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A New Method to Forecast Enrollments Using Fuzzy Time Series

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Abstract: In recent years, many methods have been proposed for forecasting enrollments based on fuzzy time series. However, the forecasting accuracy rates of the existing methods are not good enough. In this paper, we present a new method to forecast enrollments based on fuzzy time series. The proposed method belongs to the first order and time-variant methods. The historical enrollments of the university of Alabama are used to illustrate the forecasting process of the proposed method. The proposed method can get a higher forecasting accuracy rate for forecasting enrollments than the existing methods.

Keywords: fuzzy time series; fuzzy sets; fuzzified enrollments; fuzzy logical relationships.

1. Introduction

It is obvious that forecasting activities play an important role in our daily life. The classical time series methods can not deal with forecasting problems in which the values of time series are linguistic terms represented by fuzzy sets [19]. Therefore, in [13], Song and Chissom presented the theory of fuzzy time series to overcome the drawback of the classical time series methods. Based on the theory of fuzzy time series, Song et al. presented some forecasting methods [11], [13], [14], [15] to forecast the enrollments of the University of Alabama. In [1], Chen presented a method to forecast the enrollments of the University of Alabama based on fuzzy time series. It has the advantage of reducing the calculation time and simplifying the calculation process. In [7], Chen et al. used the differences of the enrollments to present a method to forecast the enrollments of the University of Alabama based on fuzzy time series. In [4], Huang extended Chen's work presented in [1] and used simplified calculations with the addition of heuristic rules to forecast the enrollments. In [3], Chen presented a forecasting method based on high-order fuzzy time series for forecasting the enrollments of the University of Alabama. In [2], Chen and Hwang presented a method based on fuzzy time series to forecast the temperature.

In [13] and [14], Song et al. used the following model for forecasting university enrollments:

$$A_i = A_{i-1} \circ R, \tag{1}$$

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where A_{i-1} denotes the fuzzified enrollments of year i - 1 represented by a fuzzy set, A_i denotes the fuzzified enrollments of year i represented by a fuzzy set, the symbol $\circ$ denotes the Max-Min composition operator, and R is a fuzzy relation formed by the fuzzified enrollments of the fuzzy time series. The forecasting method presented in [13] has the following drawbacks:

(1) It requires a large amount of computations to derive the fuzzy relation R show in formula (1).

(2) The max-min composition operations of formula (1) will take a large amount of computation time when the fuzzy relation R is very big.

In [10], Li et al. presented a dynamic neural network method for time series prediction using the KIII model. In [16], Su et al. presented a method for fusing global and local information in predicting time series based on neural networks. In [17], Sullivan et al. reviewed the first-order time-variant fuzzy time series model and the first-order time-invariant fuzzy time series model presented by Song and Chissom, where their models are compared with each other and with a time-variant Markov model using linguistic labels with probability distributions.

However, the forecasting accuracy rates of the existing fuzzy time series methods for forecasting enrollments are not good enough. In this paper, we present a new method to forecast the enrollments of the University of Alabama. The proposed method belongs to the first order and time-variant methods. It can get a higher forecasting accuracy rate for forecasting enrollments than the existing methods.

The rest of this paper is organized as follows. In Section 2, we briefly review basic concepts of fuzzy time series from [12], [13] and [14]. In Section 3, we use the theory of fuzzy time series to propose a new method to forecast the enrollments of the University of Alabama. In Section 4, we compare the forecasting results of the proposed method with the existing methods. The conclusions are discussed in Section 5.

2. Basic concepts of fuzzy time series

In [12], [13] and [14], Song et al. proposed the definition of fuzzy time series based on fuzzy sets [19]. Let U be the universe of discourse, $U = \{u_1, u_2, \ldots, u_n\}$, and let A be a fuzzy set in the universe of discourse U defined as follows:

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \cdots + f_A(u_n)/u_n, \tag{2}$$

where f_A is the membership function of A, $f_A : U \rightarrow [0, 1]$, $f_A(u_i)$ indicates the grade of membership of u_i in the fuzzy set A, $f_A(u_i) \in [0, 1]$, and $1 \leq i \leq n$.

