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An Evolutionary Based Method for Solving the Nonlinear Gripper Problem

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Abstract: This paper presents an Evolutionary Programming based (EP) solution for the nonlinear frictional gripper problem for both the isotropic and orthotropic cases. The presented method is compared with other methods that involve piecewise linearization of the friction law, and methods that are based on the solution of Nonlinear Complementarity Problem (NCP). The EP method could generate more optimal solutions than those of other methods. Numerical examples that illustrate the proposed method are presented.

Keywords: robot gripper; nonlinear complementarity problem (NCP); evolutionary programming (EP).

1. Introduction

A lot of literature has been published on the topic of multifingered robot grippers. Most of the work is concentrated on the modeling of the object-gripper system interaction, and then the development of algorithms for generating a secure grasp (see e.g. the excellent survey done by Bicchi [1]). The main three problems in grasping and manipulation are form and force closure, force feasibility and force optimization [2]. These three problems have been extensively studied by many researchers; most of them solved the problem using linearized friction law.

Number of previous works did not take into account the limits introduced by the joints actuators. However, since the problem is highly nonlinear from its nature, using nonlinear formulation is expected to guarantee stable prehensionwith less effort. In this direction, Al-Fahed Nuseirat and Stavroulakis [3] suggested a nonlinear complementarity approach (NCP). Han et al. [2] presented a linear matrix inequality (LMI) approach. Bicchi [4] proposed an iterative solution of nonlinear ordinary differential equations to solve the problem. The desired goal is to achieve a stable and firm grip of the grasped object as well as to study the grippers' potential in performing dexterous and fine manipulations tasks. Hence, the issue of optimizing the grasping forces has been of great importance. Many applications in robotics require a stable grip before any further operations of the robot can be done. The fingers forces have to be exactly balanced against any external wrench.

The normal forces have to be within friction cone also. This type of problem may be

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considered as constraint optimization problem. If the friction cone was approximated by a polyhedron then the problem can be formulated as Linear Complementarity Problem (LCP). The yielded LCP could be solved using direct algorithms such as Lemke's algorithm [5] or it could be solved using Neural Network based methods [6].

This, of course, would be on the expense of accuracy and optimality of solutions [3]. The NCP approach, on the other hand, provides a numerically solvable set of equations that lead to better results than that of Linear Complementarity Problem (LCP) [3]. However, all these analytical formulation methods and the numerical methods associated with them have certain degree of accuracy and are affected by the high nonlinearity of the constraints. In assembly task applications accuracy in grasping forces are highly required.

The need arises for techniques that can generate this type of solutions even on the expense of extra CPU time. This expense is of no importance if the application of the grasping forces was offline, i.e. the solution is not usually required right online for a given grasping task. Moreover, the availability of fast digital computers has diminished the problem of long execution time that usually accompanies EP based techniques. The main objective of this work is to use Evolutionary Programming (EP) to solve a nonlinear gripper problem.

The obtained results show that the norm of the grasp forces was better than that of the NCP approach. The EP technique considered here is based on algorithms proposed by Fogel [7]. The mechanics of real life genetics, such as selection and mutation are simulated in EP. The survivility-for-the-fittest is applied on a population of initially randomly generated solutions. The EP has the ability to be interfaced with many applications in optimization and machine learning. The nonlinearity of the solutions it offers made EP applicable in many real life applications. The stochastic nature of this technique makes it capable to escape local minima and keep its search toward global solutions.

While the nonlinearity constraints “shut down” Linear Programming methods and forces users to “linearize” their problem, this nonlinearity is the major motivation toward using techniques such as EP. The EP has the ability to handle such constraints easily. This is due to the richness of solutions EP offers and to the high flexibility it accommodates when dealing with stiff or vague situations. One may note here that the proposed approach may apply to determine force distribution in biped locomotion systems as well as in cooperating manipulators.

It should be kept in mind that the geometric nonlinearity comes from the unknown kinematics boundary condition; that is, the finger which will be in contact with the object (or the contact region on the object surface) is not known *a priori*, while the material nonlinearity comes from the friction condition. The inequality restriction on the normal contact forces is introduced by the fact that the finger and the object can only push on each other and not pull. These kinds of problems are known as unilateral contact problems. Unilateral contact problems with friction have been studied by many researchers [8], [9], [10]. They lead to quasivariational inequality problems or to nonlinear complementarity problems.

