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An Algorithm for Solving Fuzzy Maximal Flow Problems Using Generalized Trapezoidal Fuzzy Numbers

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Abstract: Kumar et al. (A new approach for solving fuzzy maximal flow problems, Lecture Notes in Computer Science, Springer-Verlag, Berlin Heidelberg 5908 (2009) 278-286) proposed a new algorithm to find the fuzzy maximal flow between source and sink by representing the flow as normal triangular fuzzy numbers. Chen (Operations on fuzzy numbers with function principal, Tamkang Journal of Management Science 6 (1985) 13-25) pointed out that in many cases it is not to possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers. There are several papers in the literature in which generalized fuzzy numbers are used for solving real life problems but to the best of our knowledge, till now no one has used generalized fuzzy numbers for solving the maximal flow problems. In this paper, the existing algorithm is modified to find fuzzy maximal flow between source and sink by representing all the parameters as generalized trapezoidal fuzzy numbers. To illustrate the modified algorithm a numerical example is solved and the obtained results are compared with the existing results. If there is no uncertainty about the flow between source and sink then the proposed algorithm gives the same result as in crisp maximal flow problems.

Keywords: Fuzzy maximal flow problem; ranking function; generalized trapezoidal fuzzy numbers

1. Introduction

The maximal flow problem is one of basic problems for combinatorial optimization in weighted directed graphs. It provides very useful models in a number of practical contexts including communication networks, oil pipeline systems and power systems. The maximal flow problem and its variations have wide range of applications and have been studied extensively. The maximal flow problem was proposed by Fulkerson and Dantzig [21] originally and solved by specializing the simplex method for the linear programming and Ford and Fulkerson [19] solved it by augmenting path algorithm. There are efficient algorithms to solve the crisp maximal flow problems [1, 2].

In the real life situations there always exist uncertainty about the parameters (e.g.: costs, capacities and demands) of maximal flow problems. To deal with such type of problems, the parameters of maximal flow problems are represented by fuzzy numbers [37] and maximal flow problems with fuzzy parameters are known as fuzzy maximal flow problems. In the literature, the numbers of papers dealing with fuzzy maximal flow problems are less. The paper by Kim and Roush [27] is one of the first on this subject.

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The authors developed the fuzzy flow theory, presenting the conditions to obtain a optimal flow, by means of definitions on fuzzy matrices. But there were Chanas and Kolodziejczyk [6,7,8] who introduced the main works in the literature involving this subject. They approached this problem using the minimum cuts technique.

In the first paper, Chanas and Kolodziejczyk [6] presented an algorithm for a graph with crisp structure and fuzzy capacities, i.e., the arcs have a membership function associated in their flow. This problem was studied by Chanas and Kolodziejczyk [7] again, in this paper the flow is a real number and the capacities have upper and lower bounds with a satisfaction function. Chanas and Kolodziejczyk [8] had also studied the integer flow and proposed an algorithm. Chanas et al. [5] studied the maximum flow problem when the underlying associated structure is not well defined and must be modeled as a fuzzy graph. Diamond [18] developed interval-valued versions of the max-flow min cut theorem and Karp-Edmonds algorithm and provide robustness estimates for flows in networks in an imprecise or uncertain environment. These results are extended to networks with fuzzy capacities and flows.

Liu and Kao [32] investigated the network flow problems in that the arc lengths of the network are fuzzy numbers. Ji et al. [25] considered a generalized fuzzy version of maximum flow problem, in which arc capacities are fuzzy variables. Hernandes et al. [22] proposed an algorithm, based on the classical algorithm of Ford-Fulkerson. The algorithm uses the technique of the incremental graph and representing all the parameters as fuzzy numbers. Kumar et al. [28] proposed a new algorithm to find fuzzy maximal flow between source and sink by using ranking function.