Let X(t) (t = …, 0, 1, 2, …) be the universe of discourse and be a subset of R, and let fuzzy set $f_i(t)$ (i = 1, 2, …) be defined in X(t). Let F(t) be a collection of $f_i(t)$ (i = 1, 2, …). Then, F(t) is called a fuzzy time series of X(t) (t = …, 0, 1, 2, …).

If F(t) is caused by F(t - 1), denoted by F(t - 1) → F(t), then this relationship can be represented by F(t) = F(t - 1) $\circ$ R(t, t - 1), where the symbol " $\circ$ " denotes the Max-Min composition operator; R(t, t – 1) is a fuzzy relation between F(t) and F(t - 1) and is called the first-order model of F(t).

Let F(t) be a fuzzy time series and let R(t, t - 1) be a first-order model of F(t). If R(t, t - 1) = R(t - 1, t - 2) for any time t, then F(t) is called a time-invariant fuzzy time series. If R(t, t - 1) is dependent on time t, that is, R(t, t - 1) may be different from R(t - 1, t - 2) for any t , then F(t) is called a time-variant fuzzy time series.

In [14], Song et al. proposed the time-variant fuzzy time-series model and forecasted the enrollments of the University of Alabama based on the model. The method for forecasting the enrollments is briefly reviewed as follows:

Step 1: Define the universe of discourse within which fuzzy sets are defined.
Step 2: Partition the universe of discourse U into several even and equal length intervals.
Step 3: Determine some linguistic values represented by fuzzy sets of the intervals of the universe of discourse.
Step 4: Fuzzify the historical enrollment data.
Step 5: Choose a suitable parameter w, where w ＞ 1, calculate $R^w(t, t - 1)$ and forecast the enrollments as follows:

$$F(t) = F(t - 1) \circ R^w(t, t - 1), \tag{3}$$

where F(t) denotes the forecasted fuzzy enrollment of year t, F(t-1) denotes the fuzzified enrollment of year t – 1, and

$$R^w(t, t - 1) = F^T(t - 2) \times F(t - 1) \cup F^T(t - 3) \times F(t - 2) \cup \ldots \cup F^T(t - w) \times F(t - w + 1), \tag{4}$$

where w is called the "model basis" denoting the number of years before t, "×" is the Cartesian product operator, and T is the transpose operator.
Step 6: Defuzzify the forecasted fuzzy enrollment using neural nets.

In [17], Sullivan et al. used the following Markov model to forecast the enrollments of the University of Alabama:

$$P'_{t+1} = P'_t \ * R_m \ , \tag{5}$$

where P_t is the vector of state probabilities at time t, P_{t+1} is the vector of state probabilities at time t + 1, R_m is the transition matrix, and "∗" is a conventional matrix multiplication operator. It is obvious that formula (5) is a time-invariant fuzzy time-series model due to the fact that it does not change with time. The other style of the Markov model is called the time-variant fuzzy time-series model as follows:

$$P'_{t+1} = P'_t \ * R^k_m, \ \ k = 1, 2, \ldots, \tag{6}$$

where R^k_m varies with time. For more details, please refer to [17].

3. A new method for forecasting enrollments using fuzzy time series

In this section, we present a new method to forecast the enrollments of the University of Alabama based on fuzzy time series. The historical enrollments of the University of Alabama are shown in Table 1 [14].

First, the proposed method defines the universe of discourse and partitions the universe of discourse into some even and equal length intervals. Then, it gets the statistical distributions of the historical enrollment data in each interval and re-divided each interval. Then, it defines linguistic values represented by fuzzy sets based on the re-divided intervals and fuzzify the historical enrollments to get fuzzified enrollments. Then, it establishes fuzzy logical relationships based on the fuzzified enrollments. Finally, it uses a set of rules to determine whether the trend of the forecasting goes up or down and to forecast the enrollments. Assume that we want to forecast the enrollment of year n, then the "difference of differences" of the enrollments between years n-1 and n-2 and between years n-2 and n-3 = (the enrollment of year n-1 - the enrollment of year n-2) - (the enrollment of year n-2 - the enrollment of year n-3). The proposed method is now presented as follows:

Step 1: Define the universe of discourse U and partition it into several even and equal length intervals $u_1, u_2, \ldots$, and u_n. For example, assume that the universe of discourse U = [13000, 20000] is partitioned into seven even and equal length intervals $u_1, u_2, u_3, u_4, u_5, u_6$ and u_7, where $u_1 = [13000, 14000]$, $u_2 = [14000, 15000]$, $u_3 = [15000, 16000]$, $u_4 = [16000, 17000]$, $u_5 = [17000, 18000]$, $u_6 = [18000, 19000]$ and $u_7 = [19000, 20000]$.

Step 2: Get a statistics of the distribution of the historical enrollments in each interval.

Sort the intervals based on the number of historical enrollment data in each interval from the highest to the lowest. Find the interval having the largest number of historical enrollment data and divide it into four sub-intervals of equal length. Find the interval having the second largest number of historical enrollment data and divide it into three sub-intervals of equal length. Find the interval having the third largest number of historical enrollment data and divide it into two sub-intervals of equal length. Find the interval with the fourth largest number of historical enrollment data and let the length of this interval remain unchanged. If there are no data distributed in an interval, then discard this interval. For example, the distributions of the historical enrollment data in different intervals are summarized as shown in Table 2 [6].

Table 1. The historical enrollments of the University of Alabama [14]

Year	Enrollments
1971	13055
1972	13563
1973	13867
1974	14696
1975	15460
1976	15311
1977	15603
1978	15861
1979	16807
1980	16919
1981	16388
1982	15433
1983	15497
1984	15145
1985	15163
1986	15984
1987	16859
1988	18150
1989	18970
1990	19328
1991	19337
1992	18876

Table 2. The distribution of the historical enrollment data [6]

Intervals	[13000,14000]	[14000,15000]	[15000,16000]	[16000,17000]	[17000,18000]	[18000,19000]	[19000,20000]
Number of historical enrollment data	3	1	9	4	0	3	2

After executing this step, the universe of discourse [13000, 2000c0] is re-divided into the following intervals [6]:

$u_{1,1}=[13000,13500]$,	$u_{1,2}=[13500,14000]$,
$u_2=[14000,15000]$,	$u_{3,1}=[15000,15250]$,
$u_{3,2}=[15250,15500]$,	$u_{3,3}=[15500,15750]$,
$u_{3,4}=[15750,16000]$,	$u_{4,1}=[16000,16333]$,
$u_{4,2}=[16333,16667]$,	$u_{4,3}=[16667,17000]$,
$u_{6,1}=[18000,18500]$,	$u_{6,2}=[18500,19000]$,
$u_7=[19000,20000]$.	

Step 3: Define each fuzzy set A_i based on the re-divided intervals and fuzzify the historical enrollments shown in Table 1, where fuzzy set A_i denotes a linguistic value of the enrollments represented by a fuzzy set, and $1 \le i \le 13$. For example, A_1=very very very very few, A_2=very very very few, A_3=very very few, A_4 = very few, A_5 = few, A_6 = moderate, A_7=many, A_8=many many, A_9= very many, A_{10}=too many, A_{11}=too many many, A_{12}=too many many many and A_{13}= too many many many many, defined as follows [6]:

$$A_1 = 1/u_{1,1} + 0.5/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_2 = 0.5/u_{1,1} + 1/u_{1,2} + 0.5/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_3 = 0/u_{1,1} + 0.5/u_{1,2} + 1/u_2 + 0.5/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_4 = 0/u_{1,1} + 0/u_{1,2} + 0.5/u_2 + 1/u_{3,1} + 0.5/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_5 = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0.5/u_{3,1} + 1/u_{3,2} + 0.5/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_6 = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0.5/u_{3,2} + 1/u_{3,3} + 0.5/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_7 = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0.5/u_{3,3} + 1/u_{3,4} + 0.5/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_8 = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0.5/u_{3,4} + 1/u_{4,1} + 0.5/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_9 = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0.5/u_{4,1} + 1/u_{4,2} + 0.5/u_{4,3} + 0/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_{10} = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0.5/u_{4,2} + 1/u_{4,3} + 0.5/u_{6,1} + 0/u_{6,2} + 0/u_7,$$

