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Modal Identification from Field Test and FEM Updating of a Long Span Cable-Stayed Bridge

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Abstract: This paper utilizes a continuous wavelet transform algorithm to identify the dynamic parameters of a cable-stayed bridge under normal traffic and environmental wind fields. The dynamic characteristics were determined by using an identification technique for the Kao Ping Hsi cable-stayed bridge. The modal parameters identified from the field vibration tests were compared with those used in the finite element analysis. The finite element model can then be refined by the experimental results. Next, a comparison between the identified results and the updating finite element results shows reasonable agreements for the first several modes in the two directions, namely, vertical, and transverse directions. Finally, the rational finite element model of Kao Ping Hsi cable-stayed bridge can be established. The finite element model obtained herein were used as the damage index for monitoring the long-term safety of the Kao Ping Hsi cable-stayed bridge under environmental loads in the future.

Keywords: cable-stayed bridge; field test; modal identification; wavelet transform.

1. Introduction

Field vibration test is the most reliable means for determining the dynamic characteristics of an existing bridge. The results obtained in a field test can be adopted to improve the finite element model used, to evaluate the rationality of the original design, or to assess the damage occurring on a bridge after a major wind or earthquake event [1]. The ambient vibration test is easy, practical, and economic. It is the most popular way to perform vibration tests because it utilizes common loadings such as wind, normal traffic and environmental noise as the input [2]. Two primary approaches are normally used to perform the system identification. Identification techniques in the frequency domain are easy and widely accepted, but they do not accurately calculate the spectra for highly damped systems. On the other hand, the system identification in time domain can provide accurate results if the measured signals are pure with low noise [3]. Over the past two decades, an advanced time-frequency algorithm method, called wavelet transform, became well-developed in both theoretical and practical aspects. Various identification methodologies for identifying modal parameters of linear systems have been developed based on discrete or continuous wavelet transforms [4, 5].

In this work, an efficient technique is proposed for identifying dynamic characteristics of a cable-stayed bridge from the data col-

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lected in an ambient vibration test with the application of the continuous wavelet transform. The linear systems of equations of motion among the measured degrees of freedom are established in the wavelet domain. The coefficients of each discrete equation can be determined in the wavelet domain through the least squares approach. Then, the modal parameters are recovered directly from these coefficients by solving an eigenvalue problem. The original finite element model is modified and accordingly refined by the identified results from the field test. Finally, a rational analytical model of the Kao Ping Hsi cable-stayed bridge can thus be established.

2. Tested bridge description and FEM model of original design

The tested bridge is the Kao Ping Hsi cable-stayed bridge which supported by an inverse Y-shaped reinforced concrete pylon. The bridge crosses the middle stream of Kao Ping Hsi connecting Tashu Village at Kaohsiung County and Chiuru Village at Pingtung County. As shown in Fig. 1, this bridge is an asymmtric cable-stayed bridge with a single tower. The asymmetric cable-stayed bridge is erected as a hybrid system, utilizing different materials for main span and side span in order to balance the force systems at both sides. Therefore, the side span was built by heavier concrete material, while the main span was built using lighter steel. A total of 30 sets of cables are used to connect the girders to the pylon in two planes.

A 3-D finite element model was constructed using commercial finite element package SAP2000 to simulate the Kao Ping Hsi cable-stayed bridge. In the finite element model, 33 beam elements are used for modeling the deck, 38 beam elements are used for the pylon, and 28 cable elements were used for the cable. The soil-structure interaction was not considered in the model. A simple hinged support was assumed at abutment (A1) and a roller support was used for the pier P2. Typical material properties for practical designs were used in this analysis. Table 1 lists the first five natural frequencies in both directions obtained from the finite element analysis.

Table 1. The modal analysis of results for the original finite element model

Mode	Vertical	Transverse
	f (Hz)	f (Hz)
1	0.318	0.646
2	0.792	1.40
3	1.046	2.16
4	1.304	2.63
5	2.06	3.02

3. Field experiment using ambient vibration test

Ambient vibration tests were performed on the Kao Ping Hsi cable-stayed bridge to determine its dynamic characteristics under normal traffic and environmental wind loads. To identify the modal shapes of the bridge, a total of 33 measuring stations on the deck was adopted to record responses in the vertical direction. Figure 1 shows that the measuring stations on the deck are setup near each cable anchor position and pier position. In addition, several measuring stations are designated between cable F101 and Pier 2 and between cable B114 and cable F114. Sixteen highly sensitive sensors of servo velocity type were used to measure simultaneously the ambient vibration response of the deck. The resolution can reach the level of 10^{-4} cm/s. The record system is a PC-based portable data acquisition system with 16 channels, which can convert analogue signals to digital data and restore the measurement data. Because of the limited number of channels that can be simultaneously used during testing, the deck was divided into three segments as shown in Fig. 1. All the responses were recorded for a period of ten minutes at a sampling rate of 100 Hz, implying that 60000 data points are recorded in each measurement. It should be noted that the points of overlap at the 11th and 12th stations of the main span, and the 21th and 22th stations of the side span, were used as the bases for linking the data recorded at two adjacent segments. Figure 2 indicates a set of typical measured data and corresponding auto-spectra from the ambient vibration test.

