International Journal of Applied Science and Engineering
2010. 8, 1: 77-98

Solving Fuzzy Bi-Criteria Fixed Charge Transportation Problem Using a New Fuzzy Algorithm

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Abstract: The bi-criteria fixed charge transportation problem is an extension of the classical transportation problem. The bi-criteria fixed charge transportation problem in a crisp environment is, often, not effective in dealing with imprecision or vagueness in the values of the problem parameters. To deal with such situations, it has been proposed that the parameters should be represented as fuzzy numbers. Hence, bi-criteria fixed charge transportation problem in fuzzy environment is considered here. In existing approaches, the programming problems in fuzzy environment are solved by converting them into crisp environment by choosing appropriate membership functions and thus, the solutions are also crisp numbers. However, in this paper, a new algorithm is proposed to solve the above said problem using a linear ranking function, without converting it into crisp environment and the solutions derived are fuzzy numbers. The algorithm is suitably illustrated with a numerical example. Numerical results are compared for both fuzzy and crisp versions of the problem.

AMS Mathematics Subject Classification: 03E72, 90C70.

Keywords: trapezoidal fuzzy number; bi-criteria fixed charge transportation problem; linear ranking function.

1. Introduction

The fixed charge transportation problem was originally formulated by Hirsch and Dantzig [1] in 1954. Several methods, thereafter, have been developed to find exact and approximate solutions of fixed charge transportation problem [2-11]. In classical transportation problem, the cost of transportation is directly proportional to the number of units transported. But in real life situation, when a commodity is transported, a fixed cost is incurred in objective function. It may represent the cost of hiring a vehicle, landing fee in an airport, setup costs for machines in a manufacturing environment etc. Many distribution problems, in practice, can be modelled as fixed charge transportation problems. For example rail, roads and trucks have invariably used a freight rate, which consists of a fixed cost and variable cost.

The real world problems are multi-objective in nature. Most of the practical transportation problems appear with two objectives known as bi-criteria transportation problems. There are two objectives – minimization of cost of transportation and minimization of time of transportation. Most of the methods for solving bi-objective transportation problems developed on giving higher priority to minimize cost than time. However, sometimes there may exist emergency situations when

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Accepted for Publication: October 26, 2010

time of transportation is more important than cost of transportation e.g. fire services, police services etc.. This led to development of procedures for minimization of time of transportation [12-14].

The crisp parameters are not effective in dealing with vagueness and impreciseness in real life problems. To address these, bi-criteria fixed-charge transportation problem in fuzzy environment is used. Zadeh [15], first, introduced the concepts of fuzzy sets to deal with real life situations. Bellman and Zadeh [16], first, introduced the fuzzy sets theory into multi-criteria analysis for effectively dealing with the imprecision, vagueness and subjectiveness of the human decision making. Since then, significant advances have been made in developing numerous methodologies and their applications to various decision problems.

Real numbers can be linearly ordered by $\geq$ or $\leq$. This type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. An efficient approach for ordering the fuzzy numbers is by the use of a ranking function $\Re : F(R) \rightarrow R$, where $F(R)$ is a set of fuzzy numbers defined on real line, which maps each fuzzy number into the real line, where a natural order exists. Thus, specific ranking of fuzzy numbers is an important procedure for decision-making in a fuzzy environment and generally, has become one of the main problems in fuzzy set theory.

The method for ranking was first proposed by Jain [17]. Yager [18] proposed four indices which may be employed for the purpose of ordering fuzzy quantities in $[0,1]$. Further references in this direction can be found in [19-22]. Ranking function is used in different areas of fuzzy optimization [23-33].

In this paper, a new algorithm using linear ranking function is proposed for solving bi-criteria fixed charge transportation problem in fuzzy environment. Existing approaches to solve the bi-criteria fixed charge transportation problem under fuzzy environment use Zimmermann approach [34] to convert the fuzzy linear programming problem in crisp linear programming problem and it is then solved to find the optimal solutions. To use existing approaches, one should have good knowledge of fuzzy linear programming problems, Zimmermann approach and methods to solve crisp linear programming problem. It is very difficult to implement the existing algorithms to programming language.

On the other hand, the proposed algorithm can be easily implemented into a programming language and there is no need of Zimmermann approach. Moreover, the proposed method is very easy to understand and apply. The optimal solutions are fuzzy numbers and by using ranking function, results can be converted into crisp numbers.

This paper is organized as follows: In Section 2, basic definitions, arithmetic operations and a ranking function are reviewed. In Section 3, the formulation of bi-criteria fixed charge transportation problem in fuzzy environment is explained. In Section 4, a new algorithm using linear ranking function is proposed. In Section 5, to illustrate the proposed algorithm, a numerical example is solved. In Section 6, results in crisp and fuzzy environment are compared. In Section 7, conclusions are discussed.

2. Preliminaries

In this section, basic definitions, arithmetic operations and ranking functions are reviewed [22, 35].

2.1. Basic definitions

In this subsection, some basic definitions are reviewed [35].

Definition 1 The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range $[0,1]$ i.e., $\mu_{\tilde{A}} : X \to [0,1]$. The assigned values indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 2 A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}} : R \to [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.
3. It is strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b,c]$.

Definition 3 A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)}, & a \le x < b \\ 1, & b \le x \le c \\ \dfrac{(d-x)}{(d-c)}, & c < x \le d \\ 0, & \text{otherwise} \end{cases}$$

where $a, b, c, d \in R$.

2.2. Arithmetic operations

In this subsection, arithmetic operations between two trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [35].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$

(ii) $\tilde{A}_1 \ \Theta \ \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$

(iii) $\lambda \otimes \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1) & \lambda < 0 \end{cases}$

2.3. Ranking function

A convenient method for comparing fuzzy numbers is by the use of ranking function [22]. A ranking function $\Re : F(R) \to R$, where $F(R)$, set of all fuzzy numbers defined on set of real numbers, maps each fuzzy number into a real number. Let $\tilde{A} = (a,b,c,d)$ be a trapezoidal fuzzy number, then $\Re(\tilde{A}) = \dfrac{a+b+c+d}{4}$.

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers, then

(i) $\tilde{A} \underset{\Re}{\ge} \tilde{B}$ if $\Re(\tilde{A}) \ge \Re(\tilde{B})$

(ii) $\tilde{A} \underset{\Re}{>} \tilde{B}$ if $\Re(\tilde{A}) > \Re(\tilde{B})$

(iii) $\tilde{A} \underset{\Re}{=} \tilde{B}$ if $\Re(\tilde{A}) = \Re(\tilde{B})$

3. Formulation of bi-criteria fixed charge transportation problem in fuzzy environment

Basu et al. [36] proposed a method to solve bi-criteria fixed charge transportation problem in crisp environment by assuming that there is no uncertainty about the values of parameters (cost, time etc.). But in real life situations, there is always some uncertainty about parameters. Several authors have represented the parameters as fuzzy numbers for solving real life problems [37-38]. To overcome this shortcoming, in this section, the problem discussed in [36] is formulated in fuzzy environment where transportation cost, transportation time and fixed cost are represented by trapezoidal fuzzy numbers and a new algorithm, using linear ranking function, is proposed for minimizing both total fuzzy cost and maximum fuzzy time of transportation simultaneously.

