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# Study on the Optimal Tilt Angle of Solar Collector According to Different Radiation Types

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**Abstract:** Resulting from the shortage of fossil fuels, solar energy plays a more and more important role in power supply in future. The amount of radiation flux incident upon a solar collector is mainly affected by the installation angle. Proper design of a collector can increase the irradiation received. In this paper, the optimal angle in Taiwan is calculated according to three different radiation types, i.e. the extraterrestrial radiation, global radiation predicted by empirical model and ten-year observation data from 1990 to 1999. Some differences among them are analyzed considering the geographic and climatic factors. The results show that the angles calculated from the extraterrestrial and predicted radiations are simply latitude-dependent and thus can be well determined, but the angles estimated from observation data vary from location to location and are generally flatter than those from other two radiation types. It tells us that the collector must be installed with a flatter tilt angle when it works in a cloudy or pollutant environment.

**Keywords:** Solar radiation; Solar collector; Clearness index; Optimal tilt angle

## 1. Introduction

Solar energy is a treasure resource in nature and plays an important role in power supply in future resulting from the shortage of fossil fuels. Solar energy can be utilized directly through a variety of devices such as solar collector or photovoltaic (PV) cell. Solar radiation incident on a collector is composed of three components, i.e. the direct beam, diffusion and reflection from the ground, which have different dependence on the slope of collector, the sum of these three components is called global radiation. Installing a collector properly can enhance its application benefit because the amount of radiation flux incident upon the collector is mainly affected by the azimuth and tilt angles that it is installed. Generally, in the northern hemisphere the best azimuth is due south (facing equator), but the tilt angle varies with factors such as the geographic latitude, climate condition, utilization period of time, etc.

It is known that the irradiation received during the day is a fundamental parameter concerning the research in meteorology, biomass energy and solar energy conversion, especially for the sizing of stand-alone PV systems. Additionally the proportion of direct beam or diffusion component within the global irradiation will affect the optimal tilt angle of collector. The optimal angle is calculated by searching for the value with which the solar radiation upon the collector is the maximum for a particular period of time.

In the past, many researchers have devoted

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to studying the optimal installation angle in the world. Saraf and Hamad [1] found the yearly optimum tilt angle in Basra, Iraq was higher than the latitude by about $8^o$. Both Gopinathan [2] and Soulayman [3] showed the optimum tilt angle is almost equal to the latitude. Nijegorodov et al. [4] proposed a set of 12 expressions for determining the monthly optimal tilt angle for solar collectors that lay between latitude $60^o$ north and $60^o$ south. Gunerhan and Hepbasli [5] found the optimal angles for solar collectors in Izmir, Turkey were in good agreement with those calculated by the expressions of Nijegorodov et al. [4]. However, Yakup and Malik [6] showed that the optimal tilt angles in Brunei Darussalam were almost less than the ones from Nijegorodov et al. [4], because the radiation pattern changes from location to location. Additionally the angles obtained for summer months are negative values suggesting that the best collector orientation is facing north in this season. Shariah et al. [7] showed that the yearly optimum angle in Jordan is less than the latitude by about $5\sim8^o$. Chow and Chan [8] analyzed the irradiation data measured at the coastal region of South China and found the collector should be mounted with a tilt angle greater than the latitude by $2.8^o$ for an annual utilization period and the optimal azimuth is southwest.

The research concerning Taiwan is seldom found in literatures. Chen et al. [9] had estimated the monthly optimal installation angle for solar cell in Chiayi, Taiwan, according to the climatology data from 1995 through 1998, by means of genetic algorithm and simulated annealing methods. However the climatic conditions elsewhere in Taiwan may differ with Chiayi. In order to build a useful database improving the application of solar energy, it is necessary to study more about what the best installation angle is around Taiwan.

In this study, the daily irradiation of horizontal surface observed in Taiwan over a 10-year period from 1990-1999, conducted by the Central Weather Bureau, was selected to analyze. The location of six meteorological stations used in this study with the related parameters such as longitude (in degree, east), latitude (degree, north) and altitude (meter, over the sea) is demonstrated in Figure 1. Meantime, the extraterrestrial radiation under air mass zero (AM0, suitable for outer space application) as well as the global radiation predicted by previously empirical model will be included for highlighting the characteristics of actual observation data. The empirical model just mentioned could be considered as the representative of worldwide average. The validity of the empirical model to Taiwan area and the effect of diffusion proportion on the tilt angle will be investigated too. The flow chart for calculating the optimal tilt angle is shown in Figure 2.

