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Fuzzy System Reliability Using Different Types of Vague Sets

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Abstract: Till now, in the literature, arithmetic operations between same types of vague sets are discussed. Also to analyze the fuzzy system reliability, it is assumed that the reliability of all components of a system follows the same membership function. However, in practical problems, such type of situations rarely occurs. Therefore, there is need of a method by which we can also find the fuzzy reliability of systems having components following different type of membership functions. In the present paper, a new algorithm has been introduced to perform various arithmetic operations between different types of vague sets. Further using the proposed algorithm, the methods are obtained to analyze the fuzzy reliability of various systems. Our study generalize the various work [6-8, 10-15] of the literature.

Keywords: Fuzzy reliability; Fuzzy sets; Vague sets.

1. Introduction

The reliability of an item is the probability that the item will perform a specified function. Traditionally, the reliability of a system behavior is fully characterized in the context of probability measures, and the outcome of the top event is certain and precise as long as the assignment of basic events are descent from reliable information. However, in real system, the information is inaccuracy and supposed to linguistic representation, the estimation of precise values of probability becomes very difficult in many cases. In order to handle the insufficient information, the fuzzy approach [1] is used to evaluate the failure rate status.

Singer [2] presented a fuzzy set approach for fault tree and the reliability analysis in which the relative frequencies of the basic events are considered as fuzzy numbers.

Cai et al. [3] pointed out that there are two fundamental assumptions in the conventional reliability theory, i.e.

(a)Binary state assumptions: the system is precisely defined as functioning or failing.

(b) Probability assumptions: the system behavior is fully characterized in the context of probability measures.

However, because of the inaccuracy and uncertainties of data, the estimation of precise values of probability becomes very difficult in many systems. Cai et al. [4] presented the following two assumptions:

(a’) Fuzzy-State assumption: the meaning of the system failure can’t be precisely defined in a reasonable way. At any time the system may be in one of the following two states: fuzzy success state or fuzzy failure state.

(b’) Possibility assumption: the system behavior can be fully characterized in the con-

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text of possibility measures.

Cai et al. [5] presented the following three forms of "fuzzy reliability theories".

(i) Profust reliability theory, based on the probability assumption and fuzzy-state assumption.

(ii) Posbist reliability theory, based on the possibility assumption and binary-state assumption.

(iii) Posfust reliability theory, based on the possibility assumption and the fuzzy-state assumption.

Cheng and Mon [6] used interval of confidence for analyzing the fuzzy system reliability. Through theoretical analysis and computational results they have shown that their proposed approach is more general and straight-forward compare to Singer [2].

Chen [7] presented a new method for analyzing the fuzzy system reliability using fuzzy number arithmetic operations and used simplified fuzzy arithmetic operations rather than complicated interval fuzzy arithmetic operations of fuzzy numbers [6] or the complicated extended algebraic fuzzy numbers [2].

Chen [8] presented a new method for fuzzy system reliability analysis based on fuzzy time series and the α-cuts arithmetic operations of fuzzy numbers.

Gau and Buehrer [9] extended the idea of fuzzy set by vague set. Chen [10] presented the arithmetic operations between vague sets. Chen [11] proposed a new method for analyzing the fuzzy system reliability based on vague sets.

Kumar et al. [12, 13] extended the concept of triangular vague set [11, 14] by idea of trapezoidal vague set and proposed new methods for analyzing the fuzzy system reliability.

So far, in the literature, arithmetic operations between same types of vague sets are discussed. Also to analyze the fuzzy system reliability, it is assumed that the reliability of all components of a system follows the same membership functions. However, in practical problems, such type of situations rarely occurs. Therefore, it is need of a method by which we can also find the fuzzy reliability of systems having components following different type of membership functions.

In the present paper, a new algorithm has been introduced to perform various arithmetic operations between different types of vague sets. Further using the proposed algorithm, the methods are obtained to analyze the fuzzy reliability of various systems. Our study generalize the various work [6-8, 10-15] of the literature.

To illustrate the above approach the fuzzy reliability of series, parallel, parallel-series and series-parallel systems all consisting of four components has been evaluated using the proposed algorithm.

2. Brief review of vague sets

In this section brief review of vague sets has been presented.

2.1. Why vague sets instead of fuzzy sets? [14]

In some circumstances, the experts can't express the fuzziness under confirmable confidence level when a new type of product is developed. It usually occur in military weapon, because the new type of weapon system is usually based on previous product, and a lot of factors would influence weapon systems operation; but these factors usually have some uncertainty and linguistic ambiguity, such as:

• Most of weapon systems are too expensive or dangerous to measure experimentally. Instead, expert opinion is used to provide the fault information, but estimates usually are uncertain and the representations are supposed to be using linguistic.

• The normal or abnormal condition of system is incomplete defined, because weapon systems can operate their functions under some limited conditions, but can not rely on 100% system reliability.

• Weapon systems are constructed from different mechanical, electronic, and special materials. We cannot rule out any possibility on system failures including power systems, nature reasons, manual mistakes, and human factors.

Therefore, we suggest using vague set instead of fuzzy sets to analyze the fuzzy reliability of such type of systems.. The vague set can solve this kind of problem when the experts just can assign the range of failure events under un-confirmable confidence level.

