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The PAPR Reduction in OFDM System with the Help of Signal Mapping Scheme

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Abstract: The non-linear distortion caused by high peak-to-average power ratio (PAPR) of transmitted signal is a major drawback in the OFDM systems. In this paper, the proposed PAPR reduction scheme is to choose the corresponding symbol patterns with a low PAPR in the expanded signal space. However, the scheme has a high complexity for the selection of symbol pattern with the smallest signal power in a larger expanded signal space. The scheme could be realized as a signal mapping from the original information symbols to the corresponding patterns. With the selection of the corresponded symbol patterns, the PAPR could be greatly reduced. According to the simulation results with the proposed scheme, while the length of the data frame is 16 and the signal expansion with adding 1 bit and 2 bits, the PAPR changing rate could be 20 percents and 16 percents of the original PAPA, respectively. However, there is no prominent improvement on PAPR reduction with more than 8 bits for using the signal expansion scheme.

Keywords: OFDM system; PAPR reduction scheme; Signal mapping scheme.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) technique has been applied into many applications such as digital audio broadcasting (DAB) systems, digital video broadcasting terrestrial TV (DVB-T) systems, asymmetric digital subscriber lines (ADSL), wireless local area networks (WLAN), broadband wireless access (BWA) networks and ultra-wideband systems [1-4]. Moreover, it could be expected to be the standard for the Fourth Generation (4G) cellular system [5].

However, OFDM signal passes through a non-linear high power amplifier (HPA) before it is transmitted over the channel. When the input signal exceeds a certain value, the output of the amplifier becomes saturated and, then, the OFDM signal results a great degradation caused by the HPA [6, 7]. The HPA is commonly considered be memoryless, which means that the current output depends on the current input only. For applications with wider signaling bandwidths, however, the memory effects of the whole system cannot be ignored. [7] Especially, OFDM signal with a high PAPR is sensitive to the non-linear distortions caused by the HPA. The non-linear

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distortion in the HPA results in such negative factors as (1) constellation warping which increase the bit error rate (BER) in the system (2) spectrum outgrowth resulting in adjacent channel interference (ACI) [6, 7]. In order to reduce the non-linear distortion, many schemes have been proposed to reduce the PAPR of transmitted signal. Among those PAPA reduction schemes, the simplest method is to use the clipping process [1, 8, 9]. However, using clipping process causes both in-band distortion and out-of-band distortion. These undesired effects could be avoided with a linear peak cancellation technique [1]. Besides, the forward-error-control coding applied scheme is another solution to the PAPR reduction [1, 10]. Moreover, based on the scrambling each OFDM symbol with different phase, the small PAPR selection is provided by symbol scrambling methods, such as selected mapping (SLM) method, partial transmitted sequence (PTS) method and subblock phase weighting (SPW) method [11-14].

In this paper, a new PAPR reduction scheme with the help of the signal mapping scheme is proposed. The corresponding symbol patterns are chosen with a low PAPR in the expanded signal space. The scheme could be realized as a signal mapping from the original information symbols to the corresponding patterns. With the selection of the corresponded symbol patterns, the PAPR could be greatly reduced. In the following section, the OFDM system with the proposed signal mapping scheme is described. The numerical analysis is shown in Section III. Finally, a conclusion is given in Section IV.

2. The OFDM System with the Signal Mapping Scheme

Basically, an OFDM system contains a series-to-parallel converter, a signal mapping scheme and a fast Fourier transform (FFT)/ inverse FFT (IFFT) module [1, 2, 4]. The series and parallel converter is considered to realize the concept of parallel data transmission to provide a high data rate transmission. *M*-ary phase shift keying (PSK) or *M*-ary quadrature amplitude modulation (QAM) mapping scheme is modulated in each parallel subcarrier to provide a different data rate service in the system. Besides, the IFFT and FFT are employed to replace the banks of sinusoidal generator for the modulation and demodulation with different carrier frequencies to reduce the complexity of OFDM modem implementation [1, 2]. Additionally, consider the signal mapping for signal space expansion, the proposed OFDM system is shown in Figure 1.