In this paper, the existing algorithm [28] is modified to find fuzzy maximal flow between source and sink by representing all the parameters as generalized trapezoidal fuzzy numbers. To illustrate the modified algorithm a numerical example is solved and the obtained results are compared with the existing results. If there is no uncertainty about the flow between source and sink then the proposed algorithm gives the same result as in crisp maximal flow problems.

This paper is organized as follows: In Section 2, advantages of generalized fuzzy numbers are discussed. In Section 3, some basic definitions, arithmetic operations and ranking function are reviewed. In Section 4, an algorithm is proposed for solving fuzzy maximal flow problems. In Section 5, to illustrate the proposed algorithm a numerical example is solved. The results are discussed in Section 6 and the conclusion is discussed in Section 7.

2. Why Generalized Fuzzy Number

Ranking of fuzzy numbers play an important role in decision making problems. Fuzzy numbers must be ranked before an action is taken by a decision maker. Jain [24] proposed the concept of ranking function for comparing normal fuzzy numbers. Chen [10] pointed out that in many cases it is not to possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers. Since then, tremendous efforts are spent and significant advances are made on the development of numerous methodologies [3,4,9,11-17,20,23, 29,31,33,35] for the ranking of generalized fuzzy numbers.

In most of the papers the generalized fuzzy numbers are converted into normal fuzzy numbers through normalization process [26] and then normal fuzzy numbers are used to solve the real life problems. Kaufmann and Gupta [26] pointed out that there is a serious disadvantage of the normalization process. Basically we have transformed a measurement of an objective value to a valuation of a subjective value, which results in the loss of information. Although this procedure is mathematically correct, it decreases the amount of information that is available in the original data, and we should avoid it.

Hsieh and Chen [23] pointed out that arithmetic operations on fuzzy numbers presented in Chen [10] does not only change the type of membership function of fuzzy numbers after arithmetic operations, but they can also reduce the troublesomeness and tediousness of arithmetic operations.

There are several papers [11-13, 15, 33, 36] in which generalized fuzzy numbers are used for solving real life problems but to the best of our knowledge, till now no one has used generalized fuzzy numbers for solving the maximal flow problems.

3. Preliminaries

In this section some basic definitions, arithmetic operations and ranking function are reviewed.

3.1. Basic Definitions

In this section, some basic definitions are reviewed.

Definition 1. [26] A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & -\infty < x \le a \\ \dfrac{(x-a)}{(b-a)}, & a \le x < b \\ 1, & b \le x \le c \\ \dfrac{(x-d)}{(c-d)}, & c < x \le d \\ 0, & d \le x < \infty \end{cases}$$

where, $a, b, c, d \in R$

Definition 2. [13] A fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & -\infty < x \le a \\ \dfrac{w(x-a)}{(b-a)}, & a \le x < b \\ w, & b \le x \le c \\ \dfrac{w(x-d)}{(c-d)}, & c < x \le d \\ 0, & d \le x < \infty \end{cases}$$

where, $a, b, c, d \in R$ and $0 < w \le 1$.

3.2. Arithmetic Operations

In this section, addition and subtraction between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [13].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(w_1, w_2))$

(ii) $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min(w_1, w_2))$

Example 1 Let $\tilde{A} = (-2,3,5,7;.3)$, $\tilde{B} = (-3, 4,10,12;.2)$ are two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A} \oplus \tilde{B} = (-5,7,15,19;\ \min(.3,.2)) = (-5,7,15,19;.2)$

(ii) $\tilde{A} \ominus \tilde{B} = (-14,-7,1,10;\ \min(.3,.2)) = (-14,-7,1,10;.2)$

Remark 1 In this paper min and max represents minimum and maximum respectively.