Consider m sources, each of which is capable of producing a certain product and supply it to n destinations. Let a_i denotes the production capacity of source i and b_j represents the quantity demanded at the destination j. Suppose $\tilde{c}_{ij}$ and $\tilde{t}_{ij}$ denote the approximate transportation cost of one unit and approximate transportation time from source i to destination j respectively. Let $\tilde{f}_i$ be the approximate fixed charge incurred at the source i, depending upon number of units transported. Then the fixed charge transportation problem in fuzzy environment may be formulated as follows:

(P) Minimize

$$\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}x_{ij}\oplus\sum_{i=1}^{m}\tilde{f}_i,\ \underset{\substack{1\le i\le m\\1\le j\le n}}{\text{maximum}}\left[\tilde{t}_{ij}:x_{ij}>0\right]\right\} \quad (1)$$

subject to

$$\sum_{j=1}^{n}x_{ij}\le a_i,\ \text{for}\ \ i=1,2,\ldots,m \quad (2)$$

$$\sum_{i=1}^{m}x_{ij}=b_j,\ \ \text{for}\ j=1,2,\ldots,n \quad (3)$$

where $a_{i1},a_{i2},\ldots,a_{ip}$ for $i=1,2,\ldots,m$ are given constants and $\tilde{f}_{il}$ for $i=1,2,\ldots,m;\ l=1,2,\ldots,p$ are given fuzzy costs and $\tilde{f}_i$ for the i^{th} source can be calculated in the following manner:

If $\sum_{j=1}^{n}x_{ij}>a_{i1}$ and less than a_{i2} i.e. if total number of units transported from i^{th} source are more than a_{i1} but less than a_{i2} then $\tilde{f}_i=\tilde{f}_{i1}$.

If $\sum_{j=1}^{n}x_{ij}>a_{i2}$ and less than a_{i3} i.e. if total number of units transported from i^{th} source are more than a_{i2} but less than a_{i3}, then $\tilde{f}_i=\tilde{f}_{i1}+\tilde{f}_{i2}$ and continue like this when $\sum_{j=1}^{n}x_{ij}>a_{ip}$, then $\tilde{f}_i=\tilde{f}_{i1}+\tilde{f}_{i2}+\ldots+\tilde{f}_{ip}$.

The problem is solved in the following way:

Minimize total fuzzy cost of transportation without considering fuzzy time of transportation. Thereafter, find maximum fuzzy time of transportation corresponding to the minimum total fuzzy cost so obtained. Subsequently, minimize total fuzzy cost of transportation after modifying fuzzy transportation costs with respect to the maximum fuzzy time obtained in the previous result. Then, find maximum fuzzy time with respect to the minimum total fuzzy cost so obtained. Repeat the process till the solution is infeasible. The procedure is known as re-optimization procedure.

4. Proposed algorithm

In this section, a new algorithm is proposed to solve the bi-criteria fixed charge transportation problem in fuzzy environment using a linear ranking function. The various steps of the proposed algorithms are as follows:

Step 1 First of all balance problem (P) by introducing dummy destination and then, split it into two separate problems (P_1) and (P_2) for solving it by re-optimization procedure where

(P_1) Minimize the total fuzzy cost $\tilde{Z}$, where

$$\tilde{Z} = \left\{ \sum_{i=1}^{m}\sum_{j=1}^{n+1} \tilde{c}_{ij}x_{ij} \oplus \sum_{i=1}^{m} \tilde{f}_i \right\}$$

subject to

$$\sum_{j=1}^{n+1} x_{ij} = a_i \text{, for } i=1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \text{, for } j=1,2,\ldots,n+1 \quad x_{ij} \geq 0$$

(P_2) Minimize the maximum fuzzy time $\tilde{\mathrm{T}}$, where

$$\tilde{\mathrm{T}} = \underset{\substack{1\le i\le m \\ 1\le j\le n+1}}{\text{maximum}}\left[\tilde{t}_{ij} : x_{ij} > 0\right]$$

subject to

$$\sum_{j=1}^{n+1} x_{ij} = a_i \text{, for } i=1,2,\ldots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \text{, for } j=1,2,\ldots,n+1 \quad x_{ij} \geq 0$$

In problems (P_1) and (P_2), the fuzzy transportation cost and fuzzy transportation time associated with the dummy cells are zero trapezoidal fuzzy numbers.

Step 2 Find a initial fuzzy basic feasible solution of the problem (P_1) with respect to fuzzy transportation costs only.

Step 3 Let B be the basis of current fuzzy basic feasible solution. Calculate the fuzzy fixed cost of the current fuzzy basic feasible solution and denote it by $\tilde{F}(\text{current})$, where

$$\tilde{F}(\text{current}) = \sum_{i=1}^{m} \tilde{f}_i$$

Step 4 Find $(\tilde{c}_{ij} \ \Theta\ \tilde{u}_i \ \Theta\ \tilde{v}_j)$, for all $(i, j) \notin B$ and denote it by $(\tilde{c}_{ij})^1$, where $\tilde{u}_i$ and $\tilde{v}_j$ are the fuzzy dual variables, for $i=1,2,\ldots,m;\ j=1,\ 2,\ldots,n+1$, where fuzzy dual variables are decision variables of the dual of the problem considered in Step 2.

Step 5 Find $\tilde{A}_{ij} = (\tilde{c}_{ij})^1 \otimes E_{ij}$ $(\text{for all}\,(i, j) \notin B)$, where $\tilde{A}_{ij}$ is the change in fuzzy cost for introducing a non-basic cell $(i, j) \notin B$ with value E_{ij} into current basis B by making reallocation.

Step 6 Find $\tilde{F}_{ij}(\text{Difference}) = \tilde{F}_{ij}(\text{NB})\ \Theta\ \tilde{F}(\text{Current})$, where $\tilde{F}_{ij}(\text{NB})$ is the fuzzy fixed cost involved for introducing a non-basic cell $(i, j) \notin B$ with values E_{ij} into current basis B by making reallocation.

Step 7 Find $\tilde{\Delta}_{ij} = \tilde{F}_{ij}(\text{Difference}) \oplus \tilde{A}_{ij}, \text{for all}\,(i, j) \notin B$.

