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A New Way to Represent the Aggregate Effects of Induction Motors at a Same Bus in a Power System

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Abstract: The paper presents a new method for representing the aggregate effects of the three-phase induction motors connected to a common bus in a power system. The proposed technique has been directly developed from the standard specifications of the considered motors. Composing the motors based on these basic and important data provided by manufacturers results in a close representation of the machines. The new method aggregates the motors by generating their aggregate specifications which are very important in power system planning, control, and operation. However, such essential information is not produced by the existing induction motor aggregation methods. The proposed method has been proved to be simple and yet accurate in aggregating induction motors in power system studies. This paper describes the derivation of the proposed technique and demonstrates its verification.

Keywords: induction motor modeling; machine specifications; power system modeling.

1. Introduction

The induction motor load is a large portion of the total system load in an electric power system. Power engineers need to study the behavior of the motors in the network to ensure their stable operation and to maintain the system's stability. Normally, a power system has many small and medium sized induction motors, which can have a significant total inertia with great effects on the system damping during some disturbances [1]. The aggregate effects of the motors, therefore, can have a strong impact on the system's stability [1]. If the motors are represented dynamically, their impact on the system's performance can be much clear. Since accurately modeling the induction motors can more closely reflect the behavior of the power system, these devices need special attention when power system studies are concerned [2-3].

A bus may have a large number of small and medium sized induction motors. However, modeling these devices in the network is feasible only if they can be represented by their aggregate machines due to the computational burden. Aggregating these motors must, therefore, be undertaken to simplify the system modeling process in power system studies.

Some methods for aggregating induction motors in power system analyses have been developed in the past [2-7]. It is interesting to know that the existing techniques possess a common feature that the aggregate equivalent of a group of induction motors is derived from their circuit parameters. Based on the power ratings of the individual motors, the electrical part of the aggregate motor is represented by an equivalent circuit of which the

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circuit parameters are determined from the circuit parameters of the individual motors. In deriving the applied load of the aggregate motor, the applied load torques of the aggregate motor and the considered motors are represented by the exponential functions or the polynomials of their rotors' angular speeds. The exponential function or the polynomial of the aggregate motor are obtained from those of the considered motors according to their power ratings.

The equivalent circuit parameters of an induction motor may be determined from its detailed design data; however, such information is not accessible to most users. On the other hand, the standard specifications of the induction motor provided by the manufacturers are usually available. Since the standard specifications of induction motors closely reflect their behavior, these data can be used to derive the aggregate effects of the machines. The specifications of the aggregate motor are very important in power system planning, control, operation, and analyses. However, such valuable information is not generated by the existing induction motor aggregation methods. This paper focuses on the aggregation of the three-phase induction motors connected to a common bus in a power system based on their standard specifications. In the process of aggregating the motors, the specifications of the aggregate motor are first derived from those of the considered motors. The applied load of the aggregate motor is then obtained from the applied loads of the individual motors.

2. Preliminary consideration

Since the proposed method has been directly developed from the standard specifications of the induction motors connected to a common bus in a power system, it is important to precisely identify these machine data. Some assumptions have been made in the derivation of the proposed method. In order to make the derivation clear, it is also essential to precisely state such assumptions.

2.1. Standard specifications of an induction motor

The standard specifications of an induction motor determine its performance. These important specifications include

(1). the power system frequency (f_{oi});
(2). the rated terminal voltage (V_{oi});
(3). the rated output real power (P_{oi});
(4). the rated input current (I_{oi});
(5). the rated power factor (pf_{oi});
(6). the rated efficiency (η_{oi});
(7). the locked-rotor input current (I_{lroi});
(8). the locked-rotor electrical torque (T_{lroi});
(9). the break-down electrical torque (T_{bdoi});
(10). the rated rotor speed (ω_{moi});
(11). the angular moment of inertia of the rotor (J_i);
(12). the number of poles (N_i).

2.2. Assumptions made in the derivation of the method

Clearly stating the assumptions makes it easier to understand the derivation of the proposed method. These assumptions are summarized as follows:

(1). the input real power of the aggregate motor is equal to the total input real power of the considered motors;
(2). the input reactive power of the aggregate motor is equal to the total input reactive power of the considered motors;
(3). the air gap power of the aggregate motor is equal to the total air gap power of the considered motors;
(4). the real power loss in the rotor winding resistance of the aggregate motor is equal to the total real power loss in the rotor winding resistances of the considered motors;
(5). the output real power of the aggregate motor is equal to the total output real power of the considered motors;

(6). the applied load of the aggregate motor is equal to the total applied load of the considered motors;
(7). the kinetic energy stored in the aggregate motor is equal to the total kinetic energy stored in the considered motors;
(8). the number of poles of the aggregate motor is equal to the number of poles of the largest motor among the considered motors;
(9). the motors are operating at their rated terminal voltages and the power system frequency;
(10). the motors are not suffered from magnetic saturation.

