

Simulation with Fuzzy Durations

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Abstract: Traditional stochastic discrete event simulation method requires the users to fit the task duration data with an empirical probability function or a theoretical probability density function. This has limited the application of discrete event simulation techniques from research stage to construction practice due to the tedious data fitting tasks required for the construction operation planners lacking of statistical background. On the other hand, usually the activity duration estimated by practitioners contains some kind of vagueness caused by estimator's subjective judgment. This type of vagueness may be more appropriate to be modeled by fuzzy numbers that can be assessed without data fitting. This paper proposes a new mechanism where fuzzy duration modeled by fuzzy numbers can be used in discrete event simulation. Consequently, the tedious data fitting tasks are eliminated but the uncertainty of task duration is still taken into account.

Keywords: fuzzy duration, discrete event simulation

1. Introduction

Discrete event simulation has gained its reputation in analyzing construction operations for several decades. Simulation experiments involve two tasks: building models representing the operations which are going to be analyzed and deciding durations of activities (i.e. time- and resource-consuming elements of a construction project) that will be used in the experiments. Therefore, the keys for conducting a successful simulation experiment should include both building correct model and measuring accurate activity duration.

Construction operations are full of uncertainties due to their nature that operations will be affected by many factors such as weather condition. Hence, it is usually that the activity duration used in simulation is stochastic for

the sake of modeling the uncertainty of duration [1]. As a result, in the construction area, extensive studies have focused on developing the patterns of probability distribution functions (PDFs) that best represent the uncertainty of activity duration [2-6].

In order to correctly select the probability distribution for activity duration, normally data-fitting processes have to be performed. The formal statistical tests such as goodness-of-fit test, then usually are required for successfully selecting an appropriate PDF for activity duration. However, construction practitioners usually do not have statistical testing skill. Hence, it limits the application of simulation methodology to practical construction application. Moreover, most of the cases, historical data for fitting PDF of activity duration can not be accessed. Thus, the deterministic duration of activities is adopted occasionally

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for running simulation and thus the uncertainty of activity duration is scarified.

On the other hand, however, the subjective judgment of activity duration from experts is usually used for the estimation of the length of duration in simulation experiments. The subjective statements used by experts in estimating task duration normally contain some sort of imprecision that is not easy to be modeled in stochastic way but is more easily to be represented in form of fuzzy data. In addition, there are two merits for adopting fuzzy data in the task of duration estimation: (1) no great amount of historical data are required; (2) no tedious data-fitting tasks are need. However, how to cope with fuzzy activity duration in running discrete event simulation is not well explored. Thus, this paper proposes the mechanism that fuzzy number can be used to represent activity duration when running discrete-event simulations. In addition, the proposed mechanism is implemented via reprogramming Construction Operation Simulation Tool (COST), which is the discrete-event simulation computer program, for facilitating the simulation with fuzzy duration. Details of this new simulation mechanism and examples demonstrating the mechanism are provided in the following sections.

2. Related research

Conventionally, modeling the uncertainty of activity duration is usually performed by selecting the PDF that best fits based on sample data. AbouRizk and Halpin [2] investigated 71 sample durations of construction activities and concluded that a flexible distribution such as beta distribution can be used for accurate modeling of construction duration data. In addition, Farid and Koning [7] confirmed that beta PDF is a better choice for closely fitting the actual truck load and travel times in earthmoving operations. Moreover, AbouRizk et al. [4] proposed the moment matching, maximum likelihood, and least-square minimization methods for facilitating the process

of fitting beta PDF. Their research concluded that least-square minimization method produces better fit compared to the other two approaches. This finding is also endorsed by Back and his colleagues [6].

However, fitting the underlying PDF with example data usually involves statistical judgment, such as goodness-of-fit techniques or heuristic procedures. Maio et al. [5] collected data related to truck performance such as payload weight, load time, haul distance, travel time, and dump time from field, and tried to fit those data with theoretical probability density functions. Their research concluded that when different goodness-of-fit techniques are adopted, different results are obtained and additionally, the test results are highly influenced by the subjectively selected data intervals.

Though the appropriate PDF can be selected via fitting data to theoretical probability models, sample data are not always available for statistical analysis. Therefore, AbouRizk et al. [8] proposed a visual interactive method for fitting beta PDF using subjective information including the minimum, maximum, mean, and variance or selected percentiles of a given activity duration. Furthermore, in order to improve the confidence of fitting beta PDF using subjective information, Fente et al. [4] proposed a methodology that based on the equipment manufacturer's machine data and a set of physical site condition to subjectively predicate the minimum, maximum, mode, and the 75th percentile activity durations to find the parameters of the beta PDF. Although when the sample data is not available, the subjective information can be used for fitting beta PDF, however, the subjective estimation of probability distributions is still accompanied with great risks [8].

The reason creating such risks is that the imprecision of subjective estimation is not easy to be modeled in stochastic way but should be more appropriate to be represented in form of fuzzy data. To prevent such risks, many applications of modeling uncertainty

with fuzzy sets are proposed after the fuzzy sets theories proposed by Zadeh [9]. The application of fuzzy sets in the scheduling can be traced to the early 1980s [10]. For example, fuzzy duration was applied in the development of fuzzy PERT with different types of membership functions [11]. In addition, Leu et al. [12] presented a genetic algorithm-based searching technique to search the fuzzy optimal project duration under resource constraints. Furthermore, the calculation of the path degree of criticality when fuzzy activity duration is used in a network has been proposed [13]. More evidences for applying fuzzy project scheduling can be found in [14].