Secure grasping of an object by a multifingered robot gripper (both the so-called “*form closure*” and “*force closure*” notions) has been investigated by many researchers [11], [12], [13], [14]. It should be noted that form closure of objects with rotational symmetry can not be achieved when friction is negligible. All these works consider a predefined contact points. Ponce and Faverjon [15] used the polytope projection method to determine the regions of contact points that yield secure grasp. In reference [16] a rule-based method that determined the optimum grip points is proposed.

The problem of frictional gripper with elastic fingers has been investigated with classical methods by Ngyuen [17] and Cutkosky [18] and by Neural Network based methods [19]. The unilateral contact method has been used in reference [11]. In this reference the gripper problem is formulated in compact form as a Linear Complementarity Problem(LCP), where the fingers are assumed to be flexible, and a piecewise linear approximation of the friction cone has been used.

2. The problem statement

The system under consideration is described in reference to coordinate system O_p attached to the palm Figure 1. Friction is assumed to exist between the fingers and the body. A unilateral contact with elastic fingers is also assumed. The equations that govern the Gripper-Object system and the constrained cost function under scope are shown in the following subsections.

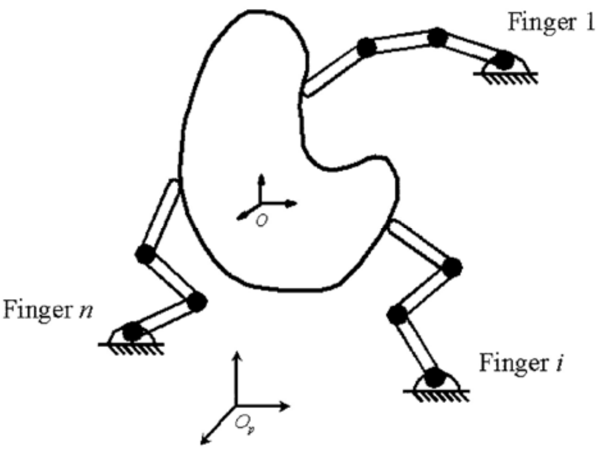

Figure 1. An object in Multifingered robot gripper

2.1. The equilibrium equations

For the object of Figure 1 all external forces and the contact forces should be in equilibrium. The equilibrium equations of the system can be written in the following form:

$$\mathbf{Gr} = \mathbf{P}, \tag{1}$$

where $\mathbf{r} = \{\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n\}^T$ is the vector of the grasping forces, $r_i = \{r_{ni}, r_{ti1}, r_{ti2}\}^T$ where r_{ni} is the normal component of the contact forces and r_{ti} are the frictional component (tangential) of the grasping forces, and $\mathbf{G}\in R^{m\times 3n}$ is the equilibrium matrix, and $\mathbf{P}\in R^m$ is the vector of the external forces applied on the object. The superscript T denotes transpose of matrix or vector. It should be noted that we choose to neglect kinematical conditions here although we assume that the unilateral contact conditions are true.

The goal of our work is mostly to find global optimum values of fingers forces. For further analysis regarding the kinematical conditions please see AL-Fahed Nuseirat and Stavroulakis [3]. The grasp forces also are subject to constraints introduced by fingers kinematics and design characteristics. These constraints are defined as follows:

$$\begin{aligned} \mathbf{J}_h{}^T\mathbf{r} &\le \tau_{max} \\ \mathbf{J}_h{}^T\mathbf{r} &\ge \tau_{min} \end{aligned} \tag{2}$$

