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In-Plane Response of a Symmetric Space Frame with Sliding Supports

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Abstract: The response of a symmetric space frame structure resting on sliding type of bearing is analysed considering all the six degree of freedom of the structure at each node. The analysis consists of two phases, a sliding phase and non-sliding phase. In non-sliding phase the horizontal stiffness of sliding bearing is considered as very large where as it is equal to zero during sliding phase. A step by step Newmark's method is adopted for the solution of dynamic equation. The analysis is used to study the response of a four story symmetric space frame structure subjected to an El Centro ground acceleration. The effects of coefficient of friction of isolation material, natural period of the structure, excitation frequency and number of storeys on response of structure are also studied when the structure is subjected to harmonic ground acceleration.

Keywords*:* sliding bearing; sinusoidal load; El Centro ground acceleration; space frame structure.

1. Introduction

Base isolation is an aseismic design approach in which the structure is protected from the damaging effects of severe earthquake forces by a mechanism which reduces the transmission of horizontal acceleration into the structure. Isolation devices are essentially classified into two types - rubber bearings and sliding bearings. Although rubber bearings have been used extensively in base isolation systems, sliding bearings have recently found increasing applications. The sliding type of bearing uses rollers or sliders between the foundation and the base of the structure. Because of the non-sliding and sliding phases exist alternatively, the dynamic behaviour of a sliding structure is highly non-linear. Yang et al. [3] studied the response of the multi degree of freedom (with one horizontal degree of freedom at each floor) structures on sliding supports using a fictitious spring to the foundation floor. The spring was assumed to be bilinear with a very large stiffness in the non-sliding phase and zero stiffness in the sliding phase. Vafai et al. [4] analysed the multi degree of freedom structure on sliding supports by replacing a fictitious spring in the model of Yang et al. [3] by a link with a rigid-perfectly plastic material. Bhasker and Jangid [1] also analysed the structure resting on sliding type of bearing assuming different equations for non-sliding phase and sliding phases. However, in all the

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above analysis the building frame is assumed as a shear frame with only one (horizontal) degree of freedom at each floor. In the present analysis the space frame structure is divided into number of elements consisting of number of columns and beams and at each node six degree of freedom (three translation and three rotations) are considered. The sliding support is modeled using a fictitious spring with two horizontal degree of freedom and these springs are attached to the base of each column. A large value of stiffness is considered in the non-sliding phase and stiffness is taken as zero in the sliding phase as proposed by Yang et al. [3].

2. Analytical modeling

The various assumptions made for analytical modeling of the isolated structural model are as follows:

1. The superstructure remains elastic during the excitation.
2. The coefficient of friction of the sliding bearing remains constant throughout the motion of the structure.
3. No overturning or tilting will occur in the super structure during sliding phase of motion.

The structure is divided into number of elements consisting of beams and columns connected at nodes. Each element is modeled using two noded frame element with six degree of freedom at each node i,e., three translations along X,Y, and Z axes and three rotations about these axes. For each element, the stiffness matrix [k], consistent mass matrix [m] and transformation matrix [T] is obtained and the mass matrix and the stiffness matrix from local direction is transformed to global direction as proposed by Paz [5]. The mass matrix and stiffness matrix of each element are assembled by direct stiffness method to obtain the overall mass matrix [M] and stiffness matrix [K] for the entire structure. The overall dynamic equation of equilibrium for the entire structure can be expressed in matrix notations as

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(t)\} \quad (1)$$

where [M], [C], and [K] are the overall mass, damping, and stiffness matrices. The damping of the superstructure is assumed as Rayleigh type and the damping matrix [C] is determined using the equation $[C]=\alpha[M]+\beta[K]$ where α and β are the Rayleigh constants. These constants can be determined easily if the damping ratio is known. $\{\ddot{u}\}$, $\{\dot{u}\}$, $\{u\}$ are the relative acceleration, velocity and displacement vectors at nodes with respect to ground. {F(t)} is the nodal load vector. $\{u\} = \{u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}, u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}, \ldots\ldots u_n, v_n, w_n, \theta_{xn}, \theta_{yn}, \theta_{zn}\}$ where n is the number of nodes. The nodal load vector is calculated using the equation

$$\{F(t)\} = -[M]\{l\}\ddot{u}_g(t)$$

where [M] is the overall mass matrix, {l} is the influence vector, $\ddot{u}_g(t)$ is the ground acceleration.

When the structure is resting on sliding type of bearing with a coefficient of friction equal to μ then the mobilized frictional force, Fx, at base will be resisted by the frictional resistance, Fs, which acts against the direction of mobilized frictional force. When the mobilized frictional force, Fx, at base is less than the frictional resistance, Fs, (i.e. $|Fx| < Fs$) the structure will not have relative movement at base and this phase of structure is known as non–sliding phase. However, when the mobilized frictional force, Fx is equal to or more than the frictional resistance, Fs (i.e. $|Fx| \geq Fs$) the structure starts sliding at base and this phase of the structure is known as sliding phase. When the structure is in sliding phase and whenever reverses its direction of motion (when the velocity at base is equal to zero) then the structure may again stop its movement at base and may enter the non-sliding phase or may slide in opposite direction. In the present analysis, sliding bearing is modeled as a fictitious spring connected to the base of each column. The conditions for sliding and non-sliding phase are duly checked at the end of each time step. When the structure is in non-sliding phase, the stiffness of the spring is assigned a very high value to prevent the movement of the structure at base where as when the structure is in sliding phase, the value of stiffness of spring is made equal to zero to allow the movement of the structure at base. Thus the stiffness of the spring may be equal to zero or very high value depending on the phase of the structure.

