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Fuzzy Reliability of a Marine Power Plant Using Interval Valued Vague Sets

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Abstract: In conventional reliability analysis, the failure probabilities of the components of a system are treated as exact values. It is often difficult to obtain data for failure probabilities under changing environmental conditions. Hence fuzzy sets are used to analyze the fuzzy system reliability, where a fuzzy number represents the reliability of each component. Chen [14] analyzed the fuzzy system reliability using vague set theory. The values of the membership and non-membership of an element, in a vague set, are represented by a real number in [0, 1]. A specialist is always uncertain about the values of the membership and non-membership of an element in a set. Hence, it is better to represent the values of the membership and non-membership of an element in a set by intervals of possible real numbers instead of real numbers. In this paper a new method has been developed for analyzing the fuzzy system reliability of a series and parallel system using interval valued trapezoidal vague sets, where the reliability of each component of each system is represented by an interval valued trapezoidal vague set defined in the universe of discourse [0, 1].The developed method has been used to analyze the fuzzy reliability of a marine power plant. The major advantage of the proposed approach (the concept of interval valued vague sets) over the existing approaches [7, 8, 14] is that the proposed approach separates the positive and negative evidence for the membership of an element in a set. Also in the proposed approach the values of the membership and non- membership of an element in a set are intervals instead of a single real number.

Keywords: fuzzy system reliability; vague set theory; fuzzy fault tree; interval valued vague sets.

1. Introduction

One of the important engineering tasks in design and development of a technical system is reliable engineering. It is well known that the conventional reliability analysis, using the probabilities, has been found to be inadequate to handle uncertainty of failure data and modeling. To overcome this problem, the concept of fuzzy approach has been used in the evaluation of the reliability of a system. Fuzzy set theory was first introduced by Zadeh [1] in 1965. Singer [2] presented a fuzzy set approach for fault tree and reliability analysis. Cai et al. [3, 4, 5] gave a different insight by introducing the possibility assumption and fuzzy state assumption to replace the probability and binary state assumptions.

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Mon et al. [7] presented a method for the fuzzy system reliability analysis for components with different membership functions via non-linear programming techniques. Chen [8] presented a method for fuzzy system reliability analysis using fuzzy number arithmetic operations. The collection of papers by Onisawa and Kacprzyk [9] presents many other different approaches for the fuzzy reliability. Cai [11] presented an introduction to system failure engineering.

Chen [14] presented a new method for analyzing the fuzzy system reliability based on vague sets.

In this paper, the concept of vague sets is extended by idea of interval valued vague sets. Also we have introduced: some definitions related to interval valued vague set, definition of interval valued trapezoidal vague set and arithmetic operations between two interval valued trapezoidal vague sets. Further, a new method has been developed for analyzing the fuzzy system reliability of a series and parallel system using interval valued trapezoidal vague sets, where the reliability of each component of each system is represented by an interval valued trapezoidal vague set defined in the universe of discourse [0,1].The developed method has been used to analyze the fuzzy reliability of a marine power plant. The proposed method can model and analyze the fuzzy system reliability in a more flexible and intelligent manner in comparison to the method given by Chen [14].

This paper is organized as follows. Sections 2, presents some definitions. Section 3, presents the arithmetic operations between two interval valued trapezoidal vague sets. Section 4, presents the fuzzy reliability calculations of a series and parallel system using interval valued trapezoidal vague sets. Section 5, presents the case study of the system. Section 6, presents the algorithm for analyzing fuzzy reliability. Section 7 presents the assumed data. Sections 8 presents the results .The conclusions are discussed in section 9.

2. Some definitions

In this section, some definitions are reviewed and some other definitions are introduced.

2.1. Definitions reviewed

We briefly review some definitions [6, 12, 14] here.

2.1.1 Definition

An interval valued fuzzy set $\tilde{F}$ (over a basic set X) is specified by a function $T_{\tilde{F}} : X \to D([0,1])$, where $D([0,1])$ is the set of all intervals within [0, 1], i.e. for all $x \in X, T_{\tilde{F}}(x)$ is an interval [μ_1, μ_2], $0 \le \mu_1 \le \mu_2 \le 1$.

2.1.2 Definition

A vague set $\tilde{V}$, in a basic set X, is characterized by a truth membership function $t_{\tilde{V}}$, $t_{\tilde{V}} : X \to [0,1]$, and a false membership function $f_{\tilde{V}}$, $f_{\tilde{V}} : X \to [0, 1]$. If the generic element of X is denoted by 'x_i' then the lower bound on the membership grade of x_i derived from evidence for x_i is denoted by $t_{\tilde{V}}(x_i)$ and the lower bound on the negation of x_i is denoted by $f_{\tilde{V}}(x_i)$, $t_{\tilde{V}}(x_i)$ and $f_{\tilde{V}}(x_i)$ both associate a real number in the interval [0,1] with each point in X ,where $t_{\tilde{V}}(x_i) + f_{\tilde{V}}(x_i) \le 1$. A vague set $\tilde{V}$ in the universe of discourse X is shown in Fig. 1.