2.2. Definition of a vague set [9]

Let X be the universe of discourse. A vague set $\tilde{V}$ over X is characterized by a truth membership function $t_{\tilde{V}}$, $t_{\tilde{V}} : X \to [0,1]$, and a false membership function $f_{\tilde{V}}$, $f_{\tilde{V}} : X \to [0, 1]$. If generic element of X is denoted by 'x_i' then the lower bound on the membership grade of x_i derived from evidence for x_i is denoted by $t_{\tilde{V}}(x_i)$ and the lower bound on the negation of x_i is denoted by $f_{\tilde{V}}(x_i)$, $t_{\tilde{V}}(x_i)$ and $f_{\tilde{V}}(x_i)$ both associate a real number in the interval [0,1] with each point x_i in X ,where $t_{\tilde{V}}(x_i) + f_{\tilde{V}}(x_i)$ ≤ 1. A vague set $\tilde{V}$ in the universe of discourse X is shown in Figure 1.

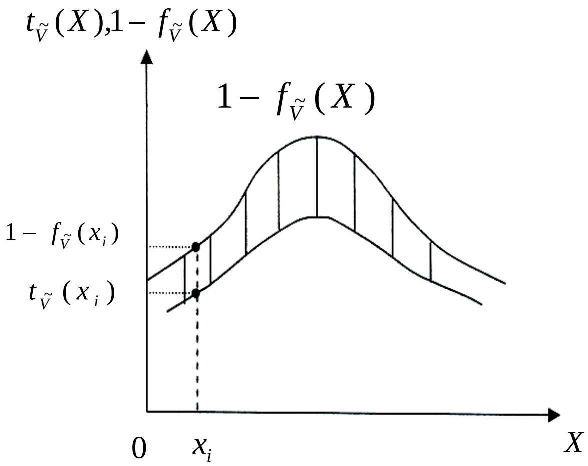

Figure 1. A vague set

when X is continuous, a vague set $\tilde{V}$ can be written as

$$\tilde{V} = \int_X [t_{\tilde{V}}(x_i), 1 - f_{\tilde{V}}(x_i)] / x_i, x_i \in X$$

when X is discrete a vague set $\tilde{V}$ can be written as

$$\tilde{V} = \sum_{i=1}^{n} [t_{\tilde{V}}(x_i), 1 - f_{\tilde{V}}(x_i)] / x_i, x_i \in X$$

3. Proposed algorithm to perform certain arithmetic operations between different types of vague sets

Till now, in the literature, no algorithm has been proposed to perform arithmetic operations between different types of vague sets.

In this section, an algorithm has been proposed to perform certain arithmetic operations between different types of vague sets $\tilde{V}_1, \tilde{V}_2, \cdots, \tilde{V}_n$.

Step 1

Find the supremum value of $t_{\tilde{V}_i}(X)$ and $1 - f_{\tilde{V}_i}(X)$, $i = 0,1,2,\ldots,n$

supremum value of $t_{\tilde{V}_i}(X) = \max_{x\in X}\{t_{V_i}(x)\}$

supremum value of $1 - f_{\tilde{V}_i}(X) = \max_{x\in X}\{1 - f_{V_i}(x)\}$.

Let the supremum value of $t_{\tilde{V}_i}(X)$ and $1 - f_{\tilde{V}_i}(X)$ are α_i and β_i respectively.

Step 2

Find the minimum value of α_i and β_i $\forall$ $i = 0,1,2,\ldots,n$. Let the minimum values are α and β respectively.

Step 3

Find the intervals for certain values of $t_{\tilde{V}_i}(X)$ lying in $[0,\alpha]$.Let the interval for $t_{\tilde{V}_i}(X) = p, 0 \le p \le \alpha$ is $[x_{i1}, x_{i2}]$, where $x_{i1}, x_{i2} \in X$.

Step 4

Find the intervals for certain values of $1 - f_{\tilde{V}_i}(X)$ lying in $[0,\beta]$. Let the interval for $1 - f_{\tilde{V}_i}(X) = q, 0 \le q \le \beta$ is $[x'_{i1}, x'_{i2}]$, where $x'_{i1}, x'_{i2} \in X$.

Step 5

Define addition, multiplication and subtraction of all vague sets for $t_{\tilde{V}}(X) = p$ as

$\left[\sum_{i=1}^{n} x_{i1}, \sum_{i=1}^{n} x_{i2}\right]$, $\left[\prod_{i=1}^{n} x_{i1}, \prod_{i=1}^{n} x_{i2}\right]$ and $\left[x_{11} - (\sum_{i=2}^{n} x_{i2}), x_{12} - (\sum_{i=2}^{n} x_{i1})\right]$ respectively,

where $\tilde{V}$ represents the resultant vague set.

Step 6

Define addition, multiplication and subtraction of all vague sets for $1 - f_{\tilde{V}}(X) = q$ as

$\left[\sum_{i=1}^{n} x'_{i1}, \sum_{i=1}^{n} x'_{i2}\right]$, $\left[\prod_{i=1}^{n} x'_{i1}, \prod_{i=1}^{n} x'_{i2}\right]$ and $\left[x'_{11} - (\sum_{i=2}^{n} x'_{i2}), x'_{12} - (\sum_{i=2}^{n} x'_{i1})\right]$ respectively.