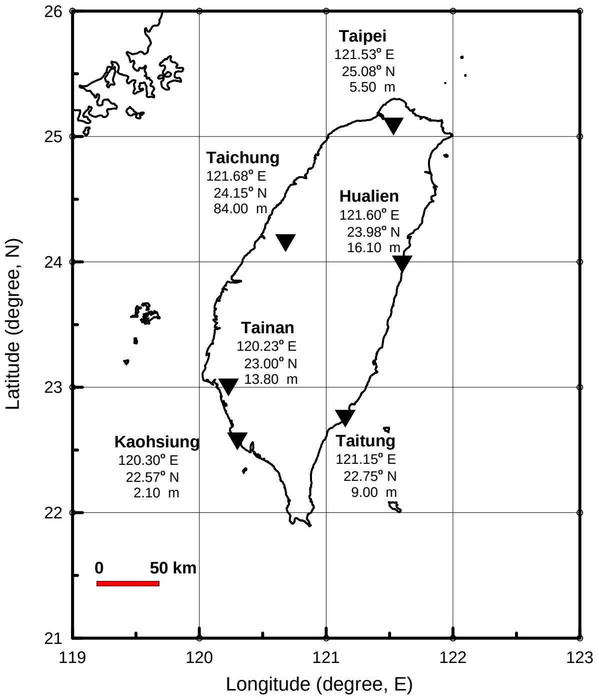

Figure1. Meteorological stations used in this study.

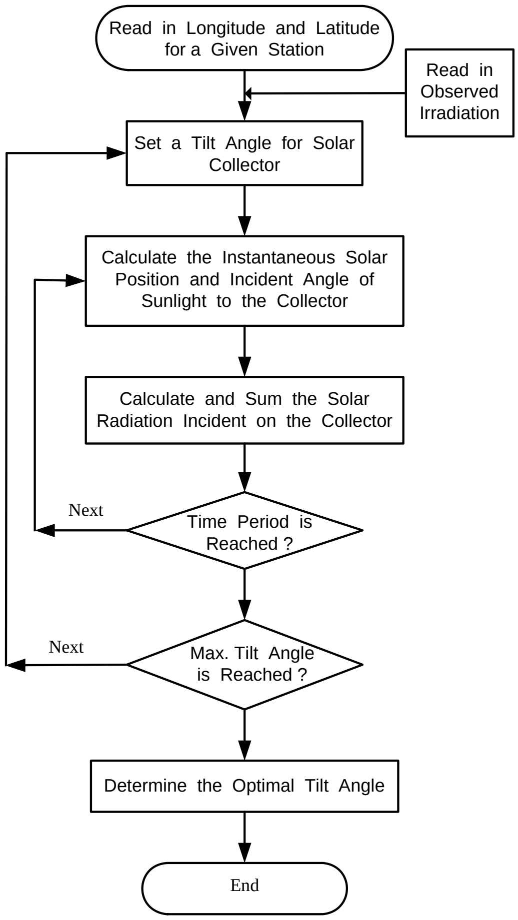

Figure 2. Flow chart for calculating optimal tilt angle.

## 2. Global Radiation Incident on Solar Collector

### 2.1. Predicted by the Empirical Model of Previous Researchers

As shown in Figure 3, the global radiation ($I_t$) incident on a south-facing collector tilted at an angle $\beta$ to the horizontal surface can be calculated according to the model of Liu and Jordan [10] as:

$$I_t = I_{cb} R_b + I_{cd}\left(1+\cos\beta\right)/2 + I_c \rho\left(1-\cos\beta\right)/2 \tag{1}$$

The tilt angle $\beta$ is positive for due south but negative for due north and zero for horizon. The geometric factor, $R_b$, is the ratio of beam radiation on the tilted surface to that on a horizontal surface as given by:

$$R_b = \cos\theta / \cos\theta_z \tag{2}$$

Where,

$$\cos\theta = \sin\theta_z \cos\psi \sin\beta + \cos\theta_z \cos\beta \tag{3}$$