When X is continuous, a vague set $\tilde{V}$ can be written as

$$\tilde{V} = \int_X \left[t_{\tilde{V}}(x_i), 1 - f_{\tilde{V}}(x_i)\right] / x_i,\ x_i \in X$$

When X is discrete a vague set $\tilde{V}$ can be written as

$$\tilde{V} = \sum_{i=1}^{n} \left[t_{\tilde{V}}(x_i), 1 - f_{\tilde{V}}(x_i)\right] / x_i, x_i \in X.$$

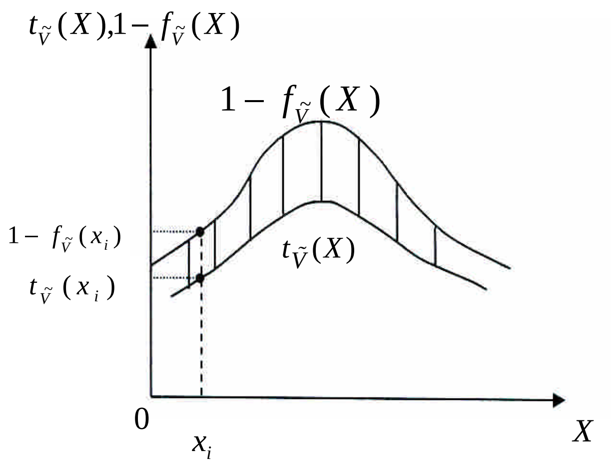


Figure 1. A vague set.

2.1.3 Definition

Let A = [a_1, a_2] and B = [b_1, b_2] be two arbitrary intervals then the minimum of A and B is represented by "MIN [A, B]" and is defined by MIN ([a_1, a_2]; [b_1, b_2]) = [min (a_1, b_1), min (a_2, b_2)]

2.1.4 Definition

The complement of an interval A = [a_1, a_2] is denoted by $\overline{A}$ and is defined by
$\overline{A}$ = [1-a_2, 1-a_1].

2.2. Definitions introduced

The definition of interval valued vague set and definitions related to interval valued vague set are introduced here.

2.2.1 Definition

An interval valued vague set $\tilde{V}$ over a basic set X is defined as an object of the form

$$\tilde{V} = <[x_i; T_{\tilde{V}}(x_i); 1-F_{\tilde{V}}(x_i)]>, x_i \in X$$

where

$T_{\tilde{V}} : X \to D[0,1]$ and $F_{\tilde{V}} : X \to D[0,1]$

are called "Truth membership function" and "False membership function" respectively and where D([0,1]) is the set of all intervals within [0, 1].

2.2.2 Definition

An interval valued vague set $\tilde{V}$ is said to be convex if and only if $\forall x_1$ and $x_2 \in X$

$$T_{\tilde{V}}\left(\lambda x_1 + (1-\lambda)x_2\right) \supseteq Min\{T_{\tilde{V}}(x_1), T_{\tilde{V}}(x_2)\}$$

$$1-F_{\tilde{V}}\left(\lambda x_1 + (1-\lambda)x_2\right) \supseteq Min\{1-F_{\tilde{V}}(x_1), 1-F_{\tilde{V}}(x_2)\}$$

where $\lambda \in [0,1]$

2.2.3 Definition

An interval valued vague number is an interval valued vague subset in the universe of discourse X that is both convex and normal.

2.2.4 Definition

An interval valued trapezoidal vague set $\tilde{V}$ (over a basic set X) can be represented by

$$\tilde{V} = < \left[(x, y, z, w); [\mu_1, \mu_2]; [\nu_1, \nu_2]\right]>$$

where $0 \le \mu_1 \le \mu_2 \le \nu_1 \le \nu_2 \le 1$

as shown in Fig. 2

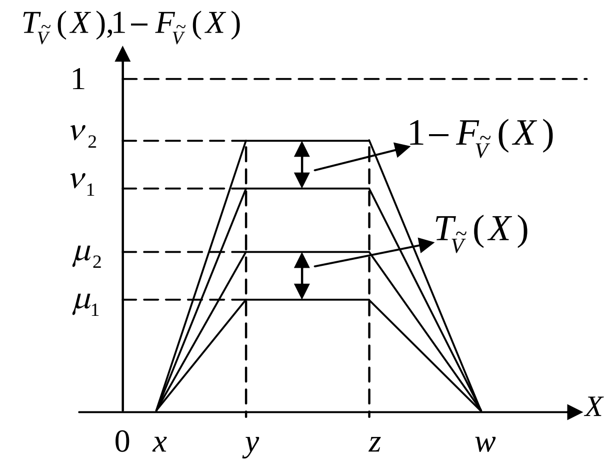


Figure 2. Interval valued trapezoidal vague set $\tilde{V}$.

3. Arithmetic operations between two interval valued trapezoidal vague sets

Certain arithmetic operations between two interval valued trapezoidal vague sets are developed here.

Let us consider two interval valued trapezoidal vague sets, $\tilde{V}_1$ and $\tilde{V}_2$, as shown in Fig. 3

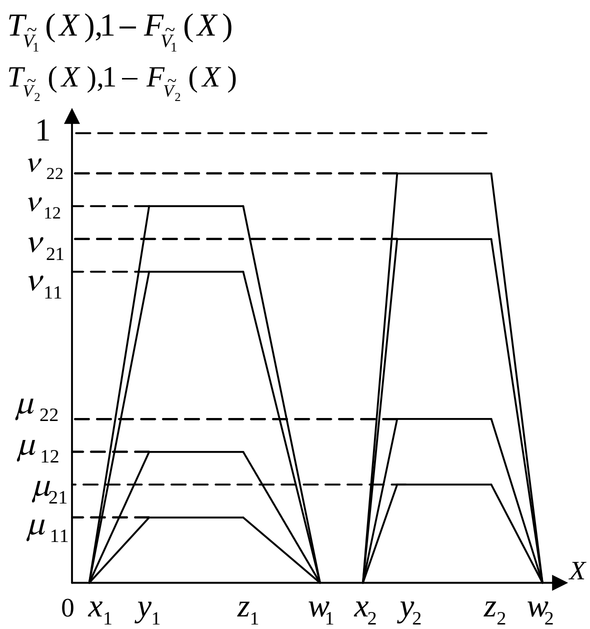


Figure 3. Interval valued trapezoidal vague sets $\tilde{V}_1$ and $\tilde{V}_2$.