Many researches are dedicated in the exploration of the use of PDFs for modeling the uncertainties of task durations using PDFs when discrete event simulation experiment is performed. However, judging from the literature reviews, the integration of a more suitable form, i.e., fuzzy sets, of modeling the subjective judgment of duration estimation with discrete event simulation is not clearly pictured. Thus, it is necessary to develop a new mechanism allowing running fuzzy duration in simulation.

3. Introduction of fuzzy numbers and simulation methodology

In this section, the triangular-shape fuzzy number used for representing fuzzy duration is introduced at first. Then, the fuzzy numbers' addition operation and defuzzification algorithms used in the advancement of simulation clock are explained. Finally, CYCLONE simulation methodology and the computer simulation program COST used for implementing the proposed mechanism are described.

3.1. Fuzzy number

Fuzzy sets theory was developed to deal with the uncertainties that are not naturally statistical [9]. Fuzzy sets are expressed by

membership functions. The membership degree of an element that is certainly belonging to or absolutely not belonging to a fuzzy set can be defined as unity or null, respectively. However, if an element partially belonging to a fuzzy set then its membership degree should be between 0 and 1.

A fuzzy set whose membership function has at least one element's membership degree that is one is called a normal fuzzy set. In addition, a fuzzy set whose membership function's membership values are strictly monotonically increasing, or monotonically decreasing, or strictly monotonically increasing then strictly monotonically decreasing with increasing values for elements is called a convex fuzzy set. If a fuzzy set is normal and convex, the fuzzy set is often termed as a fuzzy number. Membership functions of a fuzzy number can be assumed shapes or forms. One of the most widely used membership function is in triangular shape [15]. The fuzzy number with triangular shape membership function is named as triangular fuzzy number (TFN). In fact, TFN and the specific beta PDF share the same form but are based on different assumptions. For example, when the max-membership principle is used in defuzzification (explained in the following section), it is equivalent to the mean of the beta PDF. Judging from the literature reviews, beta PDFs are the better choice for fitting construction activities durations, thus TFN is used in this study for the estimation of activity duration.

A typical TFN \tilde{U} , shown in Figure 1, can be represented by a triple (a, b, c) which indicates the (lower support, core, upper support) region of university for its membership function.

Addition operations of TFNs

The general algebraic expression for the addition of two TFNs $\tilde{A}(a_1, b_1, c_1)$ and $\tilde{B}(a_2, b_2, c_2)$ is listed in Eq. (1). Additionally, this membership function of new fuzzy number generated by the addition of fuzzy number

\tilde{A} and \tilde{B} is presented in Figure 2 schematically.

$$\tilde{A} + \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2) \quad (1)$$

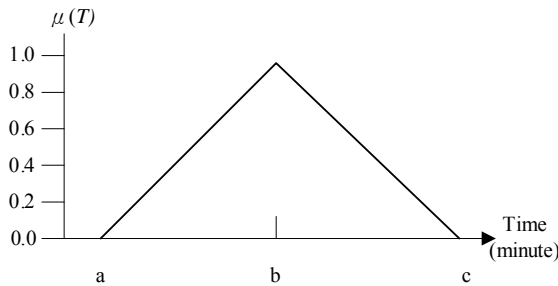


Figure 1. Triangular fuzzy number \tilde{U}

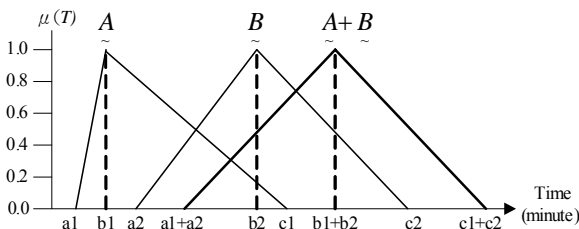


Figure 2. New fuzzy number generated by addition of two fuzzy numbers

Defuzzification of TFN

Despite the information of uncertainty can be conveyed using fuzzy numbers, decision making usually has to be crisp. For example, the final output of system performance such as production rate and the decision of which event should be removed to advance simulation clock all have to be justified based on crisp value. The process for the conversion of a fuzzy number to a crisp value is called defuzzification. Three popular defuzzification methods: max-membership principle, centroid method, and λ -cut method [15] are used in this study. Equations (2), (3), and (4) list the single outputs of a typical TFN (a, b, c) defuzzified by max-membership principle, centroid method, and λ -cut method, respectively. In addition, the schematic demonstration of

these output are shown in Figure 3. The $U(z^*)$ shown in Equations (2) and (3) represents the single output of fuzzy number \tilde{U} . However, the $U(z^*)$ described in Equation (4) represents an interval rather than a single output. Thus, when the λ -cut method is used, a random realization of activity duration has to be performed for obtaining a single output instead of an interval output.