Here

$$\mathbf{J}_h = \text{diag.}\ [\mathbf{HJ}_1, \mathbf{HJ}_2, \ldots, \mathbf{HJ}_n]$$

where τ_{max} and τ_{min} denote the vectors of the maximum and minimum torques available for the joints of the fingers, $\mathbf{J}_h$ is a $3n \times nk$ global Jacobian matrix, and k is the number of joints in each finger. Moreover constraint matrices $\mathbf{H}$ (with dimension equal to 3×6) has the following form

$$\mathrm{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Rewriting Equations 2 in compact form, we have

$$\mathbf{J}^T\mathbf{r} \le \tau, \tag{3}$$

where

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_h \\ -\mathbf{J}_h \end{bmatrix}$$

$$\tau = \begin{bmatrix} \tau_{max} \\ -\tau_{min} \end{bmatrix}$$

2.2. The orthotropic friction law and the nonlinear optimization problem

As a result of the friction of forces applied on the rigid body surface, tangential force components exist. These components are assumed to satisfy a static (or holonomic version of) Coulomb's law. For generality, the following orthotropic friction law is considered. Let the principal orthotropic axes on the tangent plane at the i-th contact point be denoted by 1 and 2 and let r_{ti1}, r_{ti2} be the components of the friction forces along these axes. The corresponding friction coefficients are denoted by μ_{i1} and μ_{i2}. The requirement that the i-th contact force must lie within the friction cone [8], [20] reads:

$$\gamma_i = |r_{ni}|^2 - \left[\left(\frac{r_{ti1}}{\mu_{i1}}\right)^2 + \left(\frac{r_{ti2}}{\mu_{i2}}\right)^2\right], \qquad \gamma_i \geq 0,$$

$$i = 1, 2, \ldots, n \tag{4}$$

where |*| denotes the norm in R^3, μ is the friction coefficient and γ_i is the friction cone(domain). If strict inequality holds in the previous equation there is no slip. Otherwise there exists a non-negative parameter λ_i [8] such that the slipping values are given by

$$y_{ti1} = -\lambda_i \frac{r_{ti1}}{\mu_{i1}^2} \text{ and } y_{ti2} = -\lambda_i \frac{r_{ti2}}{\mu_{i2}^2} \tag{5}$$

The isotropic friction law is a particular case of the above relation, and it is achieved when $\mu_{i1} = \mu_{i2} = \mu_i$. The friction law can be written in compact form as follows:

$$\mathbf{B}(\mathbf{r})\ \mathbf{r} \leq \mathbf{0} \tag{6}$$

where

$$\mathbf{B}(\mathbf{r}) = \text{diag.}[\mathbf{B}(\mathbf{r}_1), \mathbf{B}(\mathbf{r}_2), \ldots, \mathbf{B}(\mathbf{r}_n)]$$

$$\mathbf{B}(\mathbf{r}_i) = \begin{bmatrix} -r_{ni} & \dfrac{r_{ti1}}{\mu_{i1}^2} & \dfrac{r_{ti2}}{\mu_{i2}^2} \end{bmatrix}$$

As a result of the previously described constraints, the optimal fingers forces can be obtained by solving the following nonlinear programming problem

$$\textit{minimize}\ \frac{1}{2}\ \mathbf{r}^T\mathbf{r} \tag{7}$$

subject to

$$\mathbf{G}\,\mathbf{r} = \mathbf{P}$$
$$\mathbf{J}\,\mathbf{r} \leq \tau$$
$$\mathbf{B}(\mathbf{r})\ \mathbf{r} \leq \mathbf{0}$$
$$\mathbf{N}\,\mathbf{r} \leq \mathbf{0}$$

where $\mathbf{N}$ = diag [$\mathbf{N}_1, \mathbf{N}_2, \ldots, \mathbf{N}_n$] and
$\mathbf{N}_i = [-1 \quad 0 \quad 0]$ for $i = 1,2, \ldots$, n.

3. Employing the EP to solve the gripper nonlinear optimization problem

The EP requires an energy function to be considered as the objective function. The inequality constraints in 7 can be transformed to equality by adding nonnegative vectors of slack variables $\mathbf{y} = [y_1^2, y_2^2, \ldots, y_{2nk}^2]^T$, $\mathbf{z} = [z_1^2, z_2^2, \ldots, z_n^2]^T$, and $\mathbf{u} = [u_1^2, u_2^2, \ldots, u_n^2]^T$, as

$$\textit{minimize}\ \frac{1}{2}\ \mathbf{r}^T\mathbf{r}$$

subject to

$$\mathbf{G}\,\mathbf{r} = \mathbf{P}$$
$$\mathbf{J}\,\mathbf{r} + \mathbf{y} = \tau$$
$$\mathbf{B}(\mathbf{r})\ \mathbf{r} + \mathbf{z} = \mathbf{0}$$
$$\mathbf{N}\ \mathbf{r} + \mathbf{u} = \mathbf{0} \tag{8}$$