Step 7

Draw the membership functions of the resultant vague sets after finding the intervals for certain values of p (including 0 and α) and q (including 0 and β).

4. Fuzzy reliability calculations of a series, parallel, parallel-series and series-parallel systems

In this section, a new approach has been developed for analyzing the fuzzy reliability of series, parallel, parallel-series and series-parallel systems, where the reliabilities of the components of a system are represented by different types of vague sets defined on the universe of discourse [0, 1].

4.1. Series system

Consider a series system consisting of ‘n’ components as shown in Figure2. The fuzzy reliability $\tilde{R}_S = \bigotimes_{i=1}^{n} \tilde{R}_i$ of the series system shown in Figure 2 can be evaluated using the algorithm proposed in section 3 for multiplication.

where $\tilde{R}_i$ represents the reliability of the i^{th} component.

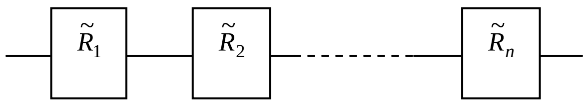

Figure 2. Series system

4.2. Parallel system

Consider a parallel system consisting of ‘n’ components as shown in Figure 3.

The fuzzy reliability $\tilde{R}_P = 1 \ominus \bigotimes_{i=1}^{n} (1 \ominus \tilde{R}_i)$ of the parallel system shown in Fig. 3 can be evaluated using the algorithm proposed in section 3 for multiplication and subtraction.

where $\tilde{R}_i$ represents the reliability of the i^{th} component.

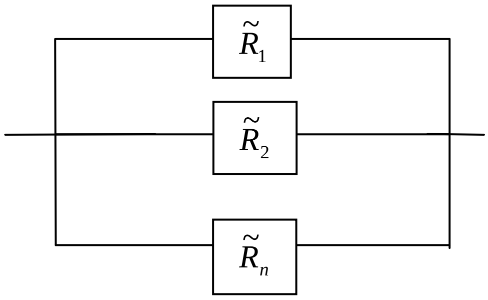

Figure 3. Parallel system

4.3. Parallel-series system

Consider a parallel-series system consisting of 'm' branches connected in parallel and each branch contains 'n' components as shown in Figure 4. The fuzzy reliability $\tilde{R}_{PS} = 1 \ominus \overset{m}{\underset{k=1}{\otimes}}(1 \ominus (\overset{n}{\underset{i=1}{\otimes}}\tilde{R}_{ki}))$ of the parallel-series system shown in Figure 4 can be evaluated using the algorithm proposed in section 3 for multiplication and subtraction.

where $\tilde{R}_{ki}$ represents the reliability of the i^{th} component at k^{th} branch.

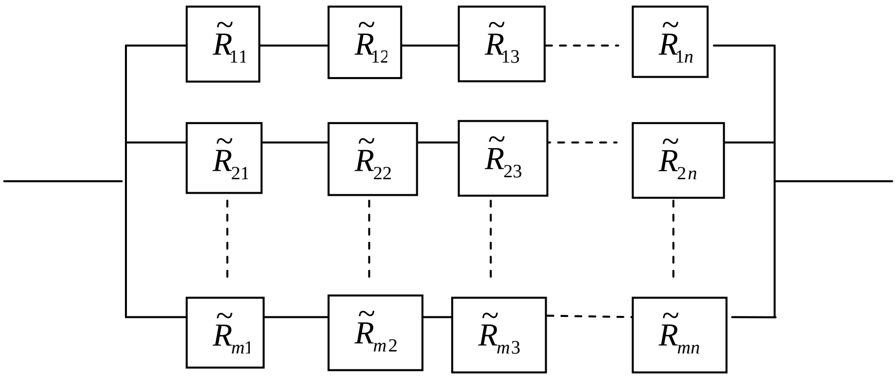

Figure 4. Parallel-series system

4.4. Series-parallel system

Consider a series-parallel system consisting of 'n' stages connected in series and each stage contains 'm' components as shown in Figure 5. The fuzzy reliability $\tilde{R}_{SP} = \overset{n}{\underset{k=1}{\otimes}}(1 \ominus \overset{m}{\underset{i=1}{\otimes}}(1 \ominus \tilde{R}_{ik}))$ of the series-parallel system shown in Figure 5 can be evaluated using the algorithm proposed in section 3 for multiplication and subtraction.

where $\tilde{R}_{ik}$ represents the reliability of the i^{th} component at k^{th} stage.

5. Numerical Examples

To illustrate the approaches developed in section 3 and section 4, fuzzy reliability $\tilde{R}_S$ of a series system (Figure 6), $\tilde{R}_P$ of a parallel system (Figure 7), $\tilde{R}_{PS}$ of a parallel-series system (Figure 8) and $\tilde{R}_{SP}$ of a series-parallel system (Figure 9) all consisting of four components has been evaluated.

Let the reliabilities of the components are represented by different types of vague sets $\tilde{R}_1, \tilde{R}_2, \tilde{R}_3$ and $\tilde{R}_4$ defined on the universe of discourse R, where R represents the crisp reliability interval [0,1].The assumed values of $\tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_4$, the expressions for calculating values of $\tilde{R}_S, \tilde{R}_P, \tilde{R}_{PS}$, $\tilde{R}_{SP}$ are presented in Table 1 and the membership functions representing $\tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_4$ are shown in Figure 10, Figure 11, Figure 12, Figure 13 respectively.