Also, during the non-sliding phase the relative acceleration ($\ddot{u}_b$) and relative velocity ($\dot{u}_b$) of the base is equal to zero and the relative displacement at base (u_b) is constant during this phase. The stiffness of the spring at base of each column are considered as very large (k_b= $1x10^{15}$ kN/m) during non-sliding phase. The dynamic equation of motion for the non-sliding phase is same as given in Eq. (1). However, [K], the stiffness matrix includes the stiffness of the structure and the stiffness of the spring (k_b, being a very large value).

During sliding phase, the stiffness of the spring at base of each column is considered as zero ($k_b = 0$) and the mobilized frictional force, Fx, under each column is equal to Fs and remains constant. Hence, the dynamic equations of motion for the structure during this phase is

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{F(t)\}-\{F_{xmax}\} \quad (2)$$

where, [K] is the stiffness of the structure and stiffness of spring with spring stiffness, k_b, equal to zero. $\{F_{xmax}\}$ is the vector with zeros at all locations except those corresponding to the horizontal degree of freedom at base of the structure. At these degrees of freedom, the vector $\{F_{xmax}\}$ will have values equal to Fs and directions opposite to the velocity vector $\{\dot{u}_b\}$.

2.1. Determination of displacements and acceleration by Newmark's method

The frictional force mobilized in the sliding system is a non-linear function and hence the response of the isolated structural system is obtained in a incremental form using Newmark's method. In this method, from the response at time t the response at time t + Δt is determined. Owing to its unconditional stability, the constant average acceleration scheme (with β=1/4 and γ=1/2) is adopted. Eq. (1) in incremental form can be written as

$$M\Delta\ddot{u}_i + C\Delta\dot{u}_i + K\Delta u_i = \Delta F_i \quad (3)$$

where Δ denotes the variations of each parameters from time t to time t + Δt, and index i indicates the i^{th} time step.

$$\Delta\dot{u}_i = \frac{2}{\Delta t}\Delta u_i - 2\dot{u}_i \quad (4)$$

$$\Delta\ddot{u}_i = \frac{4}{(\Delta t)^2}\Delta u_i - \frac{4}{\Delta t}\dot{u}_i - 2\ddot{u}_i \quad (5)$$

Substituting Eq. (4) and Eq. (5) into Eq. (3) yields

$$\hat{K}_i\Delta u_i = \Delta\hat{F}_i \quad (6)$$

where $\hat{K}_i$ and $\Delta\hat{F}_i$ are called effective stiffness matrix and effective load vector respectively. These are defined as

$$\hat{K}_i = K_i + \frac{2}{\Delta t}C + \frac{4}{(\Delta t)^2}M \quad (7)$$

$$\Delta\hat{F}_i = \Delta F_i + \left(\frac{4}{\Delta t}M + 2C\right)\dot{u}_i + 2M\ddot{u}_i \quad (8)$$

By solving Eq. (6), Δu_i is determined and subsequent values of displacement and velocity at the beginning of step (i +1) are calculated using Eq. (4) and the following two equations

$$u_{i+1} = u_i + \Delta u_i \quad (9)$$

$$\dot{u}_{i+1} = \dot{u}_i + \Delta\dot{u}_i \quad (10)$$

Accelerations are calculated based on Eq. (1) to increase the accuracy and stability of solutions. In non-linear problems, since [K] in Eq. (1) is not constant and depends on responses of the previous step, a very small time step Δt is required so as to model the transition times between phases. In the present method a time step of $\Delta t = 4x10^{-3}$ seconds is considered.

2.2. Determination of base shear and member forces

Forces in each member of the structure are obtained using the equation [k]{q}. Where [k] is the member stiffness matrix and {q} is the nodal displacement vector in local coordinate systems. The horizontal force, $F_{b,}$ at bottom node of the column in contact with the sliding bearing is the base shear under each column. Similarly the damping force at each degree of freedom can also be obtained using the equation $[C]\{\dot{u}\}$ where [C] is the overall damping matrix and $\{\dot{u}\}$ is the nodal velocity vector. The damping force corresponding to horizontal degree of freedom at base is F_d. The mobilized frictional force, Fx, under base of each column when the system is in non-sliding phase is determined using the equation

$$F_x = F_b + F_d - F \tag{11}$$

F is the applied force at base of column due to ground acceleration (ie. $F = -M_F \ddot{u}_g$, where, M_F is the base mass and $\ddot{u}_g$ is the ground acceleration). It is to be noted that the relative acceleration and velocity at base is equal to zero when the system is in non-sliding phase.