Max-membership principle

$$U(z^*) = b \quad (2)$$

Centroid method

$$U(z^*) = \frac{\left\{ \frac{(b-a)}{2} \times \left[a + \frac{2(b-a)}{3} \right] + \frac{(c-b)}{2} \times \left[b + \frac{(c-b)}{3} \right] \right\}}{\frac{(c-a)}{2}} \quad (3)$$

λ -cut method

$$a + \lambda(b - a) \leq U(z^*) \leq c + \lambda(b - c) ; 0 \leq \lambda \leq 1 \quad (4)$$

where $U(z^*)$ represents the single output of fuzzy number \tilde{U}

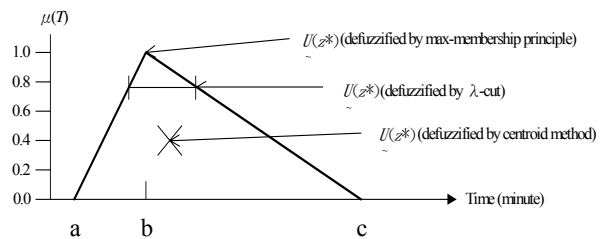


Figure 3. Defuzzification output obtained by using max-membership principle, centroid method, and λ -cut method

3.2. CYCLONE methodology and COST program

Due to its modeling elements are simple to be used, the discrete event simulation methodology CYCLONE (CYCLic Operation

NETwork) is adopted for demonstrating the new simulation mechanism. The basic modeling elements used in CYCLONE are presented in Table 1 and reader may refer to Halpin and Riggs [16] for the details. In addition, a computer program COST (Construction Operations Simulation Tool) developed based on CYCLONE is revised to implement the

mechanism. The interface of model input in COST program is given in Figure. 4. The fuzzy type duration modeled by TFN can be selected in "Duration" and the parameters of lower value, core value, and upper value can be entered. To explore the details of COST simulation system, readers may refer to the user manual of COST [17].

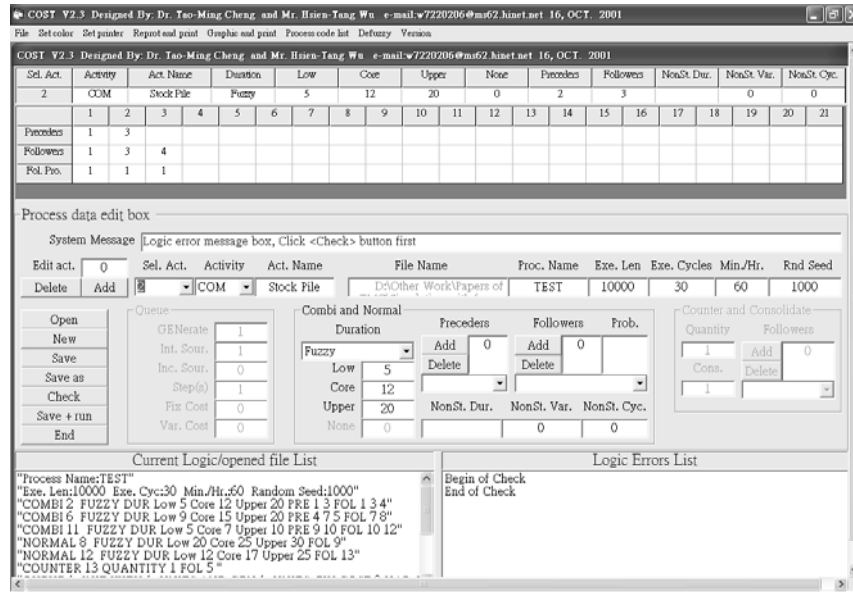








Figure 4. Fuzzy duration input dialogue boxes of COST

Table 1. CYCLONE system modeling elements

Name	Symbol	Function
Normal activity		Units arriving at Normal will be processed right away without delaying.
Combination (COMBI) activity		Units arriving at COMBI will be processed if units are available in all preceding Queue node.
Queue node		Queue provides position that allows units are delayed pending COMBI activities.
Consolidate function node		Consolidate function node performs the consolidate marking.
Counter		Counter measures the modeled system's production rate.
Arcs		Arcs show the logic that units flow from element to element.

4. Mechanism of advancing simulation clock

The key point for running discrete event simulation is how the simulation clock is advanced when different types of duration are adopted. When PDF or constant duration is used, the value of duration is crisp. There will be no difficult to determine what activity is done and should be removed during simulation process. However, when fuzzy data is used, the defuzzification operation has to be performed to determine which activity should be scheduled. The details of advancing simulation clock involving fuzzy duration are described as follows.

In discrete event simulation, the simulation clock is used to keep track of simulation time (SIM TIME) and then establish how and at what points in SIM TIME the system is reviewed to determine whether flow unit

movement should take place. CYCLONE uses EVENT List (EL) and CHRONOLOGICAL List (CL) to keep track of the end event times (EET) of scheduled activity and therefore establish the SIM TIME for a system [16]. The EL and CL are combined and shown in Table 2.

The EL files the events and the times when they are scheduled and the CL records the events and the times when they have occurred. The last event on the CL is the event which represents the current time (represented by TNOW shown in Table 2) on simulation clock. The transfer of EET (TNOW+DURATION) from EL to the CL is performed when the simulation clock is advanced. The event time selected to be transferred from EL to CL is always the earliest event that can occur. In other words, the smallest EET on EL will be moved to CL.