$$\cos\theta_z = \sin\delta \sin\phi + \cos\delta \cos\phi \cos\omega \tag{4}$$

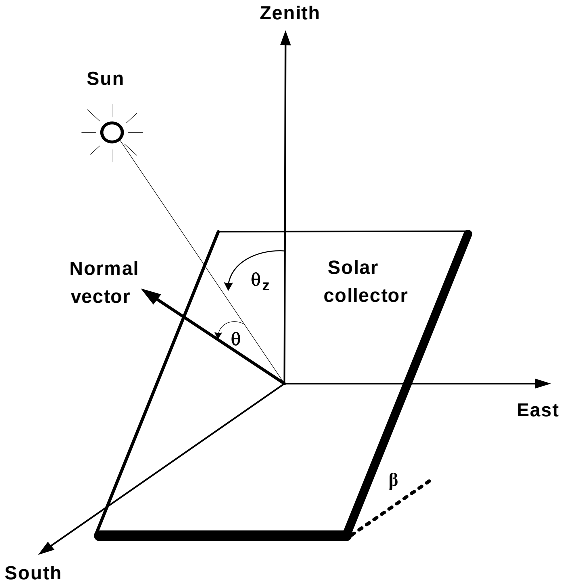

Figure 3. Geometry of south-facing solar collector.

Where, $\theta$ is the instantaneous angle between direct beam and the normal of collector, $\theta_z$ is zenith angle [11]. $\phi$ is the geographic latitude. Solar hour angle $\omega$ is 15 degrees per hour, zero at solar noon, morning negative, afternoon positive. $\psi$ is azimuth of the sun measured from due south, positive for morning, negative for afternoon. Solar declination $\delta$ is the angle between the line joining the centers of the sun and the earth and the equatorial plane:

$$\cos\psi = (\cos\theta_z \sin\phi - \sin\delta) / \sin\theta_z \cos\phi \tag{5}$$

$$\delta = 23.45\sin(2\pi(284 + dn)/365.25) \qquad (6)$$

Where, $dn$ is the day number counted from January first throughout the year (1-365). $\rho$ is the ground reflection coefficient (albedo), which is assumed to be 0.2 in this study. The clear sky radiation on horizontal surface, $I_c$, is the sum of $I_{cb}$ and $I_{cb}$:

$$I_c = I_{cb} + I_{cd} \qquad (7)$$

$I_{cb}$ is the clear sky beam radiation on horizontal surface, which can be calculated by the expression of Hottel [12]:

$$I_{cb} = I_o \tau_b \cos\theta_z \qquad (8)$$

Where, $\tau_b$ is the atmosphere transmittance for beam radiation, which is equal to the ratio of the beam radiation on horizontal surface to the air mass zero (AM0) extraterrestrial radiation, and can be approached by the following empirical formulas.

$$\tau_b = a_o + a_1 \exp(-k/\cos\theta_z) \qquad (9)$$

The constants $a_o = r_o a_o^*$, $a_1 = r_1 a_1^*$ and $k = r_k k^*$ for the standard atmosphere with 23 km visibility are calculated from the following relationship assuming that the observation altitude is less than 2.5 km:

$$\begin{aligned} a_o^* &= 0.4237 - 0.00821(6.0 - H)^2 \\ a_1^* &= 0.5055 + 0.00595(6.5 - H)^2 \qquad (10) \\ k^* &= 0.2711 + 0.01858(2.5 - H)^2 \end{aligned}$$

Where, $H$ is the altitude of observer in kilometers. The correction parameters $r_o$, $r_1$ and $r_k$ are related to climate conditions as summarized in Table 1.

Table 1. Atmospheric parameters for different climate types.

| Climate type | $r_o$ | $r_1$ | $r_k$ |
|---|---|---|---|
| Tropical | 0.95 | 0.98 | 1.02 |
| Mid-latitude summer | 0.97 | 0.99 | 1.02 |
| Sub-arctic summer | 0.99 | 0.99 | 1.01 |
| Mid-latitude winter | 1.03 | 1.01 | 1.00 |

The accumulated extraterrestrial beam radiation at AM0 for a period of time can be obtained by the integration:

$$I_O = \int S_c(1 + 0.033\cos(2\pi dn/365.25))dt \qquad (11)$$

Here, $S_c$ is solar constant (1367 W/m$^2$). The diffusion component of clear sky radiation on horizontal surface, $I_{cb}$, can be estimated using the model of Liu and Jordan [13]:

$$I_{cd} = I_o \tau_d \cos\theta_z \qquad (12)$$

$$\tau_d = 0.2710 - 0.2939\tau_b \qquad (13)$$

Where, $\tau_d$ is the atmosphere transmittance of diffusion radiation, and the accumulated extraterrestrial radiation on a tilt panel, $I_{ext}$, is given by:

$$I_{ext} = I_o \cos\theta \qquad (14)$$

### 2.2. Observed

The observation data used in the present study is given in the form of daily global irradiation on horizontal surface, i.e. the accumulated radiation during the day ($G$). For an area the instantaneous solar radiation is not available, the related methodologies could be adopted for estimation. Firstly the simple but accurate enough correlations proposed by Page [14] were applied to calculate the diffusion ($D$) and direct beam ($B$) components from the observation data ($G$) as follows:

$$D = G(1 - 1.13K_T) \tag{15}$$

$$B = G - D \tag{16}$$

Where $K_T$ is the clearness index as:

$$K_T = G / E_O \tag{17}$$

Here, $E_O$ is the irradiation received by a unit of horizontal area outside the earth's atmosphere throughout the day:

$$E_O = \frac{24}{\pi} S_C (1 + 0.033\cos(2\pi dn / 365.25)) \times (\pi / 180)(\omega_S \sin\delta \sin\phi + \cos\delta \cos\phi \sin\omega_s) \tag{18}$$

Where $\omega_s$ is the sunrise (or sunset) hour angle given by:

$$\omega_s = \cos^{-1}(-\tan\delta \tan\phi) \tag{19}$$

Secondly, the methodology called sine model was applied to spread the observed daily irradiation to any instant of daytime, because the variation of radiation intensity from sunrise to sunset within a day is very close to a sine curve [15]. The radiation at any instant is thus nearly equal to the daily irradiation multiplied by a normalized sine function coefficient $n\_sc$ as:

$$\begin{aligned} I_c &= C * n\_sc \\ I_{cb} &= B * n\_sc \\ I_{cd} &= D * n\_sc \end{aligned} \tag{20}$$

Where,

$$n\_sc = \sin\zeta_{inst} / \sum \sin\zeta_{inst} \tag{21}$$

$\zeta_{inst}$ is the instant daytime angle varied from $0^o$ (sunrise) to $180^o$ (sunset) with arbitrary increment of time. Although the daytime length changes day by day, but $\zeta_{inst}$ is always set to be $90^o$ at solar noon. Substituting Eq. (20) into Eq. (1), the observed instantaneous global radiation on a tilt panel can be obtained.

## 3. Results and Discussions

To illustrate the differences of solar radiations obtained from different sources, Figure 4 to Figure 9 shows three types of daily irradiation incident on the horizontal ground surface throughout the year. Where the tilt angle used in the calculation of Eq. (1) is zero (i.e. $\beta = 0^\circ$). The observed data is the averaged value over the ten years from 1990 to 1999, and is fitted with a sinusoidal curve by the method of least squares. The monthly optimal tilt angle of solar collector for different radiation types is demonstrated in Figure 10 to Figure 15, the corresponding clearness index for different periods is listed in Table 2.

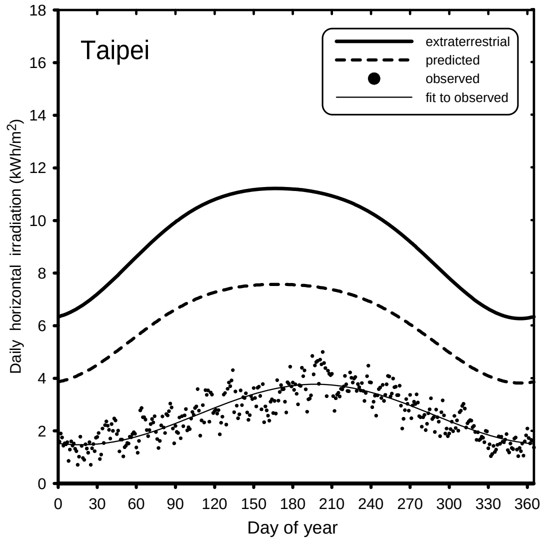

Figure 4. Daily irradiation of horizontal surface in Taipei.

Table 2. Sky clearness index calculated from predicted and observed radiations for different periods.