2.3. Determination of frictional resistance

The frictional resistance, Fs, under each column is obtained using the equation $Fs=\mu W$ where μ is the coefficient of friction of the sliding material and W is the load on column in contact with the bearing.

3. Results and discussions

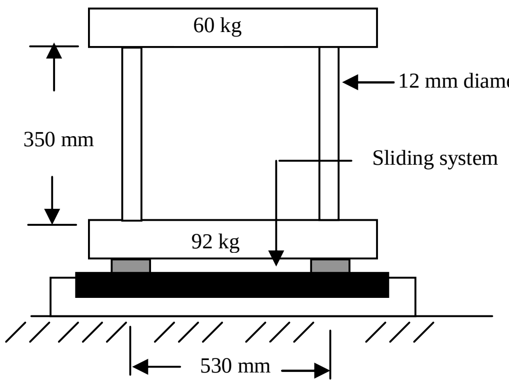
The results obtained from the analysis are compared with the results available in the literature. The analysis is also used to study the effects of frequency of ground acceleration, coefficient of friction of base material, natural period of the structure and number of storeys on response of the structure.

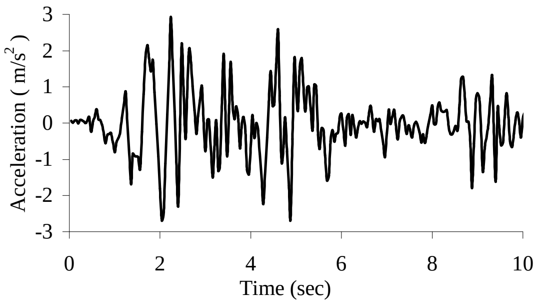
3.1. Comparison of the present analysis with results available in literature

Bhasker and Jangid [1] carried out shake table test to study the response of a structure subjected to harmonic base excitation and also developed analytical model to study the response of a structure shown in Figure 1a with sliding supports. The modulus of elasticity of the structure is equal to $2.2x10^8$ kN/m^2 (period of structure T_s = 0.1 sec) and damping ratio is equal to 2 per cent. The coefficient of friction of base material (μ) is 0.07. The structure was subjected to a harmonic ground acceleration $\ddot{u}_g = a_o \sin(\omega t)$ with $a_o = 0.2g$ and to El Centro earthquake ground acceleration. The same frame is analysed by the present analysis. The top floor acceleration and base displacement obtained from the present analysis and presented by Bhasker and Jangid [1] are shown in Figure 1b and Figure 1c. It can be observed from the Figure 1b and Figure 1c that the results obtained from the present analysis and the results presented by Bhasker and Jangid [1] are similar in terms of the response ordinates and overall characteristics.

3.2. Response of a structure resting on sliding type of supports

The response of a four story space frame structure subjected to ground acceleration due to El Centro earth quake is studied. The effect of natural period of the structure (T_s), effect of coefficient of friction of base material μ, effect of excitation frequency (ω) and effect of number of storeys on the responses of the structure subjected to sinusoidal ground acceleration is studied. To study this the natural period of the structure is selected as 0.25sec, 0.5sec, 0.75sec and 1.0 sec and the coefficient





Accelerogram of El Centro earthquake input load

Figure 1a. Model of one story plane frame structure considered by Bhasker and Jangid (2001)

