Estimate the magnetization of nano-ferrofluid with the simple pendulum’s oscillation

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Abstract: A non-destroy technology (NDT), simple pendulum’s model, to estimate the magnetization of the ferrofluid is introduced in this study. For this new testing method, the magnetic fluid, isolated in a closed volume to prevent the test sample from being exposed to ambient environment or interacted with any active medium, is suspended to the massless string to behave a simple harmonic oscillation. Thus the pollution of sample can be completely avoided during the measuring process. By regulating the input DC power voltage, various Gauss distributions of magnetic field are then axially generated along the finite solenoid immediately and then a magnetic pressure inside the ferro-particles will be induced to counteract its gravity effect. Consequently, the oscillating frequency of the simple pendulum will be slightly changed due to the loss of the apparent acceleration. From the corresponding period detected, the susceptibility of the test sample, 1.05~1.10, will be estimated within the working region of 5V~30V applied by DC power supply. That is also consistent with the original data, 1.13~1.27, provided by Matsumoto Co. and their relative errors are found to be less than 15 %.

Keywords: magnetization.

1. Introduction

For the past years, new testing technologies have been developed to investigate the physical property of the ferro – sample. However, most of them have suffered a common trouble, the testing pollution arisen by the sample interacting with the machine part or ambient air, seems to be inevitable and uneasy to handle. For example, an experiment with DC magnetic field induced from permanent magnets to predict the magnetization of ferrofluid was conducted in a rotary shaft device [1][2]. Due to the variation of viscosity caused by the rise of temperature during the rotating process, an unreasonable result was then obtained. Recall from above conventional methods based on the FHD Bernoulli theory, several relative equipments are needed to quarantine a safe and stable operation, which not only lowers down the measuring efficiency, but significantly increases the expense of the experiment. A micro/nanoscale pumping device with AC magnetic field was applied to drive the motion of ferrofluid [3][4][5]. Whether the fluid moves opposite to direction of the field (backward pumping) or moves in the same direction of the field (forward pumping) primarily depends on AC magnetic strength. The magnetic intensity produced by AC current frequency usually makes the magnetization M of the ferrofluidics to lag behind the magnetic intensity H, i.e. M is not collinear with H to produce a torque density on the nanoparticle. Under the action of this body
torque, a nanoscale flow field will be gradually formed around the magnetic particles to slow down the process of magnetization and greatly enhance the fluid viscosity. That also results a lower pumping efficiency. Present technology with the micro electromechanical system (MEMS) has been extended to detect the interaction between the magnetic field and nanoparticles [6]. Through the image photographed by SEM or TEM device, agglomeration of particles in the presence of non uniform magnetic field can be clearly captured and transferred to PC. Although the application of the technology of “MEMS” has been proven to be a more effective measurement and can be also extended to other relative fields such as nanopump, nanomixer, nanovale and nanomotor [7], the costly expense of the “MEMS” system seems to be unaffordable in local laboratory for the purpose of general investigation.

In this study, a new NDT method with simple pendulum in harmonic motion to estimate the magnetization of the ferrofluid and its susceptibility is developed. That not only has been identified to be a clean testing way with easy and safe operation, but an approximate result, meets the demand of the experiment, can be also achieved.

2. Analysis

To formulate the magnetic susceptibility $X_m$ corresponding to the oscillating period $T$, relative governing equations accompanied with several reasonable assumptions should be made as follows.

2.1 Assumptions

1. A small angle of the pendulum’s oscillation will be restricted.
2. The ferro-mass is suspended by an unstretched string whose weight can be negligible
3. Linear and collinear magnetization, $M$ is proportional and parallel to magnetic intensity $H$, will have no net hysteresis left during the magnetization process.
4. Induced magnetic field density $B$ varies along the axial direction of solenoid, but is independent to the radial distribution.