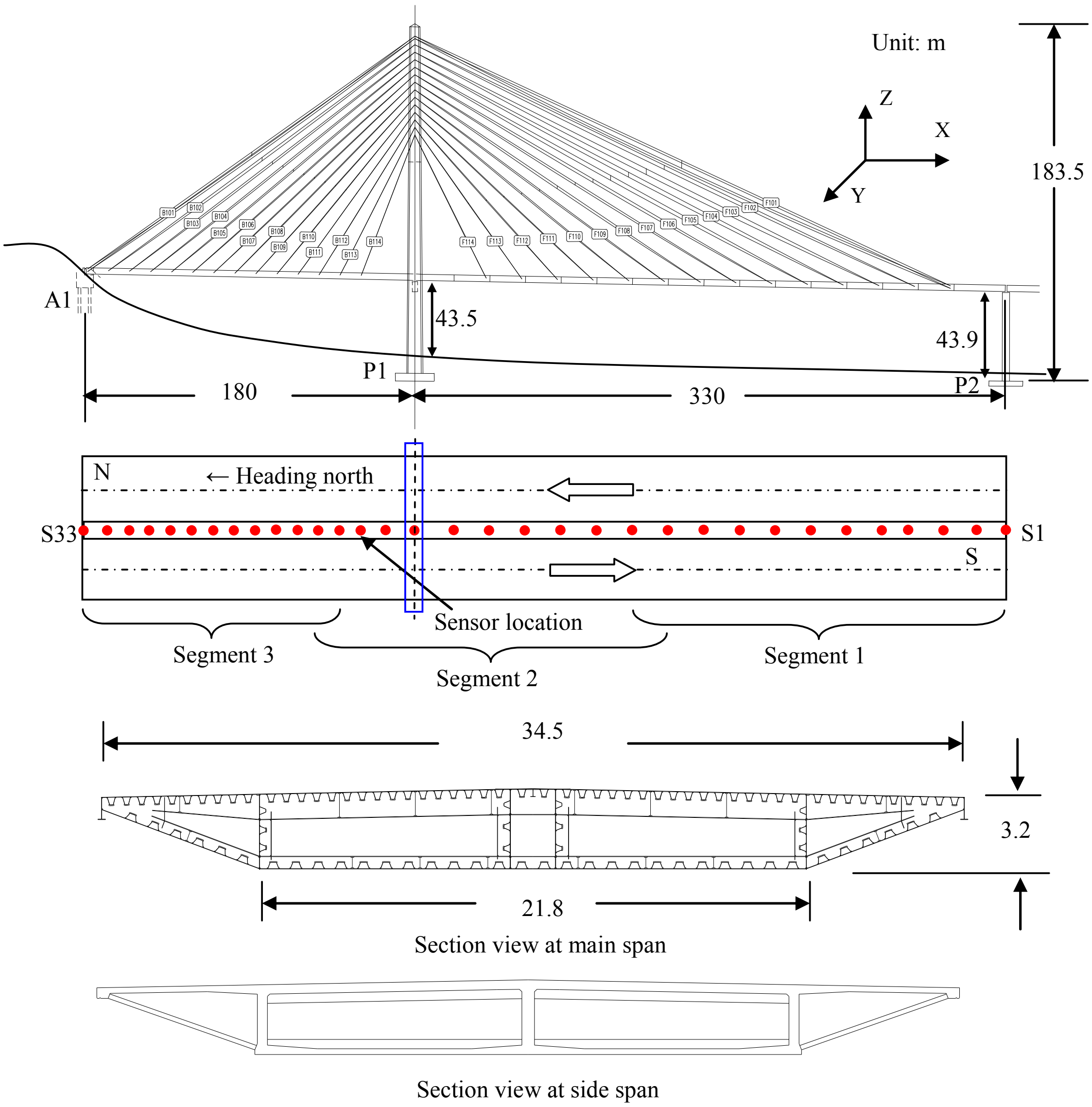
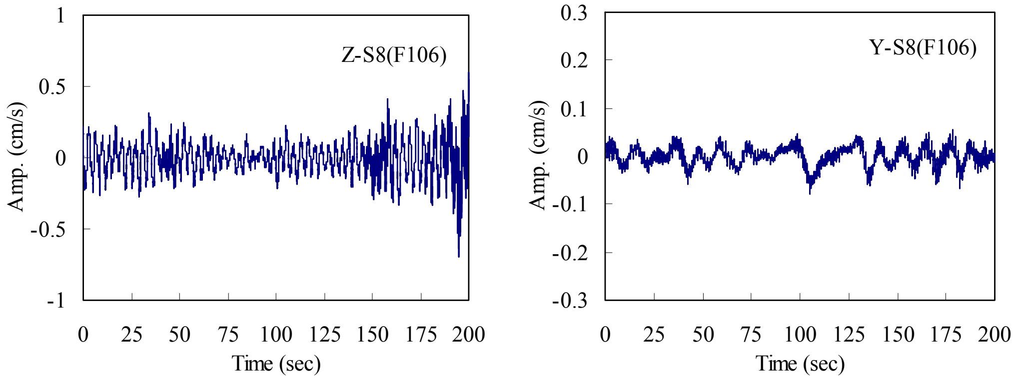
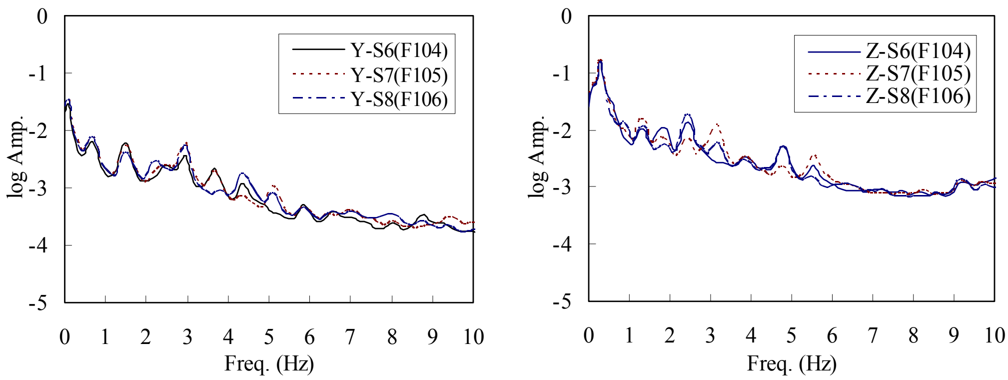