3.3. Ranking Function

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function [31], $\Re : F(R) \to R$, where

$F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

(i) $\tilde{A} \succ \tilde{B}$ iff $\Re(\tilde{A}) > \Re(\tilde{B})$

(ii) $\tilde{A} \prec \tilde{B}$ iff $\Re(\tilde{A}) < \Re(\tilde{B})$

(iii) $\tilde{A} \approx \tilde{B}$ iff $\Re(\tilde{A}) = \Re(\tilde{B})$

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$, $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers and $w = \min(w_1, w_2)$ then $\Re(\tilde{A}) = \dfrac{w(a_1 + b_1 + c_1 + d_1)}{4}$ and $\Re(\tilde{B}) = \dfrac{w(a_2 + b_2 + c_2 + d_2)}{4}$.

Remark 2 Let $\tilde{A}_i; i = 1,2,\ldots,n$ be a set of generalized trapezoidal fuzzy numbers. If $\Re(\tilde{A}_k) \le \Re(\tilde{A}_i)$, for all i, then the generalized trapezoidal fuzzy number $\tilde{A}_k$ is the minimum of $\tilde{A}_i; i = 1,2,\ldots,n$ and if $\Re(\tilde{A}_t) \ge \Re(\tilde{A}_i)$, for all i, then the generalized trapezoidal fuzzy number $\tilde{A}_t$ is the maximum of $\tilde{A}_i; i = 1,2,\ldots,n$.

4. Proposed Algorithm

Chen [10] pointed out that in many cases it is not possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers. There are several papers in the literature in which generalized fuzzy numbers are used for solving real life problems but to the best of our knowledge, till now no one has used generalized fuzzy numbers for solving the maximal flow problems. In this section, the existing algorithm [28] is modified to find fuzzy maximal flow between source and sink by representing all the parameters as generalized trapezoidal fuzzy numbers.

The proposed algorithm is a labeling technique. Since proposed algorithm is direct extension of existing algorithm [34], so it is very easy to understand and apply for solving fuzzy maximal problems occurring in real life situations. The fuzzy maximal flow algorithm is based on finding breakthrough paths with net positive flow between the source and sink nodes. Consider arc (i, j) with initial fuzzy capacities $(\overline{\tilde{fc}}_{ij}, \overline{\tilde{fc}}_{ji})$. As portions of these fuzzy capacities are committed to the flow in the arc, the fuzzy residuals (or remaining fuzzy capacities) of the arc are updated. We use the notation $(\tilde{fc}_{ij}, \tilde{fc}_{ji})$ to represent these fuzzy residuals.

For a node j that receives flow from node i, we attach a label $[\tilde{fa}_j, i]$, where $\tilde{fa}_j$ is the fuzzy flow from i node to j. The steps of the algorithm are thus summarized as follows:

Step 1 For all arcs (i, j), set the residual fuzzy capacity equal to the initial fuzzy capacity i.e., $(\tilde{fc}_{ij}, \tilde{fc}_{ji}) = (\overline{\tilde{fc}}_{ij}, \overline{\tilde{fc}}_{ji})$. Let $\tilde{fa}_1 = (\infty, \infty, \infty, \infty; 1)$ and label source 1 with $[(\infty, \infty, \infty, \infty; 1), -]$ Set $i = 1$, and go to Step 2.

Step 2 Determine S_i, the set of unlabeled nodes j that can be reached directly from node i by arcs with positive residuals (i.e., $\tilde{fc}_{ij}$ is a non-negative fuzzy number for all $j \in S_i$). If $S_i \ne \phi$, go to Step 3. Otherwise, go to Step 4.

Step 3 Determine $k \in S_i$ such that $\max_{j \in S_i}\{\Re(\tilde{fc}_{ij})\} = \Re(\tilde{fc}_{ik})$. Set $\tilde{fa}_k = \tilde{fc}_{ik}$ and label node k with $[\tilde{fa}_k, i]$. If $k = n$, the sink node has been labeled, and a breakthrough path is found, go to Step 5. Otherwise, set $i = k$, and go to Step 2.

Step 4 Backtracking. If $i = 1$, no breakthrough is possible; go to Step 6. Otherwise, let r be the node that has been labeled immediately before current node i and remove i from the set of nodes adjacent to r. Set $i = r$ and go to Step 2.