defined by

$$\tilde{V}_1 = <[(x_1, y_1, z_1, w_1);[\mu_{11}, \mu_{12}];[\nu_{11}, \nu_{12}]]>$$

$$\tilde{V}_2 = <[(x_2, y_2, z_2, w_2);[\mu_{21}, \mu_{22}];[\nu_{21}, \nu_{22}]]>$$

where

$$0 \le \mu_{11} \le \mu_{21} \le \mu_{12} \le \mu_{22} \le \nu_{11} \le \nu_{21} \le \nu_{12} \le \nu_{22} \le 1$$

The arithmetic operations between $\tilde{V}_1$ and $\tilde{V}_2$ are defined as follows

$$\tilde{V}_1 \oplus \tilde{V}_2 = <\begin{bmatrix} (x_1+x_2, y_1+y_2, z_1+z_2, w_1+w_2); \\ [\min(\mu_{11}, \mu_{21}), \min(\mu_{12}, \mu_{22})]; \\ [\min(\nu_{11}, \nu_{21}), \min(\nu_{12}, \nu_{22})] \end{bmatrix}>$$

$$(\tilde{V}_1 \Theta \tilde{V}_2) = <\begin{bmatrix} (x_1-w_2, y_1-z_2, z_1-y_2, w_1-x_2); \\ [\min(\mu_{11}, \mu_{21}), \min(\mu_{12}, \mu_{22})]; \\ [\min(\nu_{11}, \nu_{21}), \min(\nu_{12}, \nu_{22})] \end{bmatrix}>$$

$$\tilde{V}_1 \otimes \tilde{V}_2 = <\begin{bmatrix} (x_1 \times x_2, y_1 \times y_2, z_1 \times z_2, w_1 \times w_2); \\ [\min(\mu_{11}, \mu_{21}), \min(\mu_{12}, \mu_{22})]; \\ [\min(\nu_{11}, \nu_{21}), \min(\nu_{12}, \nu_{22})] \end{bmatrix}>$$

$$\tilde{V}_1 \oslash \tilde{V}_2 = <\begin{bmatrix} (x_1 / w_2, y_1 / z_2, z_1 / y_2, w_1 / x_2); \\ [\min(\mu_{11}, \mu_{21}), \min(\mu_{12}, \mu_{22})]; \\ [\min(\nu_{11}, \nu_{21}), \min(\nu_{12}, \nu_{22})] \end{bmatrix}>$$

4. Fuzzy reliability calculations of a series and parallel system using interval valued trapezoidal vague sets

In this section, a new method has been developed for the fuzzy reliability calculation of a series and parallel system using interval valued trapezoidal vague sets, where the reliability of each component of each system is represented by an interval valued trapezoidal vague set defined on the universe of discourse [0, 1].

4.1. Series system

Let us consider a series system consisting of 'n' components as shown in Fig. 4. The fuzzy reliability $\tilde{R}$ of the series system shown below can be evaluated as follows:

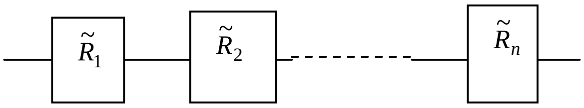


Figure 4. Series system.

$$\tilde{R} = \bigotimes_{i=1}^{n} \tilde{R}_i$$

$$=<[(\pi_{i=1}^{n} x_i, \pi_{i=1}^{n} y_i, \pi_{i=1}^{n} z_i \pi_{i=1}^{n} w_i);$$

$$[\min_{i=1}^{n}(\mu_{i1}), \min_{i=1}^{n}(\mu_{i2})];[\min_{i=1}^{n}(\nu_{i1}), \min_{i=1}^{n}(\nu_{i2})]]>$$

where

$$\tilde{R}_i =<[(x_i, y_i, z_i, w_i);[\mu_{i1}, \mu_{i2}];[\nu_{i1}, \nu_{i2}]>$$

represents the reliability of the ith component.

4.2. Parallel system

Let us consider a parallel system consisting of 'n' components as shown in Fig. 5. The fuzzy reliability $\tilde{R}$ of the parallel system can be evaluated as follows:

$$\tilde{R} = 1 \ \Theta \ \pi_{i=1}^{n}(1 \ \Theta \ \tilde{R}_i)$$

$$=<[(1-\pi_{i=1}^{n}(1-x_i), 1-\pi_{i=1}^{n}(1-y_i),$$

$$1-\pi_{i=1}^{n}(1-z_i), 1-\pi_{i=1}^{n}(1-w_i));$$

$$[\min_{i=1}^{n}(\mu_{i1}), \min_{i=1}^{n}(\mu_{i2})];$$

$$[\min_{i=1}^{n}(\mu'_{i1}), \min_{i=1}^{n}(\mu'_{i2})]]>$$

5. Case Study

A marine power plant [13] has two generators G_1 and G_2 one located at the stern and the other at bow. Each generator is connected to its respective micro switch board-1 and micro switch board-2. The distributive switch board receives the supply from the switch boards through cables C_1 and C_2 and respective junction boxes D and E .The two micro switch boards are interconnected through a long cable C_3 and the junction boxes A and B. The schematic diagram is shown in Fig. 6.

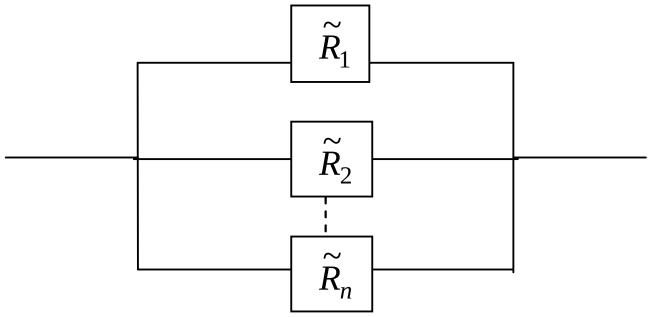


Figure 5. Parallel system.

