the reactive power of the aggregate load in the initial state, respectively;  $p_v$  and  $q_v$  are the exponents of the exponential functions of the real power and the reactive power of the aggregate load, respectively;  $P_{oi}$  and  $Q_{oi}$  are the real power and the reactive power of the *i*-th load in the initial state, respectively;  $p_{vi-cor}$  and  $q_{vi-cor}$  are the exponents of the exponential functions of the real power and the reactive power of the *i*-th load, respectively;  $V_L$  is the terminal voltage of the aggregate load;  $V_i$  is

the terminal voltage of the  $i$ -th load.

Once again, since the derivation of  $p_v$  is similar to those of  $p_\omega$ ,  $q_v$ , and  $q_\omega$ , only  $p_v$  is derived here. Eq. (18) can be rewritten as

$$f(V_L, V_i) = P_o V_L^{p_v} - \sum_{i=1}^n P_{oi} V_i^{p_{vi-cor}} \quad (20)$$

In the voltage range from  $(1 - N\Delta V)$  to  $1$  p.u. with  $N$  being an integer, taking the total derivative of  $f(V_L, V_i)$  in Eq. (20) gives

$$a_L p_v^2 + b_L p_v + c_L \approx 0 \quad (21)$$

where

$$a_L = N\Delta V_L;$$

$$b_L = -(1 + N\Delta V_L);$$

$$c_L = \sum_{i=1}^n \left( \frac{P_{oi}}{P_o} \right) \left( \frac{V_{oi}}{V_{oL}} \right) V_{oi}^{p_{vi-cor}-1} p_{vi-cor}.$$

Within the given voltage range,  $p_v$  can then be computed from Eq. (21). A similar procedure is used to obtain  $p_v$  in the voltage range from  $1$  to  $(1 + N\Delta V)$  p.u.

### 5. Generalized aggregate load model

For a given power system study, the load data may be given in different formats due to the various data acquisition methods. To accommodate the existing load data representations, the real power and the reactive power of an individual load can be expressed as

$$P_i(V_i, \omega) = f_{1i}(V_i) + f_{2i}(V_i, \omega) \quad (22)$$

$$Q_i(V_i, \omega) = g_{1i}(V_i) + g_{2i}(V_i, \omega) \quad (23)$$

where

$$f_{1i}(V_i) = \sum_{m=-h}^H k_m V_i^m;$$

$$f_{2ia}(V_i, \omega) = \sum_{j=1}^{n_{p1i}} P_{o1ij} V_i^{p_{v1ij}} \omega^{p_{\omega1ij}};$$

$$f_{2ib}(V_i, \omega) = \sum_{k=1}^{n_{p2i}} P_{o2ik} V_i^{p_{v2ik}} \omega^{p_{\omega2ik}};$$

$$f_{2i}(V_i, \omega) = f_{2ia}(V_i, \omega) + f_{2ib}(V_i, \omega);$$

$$g_{1i}(V_i) = \sum_{m=-h}^H s_m V_i^m;$$

$$g_{2ia}(V_i, \omega) = \sum_{j=1}^{n_{q1i}} Q_{o1ij} V_i^{q_{v1ij}} \omega^{q_{\omega1ij}};$$

$$g_{2ib}(V_i, \omega) = \sum_{k=1}^{n_{q2i}} Q_{o2ik} V_i^{q_{v2ik}} \omega^{q_{\omega2ik}};$$

$$g_{2i}(V_i, \omega) = g_{2ia}(V_i, \omega) + g_{2ib}(V_i, \omega);$$

$H$  and  $h$  are positive integers;  $k_m$ ,  $s_n$ ,  $P_{o1ij}$ ,  $P_{o2ik}$ ,  $Q_{o1ij}$ , and  $Q_{o2ik}$  are coefficients;  $V_i$  is the terminal voltage of the  $i$ -th load;  $n_{p1i}$  and  $n_{p2i}$  are the number of the real power exponential functions;  $n_{q1i}$  and  $n_{q2i}$  are the number of the reactive power exponential functions.  $p_{v1ij}$ ,  $p_{\omega1ij}$ ,  $p_{v2ik}$ ,  $p_{\omega2ik}$ ,  $q_{v1ij}$ ,  $q_{\omega1ij}$ ,  $q_{v2ik}$ ,  $q_{\omega2ik}$  are exponents.