Figure 1. Kao Ping Hsi cable-stayed bridge and sensor layout



(a)



(b)

Figure 2. Typical sets of recorded data for ambient test with corresponding auto-spectra in two directions: (a) time history; (b) corresponding auto-spectra

4. Modal identification

The continuous wavelet transform is applied to the measured dynamic responses of the cable-stayed bridge in the wavelet domain. The dynamic responses are discretized into a set of linear equations of motion according to the measured degrees of freedom. Several scale parameters can be adopted in the transformation to filter out the measured noise. Based on the equivalence relationship, the traditional least squares approach is proposed to determine the characteristic coefficients in the discrete domain. Consequently, the dynamic characteristics of a structure can be determined directly from such coefficients by solving the eigenvalue problem.

4.1. Continuous wavelet transform

Consider a function of time, $f(t)$, in the L^2 space, the corresponding wavelet transform can be defined as [6]

$$W_\psi f(a,b) = \langle f(t), \psi_{(a,b)}(t) \rangle = |a|^{\frac{-1}{2}} \int_R f(t) \psi^* \left(\frac{t-b}{a} \right) dt \tag{1}$$

where $\psi(t)$ is the mother wavelet function. <,> denotes inner product. The superscript * stands for the complex conjugate, respectively. a is a dilation or scale parameter, which is typically a positive real and is equivalent to of the inverse of frequency. Symbol b denotes a translation parameter, which indicates the locality of the transformation. The base function of a mother wavelet function can be written as

$$\psi_{(a,b)}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \tag{2}$$

The inverse of the wavelet transform is determined by

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{a^2} W_f(a,b) \psi\left(\frac{t-b}{a}\right) da db \tag{3}$$

where C_Ψ is the admissibility condition, which can be given by

$$C_\psi = \int_R \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega \tag{4}$$

with $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$. Basically, a mother wavelet function must satisfy the admissibility condition. The wavelet transform decomposes an arbitrary function of time *f*(*t*) into a set of functions $\psi_{a,b}(t)$ for different values *a* and *b*. Therefore, the transformation has a filtering effect because the wavelet transform decomposition alters the frequency contents of *f*(*t*) provided that the scale parameter is fixed.