| Period | Radiation | Taipei | Taichung | Tainan | Kaohsiung | Hualien | Taitung |
|---|---|---|---|---|---|---|---|
| Jan. | Predicted | 0.6178 | 0.6206 | 0.6238 | 0.6251 | 0.6210 | 0.6246 |
| | Observed | 0.2066 | 0.4028 | 0.3679 | 0.3483 | 0.3114 | 0.3925 |
| Feb. | Predicted | 0.6388 | 0.6409 | 0.6434 | 0.6441 | 0.6412 | 0.6438 |
| | Observed | 0.2253 | 0.3476 | 0.3527 | 0.3356 | 0.2823 | 0.3382 |
| Mar. | Predicted | 0.6584 | 0.6597 | 0.6612 | 0.6618 | 0.6599 | 0.6615 |
| | Observed | 0.2352 | 0.3369 | 0.3492 | 0.3533 | 0.3118 | 0.3996 |
| Apr. | Predicted | 0.6702 | 0.6709 | 0.6717 | 0.6720 | 0.6709 | 0.6718 |
| | Observed | 0.2527 | 0.3325 | 0.3424 | 0.3539 | 0.3395 | 0.3969 |
| May | Predicted | 0.6744 | 0.6748 | 0.6750 | 0.6751 | 0.6748 | 0.6751 |
| | Observed | 0.2787 | 0.3370 | 0.3493 | 0.3674 | 0.3646 | 0.4392 |
| Jun. | Predicted | 0.6750 | 0.6751 | 0.6753 | 0.6753 | 0.6752 | 0.6754 |
| | Observed | 0.2928 | 0.3479 | 0.3529 | 0.3649 | 0.4737 | 0.5133 |
| Jul. | Predicted | 0.6747 | 0.6750 | 0.6753 | 0.6753 | 0.6750 | 0.6753 |
| | Observed | 0.3567 | 0.3728 | 0.3715 | 0.3718 | 0.5695 | 0.5673 |
| Aug. | Predicted | 0.6723 | 0.6729 | 0.6736 | 0.6736 | 0.6729 | 0.6735 |
| | Observed | 0.3453 | 0.3564 | 0.3576 | 0.3603 | 0.5127 | 0.5385 |
| Sep. | Predicted | 0.6636 | 0.6647 | 0.6660 | 0.6664 | 0.6648 | 0.6662 |
| | Observed | 0.3398 | 0.3728 | 0.3713 | 0.3572 | 0.4183 | 0.4789 |
| Oct. | Predicted | 0.6464 | 0.6481 | 0.6502 | 0.6509 | 0.6484 | 0.6506 |
| | Observed | 0.2963 | 0.4112 | 0.3667 | 0.3480 | 0.3805 | 0.4718 |
| Nov. | Predicted | 0.6237 | 0.6266 | 0.6295 | 0.6307 | 0.6270 | 0.6301 |
| | Observed | 0.2907 | 0.4066 | 0.3674 | 0.3529 | 0.3939 | 0.4662 |
| Dec. | Predicted | 0.6098 | 0.6129 | 0.6166 | 0.6179 | 0.6134 | 0.6174 |
| | Observed | 0.2381 | 0.3955 | 0.3481 | 0.3432 | 0.3312 | 0.4250 |
| Yearly | Predicted | 0.6565 | 0.6575 | 0.6587 | 0.6591 | 0.6577 | 0.6589 |
| | Observed | 0.2855 | 0.3649 | 0.3579 | 0.3562 | 0.4007 | 0.4586 |
| | Observed | 11.0[a] | 17.5 | 14.9 | 13.3 | 10.8 | 13.4 |
| | Observed | 0.756[b] | 0.624 | 0.688 | 0.772 | 0.788 | 0.797 |

[a] Yearly optimal tilt angle.

[b] Coefficient of determination of fitting curve.

It is found that the extraterrestrial irradiation (AM0, without any shading) is latitude-dependent rather than longitude-dependent, the optimal angle calculated is thus very consistent to each other as long as the latitudes of stations are similar, e.g. Taichung vs. Hualien or Kaohsiung vs. Taitung. Note that the angles suitable for winter season, near 50$^{o}$, are much greater than those for other seasons due to the lowest solar elevation in this season. The negative angles appeared in May, June and July mean that the optimal orientation of collector is due north in summer, since the sun's apparent position is mostly staying in the northern hemi sky for Taiwan area in this season. Indeed the optimal angles of six stations are too consistent to distinguish them, because the latitude of island Taiwan ranges only 3.5$^{o}$ from south to north.

The same characteristics mentioned above are also found in the radiation predicted by empirical model, but the optimal angles calculated from the predicted radiation are slightly gentler than the ones from extraterrestrial radiation in winter and in summer. The clearness index calculated is about 0.66 annually for all the stations.

As for the observation data, the irradiations observed in Taiwan are generally less than the predicted irradiations that could be considered as the representative of the worldwide average. For example, in Taipei, it has the minimal irradiation among six stations due to its cloudy or rainy day or pollutants that will have a lower clearness index (near 0.29 annually) and leads to a higher ratio of diffusion within the global solar radiation. As a result the collector must be adjusted to a flatter tilt angle to collect more solar energy because the diffuse component is an isotropic radiation in the sky dome. As shown in Figure 10, the monthly angle estimated from observed data in Taipei reveals the maximum discrepancy with those from other radiation types among six stations. The optimum angle for annual period is 11.0$^{o}$, and the R-squared determination coefficient of fitting curve is 0.76.