First, source information is mapped with *M*-ary PSK scheme to obtain a serial message sequence. The message sequence $\boldsymbol{a}$ = [a_0, a_1,…, a_{K-1}] with K symbols is converted into a parallel form and called as a information symbol. In order to choose the modulated symbol patterns with low PAPRs, the information symbol is proposed to map the corresponding modulated symbol pattern $\boldsymbol{x}$ = (x_0, x_1,…, x_{N-2}, x_{N-1}) with N symbols in the expanded signal space. There are M^N patterns in the expanded signal space and M^K patterns with minimum PAPRs are chosen for mapping. The N parallel modulated symbols are transformed by an IFFT to be a data symbol, **s** = [s_0, s_1,…, s_{N-1}]. Then, the OFDM symbol is transmitted over a channel. The PAPR of the baseband OFDM transmitted symbol can be defined as

$$PAPR = \max_{0 \le n \le N-1} \frac{\left(s_n \cdot s_n^*\right)}{E\left(s_n \cdot s_n^*\right)} \tag{1}$$

where * indicates the complex conjugate, max(s_n) denotes the maximum value of s_n and E($s_n \cdot s_n^*$) is the average power of data symbol **s**. The probability density function (pdf) of power of data symbol **s**, $f_{\gamma_n}\left(\gamma_n\right)$, is

$$f_{\gamma_n}\left(\gamma_n\right) = e^{-\gamma_n} \tag{2}$$

where $\gamma_n = |s_n \cdot s_n^*|$ is the power of the n-th sample in the OFDM symbol and γ_n is an i.i.d. chi-squared distribution with 2 degrees of freedom random variable [15, 16]. At the receiver, the received symbol are converted into a parallel form and, then, transformed by a FFT to get the estimated modulated symbols. The estimated modulated symbols are mapped with corresponding demodulated scheme. Then, the estimated source information is obtained.

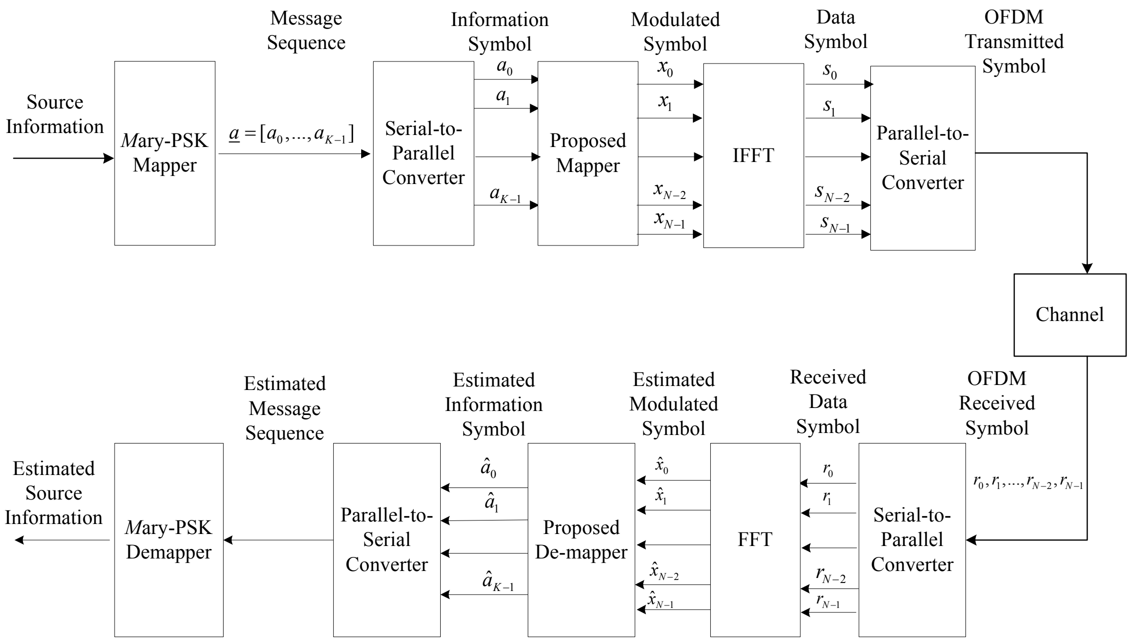

Figure 1. The block diagram of the FFT-based OFDM system

The signal expansion in the proposed mapping scheme could be considered as the transferring the original information symbols into the corresponding modulated symbol patterns, as shown in Figure 2. In the figure, there are M^K possible information symbols in the original signal space. These information symbols should be mapped into those modulated symbol patterns with lower PAPRs in the expanded signal space where there are M^N possible modulated symbol patterns. The circles with dash lines represent the symbol patterns with higher PAPRs. In the proposed mapping scheme, with the selection of the corresponded symbol patterns, the PAPR of the OFDM transmitted signal could be greatly reduced.