Applying Fourier transformation to Eq. (1) with a fined scale parameter, *a,* yields

$$\left|\hat{W}_\psi f(a,\omega)\right| = \sqrt{a}|\hat{\psi}(a\omega)| \left|\hat{f}(\omega)\right| \tag{5}$$

where $\hat{f}(\omega)$ and $\hat{W}_\psi f(a,\omega)$ are Fourier transforms of *f*(*t*) and $W_\psi f(a,b)$ with respect to *b*. According to Parseval equation [6], the wavelet transform gives the localized information of the spectrum of *f*(*t*) with a frequency window

$$\left[\frac{\omega^*}{a} - \frac{\hat{\Delta}}{a}, \frac{\omega^*}{a} + \frac{\hat{\Delta}}{a}\right] \tag{6}$$

where ω^*/a is the center of the window and $\hat{\Delta}/a$ is the half width of the window.

4.2. Estimation of Time Series model

The equations of motion of a linear system is given by

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{p(t)\} \tag{7}$$

where [M], [C], and [K] are the mass, damping, and stiffness matrices of the structural system, respectively. $\{\ddot{x}\}$, $\{\dot{x}\}$, and $\{x\}$ are the acceleration , velocity, and displacement vectors of the structure. $\{p(t)\}$ is the external force vector. The equations of motion can be accurately discretised by the impulse invariant transformation. The structural responses often only measure acceleration or velocity responses at some degrees of freedom due to limited instrumentations. The discrete equation of motion corresponding the measured degrees of freedom can be expressed

$$\{y(t)\} = \sum_{i=1,2}^{I} [\Phi]_i \{y(t-i)\} + \sum_{j=0,1}^{J} [\Theta]_j \{p(t-j)\} \tag{8}$$

where $\{y(t-i)\}$ and $\{p(t-i)\}$ are the measured responses, and the forces at *t*-*i*Δ*t*, respectively. $[\Phi]_i$ and $[\Theta]_j$ are unknown coefficient matrices. Symbols *I* and *J* denote the lags of output and input, respectively. The expression of Eq. (8) is similar to the time-series model, ARX. The ARX model equates the equations of motion as given in Eq. (7). For the problem of free decayed vibration responses, the term of external forces would vanish, thus

$$\{\bar{y}(t)\} = \sum_{i=1,2}^{I} [\Phi]_i \{\bar{y}(t-i)\} \tag{9}$$

where $\{\bar{y}(t)\}$ is the Randomdec signatures, and is similar to freely decayed signals. It should be mentioned that Eq. (9) is only valid for free vibration response.

Applying the continuous wavelet transform to Eq. (9) yields

$$W_{\{\bar{y}\}}(a,b)=\sum_{i=1}^{I}[\Phi]_i W_{\{\bar{y}\}}(a,b-i) \tag{10}$$

Carefully constructing Eq. (10) for different b yields

$$\{Y^{(0)}\}=[\widetilde{C}]\{Y\} \tag{11}$$

Where

$$[\widetilde{C}]=\{[\Phi]_1 \quad [\Phi]_2 \quad \cdots \quad [\Phi]_I\} \tag{12}$$

$$\{Y\}=\begin{Bmatrix}\{Y^{(1)}\}^T\\ \{Y^{(2)}\}^T\\ \vdots \\ \{Y^{(I)}\}^T\end{Bmatrix} \tag{13}$$

$$\{Y^{(i)}\}=W_\psi\{\bar{y}(a,bi)\} \tag{14}$$

The unknown coefficient matrix $[\widetilde{C}]$ in the overdeterminate system of Eq. (11) is calculated by using the least squares, and can be determined by

$$[\widetilde{C}]=\{Y^{(0)}\}\{Y\}^T\left(\{Y\} \quad \{Y\}^T\right)^{-1} \tag{15}$$

4.3. Identification of the modal parameter

Equation (11) is similar to the time series model, AR (Auto-Regressive model). The AR model equates the equations of motion. Hence, the dynamic characteristics of the structural system can be obtained from the coefficient matrices of AR, $[\widetilde{C}]$. The modal parameters can be determined from the eigenvalues and eigenvectors of $[G]$, which leads to the following matrix [2, 7].

$$[G]=\begin{bmatrix}0 & I & 0 & \cdots & 0\\ 0 & 0 & I & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & I\\ [\Phi]_I & [\Phi]_{I-1} & [\Phi]_{I-2} & \cdots & [\Phi]_1\end{bmatrix} \tag{16}$$