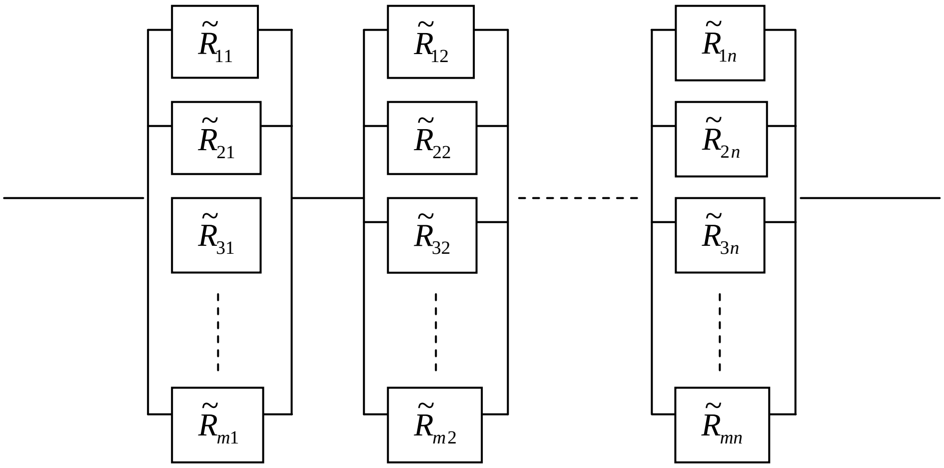

Figure 5. Series-parallel system

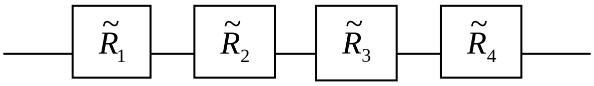

Figure 6. Series system

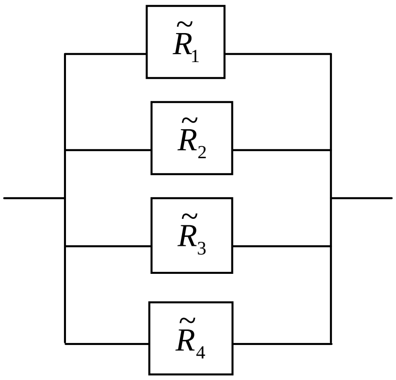

Figure 7. Parallel system

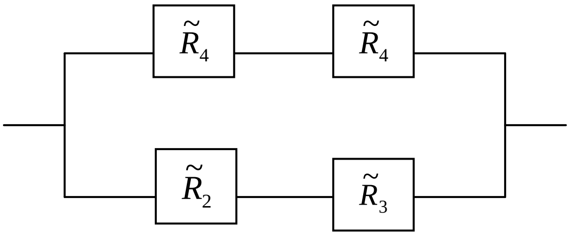

Figure 8. Parallel-series system

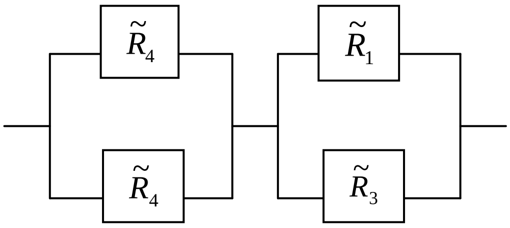

Figure 9. Series-parallel system

Table 1. Fuzzy reliability of components

Fuzzy Reliability $\tilde{R}_i$ of the ith component	Type of vague set
$\tilde{R}_1 =< [(0.2,0.4,0.6); 0.7 ; 0.8] >$	Triangular vague set [11]
$\tilde{R}_2 =< [(0..3,0.5,0.7,0.9); 0.6 ; 0.9] >$	Trapezoidal vague set [12]
$\tilde{R}_3 =< [(0.6,0.7,0.8); 0.5],[(0.5,0.7,0.9); 0.7] >$	Triangular vague set [14]
$\tilde{R}_4 =< [(0.2,0.3,0.4,0.5); 0.4],[(0.1,0.3,0.4,0.6); 0.6] >$	Trapezoidal vague set [13]
$\tilde{R}_S = \tilde{R}_1 \otimes \tilde{R}_2 \otimes \tilde{R}_3 \otimes \tilde{R}_4$	
$\tilde{R}_P = 1\Theta(1\Theta \tilde{R}_1) \otimes (1\Theta \tilde{R}_2) \otimes (1\Theta \tilde{R}_3) \otimes (1\Theta \tilde{R}_4)$	
$\tilde{R}_{PS} = 1 \ \Theta\ (1\Theta(\tilde{R}_1 \otimes \tilde{R}_2)) \otimes (1\Theta(\tilde{R}_3 \otimes \tilde{R}_4))$	
$\tilde{R}_{SP} = (1\Theta(1\Theta \tilde{R}_1) \otimes (1\Theta \tilde{R}_2)) \otimes (1\Theta(1\Theta \tilde{R}_3) \otimes (1\Theta \tilde{R}_4))$	