As shown in Figure 5, the distribution of the irradiation observed in Taichung throughout the year presents a most flat shape among six stations. The shapes of irradiations observed in Tainan and in Kaohsiung (Figure 6 and Figure 7) are most coincided with the predicted ones. Meanwhile the clearness indices calculated at these two stations have the steadiest performance, about 0.36. The irradiations observed in Hualien and in Taitung (Figure 8 and Figure 9) are significantly greater than those in western Taiwan, especially in summer, due to their mostly sunny day, because the two stations locate in the leeward side of the southwest monsoon. Therefore the optimal angles obtained from observation data in Hualien and Taitung in summer are closer to those from extraterrestrial radiation as compared to other four stations. Similar situation appears too in winter Taitung.

Generally speaking, the optimal angles obtained from observation data exhibit an obvious variation among the stations. Meanwhile the variation is bigger than those from other radiation types particularly in summer and in winter seasons. For the northern hemisphere, the sun's apparent position will reach a lower elevation in winter, thus for a station located in higher latitude, e.g. Taipei, it should have a steeper optimum angle in winter for capturing more solar energy, but it doesn't. On the contrary, station Taitung has a steeper optimum angle in winter even it locates in lower latitude. This is caused by climate or weather factors that affect the ratio of diffusion to direct beam and result in different optimum angles.

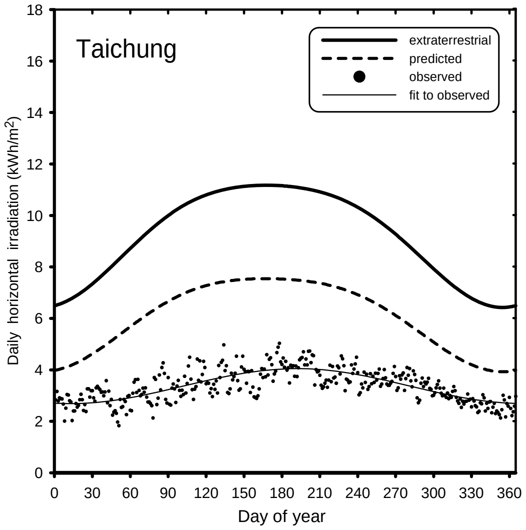

Figure 5. Daily irradiation of horizontal surface in Taichung.

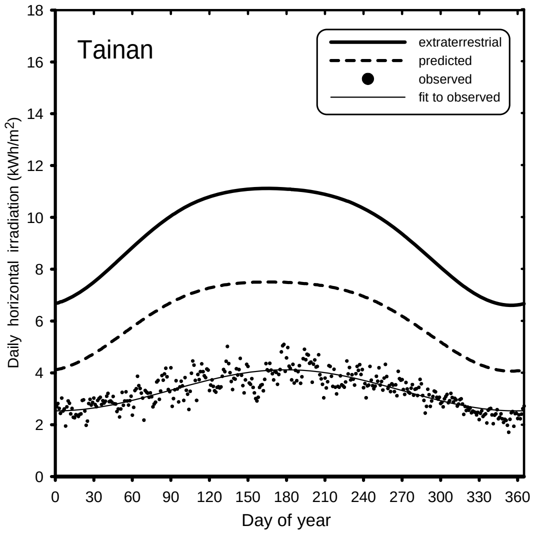

Figure 6. Daily irradiation of horizontal surface in Tainan.

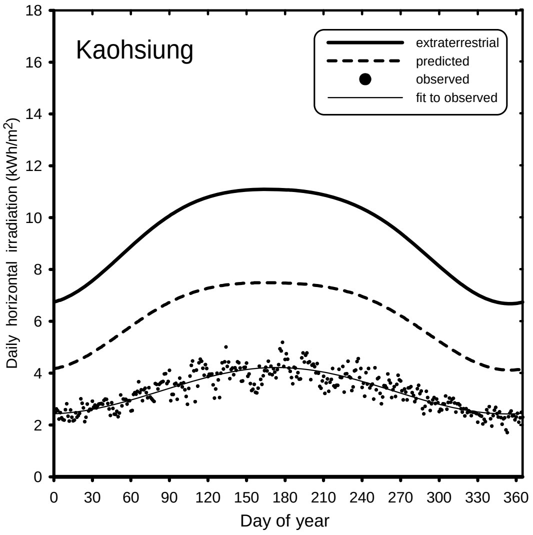

Figure 7. Daily irradiation of horizontal surface in Kaohsiung.