3. Governing equations

To estimate the magnetization of the test sample under the external magnetic field density, a small angle oscillating period of simple pendulum in (1) and (2) should be derived in advance.

$$T = 2\pi \sqrt{\frac{l}{g}}$$  \hspace{1cm} (1)

$$T' = 2\pi \sqrt{\frac{l}{g'}}$$  \hspace{1cm} (2)

where $T$, $g$ are the oscillating period of the pendulum and apparent gravity acceleration respectively. Corresponding $T'$, $g'$ are measured under the action of the external voltage applied.

Combined the equation (1) and (2), the relation between the oscillating period of pendulum and apparent gravity acceleration can be expressed in equation (3),(4) respectively.

$$\frac{g'}{g} = \left( \frac{T'}{T} \right)^2$$  \hspace{1cm} (3)

$$\Delta g = g - g' = g \left[ 1 - \left( \frac{T}{T'} \right)^2 \right] \approx g \cdot \frac{2\Delta T}{T}$$  \hspace{1cm} (4)

where $T' - T = \Delta T$

Referred to the theory of ferrohydrodynamics [8], the susceptibility of the ferro-mass $X_m$, caused by the induced magnetic pressure inside the magnetic fluid, can be resolved and arranged as in equation (5), (6) respectively.
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\[ X_m^2 + X_m = \frac{2\mu_0\Delta w}{(B_i^2 - B_o^2)A} \]  

(5)

\[ X_m = \frac{1 + \sqrt{1 + \frac{8\mu_0\Delta w}{(B_i^2 - B_o^2)A}}}{2} \]  

(6)

where \( \Delta w = m\Delta g \), \( A \) is the cross section area of the cylindrical container

4. Experimental procedure

Investigating the dependence of the external DC magnetic field on the gravity effect and oscillating period, a testing mechanism, simple pendulum, is established as shown in Fig.1 and the induced magnetic field inside the solenoid is then distributed in Fig.2. From the sketch in Fig.1 shows that five components constitute the experimental equipment, which are demonstrated as below.

1. Steel support.
2. Length of the string: 800 mm.
3. Container with ferrofluid: magnetic nanoparticles with the sizes of 10 nm~100 nm
4. Solenoid: 2800 coils are wound between the parallel acrylic disks (outer diameter \( \psi 75 \text{mm}, \) inner diameter \( \psi 50 \text{mm} \)).
5. DC power supply: 0~50V.

Once the D.C voltage is applied, a non-uniform magnetic intensity along the axis \( a-i \) of current-carry solenoid will be formed and it behaves just like a Gauss distribution in Fig.2. Due to the action of the non-uniform magnetic field lying at the axis, the induced magnetic intensity difference will be generated inside the test sample to counteract its weight and weak the apparent gravity effect. That implies the oscillating period of the simple pendulum with the ferro-sample suspended will become longer as the small angle of the harmonic motion is specified. Before we start the measuring process, several testing procedures should be undertaken as follow.

1. A solenoid is connected to DC supply.
2. Regulate the input voltage 5V~25V with each increment of 5V.
3. Measure the induced magnetic field \( B \) at the midpoint and endpoint lying on the axis of the solenoid for each input voltage.
4. Set up the experimental mechanism as illustrated in Fig.1, but the container is empty initially.
5. Repeat the step 2 and read the total duration of oscillations with 20 times, and calculate the period respectively.
6. Place the test sample into the container and duplicate the step 5.
7. Estimate the apparent gravity loss by subtracting the period in the step 5 for each input voltage.
8. Calculate the susceptibility \( X_m \) using the equation (6) and equation (5).
9. Heating the test sample up to 50°C and steps (1)~(8) are recurred for each increment of 10°C.
5. Results and Discussion

Based on the Biot-Savart law, the magnetic field intensity inside the solenoid will be linearly induced with the working current. Moreover, the current, obey by Ohm’s law, is also proportional to the external voltage applied. Be consistent with above theory, the experimental results in Fig.3 have shown that besides both magnetic fields at the midpoint and ending point of solenoid exhibit a linear relation with external voltage, the induced value at the midpoint seems to be stronger than that at the ending point, and their discrepancy will be enlarged from 2 mT, 5V to 20 mT, 25 V.