Accordingly, the proposed generalized aggregate load model is assumed to have the following real and reactive power expressions:

$$P(V_L, \omega) = F_1(V_L) + F_2(V_L, \omega) \quad (24)$$

$$Q(V_L, \omega) = G_1(V_L) + G_2(V_L, \omega) \quad (25)$$

where

$$F_1(V_L) = \sum_{i=-m}^M k_i V_L^i ;$$

$$F_2(V_L, \omega) = l_1 V_L^{p_{v1}} \omega^{p_{\omega1}} + l_2 V_L^{p_{v2}} \omega^{p_{\omega2}} ;$$

$$F_3(V_L) = \sum_{i=-n}^N s_i V_L^i ;$$

$$G_2(V_L, \omega) = r_1 V_L^{q_{v1}} \omega^{q_{\omega1}} + r_2 V_L^{q_{v2}} \omega^{q_{\omega2}} ;$$

m, M, n, and N are positive integers;  $k_i$ ,  $s_i$ ,  $l_1$ ,  $l_2$ ,  $r_1$ , and  $r_2$  are coefficients.

By the similar reasoning when aggregating loads in the previous sections,  $p_{v1}$ , which is voltage dependent, is obtained by solving the following equation:

$$a_{gR} p_{v1}^2 + b_{gR} p_{v1} + c_{gR} \approx 0 \tag{26}$$

where

$$a_{gR} = N \Delta V_L ;$$

$$b_{gR} = -(1 + N \Delta V_L) ;$$

$$c_{gR} = \sum_{i=1}^h \sum_{j=1}^{n_{pli}} \left( \frac{P_{o1ij} P_{v1ij-cor}}{\sum_{k=1}^{n_{plik}} P_{o1ik}} \right) \frac{V_{oij}}{V_{oL}} V_{v1ij-cor}^{-1} ;$$

h is the number of loads.

In the voltage range from 1 to  $(1 + N \Delta V)$  p.u.,  $p_{v1}$  can also be obtained. The values of  $p_{\omega1}$ ,  $p_{v2}$ ,  $p_{\omega2}$ ,  $q_{v1}$ ,  $q_{\omega1}$ ,  $q_{v2}$ , and  $q_{\omega2}$  can be derived by the above procedure. Similarly, the coefficients in Eqs. (24) and (25) can be approximated as

$$k_m \approx \sum_{i=-b}^B k_i \left( \frac{V_{oi}}{V_{oL}} \right)^i \tag{27}$$

$$l_1 \approx \sum_{i=1}^h \left( \frac{\sum_{j=1}^{n_{p1i}} P_{o1ij}}{\sum_{k=1}^{n_{p1i}} P_{o1ik}} \right) \tag{28}$$

$$l_2 \approx \sum_{i=1}^h \left( \frac{\sum_{j=1}^{n_{p2i}} P_{o2ij}}{\sum_{k=1}^{n_{p2i}} P_{o2ik}} \right) \tag{29}$$

$$s_n \approx \sum_{i=-b}^B s_i \left( \frac{V_{oi}}{V_{oL}} \right)^i \tag{30}$$

$$r_1 \approx \sum_{i=1}^h \left( \frac{\sum_{j=1}^{n_{q1i}} Q_{o1ij}}{\sum_{k=1}^{n_{q1i}} Q_{o1ik}} \right) \tag{31}$$

$$r_2 \approx \sum_{i=1}^h \left( \frac{\sum_{j=1}^{n_{q2i}} Q_{o2ij}}{\sum_{k=1}^{n_{q2i}} Q_{o2ik}} \right) \tag{32}$$

where b, B, and h are positive integers.

## 6. Case studies

To verify the accuracy of the proposed load model, two network simulation tests have been performed on two sample systems. Simulation test 1 has been performed on a system with the data given in Appendix [11] (please note that in this test all the loads are assumed to be connected to a common bus). In the simulation, the voltage amplitude is changed from 0.95 to 1.05 p.u., and the total real power and the total reactive power of the loads are obtained using several techniques:



(a) solving the original network without aggregating the system components (SNWAC); (b) using the load model developed with the simplified load aggregation technique (SLA) of [11]; and (c) using the proposed load model. The resulting real power and reactive power are shown in Figures 3 and 4. The results clearly indicate that as the voltage deviates from the rated value, the real power and the reactive power of the loads obtained by the simplified aggregate load model are further away from the correct values. The cause of the inaccurate results obtained by the SLA model can be traced back to its development. In the model the exponents (they are indeed voltage dependent) of the exponential functions of the real power and reactive power of a load is treated as constant while the voltage is changing.