$$A_{11} = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0.5/u_{4,3} + 1/u_{6,1} + 0.5/u_{6,2} + 0/u_7,$$

$$A_{12} = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0.5/u_{6,1} + 1/u_{6,2} + 0.5/u_7,$$

$$A_{13} = 0/u_{1,1} + 0/u_{1,2} + 0/u_2 + 0/u_{3,1} + 0/u_{3,2} + 0/u_{3,3} + 0/u_{3,4} + 0/u_{4,1} + 0/u_{4,2} + 0/u_{4,3} + 0/u_{6,1} + 0.5/u_{6,2} + 1/u_7.$$

For simplicity, the membership values of fuzzy set A_i either are 0, 0.5 or 1, where $1 \le i \le 13$. Then, fuzzify the historical enrollments shown in Table 1 based on [18] and the linguistic values of the enrollments A_1, A_2, …, A_{13}. The reason for fuzzifying the historical enrollments into fuzzified enrollments is to translate crisp values into fuzzy sets to get a fuzzy time series.

Step 4: Establish fuzzy logical relationships based on the fuzzified enrollments:

$$A_j \rightarrow A_q\ ,$$
$$A_j \rightarrow A_r\ ,$$
$$\vdots$$

where the fuzzy logical relationship "$A_j \rightarrow A_q$" denotes " if the fuzzified enrollments of year n-1 is A_j , then the fuzzified enrollments of year n is A_q ". For example, based on the fuzzify historical enrollments obtained in Step 3, we can get the fuzzy logical relationships as shown in Table 3.

Table 3. Fuzzy logical relationships [6]

$A_1 \rightarrow A_2$,	$A_2 \rightarrow A_2$,	$A_2 \rightarrow A_3$,
$A_3 \rightarrow A_5$,	$A_5 \rightarrow A_5$,	$A_5 \rightarrow A_6$,
$A_6 \rightarrow A_7$,	$A_7 \rightarrow A_{10}$,	$A_{10} \rightarrow A_{10}$,
$A_{10} \rightarrow A_9$,	$A_9 \rightarrow A_5$,	$A_5 \rightarrow A_5$,
$A_5 \rightarrow A_4$,	$A_4 \rightarrow A_4$,	$A_4 \rightarrow A_7$,
$A_7 \rightarrow A_{10}$,	$A_{10} \rightarrow A_{11}$,	$A_{11} \rightarrow A_{12}$,
$A_{12} \rightarrow A_{13}$,	$A_{13} \rightarrow A_{13}$,	$A_{13} \rightarrow A_{12}$.

Step 5: Divide each interval derived in Step 2 into four subintervals of equal length, where the 0.25-point and 0.75-point of each interval are used as the upward and downward forecasting points of the forecasting. Use the following rules to determine whether the trend of the forecasting goes up or down and to forecast the enrollment. Assume that the fuzzy logical relationship is $A_i \rightarrow A_j$, where A_i denotes the fuzzified enrollment of year n-1 and A_j denotes the fuzzified enrollment of year n, then (1) If j > i and the difference of the differences of the enrollments between years n-1 and n-2 and between years n-2 and n-3 is positive, then the trend of the forecasting will go up, and we use the following **Rule 2** to forecast the enrollments; (2) If j > i and the difference of the differences of the enrollments between years n-1 and n-2 and between years n-2 and n-3 is negative, then the trend of the forecasting will go down, and we use the following **Rule 3** to forecast the enrollments; (3) If j < i and the difference of the differences of the enrollments between years n-1 and n-2 and between years n-2 and n-3 is positive, then the trend of the forecasting will go up, and we use the following **Rule 2** to forecast the enrollments; (4) If j < i and the difference of the differences of the enrollments between years n-1 and n-2 and between years n-2 and n-3 is negative, then the trend of the forecasting will go down, and we use the following **Rule 3** to forecast the enrollments; (5) If j = i and the difference of the differences of the enrollments between years n-1 and n-2 and between years n-2 and n-3 is positive, then the trend of the forecasting will go up, and we use the following **Rule 2** to forecast the enrollments; (6) If j = i and the difference of the differences of the enrollments between years n-1 and n-2 and between years n-2 and n-3 is negative, then the trend of the forecasting will go down, and we use the following **Rule 3** to forecast the enrollments, where **Rule 1**, **Rule 2** and **Rule 3** are shown as follows:

Rule 1: When forecasting the enrollment of year 1973, there are no data before the enrollments of year 1970, therefore we are not able to calculate the difference of the enrollments between years 1971 and 1970 and the difference of the differences between years 1972 and 1971 and between years 1971 and 1970. Therefore, if |(the difference of the enrollments between years 1972 and 1971)|/2 > half of the length of the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1, then the trend of the forecasting of this interval will be upward, and the forecasting enrollment falls at the 0.75-point of this interval; if |(the difference of the enrollments between years 1972 and 1971)|/2 = half of the length of the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1, then the forecasting enrollment falls at the middle value of this interval; if |(the difference of the enrollments between years 1972 and 1971)|/2 < half of the length of the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1, then the trend of the forecasting of this interval will be downward, and the forecasting enrollment falls at the 0.25-point of the interval.

Rule 2: If (|the difference of the differences between years n-1 and n-2 and between years n-2 and n-3| × 2 + the enrollments of year n-1) or (the enrollments of year n-1 - |the difference of the differences between years n-1 and n-2 and between years n-2 and n-3| × 2) falls in the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1, then the trend of the forecasting of this interval will be upward, and the forecasting enrollment falls at the 0.75-point of the interval of the corresponding fuzzified enrollment A_j with the membership value equal to 1; if (|the difference of the differences between years n-1 and n-2 and between years n-2 and n-3|/2 + the enrollments of year n-1) or (the enrollments of year n-1 - |the difference of the differences between years n-1 and n-2 and between years n-2 and n-3|/2) falls in the interval of the corresponding fuzzified enrollment A_j with the membership value equal to 1, then the trend of the forecasting of this interval will be downward, and the forecasting value falls at the 0.25-point of the interval of the corresponding fuzzified enrollment A_j with the membership value equal to 1; if neither is the case, then we let the forecasting enrollment be the middle value of the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1.

Rule 3: If (|the difference of the differences between years n-1 and n-2 and between years n-2 and n-3|/2 + the enrollments of year n-1) or (the enrollments of year n-1 - |the difference of the differences between years n-1 and n-2 and between years n-2 and n-3|/2) falls in the interval of the corresponding fuzzified enrollment A_j with the membership value equal to 1, then the trend of the forecasting of this interval will be downward, and the forecasting enrollment falls at the 0.25-point of the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1; if (|the difference of the differences between years n-1 and n-2 and between years n-2 and n-3| × 2 + the enrollment of year n-1) or (the enrollment of year n-1 - |the difference of the differences between years n-1 and n-2 and between years n-2 and n-3| × 2) falls in the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1, then the trend of the forecasting of this interval will be upward, and the forecasting enrollment falls at the 0.75-point of the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1; if neither is the case, then we let the forecasting enrollment be the middle value of the interval corresponding to the fuzzified enrollment A_j with the membership value equal to 1.

4. A comparison of different forecasting methods

We have implemented the proposed method using visual Basic Version 6.0 on a Pentium 4 PC. Table 4 summarizes the forecasting results of the proposed method from 1972 to 1992, where the universe of discourse is divided into 13 intervals and the interval with the largest number of historical enrollment data is divided into 4 sub-intervals of equal length.