The eigenvectors of [G] correspond to the mode shapes of the structural system of interest in the form of state variables, whereas the eigenvalues of [G] relate to the natural frequencies and damping ratios. Let λ_k and $\{\psi_k\}$ represent the kth eigenvalue and eigenvector of $[G]$, respectively. The eigenvalue, λ_k, is a complex number, and can thus be expressed as a_k+ib_k. The complex conjugates of λ_k and $\{\psi_k\}$ are also an eigenvalue and eigenvector, respectively. The natural frequency and modal damping of the system, as in Eq. (16) are given by

$$\widetilde{\beta}_k=\sqrt{\alpha_k^2+\beta_k^2} \tag{17}$$

$$\xi_k=-\alpha_k/\widetilde{\beta}_k \tag{18}$$

where $\widetilde{\beta}_k$ is the pseudo-undamped circular natural frequency; ξ_k is the modal damping ratio;

$$\beta_k=\frac{1}{\Delta t}\tan^{-1}\left(\frac{b_k}{a_k}\right) \tag{19}$$

$$\alpha_k=\frac{1}{2\Delta t}\ln(a_k^2+b_k^2) \tag{20}$$

and $1/\Delta t$ is the sampling rate of measurement.

The particular composition of $[G]$ in Eq. (16) is such that its eigenvectors exhibit the following property.

$$\{\psi_k\} = (\{\psi_k\}_1^T, \lambda_k \{\psi_k\}_1^T, \lambda_k^2 \{\psi_k\}_1^T, ..., \lambda_k^{m-1} \{\psi_k\}_1^T)^T \tag{21}$$

where $\{\psi_k\}_1$ is the modal shape of the system that corresponds to the natural frequency, $\tilde{\beta}_k$. Hence, the responses measured in the structure during the field vibration test show that a continuous wavelet transform can be established. The modal parameters of such a structural system can be determined using the foregoing method.

4.4. Selection of wavelet function

For the wavelet transform, several wavelet functions can be applied to, such as the Meyer, Shannon, Morlet and Harr functions as have been studied by Huang and Su [5]. These wavelet functions can be applied to identify forced vibration data. The Meyer function is compactly supported in the frequency domain and smoothes out the discontinuity of the Shannon wavelet function. Thus, the Meyer wavelet function was selected to identify the modal parameters in this work. The Meyer wavelet (see Fig. 3) is defined in the frequency counterpart domain as below [8]

$$\hat{\psi}(\omega) = \begin{cases} (2\pi)^{-\frac{1}{2}} e^{\frac{jw}{2}} \sin\left(\frac{\pi}{2}\nu\left(\frac{3}{2\pi}|\omega| - 1\right)\right) & \frac{2\pi}{3} \le \omega \le \frac{4\pi}{3} \\ (2\pi)^{-\frac{1}{2}} e^{\frac{jw}{2}} \cos\left(\frac{\pi}{2}\nu\left(\frac{3}{2\pi}|\omega| - 1\right)\right) & \frac{4\pi}{3} \le \omega \le \frac{8\pi}{3} \\ 0 & |\omega| \notin \left[\frac{2\pi}{3}, \frac{8\pi}{3}\right] \end{cases} \tag{22}$$

where $\nu(a) = a^4(35 - 84a + 70a^2 - 20a^3)$, and $a \in [0,1]$. The Meyer wavelet and its Fourier modulus are depicted in Fig 3.

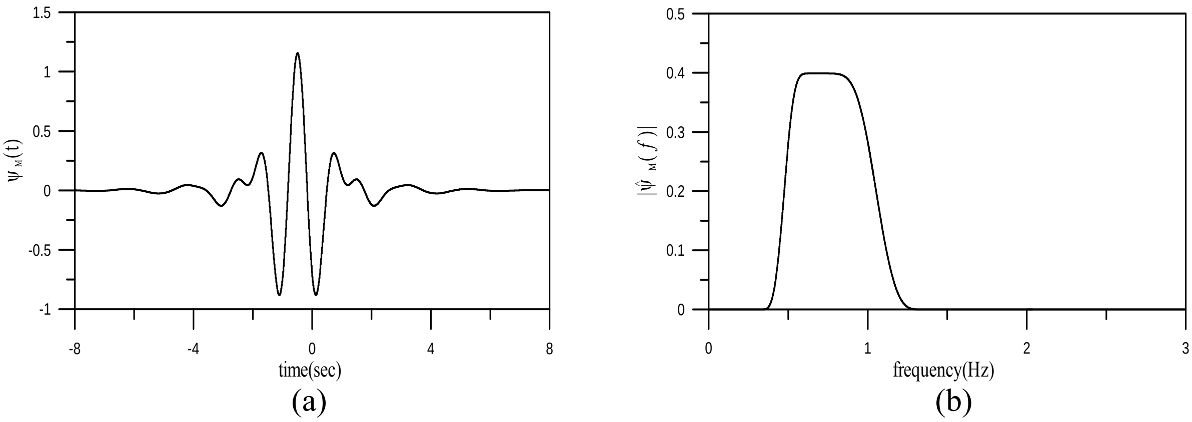


Figure 3. Meyer wavelet function and its Fourier modulus: (a) time domain; (b) frequency domain