Step 8 If $\Re(\tilde{\Delta}_{ij}) \geq 0$, then go to Step 9. Otherwise, find $\text{minimum}\left\{\tilde{\Delta}_{ij} : \Re(\tilde{\Delta}_{ij}) < 0, (i, j) \notin B\right\}$. Then x_{ij} associated with $\tilde{\Delta}_{ij}$ (whose rank is minimum) will enter into the basis. Go to Step 3.

Step 9 Let $\tilde{Z}_1$ be the minimum total fuzzy cost.

Step 10 Find $\tilde{T}_1$ where $\tilde{T}_1 = \underset{\substack{1\le i\le m \\ 1\le j\le n+1}}{\text{maximum}}\left[\tilde{t}_{ij} : x_{ij} > 0\right]$, then the corresponding pair $(\tilde{Z}_1, \tilde{T}_1)$ may be called the first fuzzy cost- time trade-off pair.

Step 11 To find 2nd fuzzy cost-time trade-off pair $(\tilde{Z}_2, \tilde{T}_2)$, change $\tilde{c}_{ij}$'s in problem (P_1) as:

$$\tilde{c}_{ij} = \begin{cases} \begin{pmatrix} M, M, \\ M, M \end{pmatrix} & \text{if } \Re(\tilde{t}_{ij}) \geq \Re(\tilde{T}_1) \\ \tilde{c}_{ij} & \text{if } \Re(\tilde{t}_{ij}) < \Re(\tilde{T}_1) \end{cases}$$

where $\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$ is sufficiently large trapezoidal fuzzy number.

Now, find fuzzy initial basic feasible solution of new problem (P_1) with respect to fuzzy transportation cost only.

(i) If total fuzzy cost is very large i.e. it depends upon M, then solution is infeasible and it is not possible to find new fuzzy cost-time trade-off pair. Consequently the process of obtaining fuzzy cost-time trade-off pairs terminates.

(ii) Otherwise apply Steps 3 to 10 to obtain $(\tilde{Z}_2, \tilde{T}_2)$.

Step 12 Third and subsequent cost-time trade-off pairs can be obtained in the same manner as is done for second fuzzy cost-time trade-off pair.

Let after q^{th} fuzzy cost-time trade-off pair, the solution is infeasible. Then the complete set of cost-time trade-off pairs is $(\tilde{Z}_1, \tilde{T}_1), (\tilde{Z}_2, \tilde{T}_2), \ldots, (\tilde{Z}_q, \tilde{T}_q)$ where $\tilde{Z}_1 < \tilde{Z}_2 < \ldots < \tilde{Z}_q$ and $\tilde{T}_1 > \tilde{T}_2 > \ldots > \tilde{T}_q$.

4.1. Advantages of proposed method

The advantages of the proposed method are:

i. Goal and parametric programming techniques are not used.

ii. The optimal solution is a fuzzy number.

iii. The proposed method is very easy to understand and apply.

iv. There is no need of using Zimmermann approach.

v. The proposed algorithm can be easily implemented into a programming language.

5. Numerical example

Now we illustrate the algorithm, proposed in Section 4, by applying it to following numerical problem to obtain the set of fuzzy cost-time trade-off pairs. Table 1 provides tableau representation of numerical problem. In this Table, rows 1-3 correspond to sources and columns 1-3 correspond to destinations. Row 4 gives demands of destinations and column 4 gives capacities of sources. Upper and lower entries of $(i, j)^{th}$ cell represent fuzzy transportation cost of one unit and fuzzy transportation time from i^{th} source to j^{th} destination respectively. For the calculations of approximate fixed costs $(\tilde{f}_i)$ at the sources, we consider that $p=3, a_{i1}=0, a_{i2}=7, a_{i3}=10, \text{for } i=1,2,3$. The fuzzy costs are given as:

$$\tilde{f}_{11} = \begin{pmatrix} 70,80, \\ 100,150 \end{pmatrix},\ \tilde{f}_{12} = \begin{pmatrix} 30,40, \\ 50,80 \end{pmatrix},\ \tilde{f}_{13} = \begin{pmatrix} 30,40, \\ 50,80 \end{pmatrix},\ \tilde{f}_{21} = \begin{pmatrix} 90,100, \\ 200,210 \end{pmatrix},\ \tilde{f}_{22} = \begin{pmatrix} 30,40, \\ 50,80 \end{pmatrix},$$

$$\tilde{f}_{23} = \begin{pmatrix} 30,40, \\ 50,80 \end{pmatrix},\ \tilde{f}_{31} = \begin{pmatrix} 100,150, \\ 200,350 \end{pmatrix},\ \tilde{f}_{32} = \begin{pmatrix} 70,80, \\ 100,150 \end{pmatrix},\ \tilde{f}_{33} = \begin{pmatrix} 30,40, \\ 50,80 \end{pmatrix}$$

For the source i, the fixed charge $\tilde{f}_i$ can be calculated in following manner:

If $\sum_{j=1}^{3} x_{ij} > 0$ and less than 7 i.e. if total number of units transported from i^{th} source are more than 0 but less than 7, then $\tilde{f}_i = \tilde{f}_{i1}$. If $\sum_{j=1}^{3} x_{ij} > 7$ and less than 10 i.e. if total number of units transported from i^{th} source are more than 7 but less than 10, then $\tilde{f}_i = \tilde{f}_{i1} + \tilde{f}_{i2}$ and if total number of units transported from i^{th} source are more than 10 i.e. $\sum_{j=1}^{3} x_{ij} > 10$, then $\tilde{f}_i = \tilde{f}_{i1} + \tilde{f}_{i2} + \tilde{f}_{i3}$.

Table 1. Tableau representation of numerical problem

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	Capacities (a_i)
1	$\begin{pmatrix}1,4,\\5,10\end{pmatrix}$ $\begin{pmatrix}5,10,\\15,30\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$ $\begin{pmatrix}3,5,\\8,16\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$ $\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	19
2	$\begin{pmatrix}1,3,\\4,8\end{pmatrix}$ $\begin{pmatrix}5,10,\\12,13\end{pmatrix}$	$\begin{pmatrix}2,4,\\6,12\end{pmatrix}$ $\begin{pmatrix}6,7,\\13,26\end{pmatrix}$	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$ $\begin{pmatrix}5,6,\\11,22\end{pmatrix}$	10
3	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$ $\begin{pmatrix}2,4,\\6,12\end{pmatrix}$	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$ $\begin{pmatrix}4,5,\\9,18\end{pmatrix}$	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$ $\begin{pmatrix}8,9,\\17,34\end{pmatrix}$	11
Demands (b_j)	5	8	15	

So, we are considering the following 3×3 bi-criteria fixed charge transportation problem in fuzzy environment.