3. Derivation of the proposed method

An induction motor experiences an electrical excitation and an applied load during its operation. The derivation of the proposed method, therefore, includes the generation of the specifications and the applied load characteristics of the aggregate motor from the data of the considered motors.

3.1. Derivation of the specifications of the aggregate motor

In order to simplify the representation of a group of three-phase induction motors connected to a common bus in a power system, they can be composed by an aggregate motor as shown in Figure 1. The power conservation is assumed to hold. Therefore, the apparent power absorbed by the aggregate motor is equal to the total apparent power absorbed by the considered motors, resulting in

$$\vec{S}_a = \sum_{i=1}^{n} \vec{S}_{bi} \tag{1}$$

where

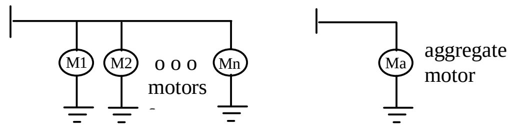
$$\vec{S}_a = \sum_{k1=0}^{2} A\left[V_o \angle\left(\theta_V - k_1\frac{2\pi}{3}\right)\right]$$

$$A = \left[I_o \angle\left(\theta_I - k_1\frac{2\pi}{3}\right)\right]^* ;$$

$$\vec{S}_{bi} = \sum_{k2=0}^{2} B\left[V_{oi} \angle\left(\theta_{Vi} - k_2\frac{2\pi}{3}\right)\right]$$

$$B = \left[I_{oi} \angle\left(\theta_{Ii} - k_2\frac{2\pi}{3}\right)\right]^* ;$$

S_a and S_{bi} are the amplitudes of the apparent power absorbed by the aggregate motor and the i-th motor, respectively; V_o and I_o are the magnitudes of the terminal voltage and the input current of the aggregate motor; θ_v and θ_I are the phase angles of the terminal voltage and the input current of the aggregate motor; V_{oi} and I_{oi} are the amplitudes of the terminal voltage and the input current of the i-th motor; θ_{vi} and θ_{Ii} are the phase angles of the terminal voltage and the input current of the i-th motor; n is the number of the motors connected to the common bus.



(a) a group of motors (b) their aggregate motor

Figure 1. Induction motors at a same bus and their aggregate representation.

Since the terminal voltages of the aggregate motor and the i-th motor are equal to the bus voltage, Eq. (1) becomes

$$I_o \cos\theta_o - jI_o \sin\theta_o = a + b \tag{2}$$

where

$$a = \sum_{i=1}^{n} I_{oi}\cos\theta_{oi} ;$$

$$b = -j\sum_{i=1}^{n} I_{oi}\sin\theta_{oi} ;$$

$$\theta_o = \theta_V - \theta_I ;$$

$$\theta_{oi} = \theta_{Vi} - \theta_{Ii} ;$$

osθ_o and cosθ_{oi} are the power factors of the aggregate and the i-th motors, respectively.

Equating the real parts and the imaginary parts on both sides of Eq. (2) gives

$$I_o \cos\theta_o = \sum_{i=1}^{n} I_{oi} \cos\theta_{oi} \quad (3)$$

$$I_o \sin\theta_o = \sum_{i=1}^{n} I_{oi} \sin\theta_{oi} \quad (4)$$

From Eqs. (3) and (4) the input current of the aggregate motor can be calculated as

$$I_o = \sqrt{\left(\sum_{i=1}^{n} I_{oi} \cos\theta_{oi}\right)^2 + \left(\sum_{i=1}^{n} I_{oi} \sin\theta_{oi}\right)^2} \quad (5)$$

Similarly, from Eqs. (3) and (5) the power factor of the aggregate motor can be derived as

$$\cos\theta_o = \frac{\sum_{i=1}^{n} I_{oi} \cos\theta_{oi}}{I_o} \quad (6)$$

The output real power of the aggregate motor equals the total output real power of the considered motors, resulting in

$$3\eta_o V_o I_o \cos\theta_o = 3\sum_{i=1}^{n} \eta_{oi} V_{oi} I_{oi} \cos\theta_{oi} \quad (7)$$

where η_o is the efficiency of the aggregate motor and η_{oi} is the rated efficiency of the i-th motor.