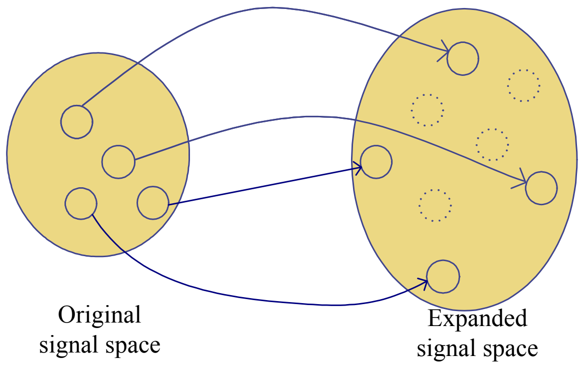

Figure 2. The concept of signal mapping in the proposed scheme

In addition, the upper bound of PAPR with the proposed scheme could be derived as bellows. Within mapping the symbol with K bits to the symbol with $K+b$ bits, the joint probability density function (pdf) is rewritten as

$$f_\gamma\left(\gamma_0,\ldots,\gamma_n,\ldots,\gamma_{K+b-1}\right)= f_\gamma\left(\gamma_0\right)\cdots f_\gamma\left(\gamma_n\right)\cdots f_\gamma\left(\gamma_{K+b-1}\right)=e^{-\sum_{n=0}^{K+b-1}\gamma_n} \tag{3}$$

In the extended signal space, there are M^{K+b} symbol patterns could be selected for the M^K dedicated symbols. Hence, the M^K minimum PAPR symbol patterns could be chosen. Then, the PAPR upper bound $\gamma_{\text{up-bound}}$ in the proposed scheme could be obtained with the Eq. (4),

$$f_\gamma\left(\gamma_0,\ldots,\gamma_n,\ldots,\gamma_{K+b-1}<\gamma_{up-bound}\right)= \iiint\int_0^{\gamma_{up-bound}}\left(e^{-\sum_{n=0}^{K+b}\gamma_n}\right)d\gamma_0 \ldots d\gamma_n \ldots d\gamma_{K+b-1}= \frac{M^K}{M^{K+b}}=\frac{1}{M^b} \tag{4}$$

because there are M^K symbol patterns should be chosen among M^{K+b} signal patterns. Besides, because γ_n is an i.i.d. random variable, Eq. (4) is satisfied.

$$f_\gamma\left(\gamma_0,\ldots,\gamma_n,\ldots,\gamma_{K+b-1}<\gamma_{up-bound}\right)= f_\gamma\begin{pmatrix}\gamma_0<\gamma_{up-bound},\ldots,\gamma_n<\gamma_{up-bound},\\ \ldots,\gamma_{K+b-1}<\gamma_{up-bound}\end{pmatrix} \tag{5}$$

Then, the PAPR upper bound $\gamma_{\text{up-bound}}$ in the proposed scheme could be obtained as

$$\int_0^{\gamma_{up-bound}}\left(e^{-\gamma_0}\right)d\gamma_0\cdots\int_0^{\gamma_{up-bound}}\left(e^{-\gamma_n}\right)d\gamma_n\cdots \int_0^{\gamma_{up-bound}}\left(e^{-\gamma_{K+b-1}}\right)d\gamma_{K+b-1}=\frac{1}{M^b} \tag{6}$$

Hence, the upper bound of PAPR, $\gamma_{\text{up-bound}}$, could be obtained as

$$\gamma_{up-bound}=\ln\left[\left(1-M^{\frac{-b}{K+b}}\right)^{-1}\right] \tag{7}$$

where $\ln(\cdot)$ denotes the logarithm function. Based on this proposed scheme, the simulations are given in the following section.