$$(P) \quad \text{Minimize} \left\{ \sum_{i=1}^{3}\sum_{j=1}^{3} \tilde{c}_{ij} x_{ij} \oplus \sum_{i=1}^{3} \tilde{f}_i,\ \underset{\substack{1\le i\le 3\\ 1\le j\le 3}}{\text{maximum}} \left[\tilde{t}_{ij} : x_{ij} > 0 \right] \right\}$$

subject to

$$\sum_{j=1}^{3} x_{ij} \le a_i, \qquad \text{for } i = 1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = b_j, \qquad \text{for } j = 1,2,3$$

$$x_{ij} \ge 0$$

Using Step 1, the above problem (P) is first balanced and then divided into two separate problems (P_1) and (P_2) for solving it by re-optimization procedure where

$$(P_1) \qquad \text{Minimize} \left\{ \sum_{i=1}^{3}\sum_{j=1}^{4} \tilde{c}_{ij} x_{ij} \oplus \sum_{i=1}^{3} \tilde{f}_i \right\}$$

subject to

$$\sum_{j=1}^{4} x_{ij} = a_i , \qquad \text{for } i=1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = b_j , \qquad \text{for } j=1,2,3,4$$

$$x_{ij} \geq 0$$

$$(P_2) \qquad \text{Minimize}\left\{ \underset{\substack{1\le i\le 3\\ 1\le j\le 4}}{\text{maximum}}\left[\tilde{t}_{ij} : x_{ij} > 0\right]\right\}$$

subject to

$$\sum_{j=1}^{4} x_{ij} = a_i , \qquad \text{for } i=1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = b_j , \qquad \text{for } j=1,2,3,4$$

$$x_{ij} \geq 0$$

Table 2 gives $\tilde{c}_{ij}$ for $i=1,2,3; j=1,2,3,4$ and Table 3 gives

So, we are considering the following 3×3 bi-criteria fixed charge transportation problem in fuzzy environment.

$$(P) \quad \text{Minimize} \left\{ \sum_{i=1}^{3}\sum_{j=1}^{3} \tilde{c}_{ij}x_{ij} \oplus \sum_{i=1}^{3} \tilde{f}_i ,\ \underset{\substack{1\le i\le 3\\ 1\le j\le 3}}{\text{maximum}}\left[\tilde{t}_{ij} : x_{ij} > 0\right]\right\}$$

subject to

$$\sum_{j=1}^{3} x_{ij} \leq a_i , \qquad \text{for } i=1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = b_j , \qquad \text{for } j=1,2,3$$

$$x_{ij} \geq 0$$

Using Step 1, the above problem (P) is first balanced and then divided into two separate problems (P_1) and (P_2) for solving it by re-optimization procedure where

$$(P_1) \qquad \text{Minimize} \left\{ \sum_{i=1}^{3}\sum_{j=1}^{4} \tilde{c}_{ij}x_{ij} \oplus \sum_{i=1}^{3} \tilde{f}_i \right\}$$

subject to

$$\sum_{j=1}^{4} x_{ij} = a_i , \qquad \text{for } i=1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = b_j , \qquad \text{for } j=1,2,3,4$$

$$x_{ij} \geq 0$$

(P_2) $$\text{Minimize}\left\{\underset{\substack{1\le i\le 3\\ 1\le j\le 4}}{\text{maximum}}\left[\tilde{t}_{ij} : x_{ij} > 0\right]\right\}$$

subject to

$$\sum_{j=1}^{4} x_{ij} = a_i\,, \qquad \text{for } i = 1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = b_j\,, \qquad \text{for } j = 1,2,3,4$$

$$x_{ij} \ge 0$$

Table 2 gives $\tilde{c}_{ij}$ for $i = 1,2,3;\ j = 1,2,3,4$ and Table 3 gives $\tilde{t}_{ij}$ for $i = 1,2,3;\ j = 1,2,3,4$.

Table 2. Fuzzy transportation cost for one unit

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	4	a_i
1	$(1,4,5,10)$	$(3,6,9,18)$	$(3,6,9,18)$	$(0,0,0,0)$	19
2	$(1,3,4,8)$	$(2,4,6,12)$	$(0,1,2,5)$	$(0,0,0,0)$	10
3	$(0,1,2,5)$	$(0,0.5,1.5,2)$	$(0,0.5,1.5,2)$	$(0,0,0,0)$	11
b_j	5	8	15	12	40

Table 3. Fuzzy transportation time

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	4	a_i
1	$(5,10,15,30)$	$(3,5,8,16)$	$(0,1,2,5)$	$(0,0,0,0)$	19
2	$(5,10,12,13)$	$(6,7,13,26)$	$(5,6,11,22)$	$(0,0,0,0)$	10
3	$(2,4,6,12)$	$(4,5,9,18)$	$(8,9,17,34)$	$(0,0,0,0)$	11
b_j	5	8	15	12	40

1st fuzzy cost-time trade-off pair:

The total fuzzy cost, which is to be minimized, is given by

$$\sum_{i=1}^{3}\sum_{j=1}^{4}\tilde{c}_{ij}x_{ij} \oplus \sum_{i=1}^{3}\tilde{f}_i$$

where

$$\tilde{f}_i = \sum_{l=1}^{3}\delta_{il}\ \tilde{f}_{il}\ \text{ for } i=1,2,3$$

$$\delta_{i1} = \begin{cases} 1, \text{ if } \sum_{j=1}^{3} x_{ij} > 0 \text{ for } i=1,2,3 \\ 0, \text{ otherwise.} \end{cases}$$

$$\delta_{i2} = \begin{cases} 1, \text{ if } \sum_{j=1}^{3} x_{ij} > 7 \text{ for } i=1,2,3 \\ 0, \text{ otherwise.} \end{cases}$$

$$\delta_{i3} = \begin{cases} 1, \text{ if } \sum_{j=1}^{3} x_{ij} > 10 \text{ for } i=1,2,3 \\ 0, \text{ otherwise.} \end{cases}$$

The initial fuzzy basic feasible solution of Table 2 with respect to fuzzy transportation cost [39] is given in Table 4. The right most column gives the fuzzy fixed cost of the current solution.

