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Comparison of Efflux Time between Cylindrical and Conical Tanks Through an Exit Pipe

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Abstract: Mathematical equations for efflux time during gravity draining of a Newtonian liquid (below its bubble point) from large open storage tanks of cylindrical and conical shapes (Where the flow in the respective tanks is essentially laminar) through an exit pipe of same length and cross sectional area (the flow in the exit pipe line being turbulent) located at the bottom of the respective storage tanks are developed. The equations are ultimately simplified and written in dimensionless forms. These equations will be of use in arriving at the minimum time required for draining the contents of the respective geometries of storage vessels. To drain the same volume of liquid, the efflux time equations so developed are compared to find out which of the tanks considered drain faster.

Keywords: Efflux time; Newtonian liquid; open storage tank; exit pipe; minimum time.

1. Introduction

Processing equipment and storage vessels used in the chemical and allied industries appear in a large variety of shapes. The reason for use of different geometrical shapes of vessels may include convenience, insulation requirements, floor space, material costs, corrosion, safety considerations etc.

Hart and Sommerfeld (1995) mentioned that the time required to drain these vessels off their liquid contents is known as efflux time and this is of crucial importance under many emergency situations besides productivity considerations [1]. They developed expressions for efflux time for annular and toriadial containers through restricted orifice. Jouse (2003) reported modeling and experimental work for efflux time during gravity draining of a Newtonian liquid through restricted orifice [2]. Vandogen and Roche.jr (1995) presented their experimental work on efflux time for a cylindrical tank through an exit pipe for the case of turbulent flow in the exit pipe[3]. The Reynolds number range considered was between 40,000 to 60,000 and exit pipe length was 1 meter. However, there is a possibility of formation of pockets of air or vapor when intense turbulent conditions prevail as in the case cited. Morison (2001) carried out modeling and simulation work for efflux time for a cylindrical tank through an exit pipe at around Reynolds number of 6,000 using computational tools [4]. The author used a roughness value of 0 in his calculations. The author also reported that the friction factor equation used is valid for Reynolds number >5000. Joye et al (2003) reported work on efflux time through an exit pipe for both laminar and turbulent flow conditions in the exit

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pipe [5]. They mentioned that turbulent flow (in the exit pipe) solutions are useful in many plant situations. They made an assumption of constant friction factor in the exit pipe line. Subbarao et al (2008 a, b) modeled the efflux time equation for draining the contents of cylindrical storage vessel through exit pipe(s) for turbulent flow in the exit pipe [6, 7]. They named the simplified form of efflux time equation as modified form of Toricelli equation and introduced a term called modified from of acceleration due to gravity g_m. They also reported that during draining of a liquid from a large cylindrical storage vessel through exit pipe, Froude number remains constant and is influenced by diameter and length of the exit pipe. They further mentioned that polymer additions influence the efflux time. Efflux time can also be influenced by the geometry of the vessel.

In the present work, mathematical equations for efflux time while draining a Newtonian liquid from a large open storage tanks of two different geometries through exit pipe are derived. The exit pipe diameter for both the geometries considered is assumed to be same and the mathematical analysis is based on macroscopic balances. Bird et al (2006) mentioned that macroscopic balances are useful for initial appraisal of an engineering problem [8]. They are sometimes used to derive approximate relations which will then be verified with experimental data for terms which have been omitted or about which there is insufficient information. The geometries considered for development of efflux time equation are

Cylindrical tank with a flat bottom
Conical tank

The expressions developed are compared to find out which of the tanks considered drains faster.

2. Development of mathematical equation for efflux time for different geometries

Suppose an open tank of given geometry (Figure1, 2) provided with an exit pipe is plugged and initially filled with a Newtonian and incompressible liquid. The liquid leaves the sation-2 when the exit pipe is unplugged. It is desired to find the time required to drain the contents of the respective storage vessels, not the exit pipe.

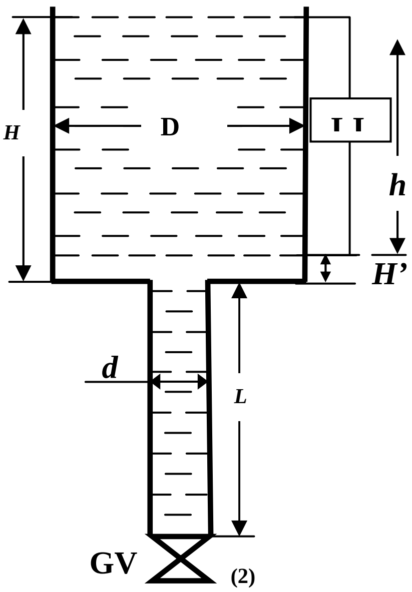

Figure1. Tank along with Exit Pipe.