5. Data processing and identification results

Each set of velocity data recorded in the ambient vibration test was processed using the random decrement technique to generate the free vibration responses. The free vibration responses obtained are called Randomdec signatures [9]. Then, the foregoing identification method was applied to these Randomdec signatures to yield the dynamic characteristics of the structure. The typical velocity-time history records with the corresponding auto-spectra are illustrated in Fig. 2. The natural frequencies and modal damping ratios of the first five modes in the vertical and transverse directions determined from the ambient vibration test are listed in Table 2 and

Table 3, respectively, while the corresponding modal shapes are shown in Fig. 4 and Fig. 5. Symbols f and ξ are natural frequency and damping ratios in those table, respectively. For such a long span cable-stayed bridge, however, the frequency range of interest lies below 2 Hz. As can be seen, the frequencies of lower modes identified in Table 2 and Table 3 correspond well with those associated with the peaks of the auto-spectra shown in Fig 2.

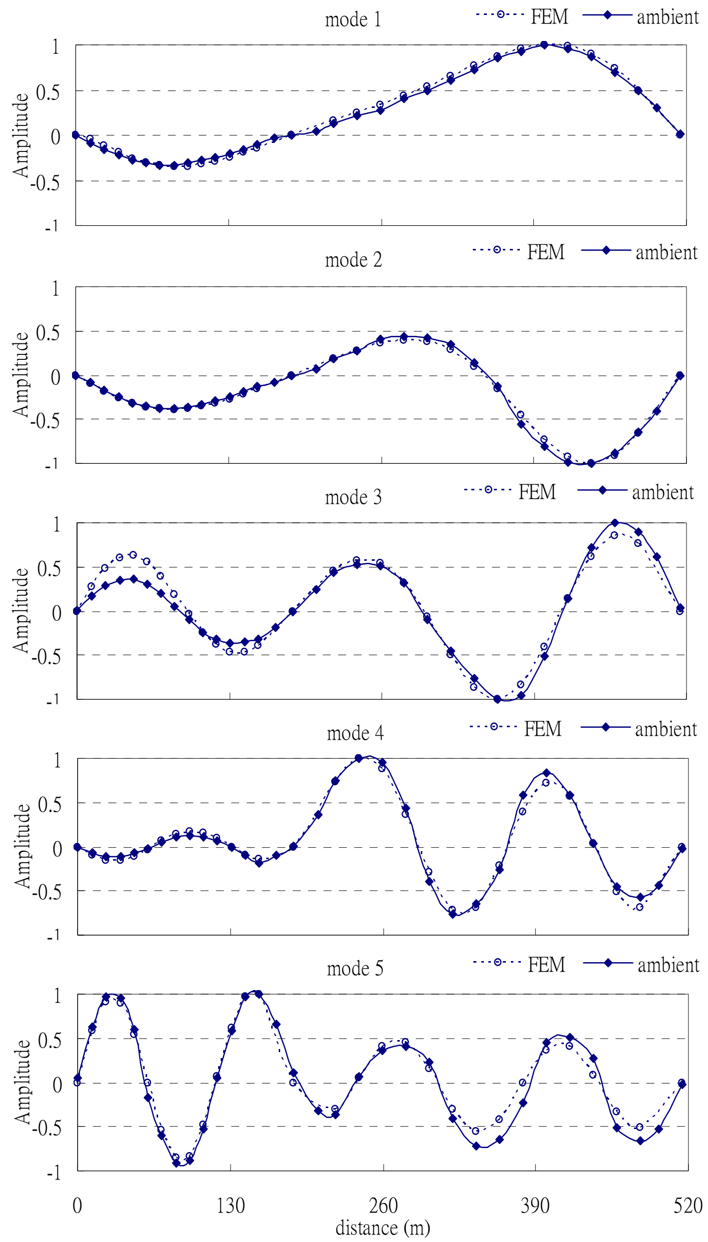


Figure 4. Comparison of identified mode shapes with finite element results in vertical direction