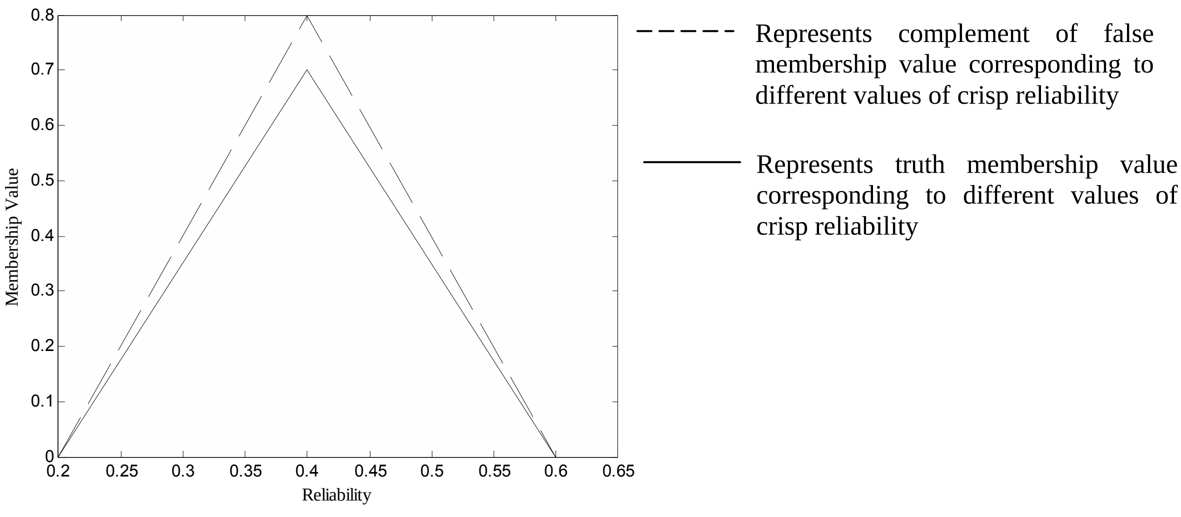

Figure 10. Membership function representing $\tilde{R}_1$

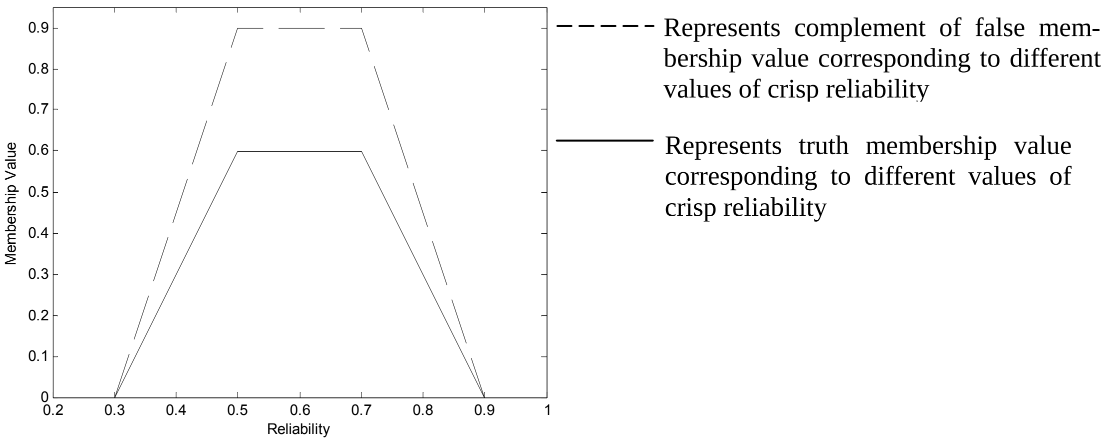

Figure 11. Membership function representng $\tilde{R}_2$

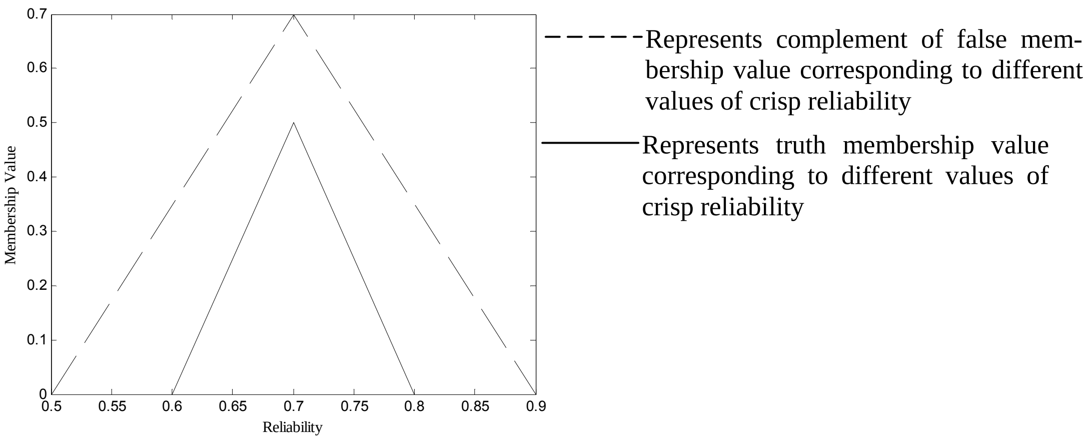

Figure 12. Membershop function representing $\tilde{R}_3$

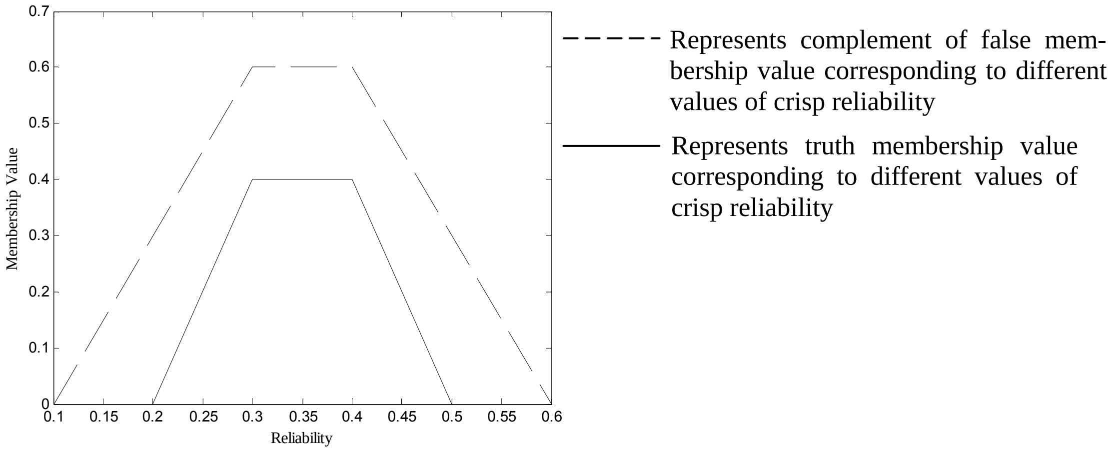

Figure 13. Menbership function representint $\tilde{R}_4$

Step 1

From the proposed values of $\tilde{R}_1, \tilde{R}_2, \tilde{R}_3$ and $\tilde{R}_4$ the values of $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3$ and β_4 are obtained as follows:

$\alpha_1 = 0.7$, $\alpha_2 = 0.6$, $\alpha_3 = 0.5$, $\alpha_4 = 0.4$ and $\beta_1 = 0.8$, $\beta_2 = 0.9$, $\beta_3 = 0.7$ $\beta_4 = 0.6$

Step 2

$\alpha = \min(0.7,0.6,0.5,0.4) = 0.4$ and $\beta = \min(0.8,0.9,0.7,0.6) = 0.6$

Step 3

The reliability intervals corresponding to reliabilities of all components have been calculated for certain values of $t_{\tilde{R}_i}(R)$ lying in $[0,0.4]$ and presented in Table 2.

Table 2. Reliability intervals for certain values of $t_{\tilde{R}_i}(R)$

Value of $t_{\tilde{R}_i}(R)$	Reliability interval corresponding to $\tilde{R}_1$	Reliability interval corresponding to $\tilde{R}_2$	Reliability interval corresponding to $\tilde{R}_3$	Reliability interval corresponding to $\tilde{R}_4$
0	[0.200,0.600]	[0.300,0.900]	[0.600,0.800]	[0.200,0.500]