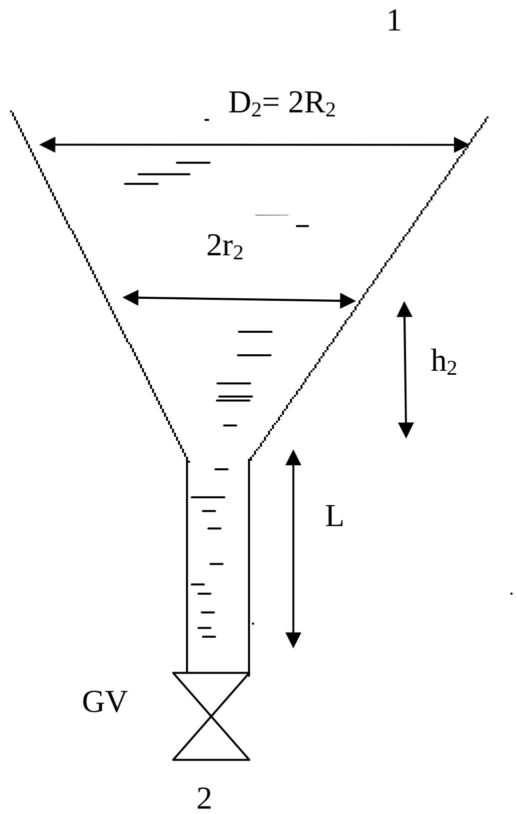

Figure 2. Conical tank along with exit pipe

Writing the mass balance equation,
Rate of mass in –Rate of mass out = Rate of mass accumulation

$$W_1 - W_2 = \frac{d}{dt}(V\rho) \tag{1}$$

For the present system $W_1 = 0$ and $W_2 = \rho V_2 \frac{\pi}{4} d^2$ Hence $-W_2 = \frac{d}{dt}(V\rho)$ (2)

The mechanical energy balance equation between station-1 and station-2 can be written as

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gZ_2 + 4f\frac{L}{2d}V_2^2 \tag{3}$$

For the present system, $P_1 = P_2$ (since top and bottom are open to atmosphere),
At any height h, $Z_1 = Z_2 + h$ Subbarao et al [6, 7] used macroscopic balances and accounted for both $4f\frac{L}{d}\frac{V_2^2}{2}$ and $\frac{V_2^2}{2}$ while simplifying the mathematical equation for efflux time. They assumed a constant friction factor while simplifying the equation. Their experiments suggested that $4f\frac{LV_2^2}{2d} >> \frac{V_2^2}{2}$. Hence neglecting $\frac{V_2^2}{2}$ allows development of an alternative equation for efflux time. With the above, Eq.3 can be written as

$$\frac{V_1^2}{2} + g(h+L) = \frac{4fLV_2^2}{2d} \tag{4}$$

Further, liquid drains very slowly (Since the diameter of tank is very large compared to the diameter of the exit pipe through which the liquid drains), $V_1 = 0$ & for turbulent flow in the exit pipe, the friction factor is given by $f = \frac{0.0791}{\mathrm{Re}^{0.25}}$ and $f = \frac{0.0791}{\left(\frac{dV_2\rho}{\mu}\right)^{0.25}}$

Eqn.4 becomes

$$V_2 = \frac{2.87 * g^{4/7}(h+L)^{4/7}(d\rho)^{1/7} d^{4/7}}{\mu^{1/7} L^{4/7}} \tag{5}$$

2.1. Development of Mathematical equation for efflux time for a cylindrical tank

For incompressible fluid, Substituting the value of V_2 from Eq.5 and $V = \frac{\pi}{4} D^2 h_1$ in Eqn.2

$$\frac{d}{dt}\left(\frac{1}{4} D_1^2 h_1\right) = -\frac{2.87 * g^{4/7}(h+L)^{4/7}(d\rho)^{1/7} d^{4/7} d^2}{\mu^{1/7} L^{4/7}\, 4} \tag{6}$$

Separating the variables and integrating between h=H_1 (t=0) and to complete draining h=0 and (t=t_1)

$$t_1 = 0.813 * \frac{D_1^2}{d^2}\left[\left(\frac{H_1}{L}+1\right)^{3/7} - 1\right] * \left(\frac{\mu d^2}{g^4 \rho}\right)^{1/7}\left(\frac{L}{d}\right) \tag{7}$$

$$\frac{t_1}{\left(\frac{\mu d^2}{\rho g^4}\right)^{1/7}} = 0.813 * \frac{D_1^2}{d^2}\left[\left(\frac{H_1}{L}+1\right)^{3/7} - 1\right] * \frac{L}{d} \tag{8}$$

$$\theta_1 = 0.813 * \frac{D_1^2}{d^2}\left[\left(\frac{H_1}{L}+1\right)^{3/7} - 1\right] * \frac{L}{d} \tag{9}$$

Where $\theta_1 = \frac{t_1}{\left(\frac{\mu d^2}{\rho g^4}\right)^{1/7}}$ (10)

2.2. Development of Mathematical equation for efflux time for a conical tank

As shown in the Figure 2, a conical tank has to be drained by means of an exit pipe (when the flow in the exit pipe is turbulent).