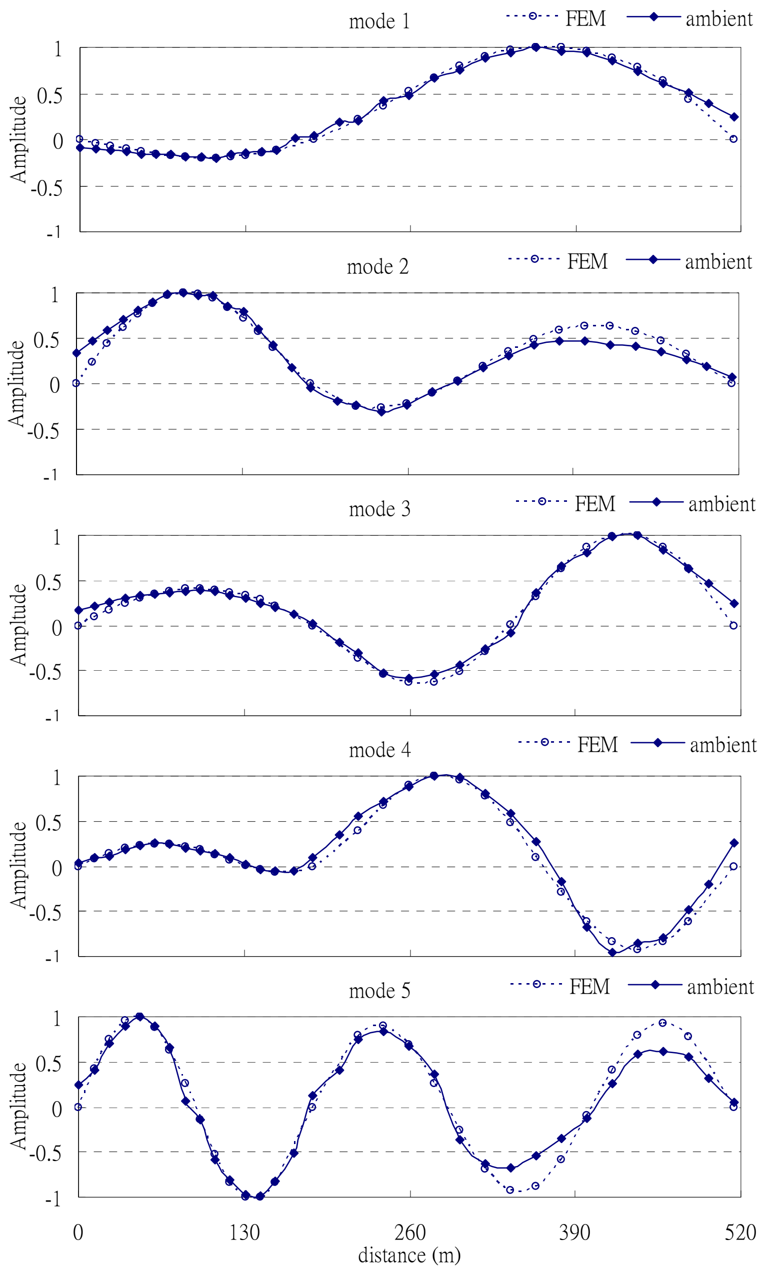


Figure 5. Comparison of identified mode shapes with finite element results in transverse direction

Table 2. Comparison of natural frequencies and damping ratios in the vertical direction

Mode	Ambient Test		FEM (updated)	MAC
	f (Hz)	ξ (%)	f (Hz)	
1	0.284	2.9	0.293	0.99
2	0.574	3.7	0.561	0.99
3	0.92	4.4	0.931	0.95
4	1.54	3.9	1.52	0.98
5	1.81	3.0	1.79	0.97

Table 3. Comparison of natural frequencies and damping ratios in the transverse direction

Mode	Ambient Test		FEM (updated)	MAC
	f (Hz)	ξ (%)	f (Hz)	
1	0.643	3.3	0.646	0.99
2	1.64	2.9	1.40	0.96
3	2.17	3.2	2.16	0.98
4	2.51	2.5	2.63	0.98
5	3.13	3.9	3.02	0.96

6. Refining FEM model

A 3-D finite element model has been established to simulate the Kao Ping Hsi cable-stayed bridge. The natural frequencies of the first five modes in the vertical and transverse directions are listed in Table 1. Comparisons with measured results in Table 2 and Table 3 reveal that the computed frequencies are significantly larger than the experimental ones for most mode. Therefore, the dynamic parameters of the original finite element model need to be updated. To obtain a more accurate analytical model, the experimental results can be used to refine the original finite element model.