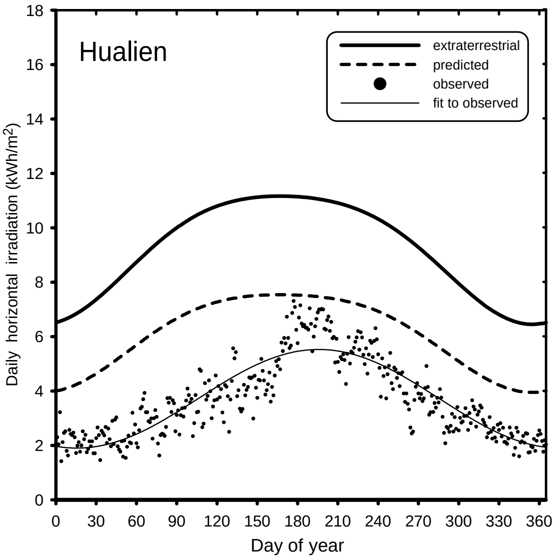

Figure 8. Daily irradiation of horizontal surface in Hualien.

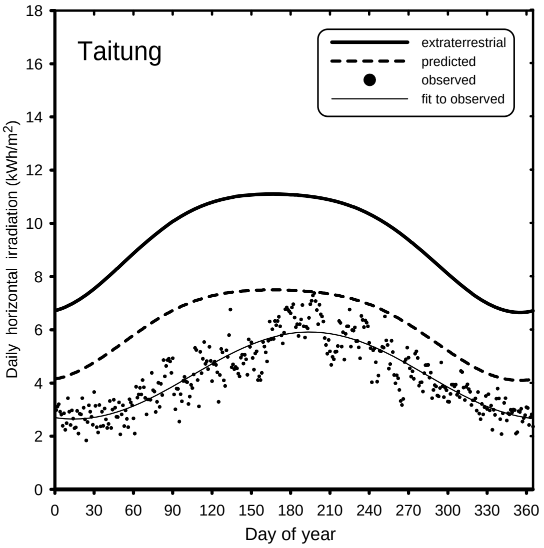

Figure 9. Daily irradiation of horizontal surface in Taitung.

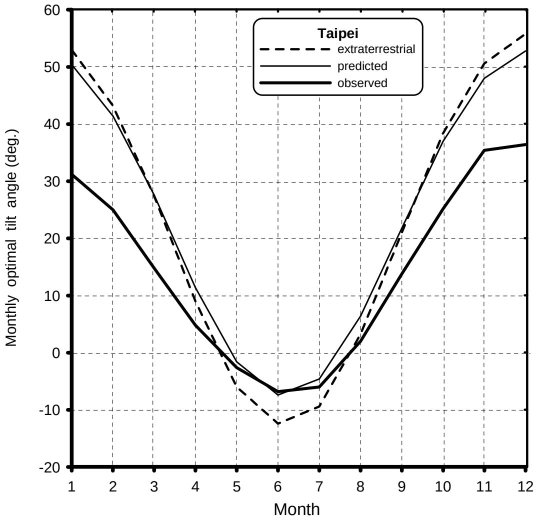

Figure 10. Monthly optimal tilt angle for Taipei.

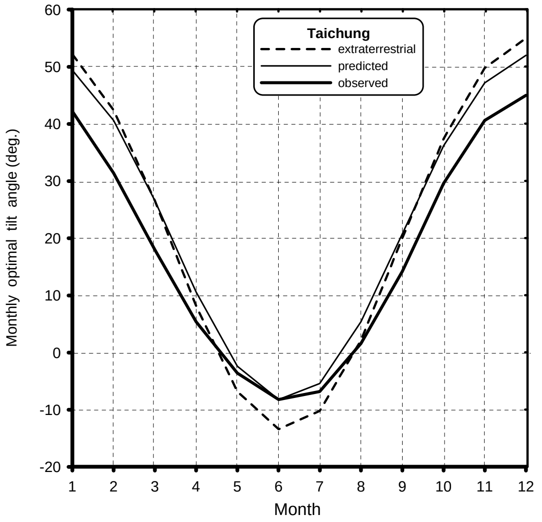

Figure 11. Monthly optimal tilt angle for Taichung.