To further evaluate the model, simulation test 2 has been conducted on the bus network shown in Figure 5 with data given in Appendix [8]. In this simulation, the voltage is changed from 0.9 to 1.1 p.u. at the rated frequency, and the total real power and the total reactive power of the loads are computed. Also, the frequency is varied from 0.9 to 1.1 p.u. at the rated voltage, and the total real power and the total reactive power of the loads are calculated. The results are again obtained by several methods: (a) solving the original network without aggregating the system components (SNWAC); (b) using the SLA model [11]; and (c) using the proposed load model. The resulting real power and reactive power are shown in Figures 6, 7, 8, and 9, respectively. The results show that the accuracy of the SLA model degrades further as the frequency deviates from the rated value. This is caused by the linear representation of frequency in the model. On the other hand, the two tests show that the results obtained with the proposed load model are in good agreement with the results obtained by solving the original networks without aggregation.

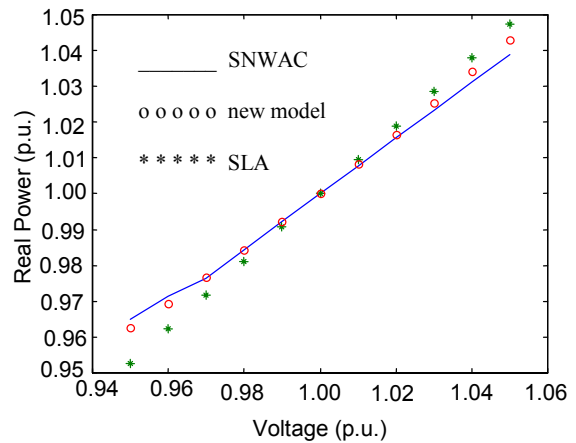


Figure 3. Real power of loads when varying voltage  $|V|$  in test 1

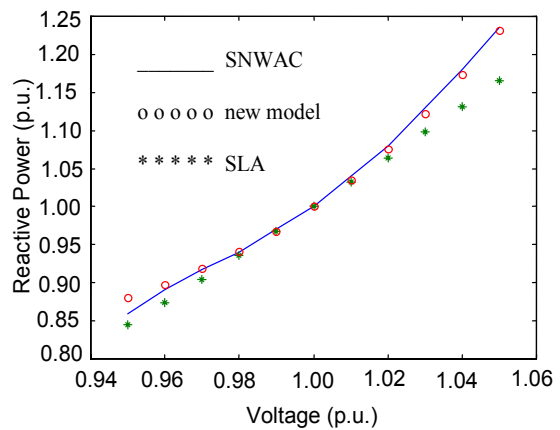


Figure 4. Reactive power of loads when varying voltage  $|V|$  in test 1

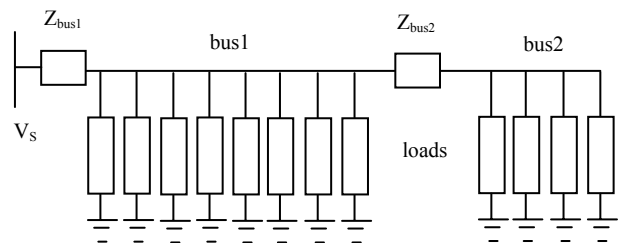
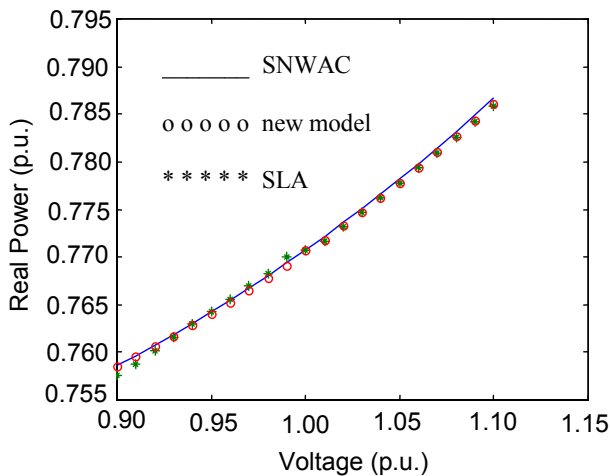