0.1	[0.229.0.571]	[0.333,0.867]	[0.620,0.780]	[0.225,0.475]
0.2	[0.257,0.543]	[0.367,0.833]	[0.640,0.760]	[0.250,0.450]
0.3	[0.286,0.514]	[0.400,0.800]	[0.660,0.740]	[0.275,0.425]
0.4	[0.314,0.486]	[0.433,0.767]	[0.680,0.720]	[0.300,0.400]

Step 4

The reliability intervals corresponding to reliabilities of all components have been calculated for certain values of $1-f_{\tilde{R}_i}(R)$ lying in $[0,0.6]$ and presented in Table 3.

Table 3. Reliability intervals for certain values of $1-f_{\tilde{R}_i}(R)$

Value of $1-f_{\tilde{R}_i}(R)$	Reliability interval corresponding to $\tilde{R}_1$	Reliability interval corresponding to $\tilde{R}_2$	Reliability interval corresponding to $\tilde{R}_3$	Reliability interval corresponding to $\tilde{R}_4$
0	[0.200,0.600]	[0.300,0.900]	[0.500,0.900]	[0.100,0.600]
0.1	[0.225.0.575]	[0.322,0.878]	[0.529,0.871]	[0.133,0.567]
0.2	[0.250,0.550]	[0.344,0.856]	[0.557,0.843]	[0.167,0.533]
0.3	[0.275,0.525]	[0.367,0.833]	[0.586,0.814]	[0.200,0.500]
0.4	[0.300,0.500]	[0.389,0.811]	[0.614,0.786]	[0.233,0.467]
0.5	[0.325,0.475]	[0.411,0.789]	[0.643,0.757]	[0.267,0.433]
0.6	[0.350,0.450]	[0.433,0.767]	[0.671,0.729]	[0.300,0.400]

Step 5

The reliability intervals of series, parallel, parallel-series and series-parallel systems (Figure 6-9) have been calculated using Table1, Table 2, Table 3 and Section 3 for certain values of $t_{\tilde{R}_i}(R)$ lying in [0,0.4] and the results are presented in Table 4.