Table 2. EVENT and CHRONOLOGICAL List

REC #	EVENT LIST				Transfer	CHRONOLOGIC LIST	
	ACT	TNOW	DURATION	END OF EVENT TIME (EET)		ACT	SIM TIME
1							
2
.
.
N							

The flows of integrating fuzzy durations into a generic simulation procedure are given in Figure 5. The procedure starts with the search of any work task that can be started. If any activity can be performed, the duration of the scheduled activity is calculated and the EET will be filed. If no further activity can be executed, the earliest schedule event will be moved to CL and the simulation clock then is advanced to EET of the transferred event. In other words, in order to transfer the next earliest scheduled event to the CL, the EETs of

all scheduled events have to be compared in order to determine the next TNOW. In conventional simulation, no matter of the duration modeled by constant value or PDF, crisp value is always concluded when work task is performed. Thus, it has no difficult to determine the next earliest scheduled event. However, when fuzzy duration is used, since the calculated EET will be also a fuzzy data, it is required to defuzzify EET to justify which the next earliest scheduled activity is.

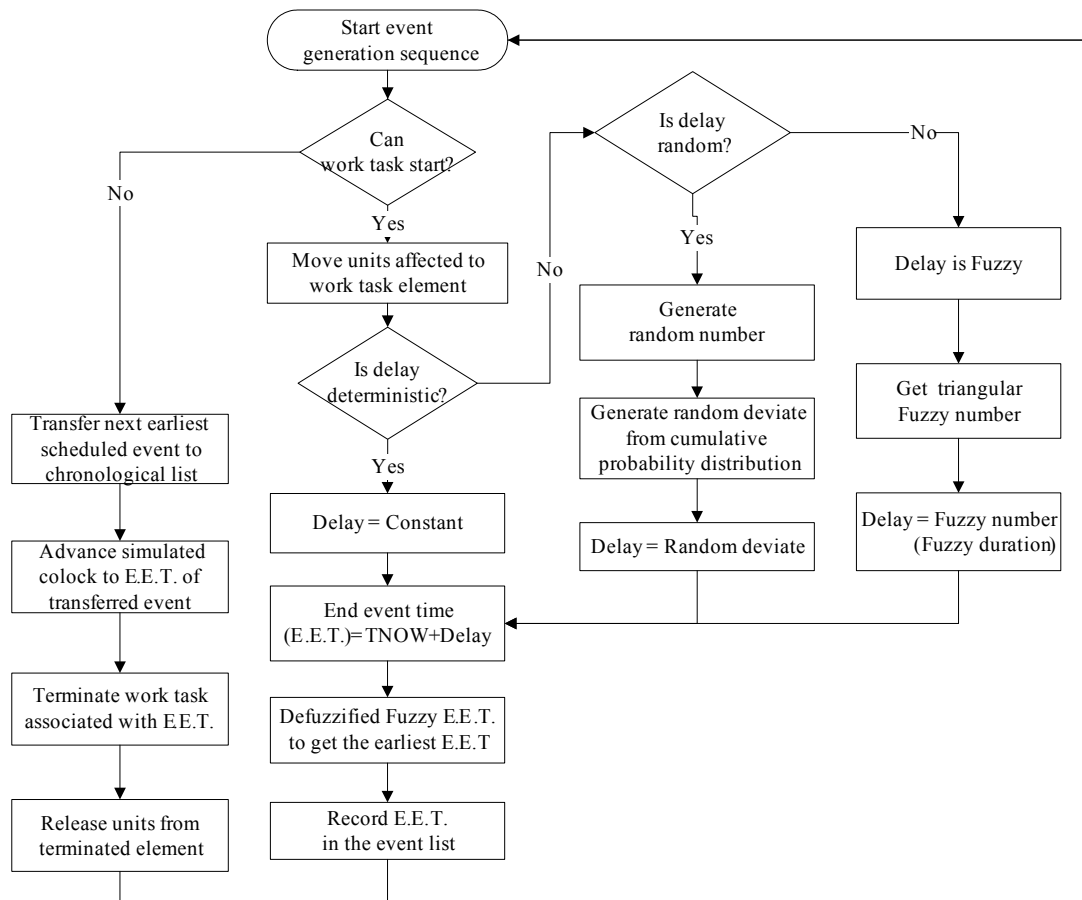


Figure 5. Simulation flow diagram (modified from [16])

In this study, the fuzzy EET is only temporally defuzzified for determining the earliest EET. Once the next earliest event time is determined, the fuzzy value of EET, rather than the crisp value of EET, will be transferred to CL. In other words, though the fuzzy EETs are defuzzified to see which the earliest is, the fuzzy value of EET is always retained for representing TNOW. The best advantage of this approach is that the feature of uncertainty in activity duration is preserved in every calculation of the simulation process. An example to illustrate this mechanism is presented in the following section.

5. Hand simulation demonstration

The step by step hand simulation processes for demonstrating the new simulation mecha-

nism is through the earthmoving operations represented by the CYCLONE model given in Figure 6. In addition, the initial resources for each queue node and fuzzy durations modeled by TFNs are given in Table 3 and Table 4, respectively.