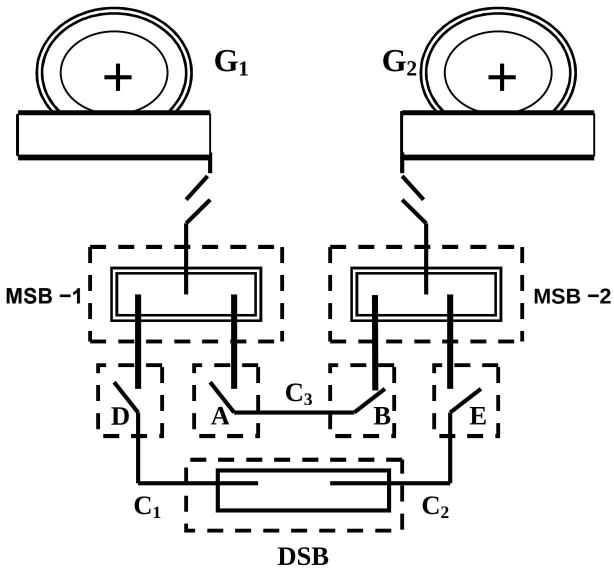


Figure 6. Marine power plant.

Let us assume that basic components subjected to failure are

(a) Generators G_1 and G_2.

(b) Micro switch board-1 (MSB-1) and Micro

switch board-2 (MSB-2).
(c) Interconnecting cable C_3 and junction boxes A and B, all are treated as one unit.
(d) Junction boxes D and E.
(e) Distributive switch board (DSB).

5.1 Notations

$\tilde{V}_1 =< [(x_1, y_1, z_1, t_1);[\mu_{11}, \mu_{12}];[\mu'_{11}, \mu'_{12}]]>,$

represents the unreliability of generator G_1.

$\tilde{V}_2 =< [(x_2, y_2, z_2, t_2);[\mu_{21}, \mu_{22}];[\mu'_{21}, \mu'_{22}]]>,$

represents the unreliability of generator G_2.

$\tilde{V}_3 =< [(x_3, y_3, z_3, t_3);[\mu_{31}, \mu_{32}];[\mu'_{31}, \mu'_{32}]]>,$

represents the unreliability of micro switch board-1.

$\tilde{V}_4 =< [(x_4, y_4, z_4, t_4);[\mu_{41}, \mu_{42}];[\mu'_{41}, \mu'_{42}]]>,$
represents the unreliability of micro switch board-2.

$\tilde{V}_5 =< [(x_5, y_5, z_5, t_5);[\mu_{51}, \mu_{52}];[\mu'_{51}, \mu'_{52}]]>,$

represents the unreliability of the junction boxes A and B.

$\tilde{V}_6 =< [(x_6, y_6, z_6, t_6);[\mu_{61}, \mu_{62}];[\mu'_{61}, \mu'_{62}]]>,$

represents the unreliability of the junction box D.

$\tilde{V}_7 =< [(x_7, y_7, z_7, t_7);[\mu_{71}, \mu_{72}];[\mu'_{71}, \mu'_{72}]]>,$

represents unreliability of the junction box E.

$\tilde{V}_8 =< [(x_8, y_8, z_8, t_8);[\mu_{81}, \mu_{82}];[\mu'_{81}, \mu'_{82}]]>,$

represents the unreliability of the distributive switch board.

$\tilde{F}_1 =< [(X_1, Y_1, Z_1, T_1);[\nu_{11}, \nu_{12}];[\nu'_{11}, \nu'_{12}]]>,$

represents the reliability of the event that there is no power supply through the junction boxes A and B.

$\tilde{F}_2 =< [(X_2, Y_2, Z_2, T_2);[\nu_{21}, \nu_{22}];[\nu'_{21}, \nu'_{22}]]>$

represents the reliability of the event that there is no power supply to micro switch board-1.

$\tilde{F}_3 =< [(X_3, Y_3, Z_3, T_3);[\nu_{31}, \nu_{32}];[\nu'_{31}, \nu'_{32}]]>$

represents the reliability of the event that there is no power supply to the junction box D.

$\tilde{F}_4 =< [(X_4, Y_4, Z_4, T_4);[\nu_{41}, \nu_{42}];[\nu'_{41}, \nu'_{42}]]>$

represents the reliability of the event that there is no power supply from the junction box D.

$\tilde{F}_5 =< [(X_5, Y_5, Z_5, T_5);[\nu_{51}, \nu_{52}];[\nu'_{51}, \nu'_{52}]]>$

represents the reliability of the event that there is no power supply through the junction boxes D and E.

$\tilde{F}_6 =< [(X_6, Y_6, Z_6, T_6);[\nu_{61}, \nu_{62}];[\nu'_{61}, \nu'_{62}]]>$

represents the reliability of the event that there is no power supply to micro switch board-2.

$\tilde{F}_7 =< [(X_7, Y_7, Z_7, T_7);[\nu_{71}, \nu_{72}];[\nu'_{71}, \nu'_{72}]]>$

represents the reliability of the event that there is no power supply to the junction box E.

$\tilde{F}_8 =< [(X_8, Y_8, Z_8, T_8);[\nu_{81}, \nu_{82}];[\nu'_{81}, \nu'_{82}]]>$

represents the reliability of the event that there is no power supply from the junction box E.

$\tilde{F}_9 =< [(X_9, Y_9, Z_9, T_9);[\nu_{91}, \nu_{92}];[\nu'_{91}, \nu'_{92}]]>$

represents the reliability of the event that no power is coming to distributive switch board.

$\tilde{F}$, represents the fuzzy unreliability of the system.