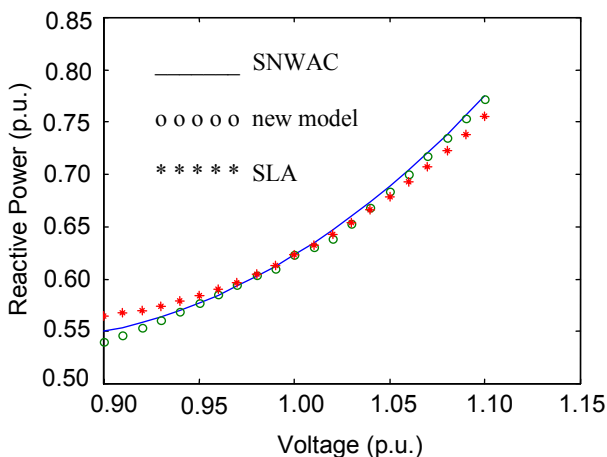
Figure 5. Simulation network in test 2

### 7. Discussions

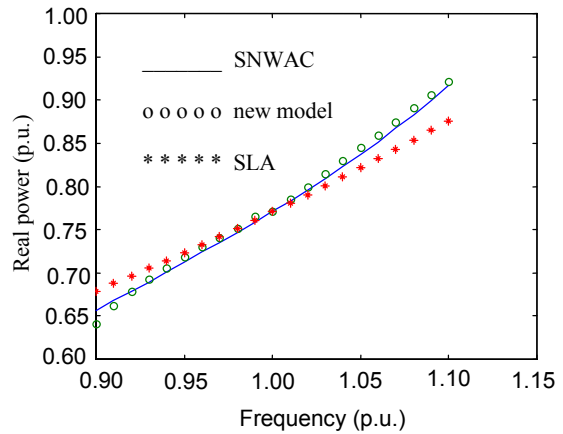
The proposed aggregate load model has broadened the voltage and frequency ranges in which electrical loads can be composed closely. In order to appreciate the performance of the model, some issues related to load aggregation need to be further explored.



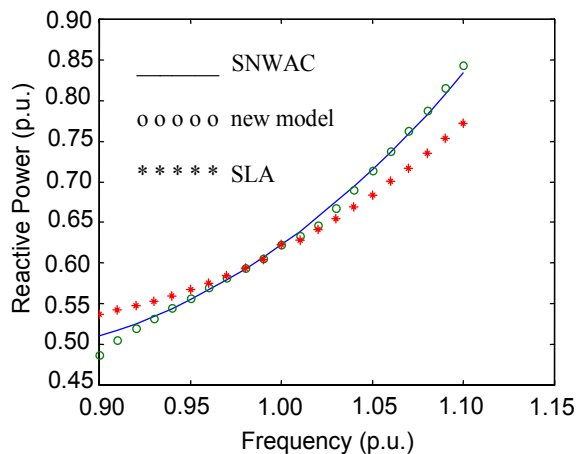
**Figure 6.** Real power of loads when varying voltage  $|V|$  in test 2



**Figure 7.** Reactive power of loads when varying voltage  $|V|$  in test 2



**Figure 8.** Real power of loads when varying frequency in test 2



**Figure 9.** Reactive power of loads when varying frequency in test 2

Normally, a practical power flow program has several different numerical solution methods including the Newton Raphson's iterative technique. In a power flow computation the bus voltage amplitudes and phase angles of a power system are calculated. The values related to each load of the network are also updated. Since the new model has been

formulated with the generalized form of the conventional load models, it can be practically used in a power flow program.

The new model is more accurate than the conventional load models because it includes more information on voltage and frequency. However, there is a trade off between the cost and the accuracy of the network simulation. Since the new model closely represents the aggregate effects of loads using the exponent adjustment scheme, it may slightly sacrifice the simulation cost. However, the optimization of the computer routine of the new model can compensate the expense of the system simulation. In industrial practice electrical engineers have different solution methods in system studies. For instance, if the accuracy is the major concern in a system study, the proposed load model should be applied. On the other hand, if the simulation cost is essential, the simplified load model may be used.

Aggregating the loads successfully by the new model depends on the structure of a power system and the types of the loads. The similar loads can be composed more accurately than dissimilar loads. In addition, the loads at a same bus are more easily and closely aggregated than the loads at different buses. If a local network does not have the control devices (such as regulated transformers, phase shifters, generators, unified power flow controllers, and switched shunts) and the network impedances are small, the overall system can be represented by the new model accurately. The size of a network that can be closely represented by the model must be determined by experiments. However, a large network can be divided into smaller sub-networks according to its structure. The so-obtained sub-networks can then be aggregated by the model separately.

The proposed model works well in the voltage range from 0.75 p.u. to 1.25 p.u. and in the frequency range from 0.90 p.u. to 1.10 p.u. In the actual power system operations, when the frequency deviates from the rated value (1 p.u.) by about 0.05 p.u., the loads of

a power system will be tripped off. As a result, the new model can closely represent the frequency dependency of the loads within the frequency range of the power system operations.

In a power system, if the network impedances are too large, aggregating its loads can be difficult due to the large voltage drops on the network impedances. When the types of the loads are too different, composing the dissimilar loads also experiences some difficulties. The proposed model is suitable for aggregating static loads. It can be used in system power flow analyses. However, it is not aimed at system transient studies.