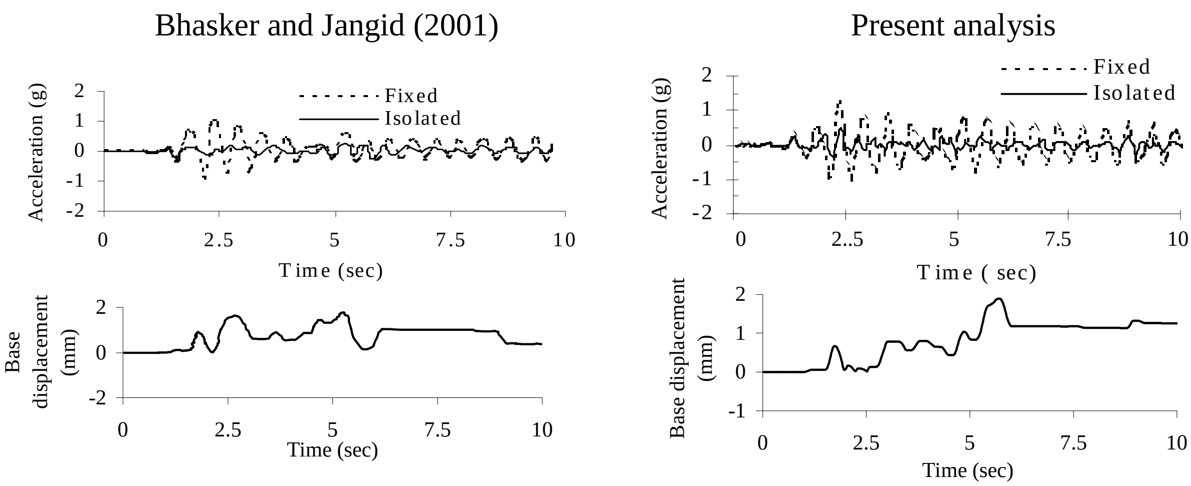


Figure 1b. Response of structure subjected to earthquake load

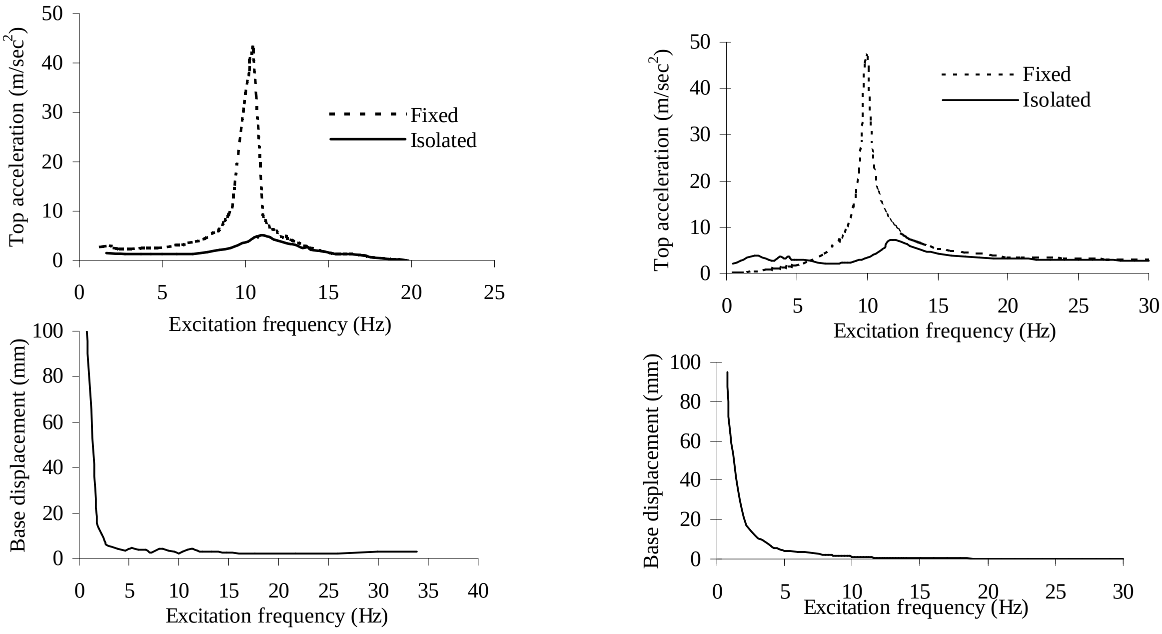


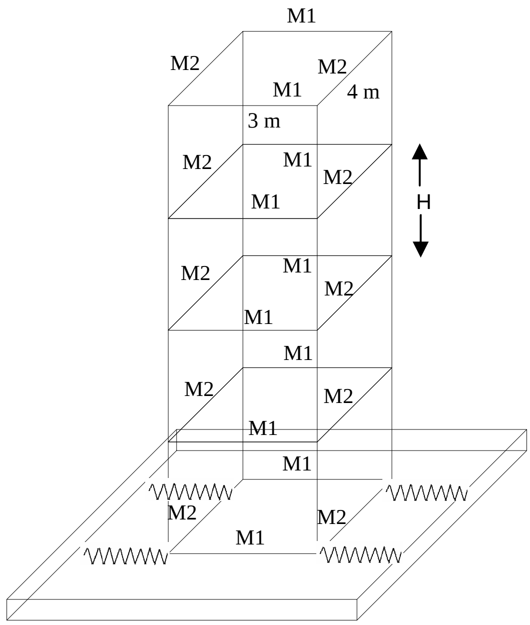
Figure 1c. Response of structure subjected to sinusoidal load

of friction of base material is varied as 0.04, 0.08, 0.12, 0.16 and 0.2. The four story space frame structure considered for the study is shown in Figure 2. The various parameters considered for the study are also shown in Figure 2. The columns are mass less and the damping ratio of the structure is equal to 5 percent. The values of Rayleigh damping constants α and β for various natural period of the structure considered for the analysis are also shown in the same figure.

Figure 3 shows the variation of acceleration at top of the structure, bending moment at base of each column, base shear under each column and base displacement with time when a structure is subjected to earthquake load due to El Centro ground acceleration. The natural period of the structure is equal to 0.5 seconds and coefficient of friction of base material is equal to 0.08. It can be observed from the figure that the acceleration**,** bending moment and base shear of the structure isolated at base are considerably less than those of the structure fixed at base. The maximum base displacement occurs at around 5.6 seconds and is equal to 53.0 mm. However the structure is shifted to a new position at the end of earthquake (at 10.0 sec). The non-sliding and sliding phases of the structure can also be seen clearly in the figure.