The hand simulation processes are recorded in Table 5. At the beginning, only activity 2 can be performed, thus the dozer unit and dirt unit are moved to activity 2. The time generated for this duration is TFN (5, 12, 20). Thus, according to the addition operation rule shown in Eq. (1), the EET should be (5, 12, 20) $((0, 0, 0) + (5, 12, 20))$. Since no other activity can be started, the EET for activity 2 is transferred to the CL and the clock is advanced to a SIM TIME of (5, 12, 20). To indicate this transfer, an arrow marker is placed in the TRANSFER column between EL and

CL. At this point, the unit in activity 2 is released, and it passes to the following element. The dozer unit and dirt unit return to the idle state at Queue nodes 1 and 3, respectively. An entity is generated and stays at Queue node 4 after releasing activity 2. This system again is examined to see what activities can be started next. At this time, activities 2 and 6 can be started. The duration for activity 2 is (5, 12, 20) minutes, added to TNOW (5, 12, 20), giving an EET equal to (10, 24, 40) for activity 2. In addition, the fuzzy duration (9, 15, 20) of activity 6 is added to TNOW, and therefore gives an EET (14, 27, 40) for activity 6. Again, no other activity can be scheduled, thus activity in EL must be transferred to CL

in order to advance SIM CLOCK and to release resources or entities. Since activities 2 and 6 are scheduled to occur, it is necessary to compare their EET to determine which should be moved to CL. Because their EET are fuzzy, hence the defuzzification operation has to be performed to decide which one is earlier. After applying centroid defuzzification method, the crisp value of EET for activities 2 and 6 are 24.7 and 27 minutes, respectively. The EET of activity 2 is earlier, therefore, this event is selected to be transferred to CL. The SIM TIME now is advanced to (10, 24, 40) minutes. The rest of calculations of hand simulation processes for first 9 events are listed in Table 5.

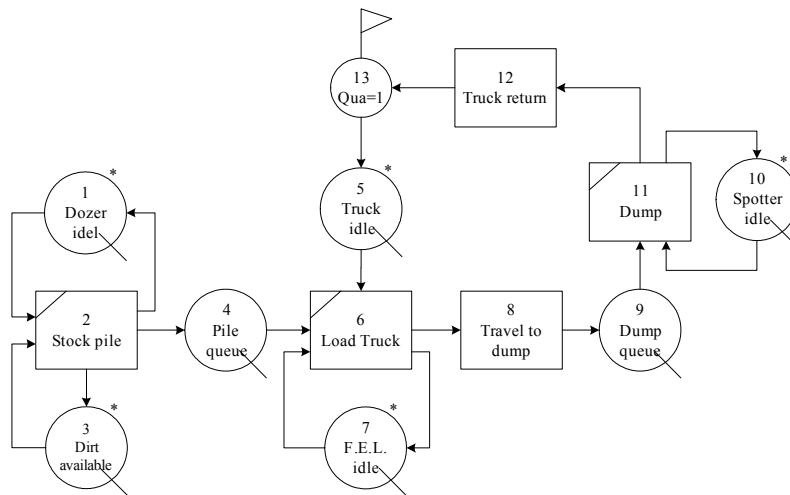


Figure 6. Earthmoving model

Table 3. Initial resource of queue nodes

Node number	Resource name	Quantity
1	Dozer	1
5	Truck	1
7	Front-End Loader	1
10	Spotter	1

Table 4. Work duration list (unit: minute)

Node number	Task name	Fuzzy number parameters		
		Lower support	Core	Upper support
2	Stock pile	5	12	20
6	Load truck	9	15	20
8	Truck travel to dump	20	25	30
11	Dump dirt	5	7	10
12	Truck return	12	17	25

Table 5. Hand simulation results

REC #	EVENT LIST					TRANSFER	CHRONOLOGIC LIST	
	ACT	TNOW	FUZZY DUR	EET			ACT	SIM TIME
				FUZZY	DEFUZZY			
1	2	(0,0,0)	(5,12,20)	(5,12,20)	12.3	→	2	(5,12,20)
2	2	(5,12,20)	(5,12,20)	(10,24,40)	24.7	→	2	(10,24,40)
2	6	(5,12,20)	(9,15,20)	(14,27,40)	27	→	6	(14,27,40)
3	2	(10,24,40)	(5,12,20)	(15,36,60)	37	→	2	(15,36,60)
4	8	(14,27,40)	(20,25,30)	(34,52,70)	52	↔	2	(20,48,80)
5	2	(15,36,60)	(5,12,20)	(20,48,80)	49.3	↔	8	(34,52,70)
6	2	(20,48,80)	(5,12,20)	(25,60,100)	61.7	↔	11	(39,59,80)
7	11	(34,52,70)	(5,7,10)	(39,59,80)	59.3	↔	2	(25,60,100)
8	12	(39,59,80)	(12,17,25)	(51,76,105)	77.3	↔	2	(30,72,120)
9	2	(25,60,100)	(5,12,20)	(30,72,120)	74	↔	12	(51,76,105)

6. Computer implementation

The computer implementation of the integration of fuzzy durations in simulation is conducted through reprogramming COST system. The modified computer program allows user to choose the max-membership principle, centroid method, or λ -cut defuzzification methods for advancing simulation clock when fuzzy duration is involved in the

simulation processes. The interface of selecting defuzzification methods for advancing simulation clock is shown in Figure 7.