Dealing with the gravity effect counteracted by the induced magnetic difference, the testing results with different temperature in various external voltages is displayed in Fig.4. Apparent gravity loss at 5V, 30 °c is just only 0.1 m/s², but the loss climbs up to 1.8 m/s² for 25 V applied. That means the apparent gravity will be significantly reduced due to the stronger magnetization of the ferrofluid while the higher voltage is specified. On the other hand, the apparent gravity loss for temperature 30°C ~50°C corresponding to voltage 5 V ~ 25 V shows that both distributions, without remarkable deviation, are nearly consistent with each other and the gravity loss arisen by induced magnetic difference seems to be independent to the rise of the temperature. Referred from above results by considering the working voltage and temperature, it can be easily found that the induced magnetic intensity, instead of higher working temperature, has a priority in determining the gravity effect of the test sample.

Corresponding to the gravity loss in Fig.4, measured oscillating period of the simple pendulum in Fig.5, greatly varies form 1.28s for 5 V to 1.70s for 25 V, is strongly influenced by the external voltage applied and the oscillating period arisen from the drop down of apparent gravity in higher voltage applied will be further extended as the calculated solution predicted in equation (2).

Subjected from the measured results of Fig.3 ~ Fig.5, the estimation of susceptibility for ferro-fluid, defined as the magnetization of ferro-particles for unit magnetic intensity applied, has been carried out and illustrated in Fig.6. In which, the degree of the alignment for dipole moments along the electric field can be well described. As to the discussion of susceptibility with external voltage in different temperature, a constant susceptibility $X_m$ , 1.05, has been kept during the working region 5V~25V, and the thermal effect loses its influence on the measured result while the temperature region of 30°C ~ 50°C is specified. From above discussion, a surprising result can be expected, i.e., the degree of magnetization for ferro-mass induced by DC voltage will be much disturbed by thermal agitation in higher temperature, which makes the alignment of dipole moment without a remarkable change. When compared with the susceptibility of test sample, 1.27, provided by Matsumoto Co., their relative errors will be not more than 15 %.

![Figure 3. Induced magnetic field vs. DC voltage](image)
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6. Conclusions

Estimated magnetic intensity inside the finite solenoid in this research is found to be proportional to the input voltage, which fits the Biot – Savart Law. Meanwhile, a stable susceptibility $X_m$ about 1.05 will be accessed by measuring the oscillating period and apparent gravity loss, and its maximum relative error approaches 15% when compared with the commercial sample data 1.13~1.27. Both physical parameters, external voltage and temperature, are believed to have important influence on the susceptibility of the testing sample. However, the voltage plays a decisive role in determining the magnetization under the use of ferro-mass in higher temperature.

The NDT testing method introduced in this study has been identified to be an easy, effective way to measure the susceptibility of the ferro-sample. It not only offers an economical and affordable model for local laboratory, but also the measured results have been identified to be compatible with the experimental data proposed by Matsumoto Co.

7. Nomenclature

- $B$: induced magnetic field [T]
- $B_i$: magnetic field induced at the center of solenoid [T]
- $B_0$: magnetic field induced at one end of solenoid [T]
- $g$: gravity acceleration [m/s$^2$]
- $g'$: apparent gravity acceleration [m/s$^2$]
- $\Delta g$: apparent gravity acceleration loss [m/s$^2$]
- $H$: magnetic field intensity [A/m]
- $l$: length of the string [m]
- $M$: the intensity of magnetization [A/m]
- $M_f$: field-average magnetization [A/m]
- $T$: period of pendulum without input voltage [s]
- $T'$: period of pendulum with input voltage [s]
- $\Delta T$: Period difference [s]
- $\Delta w$: weight loss [N]
- $\mu_0$: permeability of free space [Henery/m]
8. References