3.3. Effect of frequency of ground acceleration

The structure is subjected to a harmonic ground acceleration of intensity 2sin(ωt) m/sec^2. The variation of response with excitation frequency, ω, for a structure with natural period (T_s) equal to 0.50 sec (ω_n = 12.56



Ts (sec)	Rayleigh damping constants		Mass (kN.sec^2/m^2)		Size of column		Size of beam		Height H (m)	E (kN/m^2)
	α	β x10 $^{-3}$	M1	M2	B (m)	D (m)	B (m)	D (m)		
0.25	1.25	1.99	2	1	0.65	0.65	0.3	0.6	3	2.2x10^7
0.50	0.60	4.10	3	2	0.6	0.6	0.3	0.6	4	2.2x10^7
0.75	0.42	5.97	3	2.25	0.5	0. 5	0.3	0.5	4	2.2x10^7
1.0	0.38	6.24	5	4.5	0.5	0. 5	0.3	0.5	4	2.2x10^7

Figure 2. Model of four story space frame considered for the study

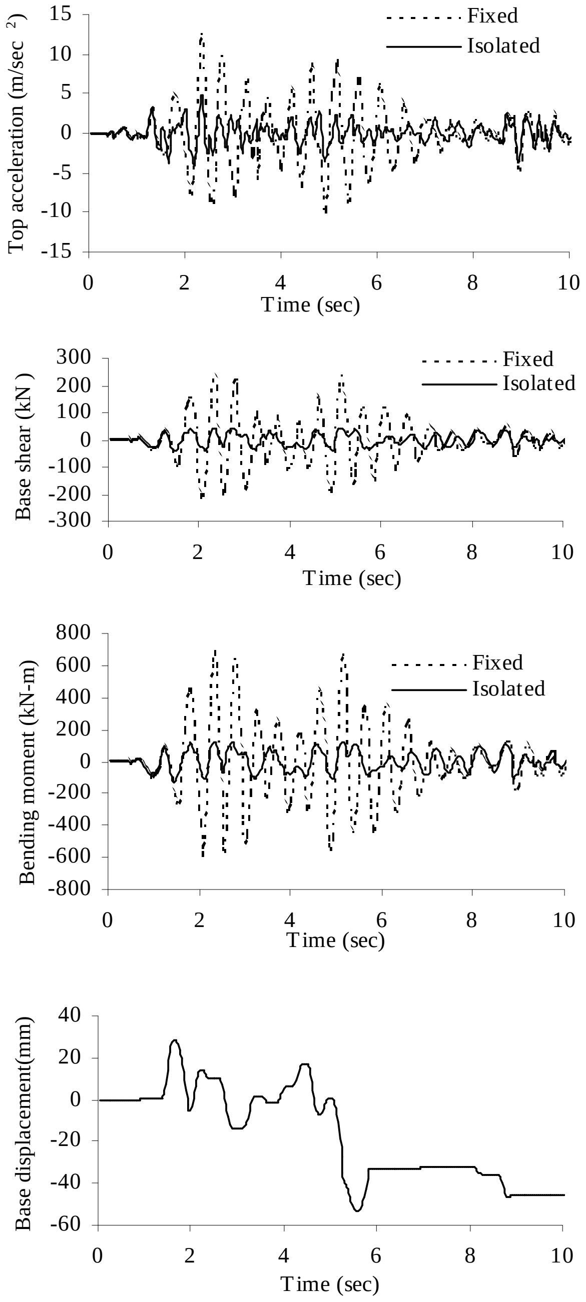


Figure 3. Response of four story space frame subjected to El Centro earthquake ground acceleration (Ts = 0.5 sec and μ = 0.08)

rad/sec) is shown in Figure 4 for various values of coefficient of friction of sliding material equal to 0.04, 0.08, 0.12, 0.16 and 0.2. The variation of response of the structure with excitation frequency for the structure fixed at base is also shown in the Figure 4. As observed from Figure 4, the acceleration, bending moment and base shear of the structure fixed at base varies with excitation frequency and shows a peak values when the frequency of excitation is equal to the natural frequency of the structure ($\omega/\omega_n = 1$) where as for the structure isolated at base the acceleration, bending moment and base shear will not vary much with variations in excitation frequency. Also, the acceleration of the structure isolated at base is higher than the acceleration of the structure fixed at base for excitation frequency less than about 7 rad/sec ($\omega/\omega_n < 0.55$). This indicates that the isolation is not effective when frequency of ground excitation is less than about half of the natural frequency of fixed base structure for the structure studied. The isolation is also not much effective when excitation frequency is more than 18 rad/sec ($\omega/\omega_n > 1.43$). The base displacement of the isolated structure increases as the excitation frequency decreases and is considerably large when excitation frequency is less than 4 rad/sec ($\omega/\omega_n = 0.318$).

3.4. Effect of coefficient of friction of base material and natural period of structure

The effect of coefficient of friction of base material and natural period of structure on response of structure is studied. The mass on beam and the sizes of column for the space frame structure corresponding to each period is shown in Figure 2. The damping matrix is obtained using Rayleigh constants α and β.