Since the terminal voltages of the aggregate motor and the i-th motor are equal to the bus voltage, from Eqs. (3) and (7) the efficiency of the aggregate motor is given by

$$\eta_o = \frac{\sum_{i=1}^{n} \eta_{oi} I_{oi} \cos\theta_{oi}}{\sum_{i=1}^{n} I_{oi} \cos\theta_{oi}} \quad (8)$$

The slip s_o of the aggregate motor and the rated slip s_{oi} of the i-th motor under the rated operating conditions are given by

$$s_o = \frac{\omega_o - \frac{N\omega_{mo}}{2}}{\omega_o} \quad (9)$$

$$s_{oi} = \frac{\omega_o - \frac{N_i \omega_{moi}}{2}}{\omega_o} \quad (10)$$

where ω_o is the angular frequency of the power system; ω_{mo} is the angular speed of the shaft of the aggregate motor; N is the number of poles of the aggregate motor; ω_{moi} is the rated angular speed of the shaft of the i-th motor; N_i is the number of poles of the i-th motor.

The air gap power of the aggregate motor is equal to the total air gap power of the considered motors. Also, the real power loss in the rotor winding resistance of the aggregate motor must equal the total real power loss in the rotor winding resistances of the individual motors. Thus the following relations hold:

$$\frac{\omega_o T_o}{\frac{N}{2}} = \sum_{i=1}^{n} \frac{\omega_o T_{oi}}{\frac{N_i}{2}} \quad (11)$$

$$\frac{s_o \omega_o T_o}{\frac{N}{2}} = \sum_{i=1}^{n} \frac{s_{oi} \omega_o T_{oi}}{\frac{N_i}{2}} \quad (12)$$

where T_o is the electrical torque of the aggregate motor; $\omega_o T_o/(N/2)$ and $s_o\omega_o T_o/(N/2)$ are the air gap power and the power loss in the rotor winding resistance of the aggregate motor, respectively; T_{oi} is the rated electrical torque of the i-th motor; $\omega_o T_{oi}/(N_i/2)$ and s_{oi}-$\omega_o T_{oi}/(N_i/2)$ are the air gap power and the power loss in the rotor winding resistance of the i-th motor, respectively.

Therefore, from Eqs. (11) and (12) the electrical torque and the rated slip of the aggregate motor can be established as

$$T_o = N\sum_{i=1}^{n} \frac{T_{oi}}{N_i} \quad (13)$$

$$s_o = \frac{\sum_{i=1}^{n} \frac{s_{oi} T_{oi}}{N_i}}{\sum_{i=1}^{n} \frac{T_{oi}}{N_i}} \tag{14}$$

When the rotor of a motor is locked, its slip becomes unity and its real input power is equal to the sum of its core loss and copper loss. Consequently, if the rotors are locked, the air gap power of the aggregate motor and that of the i-th motor become

$$P_{lro} = \frac{\omega_o T_{lro}}{\frac{N}{2}} \tag{15}$$

$$P_{lroi} = \frac{\omega_o T_{lroi}}{\frac{N_i}{2}} \tag{16}$$

where T_{lro} and T_{lroi} are the locked-rotor torques of the aggregate motor and the i-th motor, respectively.

Since the air gap power of the aggregate motor is equal to the total air gap power of the considered motors, combining Eqs. (15) and (16) gives the locked-rotor torque of the aggregate motor as

$$T_{lro} = N \sum_{i=1}^{n} \frac{T_{lroi}}{N_i} \tag{17}$$

Similarly, when the motors develop their break-down torques, the air gap power of the aggregate motor and that of the i-th motor are given by

$$P_{bdo} = \frac{\omega_o T_{bdo}}{\frac{N}{2}} \tag{18}$$

$$P_{bdoi} = \frac{\omega_o T_{bdoi}}{\frac{N_i}{2}} \tag{19}$$

where T_{bdo} is the break-down torque of the aggregate motor and T_{bdoi} is the break-down torque of the i-th motor.

The air gap power of the aggregate motor is equal to the total air gap power of the considered motors when they develop the break-down torques. From Eqs. (18) and (19) the aggregate motor's break-down torque is established as

$$T_{bdo} = N \sum_{i=1}^{n} \frac{T_{bdoi}}{N_i} \tag{20}$$

The kinetic energy stored in the aggregate motor and that stored in the i-th motor under the rated operating conditions are given by

$$W_k = \frac{J \omega_{mo}^2}{2} \tag{21}$$

$$W_{ki} = \frac{J_i \omega_{moi}^2}{2} \tag{22}$$

where W_k is the kinetic energy stored in the aggregate motor; J is the rotor's moment of inertia of the aggregate motor; W_{ki} is the kinetic energy stored in the i-th motor; J_i is the rotor's moment of inertia of the i-th motor.