Step 5 Determination of Residuals. Let $N_p = \{1, k_1, k_2, \ldots, n\}$ define the nodes of the p^{th} breakthrough path from source node 1 to sink node n. Then the maximal flow along the path is computed as

$$\widetilde{f}_p = \min\{\widetilde{fa}_1, \widetilde{fa}_{k_1}, \widetilde{fa}_{k_2}, \ldots, \widetilde{fa}_n\}$$

The residual capacity of each arc along the breakthrough path is decreased by $\widetilde{f}_p$ in the direction of the flow and increased by $\widetilde{f}_p$ in the reverse direction i.e., for nodes i and j on the path, the residual flow is changed from the current $(\widetilde{fc}_{ij}, \widetilde{fc}_{ji})$ to

(a) $(\widetilde{fc}_{ij} \ominus \widetilde{f}_p, \widetilde{fc}_{ji} \oplus \widetilde{f}_p)$ if the flow is from i to j

(b) $(\widetilde{fc}_{ij} \oplus \widetilde{f}_p, \widetilde{fc}_{ji} \ominus \widetilde{f}_p)$ if the flow is from j to i

Reinstate any nodes that were removed in Step 4. Set $i = 1$, and return to Step 2 to attempt a new breakthrough path.

Step 6 Solution.

(a) Given that m breakthrough paths have been determined, the fuzzy maximal flow in the network is

$$\widetilde{F} = \widetilde{f}_1 \oplus \widetilde{f}_2 \oplus \ldots \oplus \widetilde{f}_m$$

where m is the number of iteration to get no breakthrough.

(b) Using the initial and final fuzzy residuals of arc (i, j), $(\overline{\widetilde{fc}}_{ij}, \overline{\widetilde{fc}}_{ji})$ and $(\widetilde{fc}_{ij}, \widetilde{fc}_{ji})$ respectively, the fuzzy optimal flow in arc (i, j) is computed as follows:

Let $(\widetilde{\alpha}, \widetilde{\beta}) = (\overline{\widetilde{fc}}_{ij} \ominus \widetilde{fc}_{ij}, \overline{\widetilde{fc}}_{ji} \ominus \widetilde{fc}_{ji})$. If $\Re(\widetilde{\alpha}) > 0$, the fuzzy optimal flow from i to j is $\widetilde{\alpha}$. Otherwise, if $\Re(\widetilde{\beta}) > 0$, the fuzzy optimal flow from j to i is $\widetilde{\beta}$. (It is impossible to have both $\Re(\widetilde{\alpha})$ and $\Re(\widetilde{\beta})$ positive.)

5. Illustrative Example

In this section the proposed algorithm is illustrated by solving a numerical example.

Example 2 Consider the network shown in Figure. 1. The bidirectional fuzzy capacities are represented by generalized trapezoidal fuzzy numbers and shown on the respective arcs. For example, for arc (3,4) the flow limit is represented by generalized trapezoidal fuzzy number (0,5,15,20; .3) units from node 3 to 4. Determine the fuzzy maximal flow in this network between source 1 and sink 5.

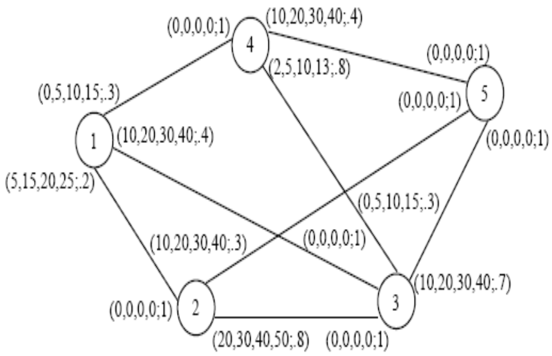

Figure1.

The algorithm is applied in the following manner:

Iteration 1

Set the initial fuzzy residual $(\widetilde{fc}_{ij}, \widetilde{fc}_{ji})$ equal to the initial fuzzy capacities $(\overline{\widetilde{fc}}_{ij}, \overline{\widetilde{fc}}_{ji})$.