The finite element model used in designing the structure must be further refined by either experimentally or using model updating methods , in which one needs to more realistically evaluate the boundary conditions and both the geometric and material characteristics of the structure. Overview the construction process of Kao Ping Hsi cable-stayed bridge, the quality of construction is satisfactory. The dynamic characteristics computed in the original FEM model are larger than the identified value of 0.5%~15% except for the second mode. The geometric and material of the structure are less sensitive than the boundary conditions to the simulated results. Therefore, only boundary conditions are modified to modulate the FEM model. As shown in Fig. 6, the transverse beams are used to simulate the supports of the bridge at the abutment A1 and the pier P2. The linking transverse system is adopted to simulate the interaction between main beam and main tower.

For comparison, the first five natural frequencies computed from the FEM updating model is also listed in Table 2 and Table 3, in which the dominant direction of vibration is determined by the maximum component of each eigenvector. The modal shapes obtained from the finite element analysis of updating model are also plotted in Fig. 4 and Fig. 5. The FEM results appear to be similar to the measured ones. To evaluate the correlation between the identified and the theoretical mode shapes, the MAC index (modal assurance criterion), which is defined as [10], was used to show the correlation between any two mode shapes of interest

$$MAC(\{\varphi_{iI}\},\{\varphi_{iA}\}) = \frac{\left|\{\varphi_{iI}\}^T\{\varphi_{iA}\}\right|^2}{\{\varphi_{iI}\}^T\{\varphi_{iI}\}\{\varphi_{IA}\}^T\{\varphi_{iA}\}} \qquad (23)$$

where $\{\varphi_{iI}\}$ is the ith mode shape identified from the wavelet transform algorithm and $\{\varphi_{iA}\}$ is the corresponding mode shape obtained by theoretical analysis. The value of MAC is between zero and one. An MAC index near unity indicates to similarity in the mode shapes of interest. An MAC index is zero when the two mode shapes are orthogonal and unrelated to each other. Table 2 and Table 3 present the MAC values for the modal shapes in Fig. 4 and Fig. 5 which are obtained from the ambient vibration test and finite element analysis. As can be seen, correlations between the two sets of modal shapes are generally good. Herein, the refined finite element model can be rationally established for the Kao Ping Hsi cable-stayed bridge.

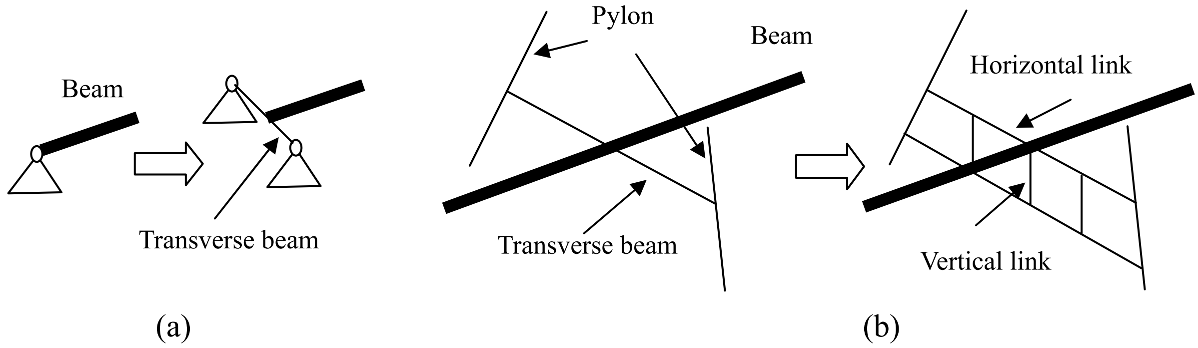


Figure 6. Boundary conditions modified for the FEM model: (a) at abutment A1 and pylon P2; (b) between main beam and main tower

7. Conclusions

This study presents a method of structural identification for the Kao Ping Hsi cable-stayed bridge by using ambient vibration testing data with the continuous wavelet transform. The modal parameters of the structural system are directly estimated from the coefficient matrices associated with the continuous wavelet transform. The first five modes in both vertical and transverse directions were identified for the bridge from the results recorded during the ambient vibration test. The results obtained in a field test were used to evaluate the rationality of the original design. The finite element model was revised by using the experimental results. Modifications on the boundary conditions of the cable-stayed bridge can be adopted to update the finite element model. A comparison of the identified results with the finite element results shows a reasonable match for the first five modes, leading to the conclusion that the proposed methodology in the structural identification of the cable-stayed bridges is applicable.

Acknowledgements

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