Applying mass balance equation for incompressible liquids

The dimensions are shown in the Figure 2. The tank is filled with a Newtonian liquid and the liquid is drained from 2. It is desired to find the efflux time required to drain the contents of the storage vessel.

$$\frac{d}{dt}\left((\frac{1}{3}\pi r_2{}^2 h_2 \right) = -\frac{2.87 * \rho * g^{4/7} (h_2 + L)^{4/7} (d\rho)^{1/7} d^{4/7} d^2 \pi}{\mu^{1/7} L^{4/7} 4} \tag{11}$$

From Figure 2 for the conical tank

$$\frac{r_2}{h_2} = \frac{R_2}{H_2} \tag{12}$$

$$r_2 = \frac{h_2 \times R_2}{H_2} \tag{13}$$

Substitute "r" value in above equation

$$\frac{d}{dt}\left(\frac{R_2{}^2}{3H_2{}^2} \pi h_2{}^2 h2 \right) = -\frac{2.87 * \rho * g^{4/7} (h_1 + L)^{4/7} (d\rho)^{1/7} \pi d^{4/7} d^2}{\mu^{1/7} L^{4/7} 4}$$

$$\frac{d}{dt}\left(\frac{1}{3} h_2{}^2 h_2 \right) = -\frac{2.87 * \rho * g^{4/7} (h_2 + L)^{4/7} (d\rho)^{1/7} d^{4/7} d^2 H_2{}^2}{\mu^{1/7} L^{4/7} D_2{}^2}$$

The above equation upon integration between the limits ($h_2=H_2$ at t=0 and $h_2=0$ at $t=t_2$) gives the following equation for efflux time

$$t_2 = 0.35 * \frac{D_1{}^2}{H_2{}^2} \left(\frac{L}{d} \right)^3 \left(\frac{\mu d^2}{\rho g^4} \right)^{1/7} \left\{ \left[\frac{7}{17} \left(1 + \frac{H_2}{L} \right)^{17/7} - 1 \right] + \left[\frac{7}{3} \left(1 + \frac{H_2}{L} \right)^{3/7} - 1 \right] - \left[\frac{7}{5} \left(1 + \frac{H_2}{L} \right)^{10/7} - 1 \right] \right\} \tag{14}$$

Defining in terms of dimensionless groups

$$\frac{t_2}{\left(\frac{\mu d^2}{\rho g^4} \right) 1/7} = \theta_2 \tag{15}$$

$$\theta_2 = 0.35 * \frac{D_2{}^2}{H_2{}^2} \left(\frac{L}{d} \right)^3 X_2 \tag{16}$$

Where

$$X_2 = \left\{ \left[\frac{7}{17} \left(1 + \frac{H_2}{L} \right)^{17/7} - 1 \right] + \left[\frac{7}{3} \left(1 + \frac{H_2}{L} \right)^{3/7} - 1 \right] - \left[\frac{7}{5} \left(1 + \frac{H_2}{L} \right)^{10/7} - 1 \right] \right\} \tag{17}$$

3. Results and Discussion

While deriving the above equations, the contraction coefficient term is neglected and the fluid motion around the respective storage tanks is also neglected.

The efflux time equations for cylindrical and conical tanks derived above are influenced only by dimensions of the tanks and physical properties of the liquid, not on the Reynolds number. They are also expected to be much more useful for the case of variable friction factor as well.

In order to drain the same volume of liquid, the mathematical equations derived are now compared to find out which of the tanks considered drain faster (i.e, whose efflux time is the lowest).