Table 4. Initial fuzzy basic feasible solution

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	4	$\tilde{f}_i$
1	$(1,4,5,10)$ (5)	$(3,6,9,18)$	$(3,6,9,18)$ (2)	$(0,0,0,0)$ (12)	$(70,80,100,150)$
2	$(1,3,4,8)$	$(2,4,6,12)$	$(0,1,2,5)$ (10)	$(0,0,0,0)$	$(120,140,250,290)$
3	$(0,1,2,5)$	$(0,0.5,1.5,2)$ (8)	$(0,0.5,1.5,2)$ (3)	$(0,0,0,0)$	$(200,270,350,580)$
$\tilde{F}(\text{current}) = \sum_{i=1}^{3}\tilde{f}_i = (390,490,700,1020)$					

Total fuzzy cost is:

$$5\otimes\begin{pmatrix}1,4,\\5,10\end{pmatrix}\oplus2\otimes\begin{pmatrix}3,6,\\9,18\end{pmatrix}\oplus10\otimes\begin{pmatrix}0,1,\\2,5\end{pmatrix}\oplus8\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus3\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus\begin{pmatrix}390,490,\\700,1020\end{pmatrix}=\begin{pmatrix}401,537.5,\\779.5,1178\end{pmatrix}$$

$$\Re\begin{pmatrix}401,537.5,\\779.5,1178\end{pmatrix}=724$$

Applying Steps 4, 5, 6 and 7, the obtained values of $\tilde{\Delta}_{ij}$ for all $(i,j)\notin B$, are shown in Table 5.

Table 5. Values of $\tilde{\Delta}_{ij}$

(i,j)	$(1,2)$	$(2,1)$	$(2,2)$	$(2,4)$	$(3,1)$	$(3,4)$
$(\tilde{C}_{ij})^1$	$(-17,-4,4,17)$	$(-11,2,8,25)$	$(-20,-2,9,29)$	$(-2,4,8,18)$	$(-9,0.5,6.5,22)$	$(1,4.5,8.5,18)$
E_{ij}	2	5	8	10	3	3
$\tilde{A}_{ij}$	$(-34,-8,8,34)$	$(-55,10,40,125)$	$(-160,-16,72,232)$	$(-20,40,80,180)$	$(-27,1.5,19.5,66)$	$(3,13.5,25.5,54)$
$\tilde{F}_{ij}$ (NB)	$(390,490,700,1020)$	$(390,490,700,1020)$	$(390,490,700,1020)$	$(330,430,550,890)$	$(390,490,700,1020)$	$(390,490,700,1020)$
$\tilde{F}_{ij}$ (Diff.)	$(-630,-210,210,630)$	$(-630,-210,210,630)$	$(-630,-210,210,630)$	$(-690,-270,60,500)$	$(-630,-210,210,630)$	$(-630,-210,210,630)$
$\tilde{\Delta}_{ij}$	$(-664,-218,218,664)$	$(-685,-200,250,755)$	$(-790,-226,282,862)$	$(-710,-230,140,680)$	$(657,208.5,229.5,696)$	$(-627,196.5,235.5,684)$
$\Re(\tilde{\Delta}_{ij})$	0	30	32	–30	15	24

Here $\Re(\tilde{\Delta}_{ij})$ is negative only at $(2,4)$ cell. Therefore entering $(2,4)$ into the basis, Table 6 is obtained.

Table 6. New fuzzy basic feasible solution

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	4	$\tilde{f}_i$
1	$\begin{pmatrix}1,4,\\5,10\end{pmatrix}$ (5)	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$ (12)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$ (2)	$\begin{pmatrix}130,160,\\200,310\end{pmatrix}$
2	$\begin{pmatrix}1,3,\\4,8\end{pmatrix}$	$\begin{pmatrix}2,4,\\6,12\end{pmatrix}$	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$ (10)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$
3	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$ (8)	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$ (3)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}200,270,\\350,580\end{pmatrix}$
					$\tilde{F}(\text{current})=\sum_{i=1}^{3}\tilde{f}_i=\begin{pmatrix}330,430,\\550,890\end{pmatrix}$

Total fuzzy cost is:

$$5\otimes\begin{pmatrix}1,4,\\5,10\end{pmatrix}\oplus 12\otimes\begin{pmatrix}3,6,\\9,18\end{pmatrix}\oplus 8\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus 3\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus\begin{pmatrix}330,430,\\550,890\end{pmatrix}=\begin{pmatrix}371,527.5,\\699.5,1178\end{pmatrix}.$$

$$\Re\begin{pmatrix}371,527.5,\\699.5,1178\end{pmatrix}=694$$

Applying Steps 4, 5, 6 and 7, the obtained values of $\tilde{\Delta}_{ij}$ for all $(i,j)\notin B$, are shown in Table 7.

Table 7. Values of $\tilde{\Delta}_{ij}$

(i,j)	$(1,2)$	$(2,1)$	$(2,2)$	$(2,3)$	$(3,1)$	$(3,4)$
$(\tilde{C}_{ij})^1$	$\begin{pmatrix}-17,-4,\\4,17\end{pmatrix}$	$\begin{pmatrix}-9,-2,\\0,7\end{pmatrix}$	$\begin{pmatrix}-18,-6,\\1,11\end{pmatrix}$	$\begin{pmatrix}-18,-8,\\-4,2\end{pmatrix}$	$\begin{pmatrix}-9,0.5,\\6.5,22\end{pmatrix}$	$\begin{pmatrix}1,4.5,\\8.5,18\end{pmatrix}$
E_{ij}	8	5	8	10	3	2
$\tilde{A}_{ij}$						

	$\begin{pmatrix}-136,-32,\\32,136\end{pmatrix}$	$\begin{pmatrix}-45,-10,\\0,35\end{pmatrix}$	$\begin{pmatrix}-144,-48,\\8,88\end{pmatrix}$	$\begin{pmatrix}-180,-80,\\-40,20\end{pmatrix}$	$\begin{pmatrix}-27,1.5,\\19.5,66\end{pmatrix}$	$\begin{pmatrix}2,9,\\17,36\end{pmatrix}$
$\tilde{F}_{ij}$ (NB)	$\begin{pmatrix}330,430,\\550,890\end{pmatrix}$	$\begin{pmatrix}420,530,\\750,1100\end{pmatrix}$	$\begin{pmatrix}420,530,\\750,1100\end{pmatrix}$	$\begin{pmatrix}390,490,\\700,1020\end{pmatrix}$	$\begin{pmatrix}330,430,\\550,890\end{pmatrix}$	$\begin{pmatrix}300,390,\\500,810\end{pmatrix}$
$\tilde{F}_{ij}$ (Diff	$\begin{pmatrix}-560,-120\\120,560\end{pmatrix}$	$\begin{pmatrix}-470,-20,\\320,770\end{pmatrix}$	$\begin{pmatrix}-470,-20,\\320,770\end{pmatrix}$	$\begin{pmatrix}-500,-60,\\270,690\end{pmatrix}$	$\begin{pmatrix}-560,-120,\\120,560\end{pmatrix}$	$\begin{pmatrix}-590,-160\\70,480\end{pmatrix}$
$\tilde{\Delta}_{ij}$	$\begin{pmatrix}-696,-152,\\152,696\end{pmatrix}$	$\begin{pmatrix}-515,-30,\\320,805\end{pmatrix}$	$\begin{pmatrix}-614,-68,\\328,858\end{pmatrix}$	$\begin{pmatrix}-680,-140,\\230,710\end{pmatrix}$	$\begin{pmatrix}-587,-118.5,\\139.5,626\end{pmatrix}$	$\begin{pmatrix}-588,-151,\\87,516\end{pmatrix}$
$\Re(\tilde{\Delta}_{ij})$	0	145	126	30	15	–34

Here $\Re(\tilde{\Delta}_{ij})$ is negative only at $(3,4)$ cell. Therefore entering $(3,4)$ into the basis, Table 8 is obtained.