Step 1 Set $\widetilde{fa}_1 = (\infty, \infty, \infty, \infty; 1)$ and label node 1 with $[(\infty, \infty, \infty, \infty; 1), -]$. Set $i = 1$.

Step 2 $S_1 = \{2, 3, 4\} (\neq \phi)$.

Step 3 $k = 3$, because $\max\{\Re(\widetilde{fc}_{12}), \Re(\widetilde{fc}_{13}), \Re(\widetilde{fc}_{14})\} = \Re(\widetilde{fc}_{13})$. Set $\widetilde{fa}_3 = \widetilde{fc}_{13} =$ (10,20,30,40;.4), and label node 3 with [(10,20,30,40;.4),1]. Set $i = 3$, and repeat Step 2.

Step 2 $S_3 = \{4, 5\}$.

Step 3 $k = 5$, because $\max\{\Re(\widetilde{fc}_{34}), \Re(\widetilde{fc}_{35})\} = \Re(\widetilde{fc}_{35})$. Set $\widetilde{fa}_5 = \widetilde{fc}_{35} =$ (10,20, 30,40;.7), and label node 5 with [(10,20, 30,40;.7),3]. Breakthrough is achieved. Go to Step 5.

Step 5 The breakthrough path is determined from the labels starting at node 5 and moving backward to node 1. The breakthrough path is $1 \to 3 \to 5$. Thus, $N_1 = \{1,3,5\}$ and $\tilde{f}_1 = \min\{\tilde{f}a_1, \tilde{f}a_3, \tilde{f}a_5\} = \min\{(\infty, \infty, \infty, \infty; 1), (10,20,30,40;.4), (10,20,30,40;.7)\} = (10,20,30,40;.4)$.

The fuzzy residual capacities along path N_1 are

$(\tilde{f}c_{13}, \tilde{f}c_{31}) =$ ((10,20,30,40;.4)Θ(10,20,30,40;.4),(0,0,0,0;1) $\oplus$ (10,20,30,40;.4)) = ((−30, $-10,10,30;.4), (10,20,30,40;.4)$)

$(\tilde{f}c_{35}, \tilde{f}c_{53}) =$ ((10,20,30,40;.7)Θ(10,20,30,40;.4),(0,0,0,0;1) $\oplus$ (10,20,30,40;.4)) = ((−30, $-10,10,30;.4), (10,20,30,40;.4)$)

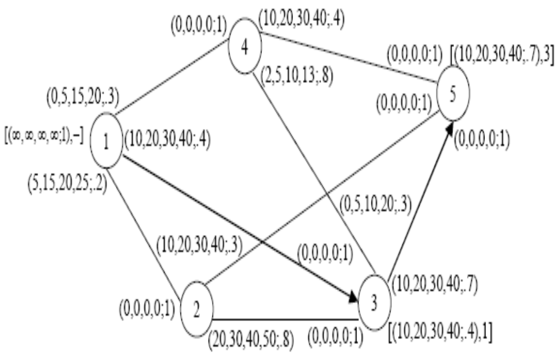

Figure2.

Iteration 2

Repeating the procedure described in the first iteration, at the starting node 1, the obtained breakthrough path is $1 \to 2 \to 3 \to 4 \to 5$ and $\tilde{f}_2 = (0,5,10,15;.3)$.

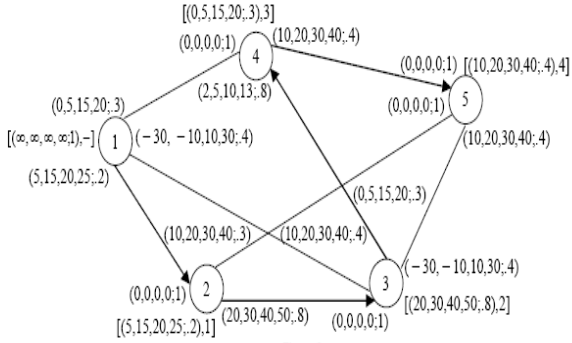

Figure3.