This nonlinear problem can be converted to the objective function (penalty function) below:

$$\mathbf{E}(\mathbf{r}) = k_1\|\mathbf{r}\|_2 + k_2\|\mathbf{Gr} - \mathbf{P}\|_2 + k_3\|\mathbf{Jr} + \mathbf{y} - \tau\|_2 + k_4\|\mathbf{B}\ (\mathbf{r})\ \mathbf{r} + \mathbf{z}\|_2 + k_5\|\mathbf{Nr} + \mathbf{u}\|_2 \tag{9}$$

where k_1, k_2, k_3, k_4, and k_5 are weighting constants.

The EP works on a population of strings where each string consists of sequence of elements of real values that are initially generated from uniform random variables generator. Each element in the string represents a normal force value applied by the fingertip. Consequently, the length of the string is equal to the length of the normal forces vector r_{ni} (i = 1, 2, …, n). The EP starts with fixed population of strings.

However, our EP uses an expanding population size strategy. This is closer to what happens in real life. When the population size reaches the maximum allowed limit, half of the members of the population are eliminated. The eliminated members are mainly the worst elements according to the objective function measure. Our EP also, has an adaptive operator for mutation. Mutation is inversely proportional to the fitness of the string.

Fitness of the string is inversely proportional to the value of the objective function. The mutation is taken from a normal distribution random variable whose deviation is also inversely proportional to the fitness of the string being mutated. The equations below show how fitness is calculated for every string in the population, and how the mutation applied on a string is also calculated:

$$\text{Fitness}(\text{string}_j) = 1 - \frac{E(r_{nj})}{\text{Max}\{E(r_{nj})\}_{\text{population}}} \quad (10)$$

where $\text{Max}\{E(r_{nj})\}_{\text{population}}$ means maximum $E(r_{nj})$ in current population. The mutation added on string associated with r_{nj} (jth normal force) is given by:

$$\text{Mutation}(\text{string}_j) = N(O, CA(\text{strings})) \quad (11)$$

where N denotes the normal distribution function, O means zero mean, C is a weighting constant, and A(strings) is the standard deviation of normal distribution N

$$A(\text{strings}) = \sum_{m=Pop-M}^{Pop} \frac{\text{Fitness}(string_j)_m}{M}$$

where $\text{Fitness}(\text{string}_j)_m$ is the average fitness of strings of generation m, m is generation index, Pop is the current generation index, and M is backward steps in generation index. The flowchart for the proposed EP algorithm is shown in Figure 2.

4. Numerical examples and discussions

Numerical examples that illustrate the application of EP in finding minimal fingers forces grip are shown in this section. The examples cover both the isotropic friction and the orthotropic friction cases. The three-fingered grasping problem in reference [3] is used to demonstrate the effectiveness of the proposed method. The configuration of the example is shown in Figure 3. The points of contact with reference to the object coordinate system are

r_1 = (0.0, 0.75, 0.75), r_2 = (0.75, 0.0, 0.75), r_3 = (0.75, 0.75, 0.0)

and the normals to the associated contact surfaces are

n_1 = (1.0, 0.0, 0.0), n_2 = (0.0, 1.0, 0.0), n_3 = (0.0, 0.0, 1.0)

The externally applied force to the object is assumed to be the object weight acting opposite tothe direction of the z-axis and is of magnitude 5. The center of mass is located at points r_c= (0.5, 0.5, 0.5). The friction coefficients are $\mu_1 = \mu_2$ =0.6. In Table 1 the contact forces and tangential friction forces components are shown. The norm of the normal forces obtained from EP method was 3.9639 while in reference [3] the obtained norm using NCP (Nonlinear Complementarity Problem) approach was 4.0494. The same example was solved using EP approach, but this time with orthotropic friction coefficients μ_1 = (0.5, 0.6, 0.5), μ_2 = (0.6, 0.5, 0.5). The obtained norm of the normal forces was 4.0489 while under the same conditions the obtained norm using NCP approach was 4.1122 [3].