The kinetic energy stored in the aggregate motor must be the total kinetic energy stored in the considered motors. From Eqs. (9), (10), (21), and (22) the rotor's moment of inertia of the aggregate motor under the rated condition can be established as

$$J = \frac{\sum_{i=1}^{n} J_i \left(\frac{1 - s_{oi}}{N_i} \right)^2}{\left(\frac{1 - s_o}{N} \right)^2} \tag{23}$$

The developed electrical torque of the aggregate motor at its rated slip can be calculated as

$$T_o = \frac{3 I_{ro}^2 R_r}{\omega_o s_o} \tag{24}$$

where T_o is the rated torque; I_{ro} is the per-phase current through the rotor winding under the rated condition; R_r is the per-phase resultant resistance of the rotor windings; s_o is the rated slip.

Similarly, when the rotor of the aggregate motor is locked, its slip becomes unity and its locked-rotor torque becomes

$$T_{lro} = \frac{3 I_{lro}^2 R_r}{\omega_o} \quad (25)$$

where T_{lro} is the locked-rotor torque; I_{lro} is the per-phase current through the rotor winding when the rotor is locked.

From Eqs. (24) and (25) we can have the following ratio of the locked-rotor torque to the rated torque of the aggregate motor:

$$\frac{T_{lro}}{T_o} = \frac{I_{lro}^2}{I_{lr}^2} s_o \quad (26)$$

From Eq. (26) the per-phase current through the rotor windings of the aggregate motor at its rated slip s_o is

$$I_{ro} = \sqrt{\frac{s_o T_o I_{lro}^2}{T_{lro}}} \quad (27)$$

The developed electrical torque of the i-th motor at its rated slip can be calculated as

$$T_{oi} = \frac{3 I_{roi}^2 R_{ri}}{\omega_o s_{oi}} \quad (28)$$

where T_{oi} is the rated torque; I_{roi} is the per-phase current through the rotor winding under the rated condition; R_{ri} is the per-phase resultant resistance of the rotor windings; s_{oi} is the rated slip.

Similarly, when the rotor of the i-th motor is locked, its slip becomes unity and its locked-rotor torque is given by

$$T_{lroi} = \frac{3 I_{lroi}^2 R_{ri}}{\omega_o} \quad (29)$$

where T_{lroi} is the locked-rotor torque; I_{lroi} is the per-phase current through the rotor windings when the rotor is locked.

From Eqs. (28) and (29) we can have the following ratio of the locked-rotor torque to the rated torque of the i-th motor:

$$\frac{T_{lroi}}{T_{oi}} = \frac{I_{lroi}^2}{I_{lro}^2} s_{oi} \quad (30)$$

From Eq. (30) the per-phase current through the rotor windings of the i-th motor at its rated slip s_{oi} becomes

$$I_{roi} = \sqrt{\frac{s_{oi} T_{oi} I_{lroi}^2}{T_{lroi}}} \quad (31)$$

The per-phase current through the rotor windings of the aggregate motor must be equal to the total per-phase current through the rotor windings of the considered motors. Thus from Eqs. (27) and (31) we can derive the locked-rotor input current of the aggregate motor as

$$I_{lro} = \sqrt{\frac{T_{lro}}{s_o T_o}} \sqrt{\sum_{i=1}^{n} \frac{s_{oi} T_{oi} I_{lroi}^2}{T_{lroi}}} \quad (32)$$

3.2. Generation of torque-speed relation

Since the torque-speed curve of an induction motor reflects its behavior, it is desirable to generate such important machine characteristics in motor aggregation. For the i-th motor, the following relation holds:

$$\frac{T_{bdoi}}{T_{oi}} \approx \frac{s_{bdoi}^2 + s_{oi}^2}{2 s_{bdoi} s_{oi}} \quad (33)$$

where s_{bdoi} is the slip of the i-th motor when it develops the break-down torque T_{bdoi}.

From Eq. (33) the slip s_{bdoi} can be calculated as

$$s_{bdoi} = \left(\frac{T_{bdoi} + \sqrt{T_{bdoi}^2 - T_{oi}^2}}{T_{oi}} \right) s_{oi} \quad (34)$$

Similarly, at any slip s_i of the i-th motor the following relation holds:

$$\frac{T_{bdoi}}{T_i} \approx \frac{s_{bdoi}^2 + s_i^2}{2 s_{bdoi} s_i} \tag{35}$$

From Eqs. (34) and (35) the slip s_i can be derived as

$$s_i = A\left(\frac{T_{bdoi} + \sqrt{T_{bdoi}^2 - T_{oi}^2}}{T_{oi}}\right) s_{oi} \tag{36}$$

where

$$A = \left(\frac{T_{bdoi} + \sqrt{T_{bdoi}^2 - T_{oi}^2}}{T_i}\right).$$

Since T_{bdoi}, T_{oi}, and s_{oi} are known, the torque-speed characteristic of the i-th motor can be generated with Eq. (36), which closely describes the relationship between the torque and speed of the motor during operation.