## 8. Conclusions

This paper has presented a new aggregate load model suitable for power system studies in which the voltage magnitude and the frequency change during the system operation. By including more information on voltage and frequency dependence, the model is valid for a wider range of variation of these quantities than conventional load models. The model accommodates load devices with different data formats, allowing the flexibility in system studies and broadening its area of application. The proposed load model has been validated by comparing simulation results obtained by the proposed aggregate model with the results obtained by solving the original system without aggregation, and the results obtained with a conventional aggregation model. The results obtained by the new model are much closer to the original network results than those obtained with the conventional model.

## Acknowledgements

The author would like to thank Dr. J. R. Marti and Dr. H. W. Dommel for their encouragement and support during the project.

**Appendix: Test data**

The data used in the tests are included in this appendix.

(1). The load data used in test 1

In the tables, inc.light stands for incandescent lights; fluor.light stands for fluorescing lights; air.cond stands for air conditioning; dryer stands for dryers; refre.freez stands for refrigerators and freezers; elect.range stands for electrical ranges; pump.fan stands for pumps and fans; space.heat stands for space heaters; tv.comp stands for television sets and computers.

Depending on the area where a load device is used, it can be an industrial load, a commercial load, or a residential load. In this test the break down of consumer types are 30% industrial (ind.), 38% commercial (com.), and 32% residential (res.).

(2). The bus system data used in test 2

$$Z_{bus1} = (0.003 + j0.03)e-2 \text{ ohm};$$

$$Z_{bus2} = (0.003 + j0.03)e-2 \text{ ohm}.$$

**Table 1.** Exponent values in test 1

Load types	Exponents			
	p <sub>v</sub>	q <sub>v</sub>	p <sub>ω</sub>	q <sub>ω</sub>
inc.light	1.55	0.00	0.00	0.00
fluor.light	0.96	7.38	1.00	-26.6
air.cond	0.20	2.30	0.90	-2.67
dryer	2.04	3.27	0.00	-2.63
refre.freez	0.77	2.50	0.53	-1.46
elect.range	2.00	0.00	0.00	0.00
pump.fan	0.08	1.60	2.90	1.80
space.heat	2.00	0.00	0.00	0.00
tv.comp	2.00	5.20	0.00	-4.60

**Table 2.** Load composition in test 1

Load types	Load fractions		
	res.	ind.	com.
inc.light	0.08	0.30	0.13
fluor.light	0.00	0.49	0.39
air.cond	0.31	0.21	0.40
dryer	0.23	0.00	0.00
refre.freez	0.13	0.00	0.00
elect.range	0.08	0.00	0.00
pump.fan	0.00	0.00	0.08
space.heat	0.17	0.00	0.00
tv.comp	0.00	0.00	0.00

**Table 3.** Power factors and coefficients of P(V) in test 2

bus no.	loads	pf	k <sub>-li</sub>
1	1	0.70	0.00
1	1	0.71	-1.45
1	1	0.71	0.17
2	1	0.70	0.00
2	1	0.71	0.17

**Table 4.** Coefficients of P(V) in test 2

bus no.	loads	k <sub>oi</sub>	k <sub>1i</sub>	k <sub>2i</sub>
1	1	2.97	-4.00	2.02
1	1	2.18	0.29	0.00
1	1	0.72	0.11	0.00
2	1	2.97	-4.00	2.02
2	1	0.72	0.11	0.00

**Table 5.** Coefficients of Q(V) in test 2

bus no.	$S_{-1i}$	$S_{0i}$
1	0.00	12.9
1	0.00	6.31
1	0.00	2.08
2	0.00	12.90
2	0.00	2.08

**Table 6.** Coefficients of Q(V) in test 2

bus no.	$S_{1i}$	$S_{2i}$	$S_{3i}$
1	-26.80	14.90	0.00
1	-15.60	10.30	0.00
1	1.63	-7.60	4.89
2	-26.80	14.90	0.00
2	1.63	-7.60	4.89

**Table 7.** Power factor and exponents of exponential functions in test 2

bus no.	loads	pf	$p_v$
1	1	0.65	0.08
1	1	0.73	0.08
1	1	0.87	0.08
1	1	0.89	0.05
1	1	0.80	0.08
2	1	0.89	0.05
2	1	0.80	0.08

**Table 8.** Exponents of exponential functions in test 2

bus no.	loads	$q_v$	$p_\omega$	$q_\omega$
1	1	1.60	2.90	1.80
1	1	1.60	1.90	1.80
1	1	1.60	2.90	1.80
1	1	0.50	1.90	1.20
1	1	1.60	2.90	1.80
2	1	0.50	1.90	1.20
2	1	1.60	2.90	1.80

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