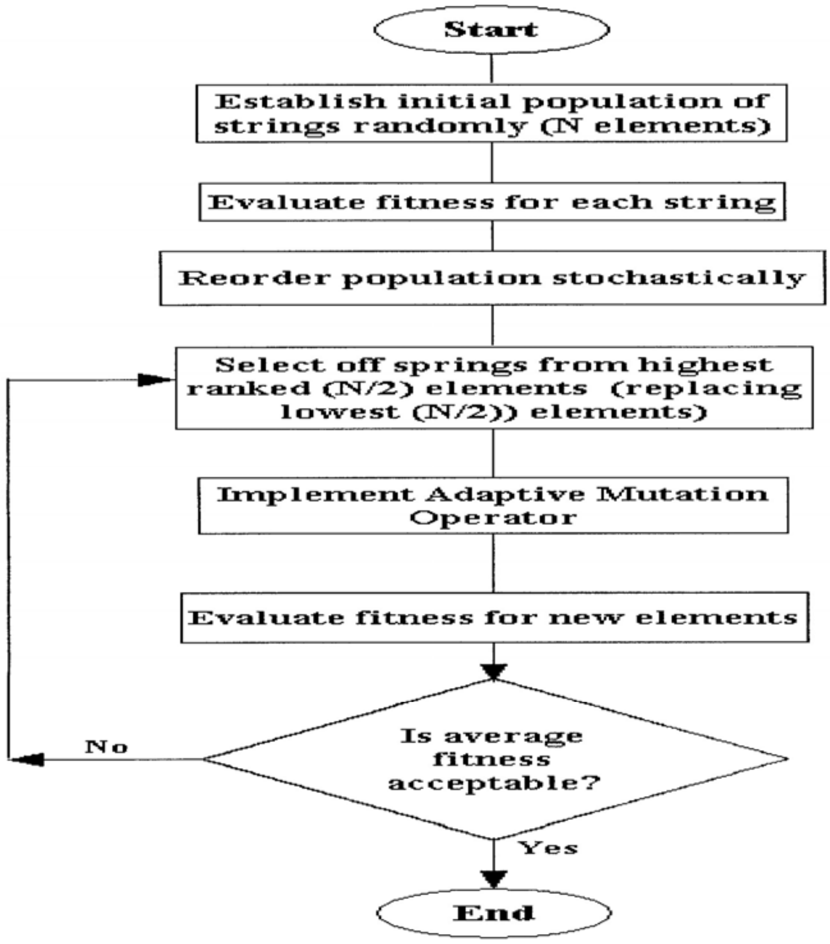

Figure 2. The flowchart of the EP used

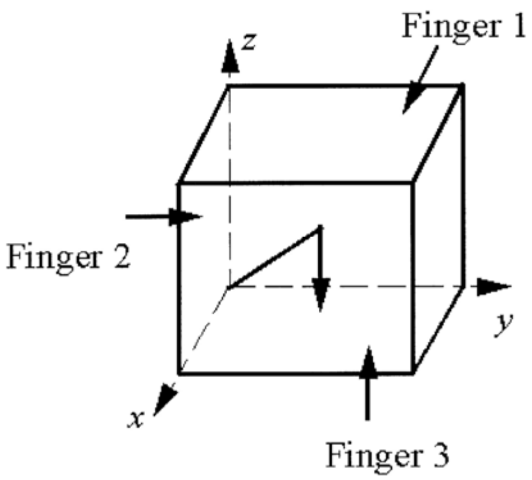

Figure 3. Configuration of the first example

Table 1. Normal contact and friction forces for the example of Figure 3 using EP

Finger	Contact force	Tangential component	Friction forces
1	1.0496	1	-0.0958
		2	0.6262
2	1.0597	1	-0.0958
		2	0.6161
3	3.6577	1	-0.9538
		2	-0.9639

As a second example a cube grasped by four fingers [12] was solved using EP approach and LCP(Linear Complementarity Problem) approach (for details about this approach see Al-Fahed and Panagiotopoulos [11]). The configuration of this example is depicted in Figure 4. The used friction coefficients were $\mu_1 = \mu_2 = 0.6$. The points of contact with reference to the object coordinate system are