$\tilde{R}$, represents the fuzzy reliability of the system.

5.2. Fault tree

The fault tree for the marine power plant (Fig. 6) is shown in Fig. 7.

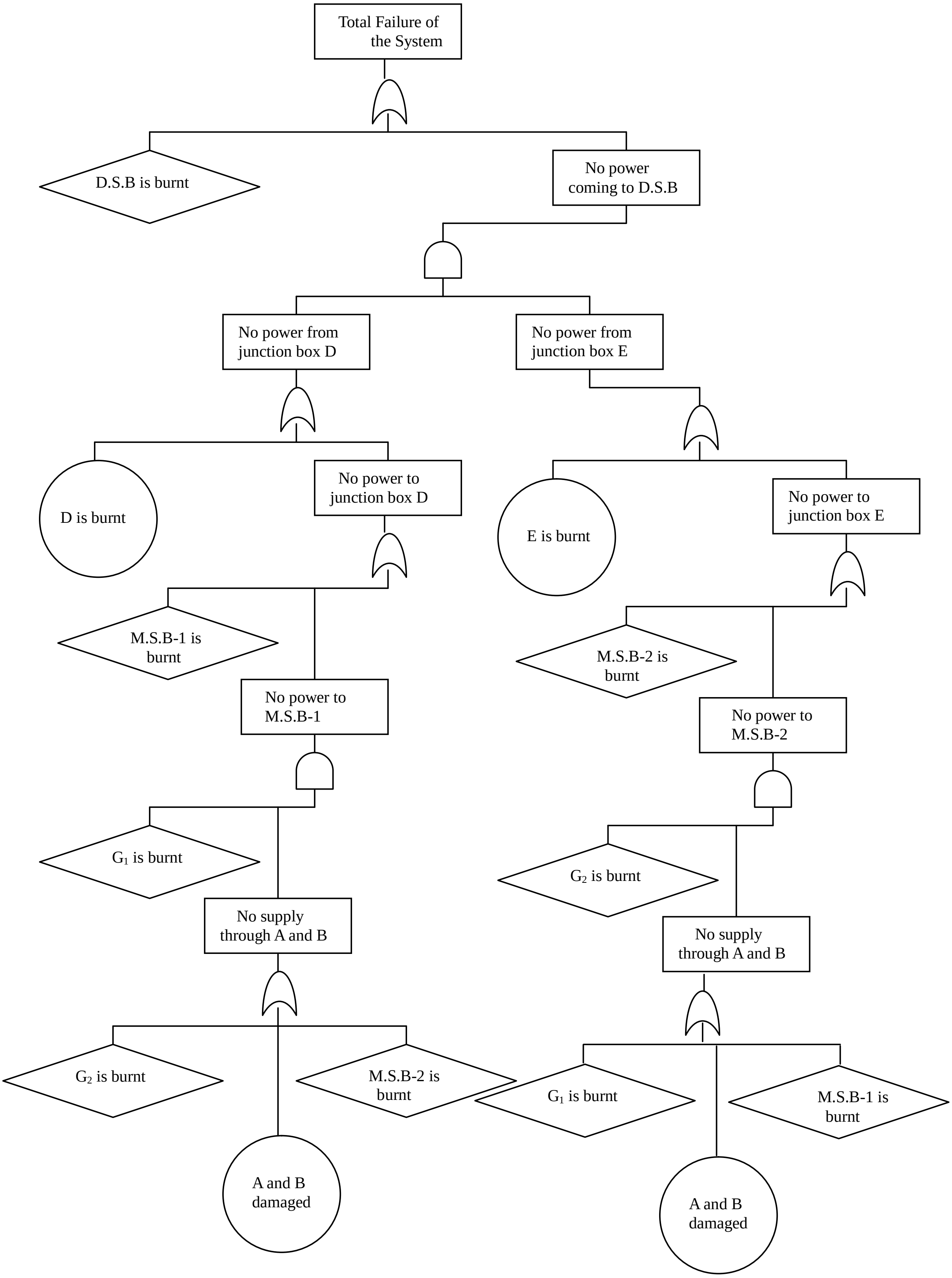


Figure 7. Fault tree of marine power plant.

5.3. Fuzzy fault tree

Using fault tree, the fuzzy fault tree (Fig. 8) is constructed for evaluating the fuzzy reliability of the above marine power plant. It is based on fuzzy logic and can handle fuzzy arithmetic with the application of fuzzy set theory.

6. Proposed algorithm

With the help of fuzzy fault tree the following algorithm is proposed for analyzing the fuzzy reliability of the above marine power plant.

Step 1

$$\tilde{F}_1 = 1\Theta(1\Theta\tilde{V}_2)\otimes(1\Theta\tilde{V}_5)\otimes(1\Theta\tilde{V}_4)$$
$$=<\left[\left(X_1,Y_1,Z_1,T_1\right);[\nu_{11},\nu_{12}];[\nu'_{11},\nu'_{12}]\right]>$$

where

$X_1 = 1-(1-x_2)(1-x_5)(1-x_4),$

$Y_1 = 1-(1-y_2)(1-y_5)(1-y_4),$

$Z_1 = 1-(1-z_2)(1-z_5)(1-z_4),$

$T_1 = 1-(1-t_2)(1-t_5)(1-t_4),$
$\nu_{11} = \min(\mu_{21},\mu_{41},\mu_{51}),$
$\nu_{12} = \min(\mu_{22},\mu_{42},\mu_{52}),$
$\nu'_{11} = \min(\mu'_{21},\mu'_{41},\mu'_{51}),$
$\nu'_{12} = \min(\mu'_{22},\mu'_{42},\mu'_{52})$