3. Numerical Analysis

At the beginning, assuming the number of subcarriers and the symbol number of information symbol to be 4 and 3 (i.e. N=4, K=3), the number of the possible information symbols in the signal space and the possible symbol patterns is 2^3 and 2^4 individually, when the BPSK scheme is used. The histogram of PAPR in the 4-IFFT BPSK OFDM system is shown in Figure 3. In the figure, there are 2^4 possible symbol patterns. The numbers of the pattern with PAPR=1, PAPR=2 and PAPR=4 are eight, four and four, individually. In the proposed scheme, the symbol patterns within the minimum 8^{th} PAPR could choose as the possible corresponding symbol patterns in the proposed algorithm. Hence, the 8 patterns with PAPR=1 are chosen for mapping. The PAPR has been reduced from 3 to 1 in the proposed scheme. Besides, in Table 1, with the increase of parameter K and N, PAPR reduction is more outstanding.

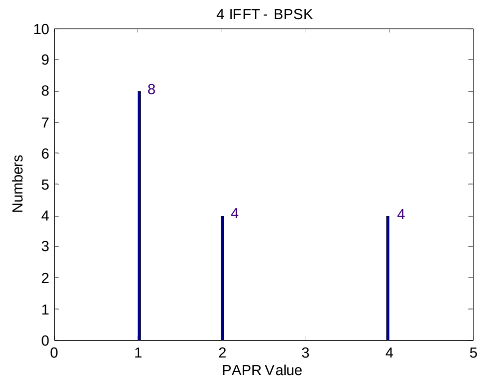

Figure 3. The histogram of PAPR in the 4-IFFT BPSK OFDM system

Table 1. PAPR of the conventional OFDM system and the OFDM system using the proposed method

K	N	Modulation scheme (M-ary PSK)	The PAPR of the conventional OFDM system with K subcarriers	The PAPR of OFDM system using the proposed method
3	4	BPSK	3	1
3	4	QPSK	3	2
7	8	BPSK	7	2
7	8	QPSK	7	2.25

In addition, this is assuming the symbol number of information symbol to be16. With adding bits, the PAPR changing rate is shown in Figure 4. The PAPR changing rate is defined as a ratio between the maximum PAPR value in the proposed scheme and the maximum one in original PAPR. Without adding any bit, the maximum PAPR value is not changed. While the length of the data frame is 16 and the signal expansion with adding 1 bit and 2 bits, the PAPR is reduced from 16 to 3.2 and 2.6, individually. Thus, the PAPR changing rate could be 20 percents and 16 percents of the original PAPA, respectively. However, there is no prominent improvement on PAPR reduction with more than 8 bits for the signal expansion.

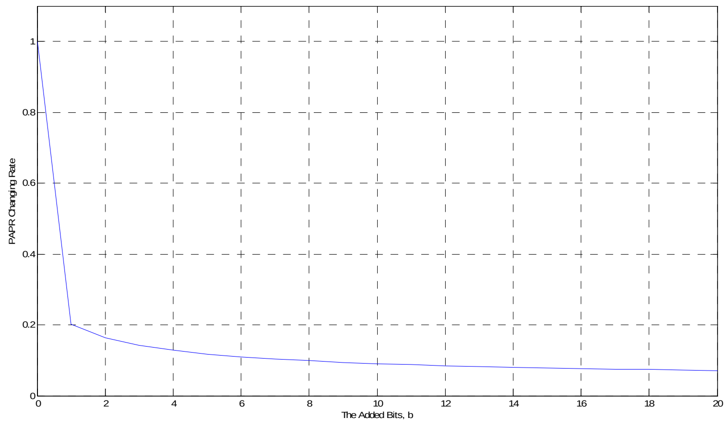

Figure 4. The PAPR changing rate in the proposed scheme

4. Conclusion

In this paper, a new scheme with signal space expansion is proposed to reduce PAPR of the OFDM transmitted signal. The scheme could be realized as a signal mapping from the original information symbols to the corresponding patterns. With the selection of the corresponded symbol patterns, the PAPR could be greatly reduced. While the length of the data frame is 3 and the signal expansion with adding 1 bit, the PAPR is reduced form 3 to 1 when the BPSK scheme is used. Besides, with the increase of parameter K and N, PAPR reduction is more outstanding. The PAPR changing rate is large while the length of the data frame is 16 and the signal expansion with adding 1 bit. However, there is no prominent improvement on PAPR reduction with more than 8 bits for the signal expansion.

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