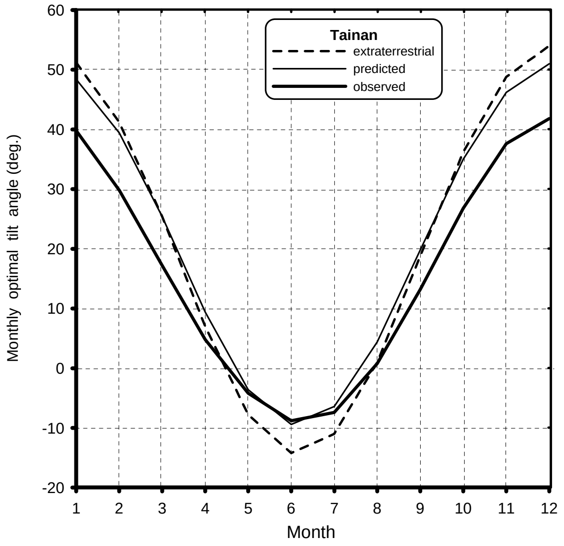

Figure 12. Monthly optimal tilt angle for Tainan.

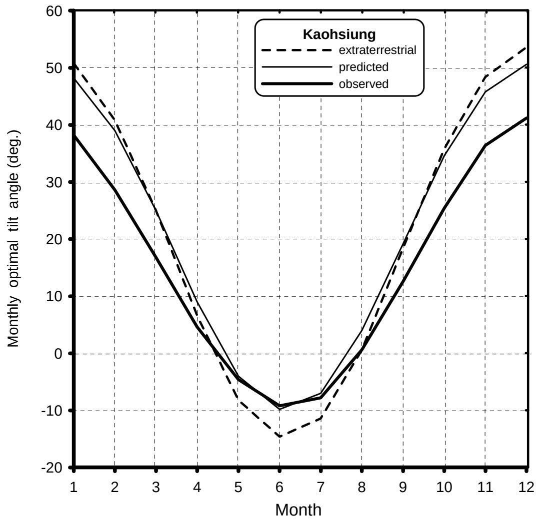

Figure 13. Monthly optimal tilt angle for Kaohsiung.

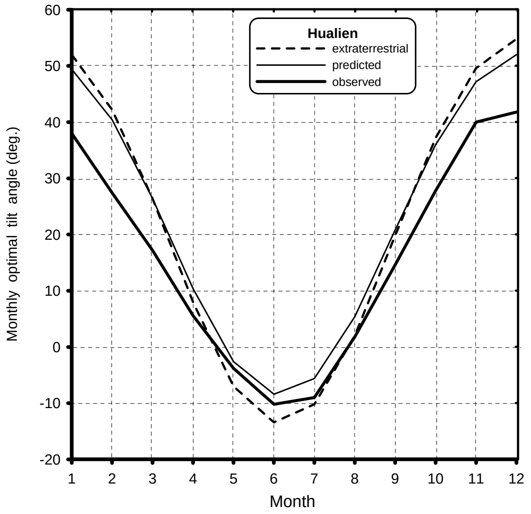

Figure 14. Monthly optimal tilt angle for Hualien.

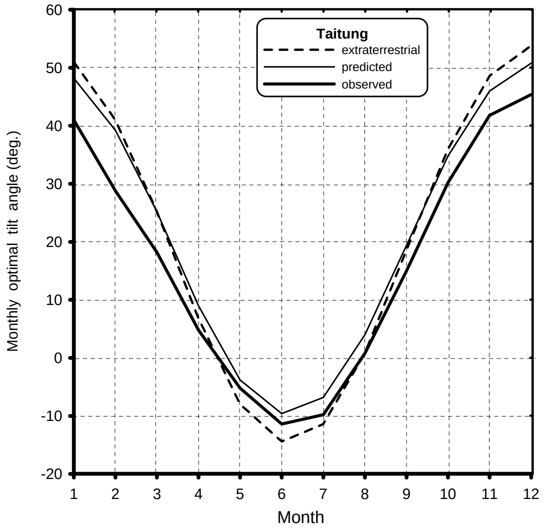

Figure 15. Monthly optimal tilt angle for Taitung.

## 4. Conclusions

In this paper, the optimal tilt angle of solar collector in Taiwan is analyzed according to three different radiation types. The results show that the angles concerning about the extraterrestrial and predicted radiations can be well determined since they are simply latitude-dependent, but the angles estimated from observation data vary significantly from location to location and are generally flatter than those from other two radiation types. For an operating environment if the clouds or pollutants are enriched the collector must be adjusted to a flatter tilt angle. The climate or weather conditions may change year by year, but the results achieved by the present study according to a set of ten-year observation data would be the best recommendation for engineers to install solar collector.

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