Table 8. New fuzzy basic feasible solution

Destination $(j)\rightarrow$ Source $(i)\downarrow$	1	2	3	4	$\tilde{f}_i$
1	$\begin{pmatrix}1,4,\\5,10\end{pmatrix}$ (5)	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$ (14)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}130,160,\\200,310\end{pmatrix}$
2	$\begin{pmatrix}1,3,\\4,8\end{pmatrix}$	$\begin{pmatrix}2,4,\\6,12\end{pmatrix}$	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$ (10)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$
3	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$ (8)	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$ (1)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$ (2)	$\begin{pmatrix}170,230,\\300,500\end{pmatrix}$
$\tilde{F}(\text{current})=\sum_{i=1}^{3}\tilde{f}_i=\begin{pmatrix}300,390,\\500,810\end{pmatrix}$					

Total fuzzy cost is:

$$5\otimes\begin{pmatrix}1,4,\\5,10\end{pmatrix}\oplus 14\otimes\begin{pmatrix}3,6,\\9,18\end{pmatrix}\oplus 8\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus 1\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus\begin{pmatrix}300,390,\\500,810\end{pmatrix}=\begin{pmatrix}347,498.5,\\664.5,1130\end{pmatrix}.$$

$\Re\begin{pmatrix}347,498.5,\\664.5,1130\end{pmatrix}=660$.

Applying Steps 4, 5, 6 and 7, the obtained values of $\tilde{\Delta}_{ij}$ for all $(i,j)\notin B$, are shown in Table 9.

Table 9. Values of $\tilde{\Delta}_{ij}$

(i,j)	$(1,2)$	$(1,4)$	$(2,1)$	$(2,2)$	$(2,3)$	$(3,1)$
$(\tilde{C}_{ij})^1$	$\begin{pmatrix}-17,-4,\\4,17\end{pmatrix}$	$\begin{pmatrix}-18,-8.5,\\-4.5,-1\end{pmatrix}$	$\begin{pmatrix}-8,2.5,\\8.5,25\end{pmatrix}$	$\begin{pmatrix}-17,-1.5,\\9.5,29\end{pmatrix}$	$\begin{pmatrix}-17,-3.5,\\4.5,20\end{pmatrix}$	$\begin{pmatrix}-9,0.5,\\6.5,22\end{pmatrix}$
E_{ij}	8	2	1	8	1	1
$\tilde{A}_{ij}$	$\begin{pmatrix}-136,-32,\\32,136\end{pmatrix}$	$\begin{pmatrix}-36,-17,\\-9,-2\end{pmatrix}$	$\begin{pmatrix}-8,2.5,\\8.5,25\end{pmatrix}$	$\begin{pmatrix}-136,-12,\\76,232\end{pmatrix}$	$\begin{pmatrix}-17,-3.5,\\4.5,20\end{pmatrix}$	$\begin{pmatrix}-9,0.5,\\6.5,22\end{pmatrix}$
$\tilde{F}_{ij}(\text{NB})$	$\begin{pmatrix}300,390,\\500,810\end{pmatrix}$	$\begin{pmatrix}330,430,\\550,890\end{pmatrix}$	$\begin{pmatrix}390,490,\\700,1020\end{pmatrix}$	$\begin{pmatrix}350,450,\\650,950\end{pmatrix}$	$\begin{pmatrix}390,490,\\700,1020\end{pmatrix}$	$\begin{pmatrix}300,390,\\500,810\end{pmatrix}$
$\tilde{F}_{ij}(\text{Diff}$	$\begin{pmatrix}-510,-110,\\110,510\end{pmatrix}$	$\begin{pmatrix}-480,-70,\\160,590\end{pmatrix}$	$\begin{pmatrix}-420,-10,\\310,720\end{pmatrix}$	$\begin{pmatrix}-460,-50,\\260,650\end{pmatrix}$	$\begin{pmatrix}-420,-10,\\310,720\end{pmatrix}$	$\begin{pmatrix}-510,-110,\\110,510\end{pmatrix}$
$\tilde{\Delta}_{ij}$	$\begin{pmatrix}-646,-142,\\142,646\end{pmatrix}$	$\begin{pmatrix}-516,-87,\\151,588\end{pmatrix}$	$\begin{pmatrix}-428,-7.5\\318.5,745\end{pmatrix}$	$\begin{pmatrix}-596,-62,\\336,882\end{pmatrix}$	$\begin{pmatrix}-437,-13.5\\314.5,740\end{pmatrix}$	$\begin{pmatrix}-519,-109.5\\116.5,532\end{pmatrix}$
$\Re(\tilde{\Delta}_{ij})$	0	34	157	140	151	5

Here $\Re(\tilde{\Delta}_{ij})$ is positive for all $(i,j)\notin B$ So, Table 8 gives optimal solution. So, $5\otimes\begin{pmatrix}1,4,\\5,10\end{pmatrix}\oplus 14\otimes\begin{pmatrix}3,6,\\9,18\end{pmatrix}\oplus 8\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus 1\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus\begin{pmatrix}300,390,\\500,810\end{pmatrix}=\begin{pmatrix}347,498.5,\\664.5,1130\end{pmatrix}$ is minimum total fuzzy cost. Now, corresponding to optimal Table 8, Table of fuzzy transportation time is:

$\tilde{T}_1=\text{max imum}\left\{\begin{pmatrix}5,10,\\15,30\end{pmatrix},\begin{pmatrix}0,1,\\2,5\end{pmatrix},\begin{pmatrix}0,0,\\0,0\end{pmatrix},\begin{pmatrix}8,9,\\17,34\end{pmatrix},\begin{pmatrix}4,5,\\9,18\end{pmatrix}\right\}=\begin{pmatrix}8,9,\\17,34\end{pmatrix}$ is maximum fuzzy time.