3.3. Slip and moment of inertia in any operating state

The slip of the aggregate motor which is derived in Eq. (14) is the slip under the rated operating condition. However, the considered motors may be operating under other conditions as well. In order to closely represent the behavior of the motors, the slip of the aggregate motor in other operating states must be derived. To calculate the slip of the aggregate motor in any operating state, the torque-speed relation of the considered motors must be first generated with Eq. (36).

s_i of the i-th motor is a function of its electrical torque T_i which can be obtained from its generated torque-speed relation. Consequently, the slip s of the aggregate motor under any operating condition can be obtained by replacing s_{oi} with s_i and T_{oi} with T_i in Eq. (14) as

$$s = \frac{\sum_{i=1}^{n} \frac{s_i T_i}{N_i}}{\sum_{i=1}^{n} \frac{T_i}{N_i}} \tag{37}$$

Similarly, the moment of inertia of the aggregate motor given in Eq. (23) is under the rated condition. In order to closely represent the considered motors, the moment of inertia of the aggregate motor in any operating state must be derived. Since the slip of the aggregate motor under any operating condition is given in Eq. (37), its moment of inertia which is a function of slip can obtained by replacing s_{oi} with s_i and s_o with s in Eq. (23) as

$$J = \frac{\sum_{i=1}^{n} J_i \left(\frac{1-s_i}{N_i}\right)^2}{\left(\frac{1-s}{N}\right)^2} \tag{38}$$

3.4. Derivation of the aggregate applied load

The applied load of the aggregate motor and that of the i-th motor are assumed to have the following expressions:

$$T_m = a A^2 + bA + c \tag{39}$$

$$T_{mi} = a_i A_i^2 + b_i A_i + c_i \tag{40}$$

where

$$A = \frac{2\omega_o (1 - s_o)}{N};$$

$$A_i = \frac{2\omega_o (1 - s_{oi})}{N_i};$$

a, b, c, a_i, b_i, and c_i are coefficients; T_m is the applied load toque of the aggregate motor; T_{mi} is the applied load toque of the i-th motor.

The load applied to the aggregate motor must be the total load applied to the considered motors, resulting in

$$T_m = \sum_{i=1}^{n} T_{mi} \tag{41}$$

Equating the coefficients of ω_o^2, ω_o^1 (it is equal to ω_o), and ω_o^0 (it is equal to 1) on both sides of Eq. (41) gives

$$a = \sum_{i=1}^{n} \frac{N^2 a_i (1 - s_{oi})^2}{N_i^2 (1 - s_o)^2} \tag{42}$$

$$b = \sum_{i=1}^{n} \frac{N b_i (1 - s_{oi})}{N_i (1 - s_o)} \tag{43}$$

$$c = \sum_{i=1}^{n} c_i \tag{44}$$

4. Case simulation

In order to verify the proposed method, a series of simulation tests have been conducted. One of these tests on three-phase systems is presented here. The single-line diagram of the network is shown in Figure 2(a) with the data given in the Appendix [8]. The conditions for motor tripping states were not considered in the simulation test. The motors were, therefore, connected to the common bus in the test system throughout the test.

To obtain the aggregate representation of the motors at the common bus shown in Figure 2(a), the specifications and the applied load characteristics of the aggregate motor were generated using the proposed method. In the simulation test the original network without aggregating the motors shown in Figure 2(a) was first solved for the desired system quantities. Then the motors were aggregated and the simplified network shown in Figure 2(b) was solved for the same system quantities. The system quantities obtained without aggregating the motors (WOA) and with aggregating the motors (WA) in the simulation were compared. In the simulation test, a three-phase-to-ground fault was applied at time = 0.02 second to the circuit by closing switch sw2 in each phase and the fault was cleared at time = 0.09 second. The currents of phase-a, phase-b, and phase-c are shown in Figures 3, 4, and 5. In addition, Figures 6 and 7 give the real power and the reactive power of phase-a. Similarly, Figures 8 and 9 show the real power and the reactive power of phase-b. Finally, Figures 10 and 11 give the real power and the reactive power of phase-c. The phase currents, the phase real power, and the phase reactive power obtained without aggregating the motors and by the proposed method are in good agreement.