$r_1 = (2.0, 0.0, 0.0)$, $r_2 = (0.0, 1.5, 0.0)$,
$r_3 = (0.0, 0.0, 2.0)$, $r_4 = (1.2, -2.0, 0.0)$

and the normals to the associated contact surfaces are

$n_1 = (-1.0, 0.0, 0.0)$, $n_2 = (0.0, -1.0, 0.0)$,
$n_3 = (0.0, 0.0, -1.0)$, $n_4 = (0.0, 1.0, 0.0)$

With external forces $\mathbf{P} = (-0.2, -1.0, -2.0, -0.2, -0.3, -0.2)^T$. In Tables 2 and 3, the contact forces, tangential friction forces components, obtained using LCP and EP approaches are shown, respectively. The norm of normal forces, for both the LCP and the EP approaches was 1.7072 and 1.6208. The same simulations for both cubes were repeated, but this time with orthotropic conditions.

For the example of Figure 3 these conditions were $\mu_1 = (0.5, 0.6, 0.5)$, $\mu_2 = (0.6, 0.5, 0.5)$. Table 4 shows the forces values obtained using EP approach. Tables 5 shows the results for the same example using LCP method. The obtained norms of the normal contact forces from both methods were 4.0265 and 4.0489 respectively.

The orthotropic conditions used for the example of Figure 4 were $\mu_1 = (0.5, 0.8, 0.5, 0.8)$, $\mu_2 = (0.8, 0.5, 0.8, 0.5)$. Tables 6 and 7 show forces values for both approaches LCP and EP. The norms of the normal forces were 1.7918 and 1.7146 respectively.

5. Conclusions and discussions

The ultimate goal of this work is to present the capabilities of evolutionary based methods in finding most ultimate solutions for the gripper problem. Although the NCP far outperforms the other LCP (Linear Complementarity Problem) formulation methods [11], the EP showed superiority over the NCP in both isotropic and orthotropic cases.

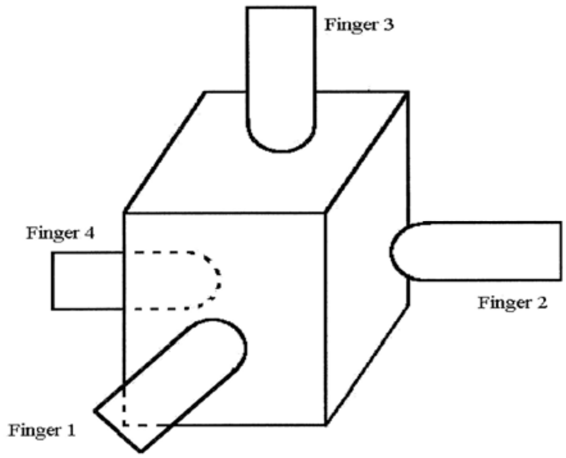

Figure 4. Configuration of the second example

Table 2. Normal contact and friction forces for the example of Figure 4 using LCP method

Finger	Contact force	Tangential component	Friction forces
1	0.1741	1	0.0
		2	0.0
2	0.9251	1	-0.2000
		2	-0.5128
3	1.4124	1	-0.1949
		2	-0.2098
4	0.1349	1 2	-0.0310 0.0748

Table 3. Normal contact and friction forces for the example of Figure 4 using EP method

Finger	Contact force	Tangential component	Friction forces
1	0.1350	1	-0.0368
		2	-0.0751
2	0.9267	1	-0.2098
		2	-0.5142
3	1.3141	1	-0.2825
		2	-0.1892
4	0.1526	1	0.0076
		2	0.0965

Table 4. Normal contact and friction forces for the example of Figure 3 with orthotropic conditions using EP method

Finger	Contact force	Tangential component	Friction forces
1	1.0899	1	-0.0570
		2	0.6371
2	1.0931	1	-0.0570
		2	0.6340
3	3.7189	1	-1.0329
		2	-1.0360

Table 5. Normal contact and friction forces for the example of Figure 3 with orthotropic conditions using LCP method

Finger	Contact force	Tangential component	Friction forces
1	0.9583	1	0.0
		2	0.7083
2	1.1400	1	0.0
		2	0.5266
3	3.7651	1	-0.9583
		2	-1.1400

It is worth mentioning that the EP and NCP methods are nonlinear techniques that do not depend on re-defining the problem as a linear approximation and then finding the solutions.