Step 6

The reliability intervals of series, parallel, parallel-series and series-parallel systems (Figure 6-9) have been calculated using Table1, Table 2, Table 3 and Section 3 for certain values of $1-f_{\tilde{R}_i}(R)$ lying in [0,0.6] and the results are presented in Table 5.

Table 4. Reliability intervals for certain values of $t_{\tilde{R}_i}(R)$

Values of $t_{\tilde{R}_i}(R)$	Reliability interval corresponding to $\tilde{R}_S$	Reliability interval corresponding to $\tilde{R}_P$	Reliability interval corresponding to $\tilde{R}_{PS}$	Reliability interval corresponding to $\tilde{R}_{SP}$
0	[0.007,0.216]	[0.821,0.996]	[0.173,0.724]	[0.299,0.864]
0.1	[0.011.0.183]	[0.849,0.993]	[0.205,0.682]	[0.343,0.834]
0.2	[0.015,0.155]	[0.873,0.990]	[0.239,0.640]	[0.387,0.802]
0.3	[0.021,0.129]	[0.894,0.985]	[0.275,0.596]	[0.431,0.768]
0.4	[0.028,0.107]	[0.913,0.980]	[0.312,0.553]	[0.474,0.732]

Table 5. Reliability intervals for certain values of $1 - f_{\tilde{R}_i}(R)$

Values Of $1 - f_{\tilde{R}_i}(R)$	Reliability interval corresponding to $\tilde{R}_S$	Reliability interval corresponding to $\tilde{R}_P$	Reliability interval corresponding to $\tilde{R}_{PS}$	Reliability interval corresponding to $\tilde{R}_{SP}$
0	[0.003,0.292]	[0.748,0.998]	[0.107,0.788]	[0.242,0.922]
0.1	[0.005.0.249]	[0.785,0.997]	[0.138,0.749]	[0.281,0.895]
0.2	[0.008,0.212]	[0.818,0.995]	[0.171,0.709]	[0.321,0.867]
0.3	[0.012,0.178]	[0.848,0.993]	[0.206,0.666]	[0.362,0.835]
0.4	[0.017,0.149]	[0.873,0.989]	[0.243,0.624]	[0.403,0.802]
0.5	[0.023,0.123]	[0.896,0.985]	[0.282,0.580]	[0.445,0.767]
0.6	[0.031,0.101]	[0.915,0.979]	[0.322,0.536]	[0.486,0.730]

Step 7

The membership functions representing the fuzzy reliabilities of series, parallel, parallel-series and series-parallel systems (Figure 6-9) are shown in Figure 14, Figure 15, Figure 16 and Figure 17 respectively.