In addition, the new COST system provides results of different λ -cut level for facilitating the establishment of subjective interval of system performance output such as production rate. The interface of this output is given in Figure 8. User only has to click-and-hold the left button of mouse and can move the

pointer up and down to see the system performance generated under any λ -cut value.

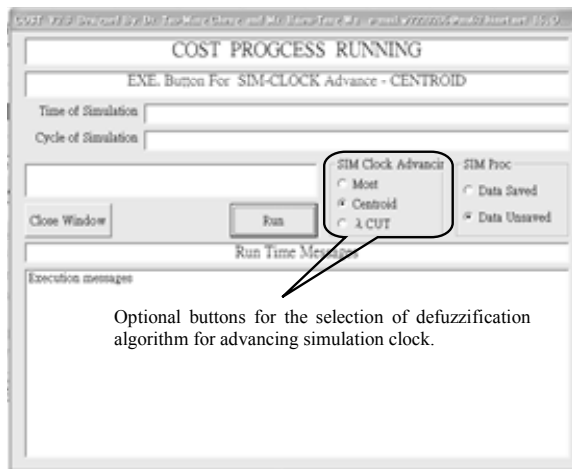


Figure 7. The interface of selection of defuzzification methods for advancing simulation clock

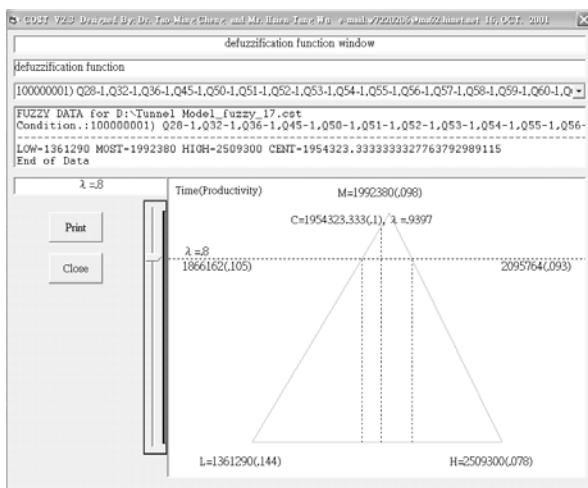


Figure 8. Interface of λ -cut output for system production rate in COST program

7. Examples

Two examples are presented in this study to demonstrate this new simulation mechanism. The first one is the same example as described in the section of “hand simulation demonstration”. The results after simulating 100 cycles are indicated in Table 6. Max-membership principle and centroid de-

fuzzification methods are used for advancing simulation clock and together with λ -cut method are used for defuzzifying the final system production rate. The system production rates are 0.932 (cyc/hr) and 0.917 (cyc/hr) when max-membership principle and centroid method are used for defuzzifying the final output, respectively. In this case, the defuzzification method used in advancing simulation clock does not affect the final output of system production rate. It can be explained that during the simulation processes, always the same activity are removed to CL no matter which defuzzification method is used in advancing simulation clock. In addition, different λ -cut levels which are 0.8, 0.5, and 0^+ are adopted to predict the range of possible production rate for the system. For example, when λ -cut value is 0.5, the system production rate is expected to be between 0.8 (cyc/hr) and 1.085 (cyc/hr).

Table 6. Production rate output for the first example (unit: cycles/hour)

Defuzzification methods used in final output of system production rate	Defuzzification methods used in advancing simulation clock	
	Max-membership principle	Centroid
Max-membership principle	0.932	0.932
Centroid	0.917	0.917
λ -cut ($\lambda=0.8$)	0.874 ~ 0.988	
λ -cut ($\lambda=0.5$)	0.800 ~ 1.085	
λ -cut ($\lambda=0^+$)	0.700 ~ 1.300	

The second example is a real project involving the tunneling operations. Its CYCLONE model is illustrated in Figure 9. This project is to construct a twin-tube tunnel which is approximately 5 kilometers long through the Pa-Guah-Shan Mountain where is located in central area of Taiwan. The tunnel was advanced by adopting New Austria Tunneling Method (NATM). Each advanced section is 0.8-1.2 meters long. Details of the operation sequences are depicted in the following steps:

- (1) The tunnel operation process is begun from digging the front face.
- (2) Shotcrete is then applied to the tunnel face.
- (3) After the shotcrete is applied, the wiremesh is placed. In addition, the steel rib cage is assembled and placed.
- (4) After the steel rib is placed, the second application of shotcrete is applied.
- (5) Wire mesh is placed for the second time.
- (6) The sidewall bolting for the next section is performed.
- (7) A final application of shotcrete is applied in order to finish the surface of the face.
- (8) Repeat steps 3-7 for 4 sections before the temporary heading invert is placed.
- (9) Replacement for earthwork.
- (10) Restart for new section.

The resources required for the tunneling operations are given in Table 7. In addition, three experts with 7, 17, and 23 years experience in NATM tunneling construction are interviewed for the extraction of their opinion to establish the TFNs to express the fuzzy durations. These TFNs are recorded in Table 8.