Table 4. Actual enrollments and forecasting enrollments of the University of Alabama

Year	Enrollments	Trend of the forecasting	Forecasting enrollments
1971	13055		
1972	13563	Middle value	13750
1973	13867	Upward; 0.75-point	13875
1974	14696	Upward; 0.75-point	14750
1975	15460	Middle Value	15375
1976	15311	Downward; 0.25-point	15312.5
1977	15603	Middle Value	15625
1978	15861	Downward; 0.25-point	15812.5
1979	16807	Middle Value	16833.5
1980	16919	Middle Value	16833.5
1981	16388	Downward; 0.25-point	16416.25
1982	15433	Middle Value	15375
1983	15497	Middle Value	15375
1984	15145	Middle Value	15125
1985	15163	Middle Value	15125
1986	15984	Upward; 0.75-point	15937.5
1987	16859	Middle Value	16833.5
1988	18150	Middle Value	18250
1989	18970	Upward; 0.75-point	18875
1990	19328	Downward; 0.25-point	19250
1991	19337	Downward; 0.25-point	19250
1992	18876	Upward; 0.75-point	18875

In the following, we use the mean square error (MSE) to compare the forecasting results of different forecasting methods, where the mean square error is calculated as follows:

$$MSE = \frac{\sum_{i=1}^{n} (Actual\_Enrollment_i - Forecasted\_Enrollment_i)^2}{n}, \tag{7}$$

where $Actual\_Enrollment_i$ denotes the actual enrollment of year i, and $Forecasted\_Enrollment_i$ denotes the forecasting enrollment of year i. In Table 5, we compare the forecasting results of the proposed method with that of the existing methods.

Table 5. A comparison of the forecasting results of different forecasting methods

Year	Enrollments	Song and Chissom's method [13]	Song and Chissom's method [14]	Chen's method [1]	Hwang, Chen and Lee's method [7]	Huarng's method [4]	Chen's method [3]	The Proposed method (13 intervals; the interval with the largest number of enrollment data is divided into 4 sub-intervals)
1971	13055							
1972	13563	14000		14000		14000		13750
1973	13867	14000		14000		14000		13875
1974	14696	14000		14000		14000	14500	14750
1975	15460	15500	14700	15500		15500	15500	15375
1976	15311	16000	14800	16000	16260	15500	15500	15312.5
1977	15603	16000	15400	16000	15511	16000	15500	15625
1978	15861	16000	15500	16000	16003	16000	15500	15812.5
1979	16807	16000	15500	16000	16261	16000	16500	16833.5
1980	16919	16813	16800	16833	17407	17500	16500	16833.5
1981	16388	16813	16200	16833	17119	16000	16500	16416.25
1982	15433	16789	16400	16833	16188	16000	15500	15375
1983	15497	16000	16800	16000	14833	16000	15500	15375
1984	15145	16000	16400	16000	15497	15500	15500	15125
1985	15163	16000	15500	16000	14745	16000	15500	15125
1986	15984	16000	15500	16000	15163	16000	15500	15937.5
1987	16859	16000	15500	16000	16384	16000	16500	16833.5
1988	18150	16813	16800	16833	17659	17500	18500	18250
1989	18970	19000	19300	19000	19150	19000	18500	18875
1990	19328	19000	17800	19000	19770	19000	19500	19250
1991	19337	19000	19300	19000	19928	19500	19500	19250
1992	18876		19600	19000	19537	19000	18500	18875
MSE		423027	775687	407507	321418	226611	86694	5353

From Table 5, we can see that when the number of intervals in the universe of discourse is 13 and the interval with the largest number of enrollment data is divided into 4 sub-intervals, the MSE of the forecasting results of the proposed method is smaller than that of the existing methods. That is, the proposed method can get a higher forecasting accuracy rate for forecasting enrollments than the existing methods.

5. Conclusions

In this paper, we have presented a new method for forecasting the enrollments of the University of Alabama using fuzzy time series. The proposed method belongs to the first order and time-variant methods. From Table 5, we can see that the MSE of the forecasting results of the proposed method is smaller than that of the existing methods. That is, the proposed method gets a higher forecasting accuracy rate for forecasting enrollments than the existing methods. In the future, we will extend the proposed method to deal with other forecasting problems based on fuzzy time series. We also will develop a new method for forecasting enrollments based on fuzzy neural networks to get a higher forecasting accuracy rate.

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