Previous work [3], showed that linearizing the constraints may result in solutions that are quite far from the actual optimal solutions. Many applications are very sensitive to the forces applied by the fingers on the rigid body surface. In these applications, 5 % variation or even less than that affects the balance of the grasped rigid body.

The flexibility of the EP and its ability to "dig" deep into the search space of the forces enables it to come up with near optimum solutions. The continuous evolution of the strings pool that is always updated by mutation and reproduction operators is the main reason for the richness of solutions. The NCP is solved with PATH algorithm which is a deterministic nonlinear optimization algorithm [21]. Deterministic techniques take one direction to an optimal solution. Sometimes, due to the numerical nature of the algorithm and the shape of the search space, the numerical algorithms settle in a region that is not the most optimal. The deterministic operators of that algorithm can not allow it to skip that region.

The Tables from 1 - 7 show solid evidence of better solutions generated by EP. We have to admit, however, that EP is much more computationally costly than the PATH algorithm when solving the NCP. There is no need to present figures to demonstrate that since the PATH algorithm is clearly much less costly. This is a general problem with all evolutionary based methods.

Fortunately, many gripper applications are off-line applications. That is, the minimal gripper fingers forces are calculated only one time and then used the rest of the time. If the external force varies from time to time, then we can use the EP to find some minimal gripper fingers forces that achieve the balance momentarily, and then continue running the EP until most minimum force is found. The most minimum (optimum) forces are theoretically detected by calculating the standard deviation of the last 5 minimal gripper forces generated by EP. If the standard deviation was equal or close to zero, EP quits, and string with highest fitness in the population gives the most minimum forces.

Observing the figures in the Tables 1 - 7, we find that the minimal forces in the case of isotropic friction are less than those for orthotropic friction. This is due to the fact that the average of friction coefficients values are less in the case of orthotropic than those used with isotropic case.

Higher friction coefficients imply less fingers forces are required to secure the grip. In all cases, it was faster and easier to generate minimal values for grips when higher values of friction coefficients were used. In the case with the cube of the first example it is shown that three fingers are enough to grasp the cube, and for both the isotropic and orthotropic cases the EP provided less minimal values for the normal contact forces by around 2.5 %. In the case of second example of the cube with four fingers and for both isotropic and orthotropic cases, figures in the tables show that EP provided solutions with minimal normal contact forces with around 5 % less than those provided by NCP algorithm.

As we mentioned earlier, these savings in minimal forces, although relatively small, can be very useful from engineering point of view especially if the number of fingers used and the values of forces were large. This, consequently, means more savings in energy and better chance in protecting the grasped body from any possible damage. Figure 5 shows the process of evolving solutions by EP. To study this problem we picked the orthotropic case of the second example.

The upper graph in the Figure shows at each point a minimal norm of total forces (normal and tangential) used by the gripper to grasp the object.

The lower graph shows at each point a minimal norm of only normal forces used by the gripper to grasp the object. Each point in both graphs is a possible solution. However, the last point in each graph represents the most minimum solution.

In general, solutions generated by EP tend to converge faster at the beginning toward the most minimal one. As number of iterations increases, EP starts to move slower toward the most minimal point until it tends to settle. There, EP hardly moves anywhere else. It is clear from the graphs also that there is constant difference in value between the norm of total forces and the norm of normal forces.

Solutions at earlier stages have the same difference between total and normal forces compared to solutions at final stages. This emphasizes that this difference depends on the properties of the grasped body itself and not merely on the values of generated forces themselves.

A. Analysis and proof that objective function of EP has optimal solution

This appendix is concerned with the solution of the minimization problem stated by Equations 8.

The lagrangian function for this problem can be constructed as

$$\mathbf{L}(\mathbf{r}, \mathbf{y}, \mathbf{z}, \mathbf{u}, \boldsymbol{\lambda}, \boldsymbol{\eta}, \boldsymbol{\gamma}, \boldsymbol{\rho}) = \frac{1}{2}\mathbf{r}^T \mathbf{r} + \boldsymbol{\lambda}^T (\mathbf{Gr} - \mathbf{P}) + \boldsymbol{\eta}^T (\mathbf{Jr} + \mathbf{y} - \boldsymbol{\tau}) + \boldsymbol{\gamma}^T (\mathbf{B}(\mathbf{r})\, \mathbf{r} + \mathbf{z}) + \boldsymbol{\rho}^T (\mathbf{Nr} + \mathbf{u}) \quad (12)$$

where **λ**, **η**, **γ**, and **ρ** are vectors of Lagrangian multipliers.