There are two steps in the process of verifying the practical capability of the new simulation mechanism. The first step is to obtain the system production rate by observing the actual production rate for the sake of comparing the effect of running fuzzy durations. The second step is to use fuzzy durations to obtain the simulated production rate. After observing 155 meters long tunneling advancement rate, the actual production rate was averaged as 10.56 (hours/meter) which indicates that advancing 1 meter takes more than 10 hours. In addition, the 95% confidence interval of production rate is between 7.56 and 13.56 hours per meter.

The simulation results are shown in Table 9. The system production rate shows slight difference when max-membership principle and centroid defuzzification methods are used for defuzzifying the final output. Experts with

more experience tend to be more accurate in the predication of production rate.

In addition, the output assessed by the expert with 17-year-experience is used for testing the result of λ -cut. For instance, while setting λ -cut value to 0.5 and 0^+ , the interval for system output is between 10.31 and 13.12 and from 9.01 to 14.71 hours per meter, respectively. Compared to the actual data of 95% confidence interval which are between 7.56 and 13.56 hours per meter, the results show some degree of certainty for the estimation of production rate.

8. Conclusions

Simulation should incorporate the uncertainties in task durations that are due to uncertainties in the project environment. Traditionally, these uncertainties are modeled using PDFs by data fitting processes and are highly influenced by the availability of historical data. A new simulation mechanism integrating the triangular fuzzy numbers with simulation is proposed in this study. This approach allows experts estimate task duration subjectively while data fitting process can not be performed. Meanwhile, the ambiguity of task duration caused from expert's judgment is maintained while using fuzzy numbers. The effects of different defuzzification methods for advancing simulation clock and outputting system performance are compared. Examples are illustrated how the proposed mechanism is applied. It is found that the results generated are meaningful and show some degree of certainty for the estimation of production rate. In addition, the reprogrammed computer simulation system, COST, simplifies the processes of simulation with fuzzy durations modeled by triangular shape membership fuzzy numbers. However, it should also be noticed that when fuzzy numbers are used, the merit of random realization of the activity duration during the simulation while using PDF is lost.