The various values of α and β obtained using the first two natural frequencies of the structure for 5 percent damping ratio is also listed in Figure 2. It is noted that the values of α will decrease whereas the values of β will increase with increase in natural period for the structure considered for the analysis. The structure is subjected to harmonic ground acceleration of intensity $2\sin(\omega t)$ m/sec^2 and the

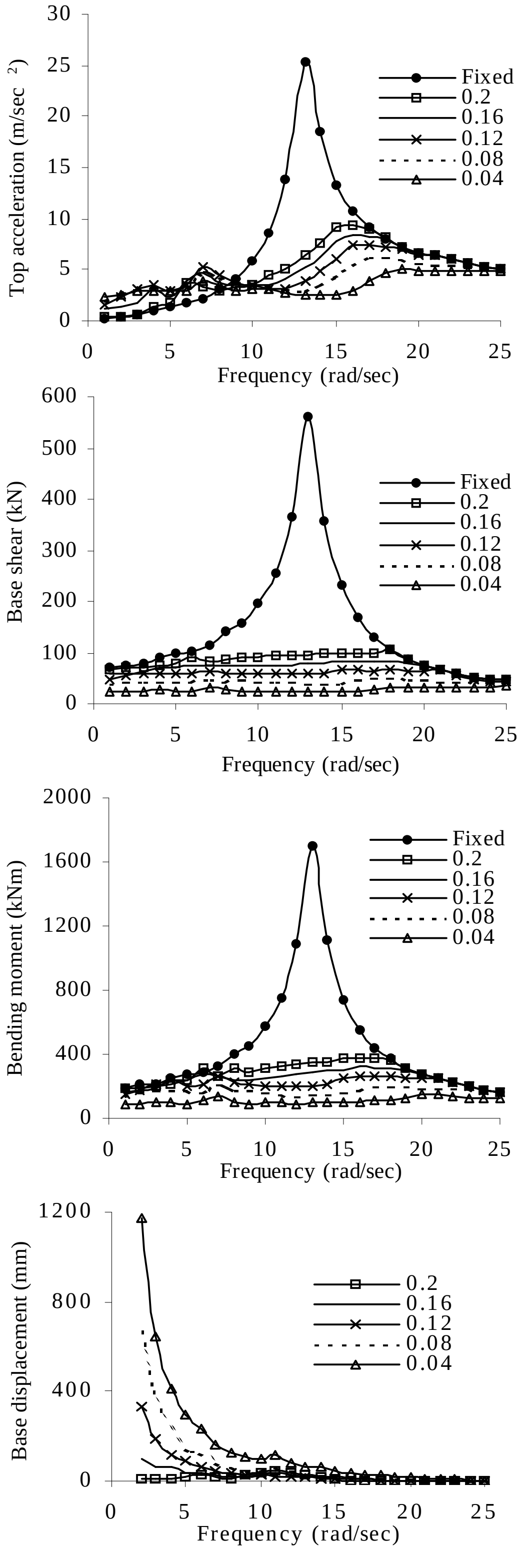


Figure 4. Variation of response with excitation frequency

maximum response of the structure is obtained for excitation frequency, ω, ranged from 2 rad/sec to 30 rad/sec. The response versus natural period plots for various coefficient of friction of base material are shown in Figure 5. However, the base displacement is obtained at excitation frequency, ω, equal to 2 rad/sec. The maximum acceleration for the fixed base structure and isolated structure remains basically constant with increase in natural period of the structure. As can be expected, the acceleration decreases with decrease in coefficient of friction of base material. The bending moment and base shear increase with increase in coefficient of friction of base material. The displacement at base increases with increase in natural period of the structure and decreases with increase in coefficient of friction of base material. Figure 6 shows the variation of response ratio with natural period for various values of coefficient of friction of base material. The response ratio is defined as the ratio of the response of the structure isolated at base to the response of the structure fixed at base. A response ratio less than 1.0 indicates that the isolation is effective. From Figure 6, it can be observed that the response ratios for acceleration and bending moment increase slightly with increase in natural period of the structure where as the base shear remains almost constant with increase in natural period of the structure. The response ratios for acceleration, bending moment and base shear also increase considerably with increase in coefficient of friction of base material. All response ratios for top displacement are more than 1.0 which indicates that the displacements of the isolated structure are larger as can be expected.

3.5. Effect of number of storeys on response of structure

The effect of number of storeys on response of the structure resting on sliding bearing is studied. To study this, structures