Step 2

$$\tilde{F}_2 = \tilde{V}_1 \otimes \tilde{F}_1$$
$$=<\left[\left(X_2,Y_2,Z_2,T_2\right);[\nu_{21},\nu_{22}];[\nu'_{21},\nu'_{22}]\right]>$$

where

$X_2 = x_1 \times X_1$ $,Y_2 = y_1 \times Y_1,$
$Z_2 = z_1 \times Z_1,$ $T_2 = t_1 \times T_1$

$\nu_{21} = \min(\mu_{11},\mu_{21},\mu_{41},\mu_{51}),$
$\nu_{22} = \min(\mu_{12},\mu_{22},\mu_{42},\mu_{52}),$
$\nu'_{21} = \min(\mu'_{11},\mu'_{21},\mu'_{41},\mu'_{51}),$
$\nu'_{22} = \min(\mu'_{12},\mu'_{22},\mu'_{42},\mu'_{52})$

Step 3

$$\tilde{F}_3 = 1\Theta(1\Theta\tilde{V}_3)\otimes(1\Theta\tilde{F}_2)$$

$$=<\left[\left(X_3,Y_3,Z_3,T_3\right);[\nu_{31},\nu_{32}];[\nu'_{31},\nu'_{32}]\right]>$$

where
$X_3 = 1-(1-x_3)(1-X_2),$ $Y_3 = 1-(1-y_3)(1-Y_2),$
$Z_3 = 1-(1-z_3)(1-Z_2),$ $T_3 = 1-(1-t_3)(1-T_2),$
$\nu_{31} = \min\ (\mu_{11},\mu_{21},\mu_{31},\mu_{41},\mu_{51}),$
$\nu_{32} = \min\ (\mu_{12},\mu_{22},\mu_{32},\mu_{42},\mu_{52}),$
$\nu'_{31} = \min\ (\mu'_{11},\mu'_{21},\mu'_{31},\mu'_{41},\mu'_{51}),$
$\nu_{32} = \min\ (\mu'_{12},\mu'_{22},\mu'_{32},\mu'_{42},\mu'_{52})$

Step 4

$$\tilde{F}_4 = 1\Theta(1\Theta\tilde{V}_6)\otimes(1\Theta\tilde{F}_3)$$

$$=<\left[(X_4,Y_4,Z_4,T_4);[\nu_{41},\nu_{42}];[\nu'_{41},\nu'_{42}]\right]>$$

where
$X_4 = 1-(1-x_6)(1-X_3),$
$Y_4 = 1-(1-y_6)(1-Y_3),$
$Z_4 = 1-(1-z_6)(1-Z_3),$
$T_4 = 1-(1-t_6)(1-T_3),$
$\nu_{41} = \min(\mu_{11},\mu_{21},\mu_{31},\mu_{41},\mu_{51},\mu_{61}),$
$\nu_{42} = \min(\mu_{12},\mu_{22},\mu_{32},\mu_{42},\mu_{52},\mu_{62}),$
$\nu'_{41} = \min(\mu'_{11},\mu'_{21},\mu'_{31},\mu'_{41},\mu'_{51},\mu'_{61}),$
$\nu'_{42} = \min(\mu'_{12},\mu'_{22},\mu'_{32},\mu'_{42},\mu'_{52},\mu'_{62})$

Step 5

$$\tilde{F}_5 = 1\Theta(1\Theta\tilde{V}_1)\otimes(1\Theta\tilde{V}_5)\otimes(1\Theta\tilde{V}_3)$$

$$=<\left[(X_5,Y_5,Z_5,T_5);[\nu_{51},\nu_{52}];[\nu'_{51},\nu'_{52}]\right]>$$

where

$X_5=1-(1-x_1)(1-x_5)(1-x_3)$,
$Y_5=1-(1-y_1)(1-y_5)(1-y_3)$,
$Z_5=1-(1-z_1)(1-z_5)(1-z_3)$,
$T_5=1-(1-t_1)(1-t_5)(1-t_3)$,

$\nu_{51}=\min(\mu_{11},\mu_{31},\mu_{51})$,
$\nu_{52}=\min(\mu_{12},\mu_{32},\mu_{52})$,
$\nu'_{51}=\min(\mu'_{11},\mu'_{31},\mu'_{51})$,
$\nu'_{52}=\min(\mu'_{12},\mu'_{32},\mu'_{52})$

Step 6

$$\tilde{F}_6=\tilde{V}_2\otimes\tilde{F}_5$$
$$=<\left[\left(X_6,Y_6,Z_6,T_6\right);[\nu_{61},\nu_{62}];[\nu'_{61},\nu'_{62}]\right]>$$

$X_6=x_2\times X_5,\quad Y_6=y_2\times Y_5$,
$Z_6=z_2\times Z_5,\quad T_6=t_2\times T_5$,
$\nu_{61}=\min(\mu_{11},\mu_{21},\mu_{31},\mu_{51})$,
$\nu_{62}=\min(\mu_{12},\mu_{22},\mu_{32},\mu_{52})$,
$\nu'_{61}=\min(\mu'_{11},\mu'_{21},\mu'_{31},\mu'_{51})$,
$\nu'_{62}=\min(\mu'_{12},\mu'_{22},\mu'_{32},\mu'_{52})$

Step 7

$$\tilde{F}_7=1\Theta(1\Theta\tilde{V}_4)\otimes(1\Theta\tilde{F}_6)$$
$$=<\left[\left(X_7,Y_7,Z_7,T_7\right);[\nu_{71},\nu_{72}];[\nu'_{71},\nu'_{72}]\right]>$$

where
$X_7=1-(1-x_4)(1-X_6),\quad Y_7=1-(1-y_4)(1-Y_6)$,
$Z_7=1-(1-z_4)(1-Z_6),\quad T_7=1-(1-t_4)(1-T_6)$
$\nu_{71}=\min(\mu_{11},\mu_{21},\mu_{31},\mu_{41},\mu_{51})$,
$\nu_{72}=\min(\mu_{12},\mu_{22},\mu_{32},\mu_{42},\mu_{52})$,
$\nu'_{71}=\min(\mu'_{11},\mu'_{21},\mu'_{31},\mu'_{41},\mu'_{51})$,
$\nu'_{72}=\min(\mu'_{12},\mu'_{22},\mu'_{32},\mu'_{42},\mu'_{52})$