The 1st fuzzy cost-time trade-off pair is $\left(\tilde{Z}_1,\tilde{T}_1\right)=\left(\begin{pmatrix}347,498.5,\\664.5,1130\end{pmatrix},\begin{pmatrix}8,9,\\17,34\end{pmatrix}\right)$

Table 10. Fuzzy transportation time

Destination $(j)\rightarrow$ Source $(i)\downarrow$	1	2	3	4	a_i
1	$\begin{pmatrix}5,10,\\15,30\end{pmatrix}$ (5)	$\begin{pmatrix}3,5,\\8,16\end{pmatrix}$	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$ (14)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	19
2	$\begin{pmatrix}5,10,\\12,13\end{pmatrix}$	$\begin{pmatrix}6,7,\\13,26\end{pmatrix}$	$\begin{pmatrix}5,6,\\11,22\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$ (10)	10
3	$\begin{pmatrix}2,4,\\6,12\end{pmatrix}$	$\begin{pmatrix}4,5,\\9,18\end{pmatrix}$ (8)	$\begin{pmatrix}8,9,\\17,34\end{pmatrix}$ (1)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$ (2)	11
b_j	5	8	15	12	40

2nd fuzzy cost-time trade-off pair:

The fuzzy costs, shown in Table 2, are modified using Step 11 and are shown in Table 11.

Table 11. Modified fuzzy costs

Destination $(j)\rightarrow$ Source $(i)\downarrow$	1	2	3	4	a_i
1	$\begin{pmatrix}1,4,\\5,10\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	19
2	$\begin{pmatrix}1,3,\\4,8\end{pmatrix}$	$\begin{pmatrix}2,4,\\6,12\end{pmatrix}$	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	10
3	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$	$\begin{pmatrix}\mathrm{M},M,\\M,M\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	11
b_j	5	8	15	12	40

The fuzzy initial basic feasible solution of new problem $\left(P_1\right)$ is shown in Table 12.

Table 12. Initial fuzzy basic feasible solution

Destination $(j) \rightarrow$ Source $(i) \downarrow$	**1**	**2**	**3**	**4**	$\tilde{f}_i$
1	$\begin{pmatrix}1,4,\\5,10\end{pmatrix}$ (2)	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$ (5)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$ (12)	$\begin{pmatrix}70,80,\\100,150\end{pmatrix}$
2	$\begin{pmatrix}1,3,\\4,8\end{pmatrix}$	$\begin{pmatrix}2,4,\\6,12\end{pmatrix}$	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$ (10)	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}120,140,\\250,290\end{pmatrix}$
3	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$ (3)	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$ (8)	$\begin{pmatrix}M,M,\\M,M\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	$\begin{pmatrix}200,270,\\350,580\end{pmatrix}$
$\tilde{F}(\text{current})=\sum_{i=1}^{3}\tilde{f}_i=\begin{pmatrix}390,490,\\700,1020\end{pmatrix}$					

Total fuzzy cost is:

$$2\otimes\begin{pmatrix}1,4,\\5,10\end{pmatrix}\oplus 5\otimes\begin{pmatrix}3,6,\\9,18\end{pmatrix}\oplus 10\otimes\begin{pmatrix}0,1,\\2,5\end{pmatrix}\oplus 8\otimes\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}\oplus 3\otimes\begin{pmatrix}0,1,\\2,5\end{pmatrix}\oplus\begin{pmatrix}390,490,\\700,1020\end{pmatrix}=\begin{pmatrix}407,545,\\793,1211\end{pmatrix}.$$

Since total fuzzy cost does not depend upon M . So, 2nd efficient solution exists.

Applying steps 3 to 10, we get the 2nd fuzzy cost-time trade-off pair $(\tilde{Z}_2,\tilde{T}_2)=\left(\begin{pmatrix}349,501,\\669,1141\end{pmatrix},\begin{pmatrix}5,10,\\15,30\end{pmatrix}\right)$

3rd fuzzy cost-time trade-off pair:

The fuzzy costs, shown in Table 2, are modified using Step 11 and are shown in Table 13.

Table 13. Modified fuzzy costs

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	4	a_i
1	$\begin{pmatrix}M,M,\\M,M\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$	$\begin{pmatrix}3,6,\\9,18\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	19
2	$\begin{pmatrix}1,3,\\4,8\end{pmatrix}$	$\begin{pmatrix}2,4,\\6,12\end{pmatrix}$	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	10
3	$\begin{pmatrix}0,1,\\2,5\end{pmatrix}$	$\begin{pmatrix}0,0.5,\\1.5,2\end{pmatrix}$	$\begin{pmatrix}M,M,\\M,M\end{pmatrix}$	$\begin{pmatrix}0,0,\\0,0\end{pmatrix}$	11
b_j	5	8	15	12	40

The fuzzy initial basic feasible solution of new problem (P_1) is shown in Table 14.

Table 14. Initial fuzzy basic feasible solution

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	4	$\tilde{f}_i$
1	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix}$ (2)	$\begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix}$ (5)	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}$ (12)	$\begin{pmatrix} 70,80, \\ 100,150 \end{pmatrix}$
2	$\begin{pmatrix} 1,3, \\ 4,8 \end{pmatrix}$	$\begin{pmatrix} 2,4, \\ 6,12 \end{pmatrix}$	$\begin{pmatrix} 0,1, \\ 2,5 \end{pmatrix}$ (10)	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}$	$\begin{pmatrix} 120,140, \\ 250,290 \end{pmatrix}$
3	$\begin{pmatrix} 0,1, \\ 2,5 \end{pmatrix}$ (5)	$\begin{pmatrix} 0,0.5, \\ 1.5,2 \end{pmatrix}$ (6)	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}$	$\begin{pmatrix} 200,270, \\ 350,580 \end{pmatrix}$
					$\tilde{F}(\text{current}) = \sum_{i=1}^{3} \tilde{f}_i = \begin{pmatrix} 390,490, \\ 700,1020 \end{pmatrix}$

Total fuzzy cost is:

$$2 \otimes \begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix} \oplus 5 \otimes \begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix} \oplus 10 \otimes \begin{pmatrix} 0,1, \\ 2,5 \end{pmatrix} \oplus 6 \otimes \begin{pmatrix} 0,0.5, \\ 1.5,2 \end{pmatrix} \oplus 5 \otimes \begin{pmatrix} 0,1, \\ 2,5 \end{pmatrix} \oplus \begin{pmatrix} 390,490, \\ 700,1020 \end{pmatrix} = \begin{pmatrix} 411,550, \\ 802,1233 \end{pmatrix}$$

Since total fuzzy cost does not depend upon M. So, 3rd efficient solution exists.
Similarly, applying Steps 3 to 10, we get the 3rd fuzzy cost-time trade-off pair.

$$\left(\tilde{Z}_3, \tilde{T}_3\right) = \left(\begin{pmatrix} 357,511, \\ 687,1185 \end{pmatrix}, \begin{pmatrix} 4,5, \\ 9,18 \end{pmatrix} \right)$$

4th fuzzy cost-time trade-off pair:

The fuzzy costs, shown in Table 2, are modified using Step 11 and are shown in Table 15.