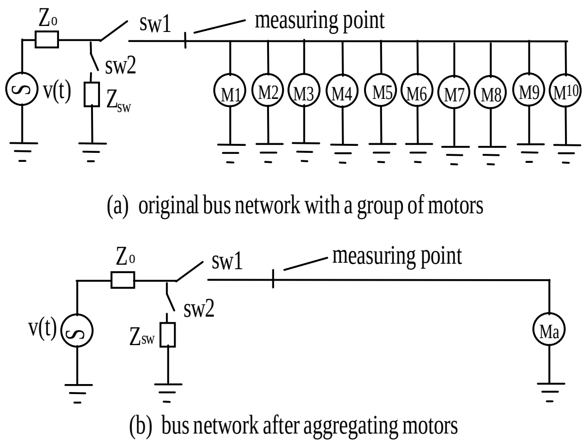


Figure 2. System used in the test.

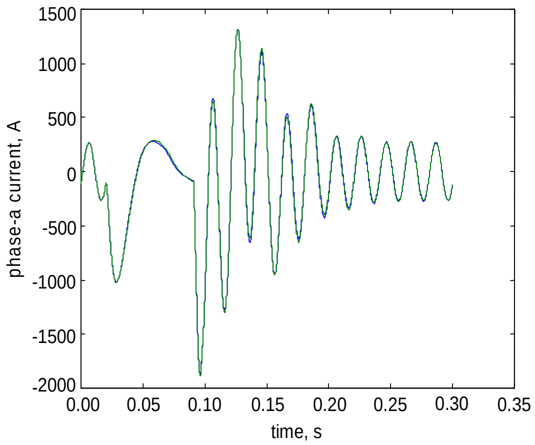


Figure 3. Phase-a current in the test.

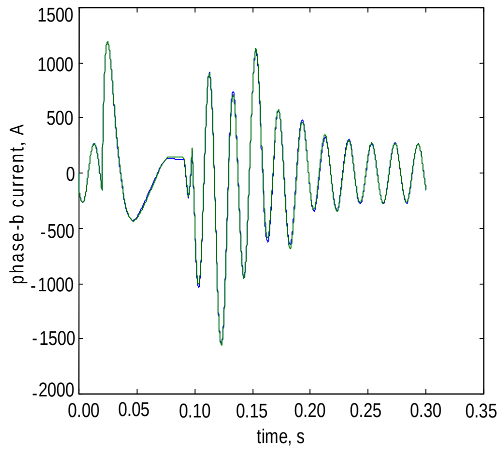


Figure 4. Phase-b current in the test.

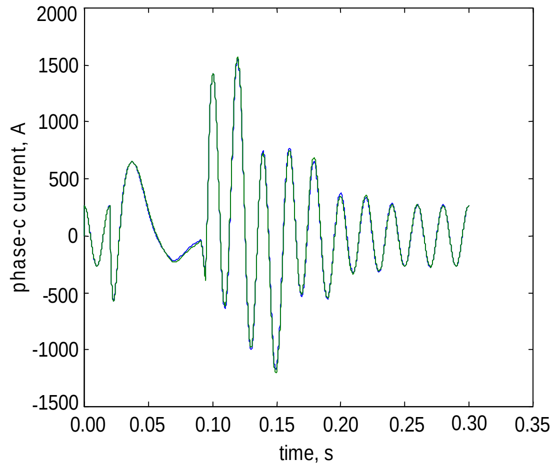


Figure 5. Phase-c current in the test.

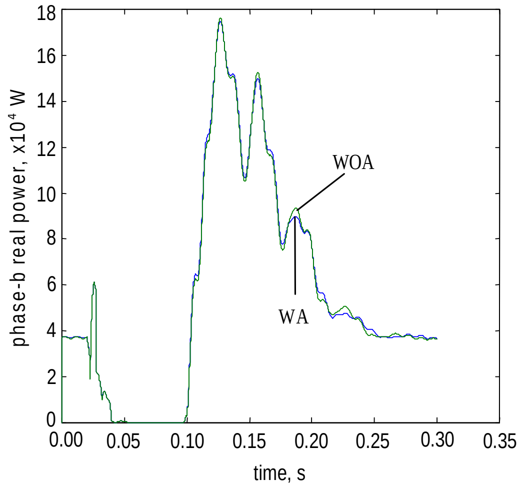


Figure 8. Phase-b real power in the test.