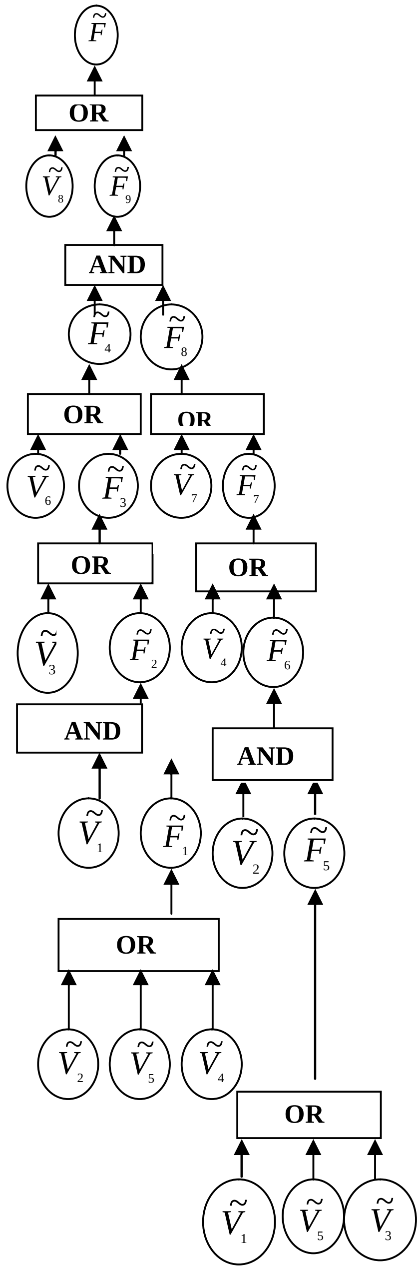


Figure 8. Fuzzy fault tree.

Step 8

$$\tilde{F}_8=1\Theta(1\Theta\tilde{V}_7)\otimes(1\Theta\tilde{F}_7)$$
$$=<\left[\left(X_8,Y_8,Z_8,T_8\right);[\nu_{81},\nu_{82}];[\nu'_{81},\nu'_{82}]\right]>$$

where

$X_8 = 1-(1-x_7)(1-X_7),\ \ Y_8 = 1-(1-y_7)(1-Y_7),$
$Z_8 = 1-(1-z_7)(1-Z_7),\ \ T_8 = 1-(1-t_7)(1-T_7)$
$v_{81} = \min(\mu_{11}, \mu_{21}, \mu_{31}, \mu_{41}, \mu_{51}, \mu_{71}),$
$v_{82} = \min(\mu_{12}, \mu_{22}, \mu_{32}, \mu_{42}, \mu_{52}, \mu_{72}),$
$v'_{81} = \min(\mu'_{11}, \mu'_{21}, \mu'_{31}, \mu'_{41}, \mu'_{51}, \mu'_{71}),$
$v'_{82} = \min(\mu'_{12}, \mu'_{22}, \mu'_{32}, \mu'_{42}, \mu'_{52}, \mu'_{72})$

Step 9

$$\tilde{F}_9 = \tilde{F}_4 \otimes \tilde{F}_8$$
$$=< \left[\left(X_9, Y_9, Z_9, T_9 \right); [v_{91}, v_{92}]; [v'_{91}, v'_{92}] \right] >$$

where
$X_9 = X_4 \times X_8,\ \ Y_9 = Y_4 \times Y_8,$
$Z_9 = Z_4 \times Z_8,\ \ T_9 = T_4 \times T_8,$
$v_{91} = \min(\mu_{11}, \mu_{21}, \mu_{31}, \mu_{41}, \mu_{51}, \mu_{61}, \mu_{71}),$
$v_{92} = \min(\mu_{12}, \mu_{22}, \mu_{32}, \mu_{42}, \mu_{52}, \mu_{62}, \mu_{72}),$
$v'_{91} = \min(\mu'_{11}, \mu'_{21}, \mu'_{31}, \mu'_{41}, \mu'_{51}, \mu'_{61}, \mu'_{71}),$
$v'_{92} = \min(\mu'_{12}, \mu'_{22}, \mu'_{32}, \mu'_{42}, \mu'_{52}, \mu'_{62}, \mu'_{72})$

Step 10

$$\tilde{F} = 1 \Theta (1 \Theta \tilde{V}_8) \otimes (1 \Theta \tilde{F}_9)$$

$$\begin{aligned} =< \Big[\big(& 1-(1-x_8)(1-X_9), 1-(1-y_8)(1-Y_9), \\ & 1-(1-z_8)(1-Z_9), 1-(1-t_8)(1-T_9) \big); \\ & [\min(\mu_{11}, \mu_{21}, \mu_{31}, \mu_{41}, \mu_{51}, \mu_{61}, \mu_{71}, \mu_{81}); \\ & \min(\mu_{12}, \mu_{22}, \mu_{32}, \mu_{42}, \mu_{52}, \mu_{62}, \mu_{72}, \mu_{82})]; \\ & [\min(\mu'_{11}, \mu'_{21}, \mu'_{31}, \mu'_{41}, \\ & \quad \mu'_{51}, \mu'_{61}, \mu'_{71}, \mu'_{81}); \\ & \min\ (\mu'_{12}, \mu'_{22}, \mu'_{32}, \mu'_{42}, \\ & \quad \mu'_{52}, \mu'_{62}, \mu'_{72}, \mu'_{82})] \Big] > \end{aligned}$$