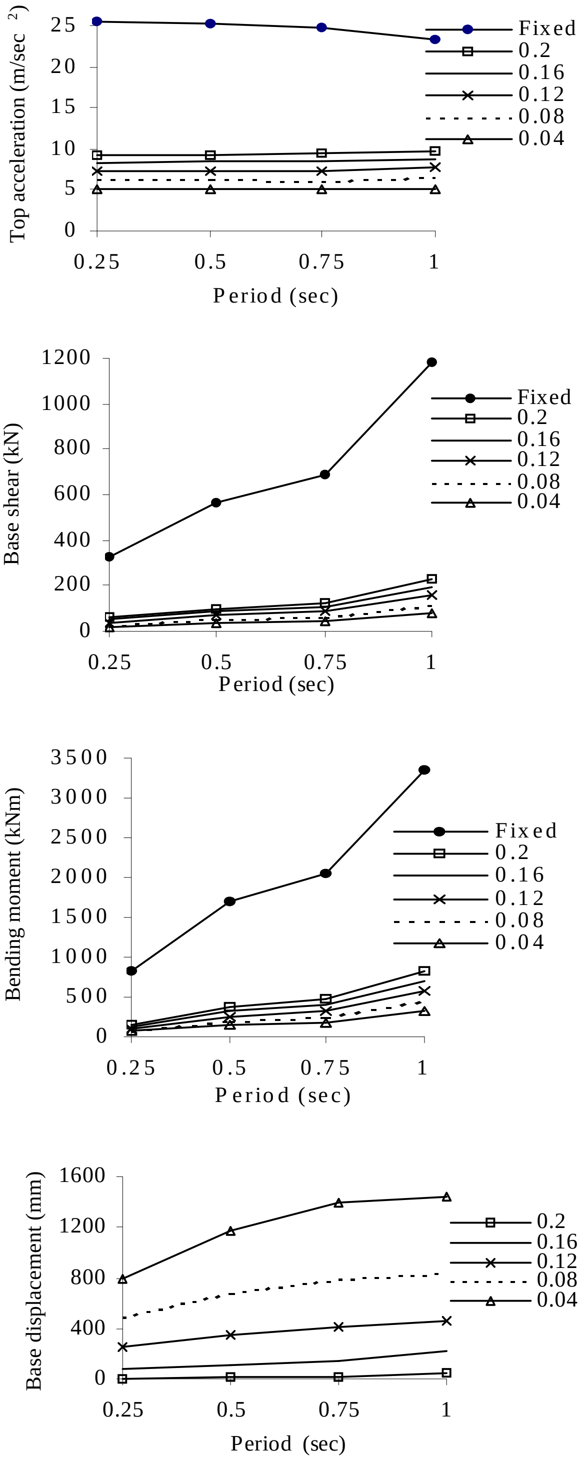


Figure 5. Variation of maximum responses with natural periods (frequency of harmonic excitations ranged from 2 rad/sec to 30 rad/sec)

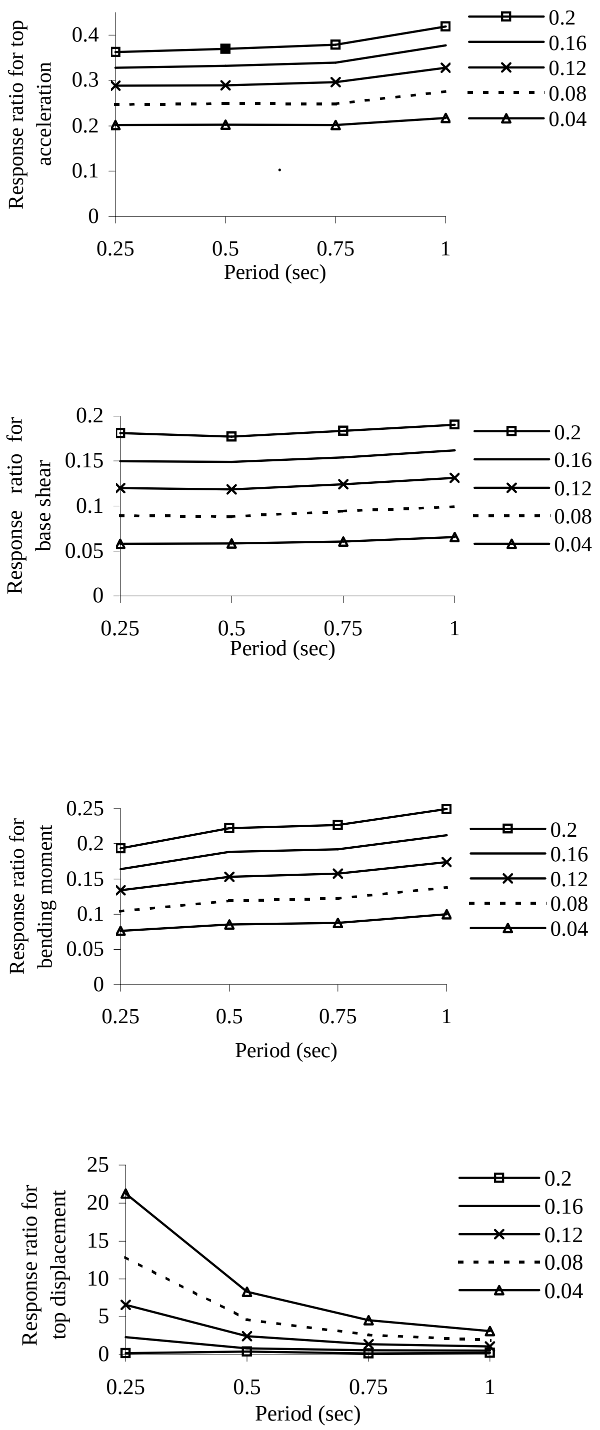


Figure 6. Variation of response ratios with natural periods (corresponding to Figure 5)