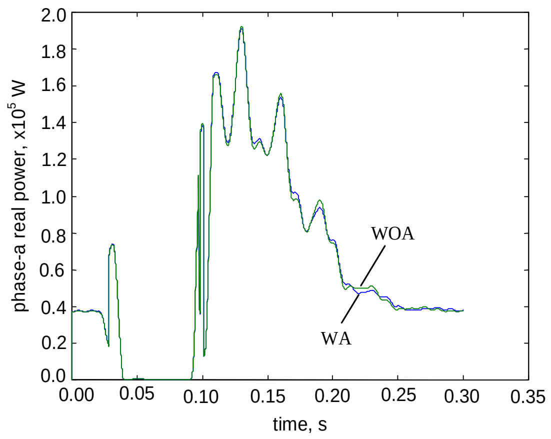


Figure 6. Phase-a real power in the test.

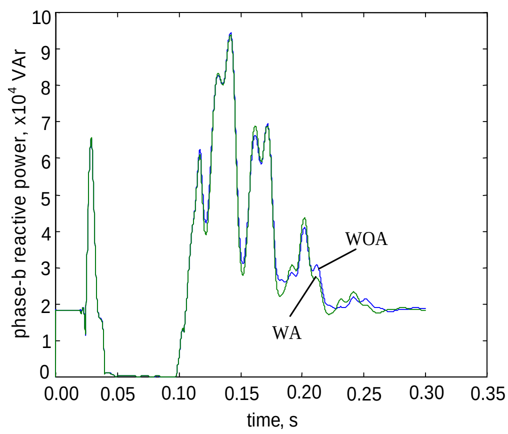


Figure 9. Phase-b reactive power in the test.

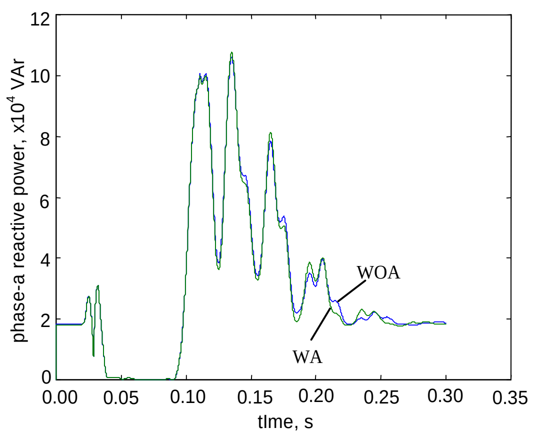


Figure 7. Phase-a reactive power in the test.

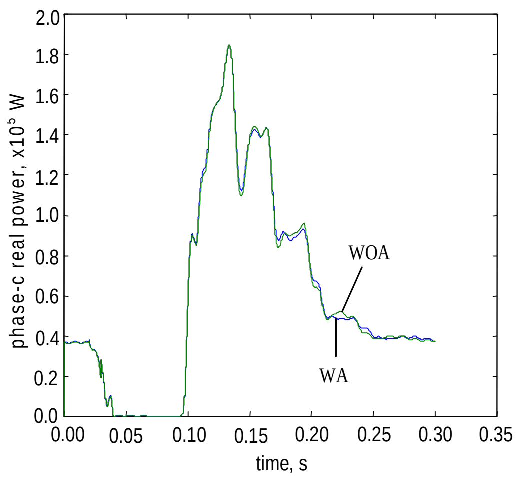


Figure 10. Phase-c real power in the test.

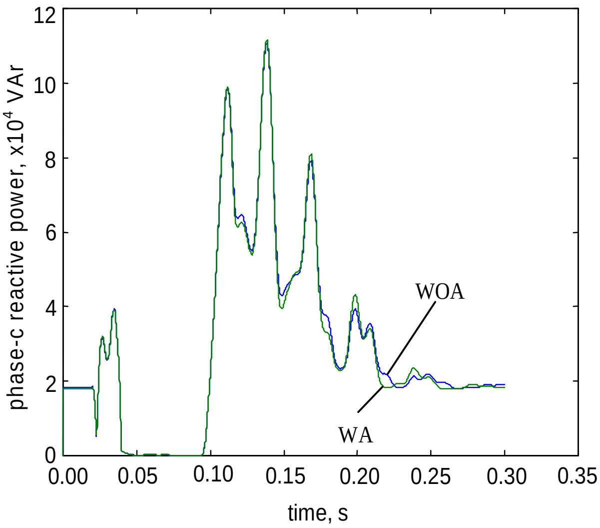


Figure 11. Phase-c reactive power in the test.

5. Discussion

The simulation test results have proved that the proposed method can be used to provide a close aggregate representation of a group of three-phase induction motors connected to a common bus in power system studies. The proposed method has been directly developed from the principle of conservation of power. The specifications of the aggregate motor, which determine the behavior of the device, are derived from those of the individual motors. The so-obtained specifications closely reflect the behavior of the aggregate motor and precisely represent the total influence of the considered motors. Consequently, the proposed method is able to provide system engineers with the most valuable information in power system design and studies. This new method is a powerful technique in power network reduction when studying large power systems.