Iteration 3

The obtained breakthrough path is $1 \to 4 \to 3 \to 2 \to 5$ and $\tilde{f}_3 = (0,5,15,20;.3)$.

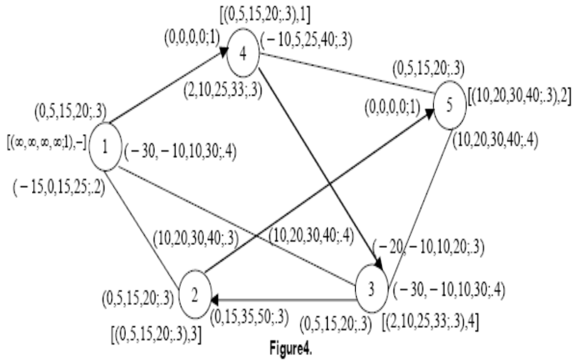

Figure4.

Iteration 4

The obtained breakthrough path is $1 \to 2 \to 3 \to 4 \to 5$ and $\tilde{f}_4 = (-15,0,15,25;.2)$.

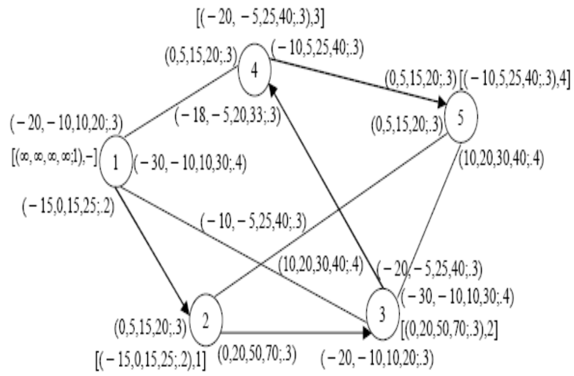

Figure5.

Iteration 5

More iterations are not possible after 4^{th} iteration because there is no way out to reach at sink from source.

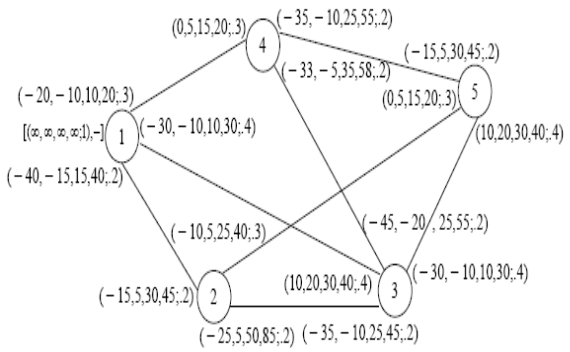

Figure6.

Step 6 Fuzzy maximal flow in the network is $\widetilde{F} = \widetilde{f}_1 \oplus \widetilde{f}_2 \oplus \widetilde{f}_3 \oplus \widetilde{f}_4 =$ (10,20,30,40;.4) $\oplus$ (0,5,15,20;.3) $\oplus$ (0,5,15,20;.3) $\oplus$ (−15,0,15, 25;.2) = (−5,30,75,105;.2). The fuzzy flow in the different arcs is computed by subtracting the last fuzzy residuals $(\widetilde{fc}_{ij}, \widetilde{fc}_{ji})$ from the initial fuzzy capacities $(\widetilde{fc}_{ij}, \widetilde{fc}_{ji})$ and is shown in Table 1.