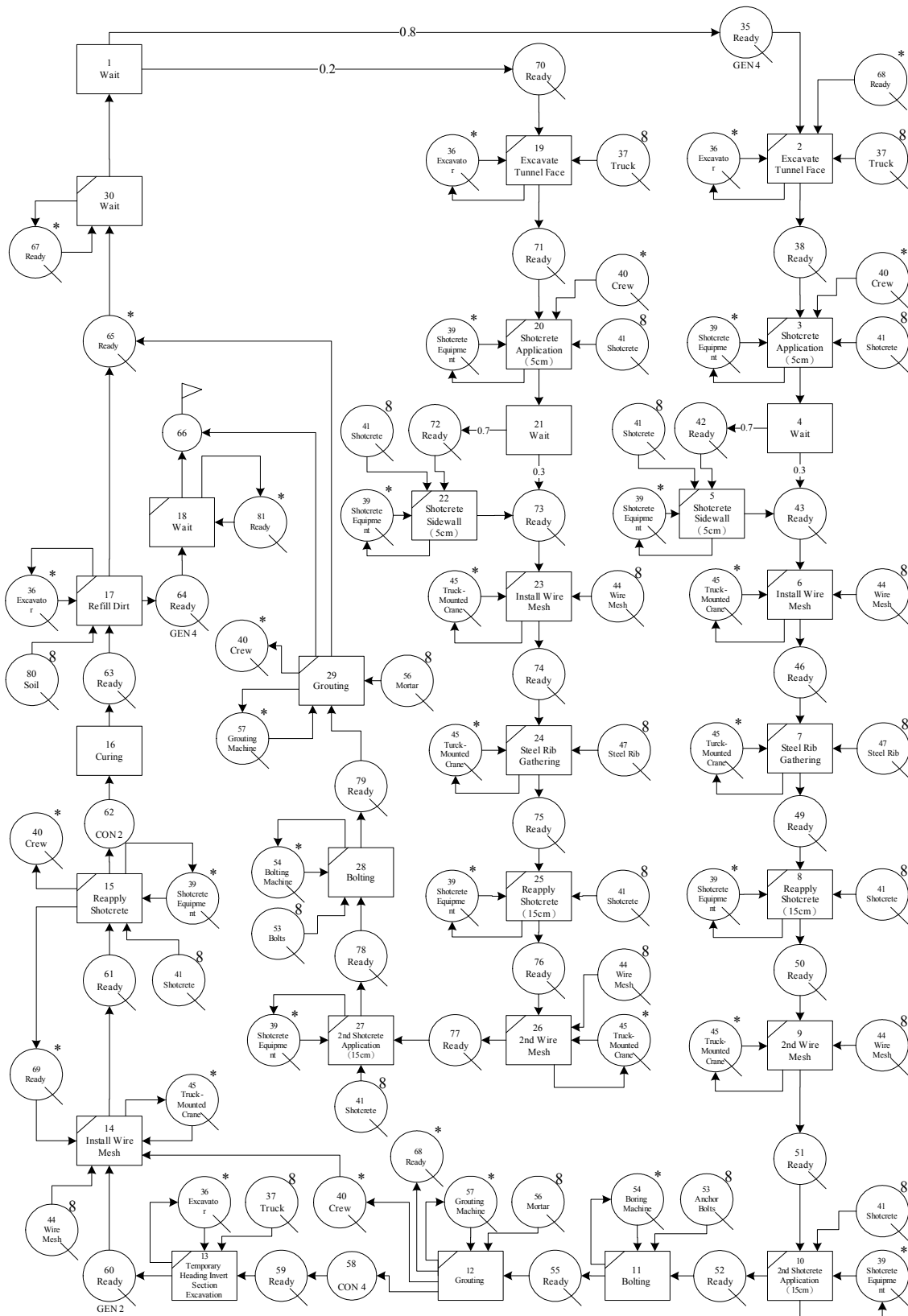


Figure 9. CYCLONE model for tunneling operations

Table 7. Resources for the tunneling operations CYCLONE model

Queue	Resource	QUA.	Resource Description
40	Work Crew	1	Three crews rotate on a 3 or 5-day schedule The first crew will work a 12-hour shift during the day, the second will work the night shift and the third will rest.
36	Excavator	1	EX-200 Excavator
53	Anchor Bolts	∞	There are 4m and 6m bolts that are rotated every two lengths. The 4m bolts are inserted, then the 6m bolts, etc.
41	Ready-Mix Concrete	∞	Type I Portland Cement
47	Steel Rib	∞	An infinite amount is assumed in the model.
57	Boring Machine	1	RCP-200E Boring Machine
37	20 Ton Dump Truck	∞	An infinite amount is assumed in the model.
56	Dry Ready-Mix	∞	For the Shotcrete Machine
39	Shotcrete Equipment	1	AL-260 Shotcrete Machine and AL-405 Add Mix Machine
44	Wiremesh	∞	An infinite amount is assumed in the model.
45	Truck-Mounted-Crane	1	2.8 Truck-Mounted Crane
80	Earth Refill	∞	An infinite amount is assumed in the model.
176	Grouting Machine	1	Grouting Machine with a 2 bag Capacity

Table 8. Fuzzy duration obtained from interviewing experts

Act.	Act. Name	Data acquired form expert #1	Data acquired form expert #2	Data acquired form expert #3
		with 7 years experience	with 17 years experience	with 23 years experience
2	Excavate Tunnel Face	(110,150,200)	(120,160,210)	(150,180,240)
3	Shotcrete Application (5cm)	(20,30,40)	(40,60,80)	(60,70,80)
5	Shotcrete Sidewall (5cm)	(20,25,40)	(20,30,40)	(20,25,40)
6	Install Wire Mesh	(20,30,40)	(20,30,40)	(20,30,45)
7	Steel Rib Gathering	(50,60,70)	(30,60,90)	(35,50,70)
8	Reapply Shotcrete (15cm)	(50,70,90)	(60,70,100)	(60,80,90)
9	2 nd Wire Mesh	(20,25,30)	(20,30,40)	(20,25,30)
10	2 nd Shotcrete Application (15cm)	(20,30,40)	(20,30,40)	(20,30,40)
11	Bolting	(60,80,100)	(70,85,110)	(60,80,100)
12	Grouting	(50,60,70)	(50,60,70)	(50,60,70)
13	Temporary Heading Invert Section Excavation	(50,60,70)	(70,80,100)	(75,90,120)
14	Install Wire Mesh	(20,25,30)	(20,30,40)	(15,20,30)
15	Reapply Shotcrete	(40,45,60)	(50,60,70)	(40,45,60)
16	Curing	(90,120,140)	(90,120,140)	(100,120,140)
17	Refill Dirt	(30,40,50)	(30,40,60)	(30,40,50)
19	Excavate Tunnel Face	(110,150,200)	(120,160,210)	(150,180,240)
20	Shotcrete Application (5cm)	(20,30,40)	(40,60,80)	(60,70,80)
22	Shotcrete Sidewall (5cm)	(20,25,40)	(20,30,40)	(20,25,40)
23	Install Wire Mesh	(20,30,40)	(20,30,40)	(20,30,45)
24	Steel Rib Gathering	(50,60,70)	(30,60,90)	(35,50,70)
25	Reapply Shotcrete (15cm)	(50,70,90)	(60,70,100)	(60,80,90)
26	2 nd Wire Mesh	(20,25,30)	(20,30,40)	(20,25,30)
27	2 nd Shotcrete Application (15cm)	(20,30,40)	(20,30,40)	(20,30,40)
28	Bolting	(60,80,100)	(70,85,110)	(60,80,100)
29	Grouting	(50,60,70)	(50,60,70)	(50,60,70)

Table 9. Production rate output for the second example

Item	System production rate estimated by expert #1		System production rate estimated by expert #2		System production rate estimated by expert #3	
	Max. principle	Centroid	Max. principle	Centroid	Max. principle	Centroid
Simulated production rate	8.93	9.01	10.98	11.24	10.31	10.53
Deviation ((simulated prod. rate – actual average prod. rate)/ actual average prod. rate*100%)	15.4 %	14.7 %	4.0%	6.4 %	2.4 %	0.3 %

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