7. Data

The following data in terms of interval valued trapezoidal vague sets are assumed for analyzing the fuzzy reliability of the above marine power plant:

$\tilde{V}_1$ =< [(0.003, 0.006, 0.013, 0.026); [0. 85, 0.92]; [0.94, 0.98]]>

$\tilde{V}_2$ =< [(0.006, 0.008, 0.015, 0.028); [0. 85, 0.89]; [0.92, 0.97]]>

$\tilde{V}_3$ =< [(0.011, 0.022, 0.033, 0.046); [0. 82, 0.91]; [0.93, 0.96]]>

$\tilde{V}_4$ =< [(0.012, 0.024,0 .040, 0.059); [0. 84, 0.91]; [0.94, 0.98]]>

$\tilde{V}_5$ =< [(0.021, 0.032, 0.053, 0.084); [0. 87, 0.92]; [0.93, 0.98]]>

$\tilde{V}_6$ =< [(0.041, 0.062, 0.093, 0.134); [0. 81, 0.90]; [0.93, 0.99]]>

$\tilde{V}_7$ =< [(0.052, 0.083, 0.140, 0.175); [0. 85, 0.90]; [0.92, 0.98]]>

$\tilde{V}_8$ =< [(0.111, 0.132, 0.163, 0.184); [0. 86, 0.92]; [0.94, 0.97]]>

8. Results

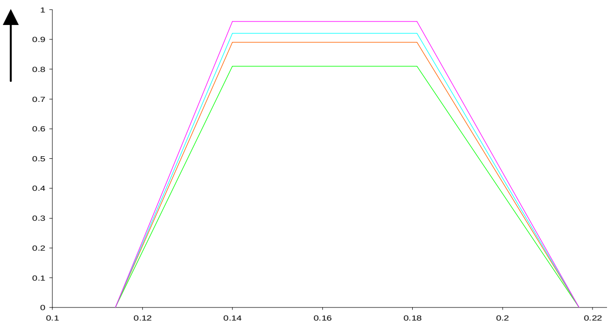
The fuzzy unreliability $\tilde{F}$ and the fuzzy reliability, $\tilde{R} = 1 \Theta \tilde{F}$, of the above marine power plant has been computed using the above data and the proposed algorithm and obtained as the following interval valued trapezoidal vague sets

$\tilde{F}$ =< [(0.114, 0.140, 0.181, 0.217); [0. 81, 0.89]; [0.92, 0.96]]>
and

$\tilde{R}$ =< [(0.783, 0.819, 0.860, 0.886); [0. 81, 0.89]; [0.92, 0.96]]>

Interval valued trapezoidal vague sets representing $\tilde{F}$ and $\tilde{R}$ are shown in Figs. 9 and 10, respectively.

$T_{\tilde{F}}(r)$,
$1 - F_{\tilde{F}}(r)$



Unreliability (r)

Lines from top to bottom:

Represents supremum value of the interval $1 - F_{\tilde{F}}(r_i)$ **for each** $r_i \in r$

Represents infimum value of the interval $1 - F_{\tilde{F}}(r_i)$ **for each** $r_i \in r$

Represents supremum value of the interval $T_{\tilde{F}}(r_i)$ **for each** $r_i \in r$

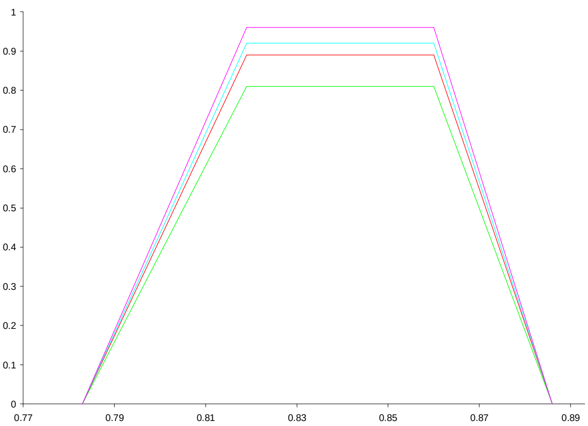
Represents infimum value of the interval $T_{\tilde{F}}(r_i)$ **for each** $r_i \in r$

Figure 9. An interval valued trapezoidal vague set representing $\tilde{F}$.

9. Conclusion

In this paper, a new method has been developed for analyzing the fuzzy system reliability of a series and parallel system using interval valued trapezoidal vague sets, where the reliability of each component of each system is represented by an interval valued trapezoidal vague set defined in the universe of discourse [0, 1]. The developed method has been used to analyze the fuzzy reliability of a marine power plant. The major advantage of using interval valued vague sets, over fuzzy sets and vague sets, is that interval valued vague sets separate the positive and negative evidence for the membership of an element in a set and also the values of membership and non membership of an element in a set are intervals instead of a single real number. Interval valued trapezoidal vague sets are used which is very effective in representing the failure possibility of the basic events under fuzzy environment.

$T_{\tilde{R}}(r)$,
$1 - F_{\tilde{R}}(r)$



Reliability (r)

Lines from top to bottom:

Represents supremum value of the interval $1 - F_{\tilde{R}}(r_i)$ **for each** $r_i \in r$

Represents infimum value of the interval $1 - F_{\tilde{R}}(r_i)$ **for each** $r_i \in r$

Represents supremum value of the interval $T_{\tilde{R}}(r_i)$ **for each** $r_i \in r$

Represents infimum value of the interval $T_{\tilde{R}}(r)$ **for each** $r_i \in r$

Figure 10. An interval valued trapezoidal vague set representing $\tilde{R}$.

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