Table 1. Fuzzy optimal flow in different arcs

Arc	$((\widetilde{fc}_{ij} \ominus \widetilde{fc}_{ij}),$ $(\widetilde{fc}_{ji} \ominus \widetilde{fc}_{ji}))$	$(\Re(\widetilde{fc}_{ij} \ominus \widetilde{fc}_{ij}),$ $\Re(\widetilde{fc}_{ji} \ominus \widetilde{fc}_{ji}))$	Fuzzy flow amount	Direction
(1,2)	((−35,0,35,65;.2), (−45,−30,−5,15;.2))	(3.25, −3.25)	(−35,0,35,65;.2)	$1 \rightarrow 2$
(1,3)	((−20,10,40,70;.4), (−40,−30,−20,−10;.4))	(10, −10)	(−20,10,40,70;.4)	$1 \rightarrow 3$
(1,4)	((−20,−5,25,40;.3), (−20,−15,−5,0;.3))	(3, −3)	(−20,−5,25,40;.3)	$1 \rightarrow 4$
(2,3)	((−65,−20,35,75;.2), (−45,−25,10,35;.2))	(1.25, −1.25)	(−65,−20,35,75;.2)	$2 \rightarrow 3$
(2,5)	((−30,−5,25,50;.3), (−20,−15,−5,0;.3))	(3, −3)	(−30,−5,25,50;.3)	$2 \rightarrow 5$
(3,4)	((−55,−20,35,65;.2), (−56,−30,15,46;.2))	(1.25, −1.25)	(−55,−20,35,65;.2)	$3 \rightarrow 4$
(3,5)	((−20,10,40,70;.4), (−40,−30,−20,−10;.4))	(10, −10)	(−20,10,40,70;.4)	$3 \rightarrow 5$
(4,5)	((−45,−5,40,75;.2), (−45,−30,−5,15;.2))	(3.25, −3.25)	(−45,−5,40,75;.2)	$4 \rightarrow 5$

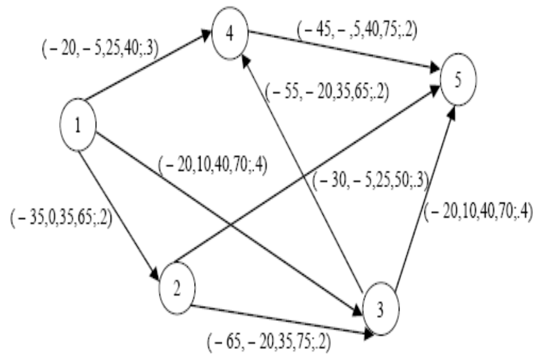

Figure7.

6. Results and Discussion

The fuzzy optimal flow is $\widetilde{F} = (-5,30,75,105;.2)$. The obtained result can be explained as follows:

(i) According to decision maker the amount of flow between source and sink is greater than -5 and less than 105 units.

(ii) The overall level of satisfaction of the decision maker about the statement that the fuzzy maximal flow will be 30 to 75 units is 20%.

(iii) The overall level of satisfaction of the decision maker for the remaining amount of flow can be obtained as follows:

Let x represents the amount of flow, then the overall level of satisfaction of the decision maker for $x = \mu_{\widetilde{F}}(x) \times 100$,

$$\mu_{\widetilde{F}}(x) = \begin{cases} 0, & -\infty < x \le -5 \\ \dfrac{.2(x+5)}{35}, & -5 \le x < 30 \\ .2, & 30 \le x \le 75 \\ \dfrac{.2(x-105)}{(-30)}, & 75 < x \le 105 \\ 0, & 105 \le x < \infty \end{cases}$$

7. Conclusion and Future Work

In this paper, an algorithm is proposed for finding the fuzzy optimal flow of fuzzy maximal flow problems in which the flows are represented by generalized trapezoidal fuzzy numbers. A numerical example is solved to illustrate the proposed algorithm. The proposed algorithm is very easy to understand and to apply for solving the fuzzy maximal flow problems occurring in real life situations.

The concept of finding the fuzzy optimal solution of fuzzy maximal flow problems, presented in this paper, is quite general in nature and can be extended to solve the other network flow problems like shortest path problems, critical path method etc. Also, the concept of vague set [30] can be used to develop new algorithms for solving network flow problems.

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