The new method is more accurate than the existing motor aggregation methods because it directly composes motors from their standard specifications. The existing motor aggregation methods compose motors from their circuit parameters and load characteristics based on their power ratings. However, the power ratings of the motors may not truly reflect their shares of the aggregate impact on the system's performance due to the fact that many other factors may influence the aggregate effect of the machines.

Although the circuit parameters can be generated from the detailed design information of the motors, these design data normally are not accessible to most users due to the ownership of the machines or other reasons. Consequently, sometimes aggregating motors based on their circuit parameters can be very difficult or even impossible to be conducted. The best alternative a user can have is to guess the circuit parameters of some motors in system studies, resulting in inaccurate results.

Aggregating the motors successfully by the new method depends on the types of induction motors. Since similar motors have similar specifications, their aggregate effects can be closely represented. On the other hand, if some of the motors were much larger than others, accurately representing the behavior of the machines could be challenging. However, this situation can be overcome by carefully grouping the motors based on their specifications and representing the different motor groups by different aggregate motors.

6. Conclusions

The paper has presented a new method for aggregating the three-phase induction motors connected to a common bus in a power system based on their standard specifications. Composing the motors with their specifications is simple and yet gives a close aggregate representation of the devices. The proposed method provides the aggregate specifications of the motors, which are very essential in power system design, studies, and operation. The validity of the proposed method has been proved by comparing the simulation results of a test system obtained without composing the motors and by the proposed method.

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Appendix: Test data

Table 1. First set of machine specifications.

Motor	P_{oi} (HP)	T_{lroi} (p.u.)	T_{bdoi} (p.u.)
M1	6.3	1.9	2.6
M2	6.3	1.9	2.6
M3	7.5	2.0	2.6
M4	7.5	2.0	2.6
M5	8.8	2.3	2.7
M6	8.8	2.3	2.7
M7	10.0	2.6	3.0
M8	10.0	2.6	3.0
M9	12.9	2.0	2.3
M10	12.9	2.0	2.3
Ma	91.0	2.17	2.62

Table 2. Second set of machine specifications.

Motor	I_{lroi} (p.u.)	pf_{oi}	η_{oi}
M1	6.2	0.90	0.86
M2	6.2	0.90	0.86
M3	6.5	0.91	0.88
M4	6.5	0.91	0.88
M5	7.1	0.91	0.88
M6	7.1	0.91	0.88
M7	7.3	0.83	0.89
M8	7.3	0.83	0.89
M9	5.8	0.86	0.88
M10	5.8	0.86	0.88
Ma	6.51	0.88	0.88

Table 3. Third set of machine specifications.

Motor	V_{oi} (volt)	I_{oi} (A)	f_{oi} (Hz)
M1	380.0	9.0	50.0
M2	380.0	9.0	50.0
M3	380.0	10.5	50.0
M4	380.0	10.5	50.0
M5	380.0	12.3	50.0
M6	380.0	12.3	50.0
M7	380.0	15.4	50.0
M8	380.0	15.4	50.0
M9	380.0	19.1	50.0
M10	380.0	19.1	50.0
Ma	380.0	132.3	50.0

Table 4. Fourth set of machine specifications.

Motor	N_i	S_{oi}	J_i (kg.m)
M1	2	2.33	1.4e-2
M2	2	2.33	1.4e-2
M3	2	2.17	1.9e-2
M4	2	2.17	1.9e-2
M5	2	2.00	1.9e-1
M6	2	2.00	1.9e-1
M7	2	1.67	3.3e-2
M8	2	1.67	3.3e-2
M9	2	2.33	3.3e-2
M10	2	2.33	3.3e-2
Ma	2	2.09	2.38e-1

Table 5. Applied load coefficients.

Motor	Load coefficients		
M1	a1 = 0.0	b1 = 0.048	c1 = 0.0
M2	a2 = 0.0	b2 = 0.048	c2 = 0.0
M3	a3 = 0.0	b3 = 0.057	c3 = 0.0
M4	a4 = 0.0	b4 = 0.057	c4 = 0.0
M5	a5 = 0.0	b5 = 0.067	c5 = 0.0
M6	a6 = 0.0	b6 = 0.067	c6 = 0.0
M7	a7 = 0.0	b7 = 0.076	c7 = 0.0
M8	a8 = 0.0	b8 = 0.076	c8 = 0.0
M9	a9 = 0.0	b9 = 0.098	c9 = 0.0
M10	a10 = 0.0	b10 = 0.1	c10 = 0.0
Ma	a = 0.0	b = 0.692	c = 0.0