with one story, two stories, three stories and four stories are considered. All parameters except for the size of the column are kept same for all cases. (i.e. the models have the same mass on each beam but different column sizes). The mass on the beam is equal to 3 kN.sec^2/m^2 and 2 kN.sec^2/m^2 (as shown in Figure 2 for Ts = 0.5 sec) and the various sizes of the column considered for the analysis to obtain identical natural period, that is Ts equal to 0.5 sec, are 0.25 m x 0.25 m for one story, 0.33 m x 0.33 m for two story, 0.43 m x 0.43 m for three story and 0.6 m x 0.6 m for four stories. The values of Rayleigh constants (α = 0.60 and β = 0.0041) are same for all models considered for the analysis. The variation of response ratios with number of storeys for various values of coefficient of friction of base material is shown in Figure 7. As can be seen in Figure 7, the response ratios for acceleration, bending moment and base shear decreases slightly when number of story is increased from one to two and remains almost constant with further increase in number of storyes. Hence, it is concluded that the response ratios for acceleration, bending moment and base shear are not significantly affected by the number of storeys used in the FEM model. Besides, the base displacement of the structure remains constant for different number of storeys. Therefore, effect of number of storyes on responses of building type structures is relatively insignificant in relation to the most important factor that is frictional coefficient of base materials.

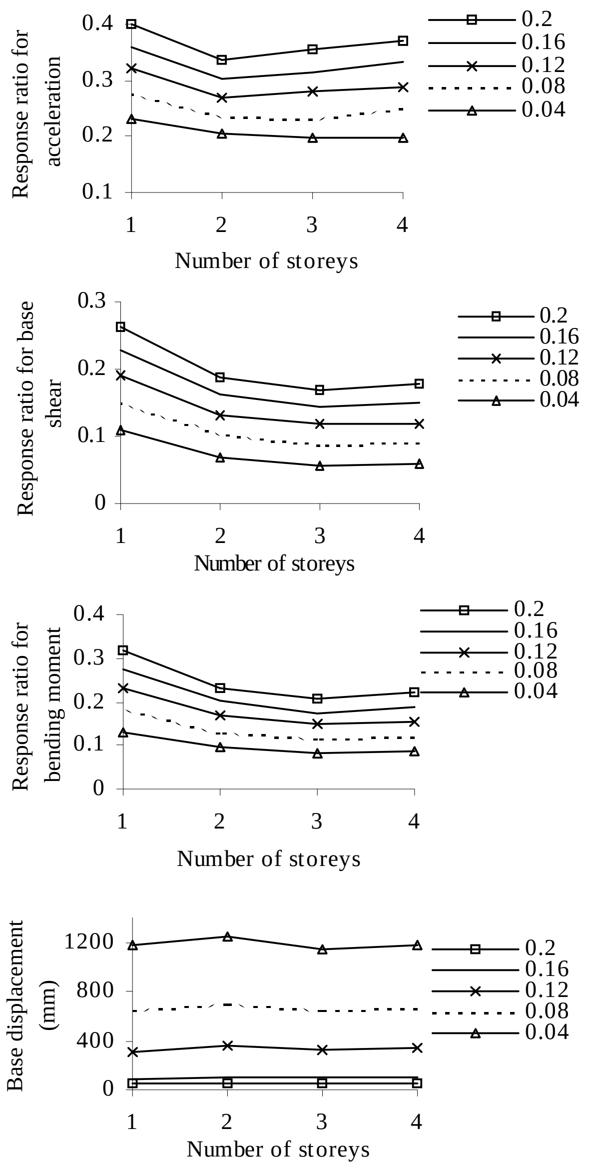


Figure 7. Variation of response ratios with number of storeys

4. Conclusions

The response of a four story space frame structure resting on sliding type of bearing is analyzed considering all the six degree of freedom at each node. The response of the structure fixed at base is compared with the response of the structure isolated at base. Based on the analysis the following conclusions are drawn.

1. The response of the structure isolated at base is considerably less than the response of the structure fixed at base.

2. The response of the structure fixed at base varies with excitation frequency and it shows a peak response when frequency of excitation is equal to the natural frequency of the structure where as for the structure isolated at base the responses do not sig-

nificantly vary with variations in excitation frequency.

3. As the coefficient of friction of base material decreases, the acceleration, bending moment and base shear of the structure decrease while the base displacement increases.
4. When comparing to the responses of base-fixed structure, response ratios of the maximum top accelerations for base – isolated structures are about 2μ for relatively large frictional coefficients (μ = 0.2) and can be as high as 5μ for very low frictional coefficients (μ = 0.04). The response ratios for the maximum base forces can also be reduced to less than 1.2μ for high values of μ and would be higher for smaller values of μ, as high as 2μ for μ equal to 0.04.
5. The maximum displacement of base isolated structures increases with increase in natural period of the structure. However, when expressed in terms of response ratios, the magnification of the maximum top displacement due to base isolation would be decreased with increasing in the natural period of structures.
6. Generally speaking, response ratios for acceleration and bending moment increase slightly whereas base shear remains almost constant with increases in natural period of the structure. The response ratio for displacement decreases with increase in natural period of the structure.
7. The response ratios for acceleration, bending moment and base shear are not significantly affected by the number of storeys used in the model. Besides, the response ratio for base displacement of the structure remains constant for different number of storeys.

Acknowledgements

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