According to classical optimization theory [22], [23], the stationary points of the Lagrangian function can be found by solving the following Euations:

$$\frac{\partial L}{\partial r} = \mathbf{r} + \mathbf{G}^T \boldsymbol{\lambda} + \mathbf{J}^T \boldsymbol{\eta} + \left(\frac{\partial}{\partial r}(\mathbf{B}\ (\mathbf{r})\ \mathbf{r})\right)^T \boldsymbol{\gamma} + \mathbf{N}^T \boldsymbol{\rho} = \mathbf{0} \quad (13)$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{Gr} - \mathbf{P} = \mathbf{0} \quad (14)$$

$$\frac{\partial L}{\partial \eta} = \mathbf{Jr} + \mathbf{y} - \boldsymbol{\tau} = \mathbf{0} \quad (15)$$

$$\frac{\partial L}{\partial \gamma} = \mathbf{B}(\mathbf{r})\ \mathbf{r} + \mathbf{z} = \mathbf{0} \quad (16)$$

$$\frac{\partial L}{\partial y} = \mathbf{2x} = \mathbf{0} \quad (17)$$

$$\frac{\partial L}{\partial z} = \mathbf{2v} = \mathbf{0} \quad (18)$$

$$\frac{\partial L}{\partial u} = \mathbf{2w} = \mathbf{0} \quad (19)$$

where $\mathbf{x} = [\eta_1\ y_1, \eta_2\ y_2, \ldots, \eta_{2nk}\ y_{2nk}]^T$, $\mathbf{v} = [\gamma_1\ z_1, \gamma_2\ z_2, \ldots, \gamma_{2nk}\ z_{2nk}]^T$, and $\mathbf{w} = [\rho_1\ u_1, \rho_2\ u_2, \ldots, \rho_{2nk}\ u2nk]^T$.

Equations 14 - 19 are the necessary conditions that ensure the relative minimum of our problem. This relative minimum is global minimum since the cost function is convex (quadratic energy function). Hence, Equations 8 have a minimal solution, and that solution is global optimum.

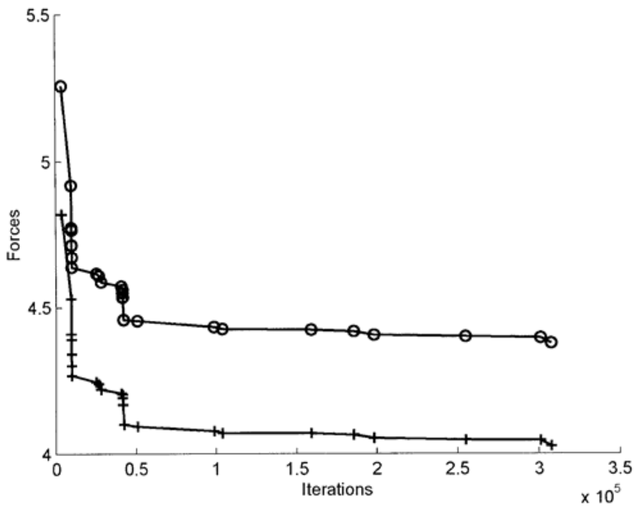

Figure 5. Minimal grasp forces for orthotropic case of second example

Table 6. Normal contact and friction forces for the example of Figure 4 under orthotropic conditions using LCP method

Finger	Contact force	Tangential component	Friction forces
1	0.1572	1	0.0
		2	0.0
2	0.9416	1	-0.1760
		2	-0.4350
3	1.5120	1	-0.1818
		2	-0.1732
4	0.1148	1	-0.0369
		2	0.0530

Table 7. Normal contact and friction forces for the example of Figure 4 under orthotropic conditions using EP method

Finger	Contact force	Tangential component	Friction forces
1	0.0697	1	-0.0043
		2	-0.0414
2	0.8744	1	-0.1296
		2	-0.4628
3	1.4699	1	-0.2064
		2	-0.2212
4	0.0999	1	-0.0535
		2	0.0259

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