Table 15. Modified fuzzy costs

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	4	a_i
1	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix}$	$\begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix}$	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}$	19
2	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}$	10
3	$\begin{pmatrix} 0,1, \\ 2,5 \end{pmatrix}$	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}$	11
b_j	5	8	15	12	40

The fuzzy initial basic feasible solution of new problem (P_1) is shown in Table 16.

Table 16. Initial fuzzy basic feasible solution

Destination $(j) \rightarrow$ Source $(i) \downarrow$	1	2	3	4	$\tilde{f}_i$
1	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix}(4)$	$\begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix}(15)$	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}$	$\begin{pmatrix} 130,160, \\ 200,310 \end{pmatrix}$
2	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}(10)$	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}$
3	$\begin{pmatrix} 0,1, \\ 2,5 \end{pmatrix}(5)$	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}(4)$	$\begin{pmatrix} M, M, \\ M, M \end{pmatrix}$	$\begin{pmatrix} 0,0, \\ 0,0 \end{pmatrix}(2)$	$\begin{pmatrix} 170,230, \\ 300,500 \end{pmatrix}$
					$\tilde{F}(\text{current}) = \sum_{i=1}^{3} \tilde{f}_i = \begin{pmatrix} 300,390, \\ 500,810 \end{pmatrix}$

Total fuzzy cost is:

$$4 \otimes \begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix} \oplus 15 \otimes \begin{pmatrix} 3,6, \\ 9,18 \end{pmatrix} \oplus 5 \otimes \begin{pmatrix} 0,1, \\ 2,5 \end{pmatrix} \oplus 4 \otimes \begin{pmatrix} M, M, \\ M, M \end{pmatrix} \oplus \begin{pmatrix} 300,390, \\ 500,810 \end{pmatrix} = \begin{pmatrix} 4M+357, 4M+509, \\ 4M+681, 4M+1177 \end{pmatrix}.$$

Since total fuzzy cost depends upon M. So, the solution is infeasible. It is not possible to findfuzzy cost-time trade-off pair. So, the algorithm terminates here.
After 3rd fuzzy cost-time trade-off pair, the solution is infeasible. So, we have three cost-time trade-off pair shown in Table 17.

Table 17. Three cost-time trade-off pair

Fuzzy cost-time trade-off pair	Total fuzzy cost	Maximum fuzzy time.
$(\tilde{Z}_1,\tilde{T}_1)$	$\begin{pmatrix}347,498.5,\\664.5,1130\end{pmatrix}$	$\begin{pmatrix}8,9,\\17,34\end{pmatrix}$
$(\tilde{Z}_2,\tilde{T}_2)$	$\begin{pmatrix}349,501,\\669,1141\end{pmatrix}$	$\begin{pmatrix}5,10,\\15,30\end{pmatrix}$
$(\tilde{Z}_3,\tilde{T}_3)$	$\begin{pmatrix}357,511,\\687,1185\end{pmatrix}$	$\begin{pmatrix}4,5,\\9,18\end{pmatrix}$

6. Results

To compare the results in fuzzy and crisp environment, the problem formulated in Section 3, is solved in both fuzzy and crisp environment.

The results obtained in fuzzy environment can be explained as follows:

1) The total cost for first cost-time trade-off pair (Z_1,T_1) lies between 347 units and 1130 units and maximum time lies between 8 units and 34 units.
2) Maximum numbers of experts are in favour that total cost will be between 498.5 and 664.5 units and maximum time will be between 9 and 17 units.
3) The values of $\mu_{\tilde{Z}_1}(x)$ corresponds to different values of total cost $x\in[347,1130]$ can be evaluated as follows:

$$\mu_{\tilde{Z}_1}(x)=\begin{cases}0, & -\infty<x\le 347\\ \dfrac{x-347}{151.5}, & 347\le x<498.5\\ 1, & 498.5\le x\le 664.5\\ \dfrac{1130-x}{465.5}, & 664.5<x\le 1130\\ 0, & 1130\le x<\infty\end{cases}$$

The values of $\mu_{\tilde{T}_1}(t)$ corresponds to different values of maximum time $t\in[8,34]$ can be evaluated as follows:

$$\mu_{\tilde{T}_1}(t)=\begin{cases}0, & -\infty<t\le 8\\ \dfrac{t-8}{1}, & 8\le t<9\\ 1, & 9\le t\le 17\\ \dfrac{34-t}{17}, & 17<t\le 34\\ 0, & 34\le t<\infty\end{cases}$$

Table 18. Efficient solutions of the numerical problem in crisp and fuzzy environment

Cost-time trade-off in crisp environment			Cost-time trade-off in fuzzy environment		
Cost-time trade-off pair	Total cost (Z)	Time (T)	Fuzzy cost-time trade-off pair	Total fuzzy cost $(\tilde{Z})$	Fuzzy time $(\tilde{T})$
(Z_1, T_1)	660	17	$(\tilde{Z}_1, \tilde{T}_1)$	$(347, 498.5, 664.5, 1130)$ Rank = 660	$(8, 9, 17, 34)$ Rank = 17
(Z_2, T_2)	665	15	$(\tilde{Z}_2, \tilde{T}_2)$	$(349, 501, 669, 1141)$ Rank = 665	$(5, 10, 15, 30)$ Rank = 15
(Z_3, T_3)	685	9	$(\tilde{Z}_3, \tilde{T}_3)$	$(357, 511, 687, 1185)$ Rank = 685	$(4, 5, 9, 18)$ Rank = 9

Similarly, the results can be explained for 2nd and 3rd cost-time trade-off pairs.

Table 18 compares the results obtained in crisp and fuzzy environment. It is obvious, the results obtained by using the proposed approach represent the solution in more realistic manner. If there is no uncertainty about any parameter, then results of proposed approach will be same as obtained in crisp environment [36].

7. Conclusion

In this paper, a new algorithm is proposed for finding cost-time trade-off pairs of fuzzy bi-criteria fixed charge transportation problems occurring in real life situations. The algorithm is very easy to understand and simple to apply on real life problems and the obtained results are more flexible than existing approach [36].

The method proposed in this paper can be used to solve transportation problems like solid transportation problems, multi-objective transportation problems etc. Also, the concept of vague set [40] can be used to develop new algorithms for solving bi-criteria fixed charge transportation problems.

Acknowledgments

The authors would like to thank the Editor-in-Chief and anonymous referees for the various suggestions which have led to an improvement in both the quality and clarity of the paper. Dr. Amit Kumar acknowledges the innocent blessings of Mehar without which it was not possible to think of the idea proposed in this manuscript. Mehar is a lovely daughter of Parmpreet Kaur (A research scholar under the supervision of Dr. Amit Kumar).

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