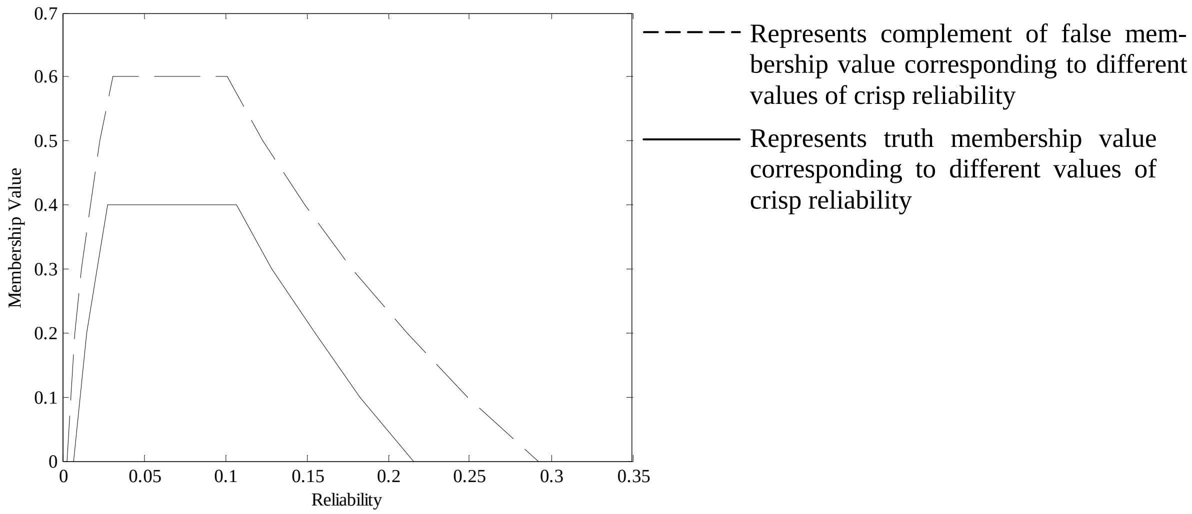

Figure 14. Membershop function representing $\tilde{R}_S$

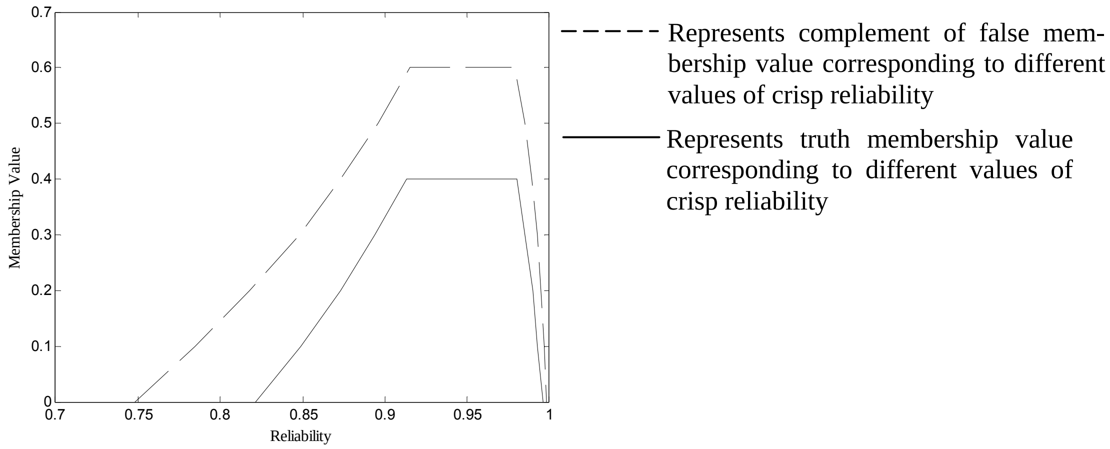

Figure 15. Membership function representing $\tilde{R}_P$

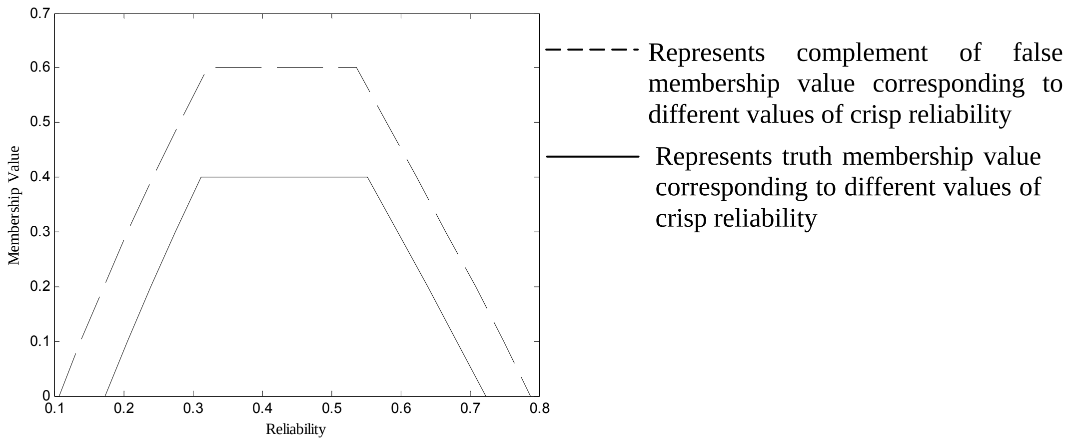

Figure 16. Membership function representing $\tilde{R}_{PS}$

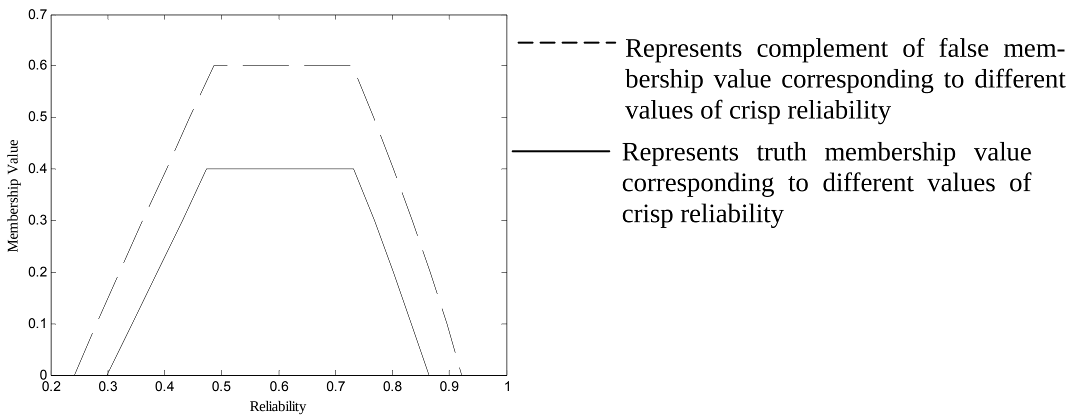

Figure 17. Membership function representing $\tilde{R}_{SP}$

6. Conclusions

Till now, in the literature there is no algorithm to perform arithmetic operations between different types of vague sets.

In this paper, we have proposed an algorithm to perform various arithmetic operations between different types of vague sets. Several known results [10-14, 16] are the particular cases of the proposed algorithm.

Further, a new approach has been developed for analyzing the fuzzy reliability of series, parallel, parallel-series and series-parallel systems. In the proposed approach the reliabilities of the components of a system have been represented by different types of vague sets so it is more flexible than the ones presented in [11-14, 16].

Moreover, in this paper vague sets are used to analyze the fuzzy system reliability which generalizes the approaches presented in [6-8, 15].

One can also use our proposed algorithm to analyze the fuzzy reliability of a non series-parallel system.

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