3.1. Comparison of Efflux time of Cylinder with that of Cone

The efflux time equation for cylindrical tank and that of conical tank are compared by taking ratio of $\frac{\theta_1}{\theta_2}$. This is obtained by dividing Eq.9 with Eq.16

$$\frac{\theta_1}{\theta_2}=\frac{2.32D_1^{\,2}*\left(\frac{H_2}{L}\right)^2\left[\left(1+\frac{H_1}{L}\right)^{3/7}-1\right]}{D_2^{\,2}\left\{\left[\frac{7}{17}*(1+\frac{H_2}{L})^{17/7}-1\right]+\left[\frac{7}{3}*(1+\frac{H_2}{L})^{3/7}-1\right]-\left[\frac{7}{5}*(1+\frac{H_2}{L})^{10/7}-1\right]\right\}} \tag{18}$$

When this ratio is >1, it can be concluded that conical tank drains faster than a cylinder.

When same volume of liquid is to be drained, i.e $\frac{\pi}{4}D_1^{\,2}H_1=\frac{1}{3}\pi R_2^{\,2}H_2$ (19)

The equation suggests that the volume of liquid in both the tanks is influenced by both the diameter and height of liquid in the tank. This gives rise to the following two cases.

(a) When the diameter of cylinder is same as maximum diameter of cone i.e ($D_1=2R_2=D_2$)

In this case, Eq19 becomes $H_1=\frac{H_2}{3}$ and eq.18 can be written in terms of H_2 and L as

$$\frac{\theta_1}{\theta_2}=\frac{2.32*\left(\frac{H_2}{L}\right)^2\left[\left(1+\frac{H_2}{3L}\right)^{3/7}-1\right]}{\left\{\left[\frac{7}{17}*((1+\frac{H_2}{L})^{17/7}-1)\right]+\left[\frac{7}{3}*((1+\frac{H_2}{L})^{3/7}-1)\right]-\left[\frac{7}{5}*((1+\frac{H_2}{L})^{10/7}-1)\right]\right\}} \tag{20}$$

Since the ratio is a function of H_2/L and H_2/L is always >0 for all values of H & L, a plot of $\frac{H_2}{L}$ vs $\frac{\theta_1}{\theta_2}$ is shown in figure 3.

The plot suggests that the ratio is >1 for all values of H_2/L suggesting the efflux time for cylindrical tank is greater than conical tank. Hence conical tank drains faster than a cylindrical tank when the maximum diameter of cone is same as that of cylinder.

The plot also suggests that the ratio increases as H_2/L increases. H_2/L can be increased by keeping L constant (Figure 5) and increasing H_2 or by keeping H_2 constant and decreasing L (figure 6)

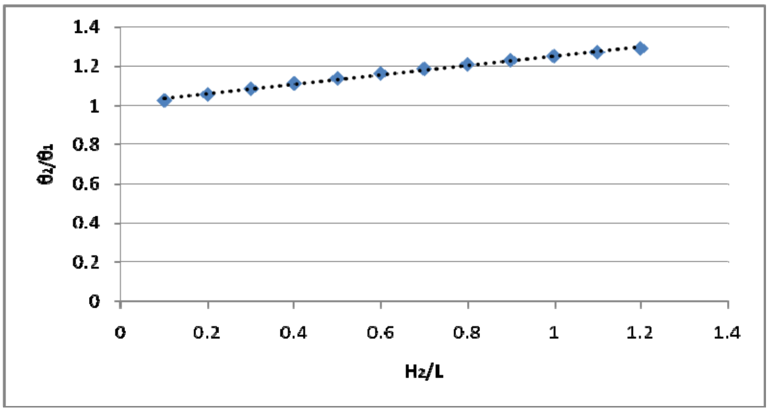

Figure 3. Efflux time ratio for different values of (H_2/L)>0)) ($D_1=D_2$)

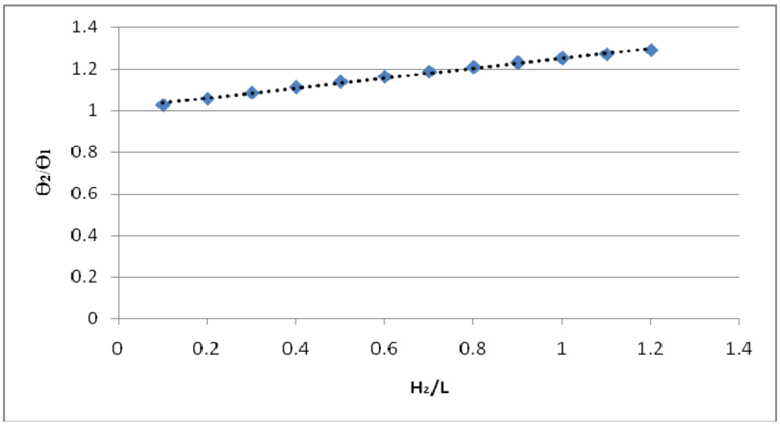

Figure 4. Efflux time ratio for different values of (H_2/L)>0, L fixed)) ($D_1=D_2$)

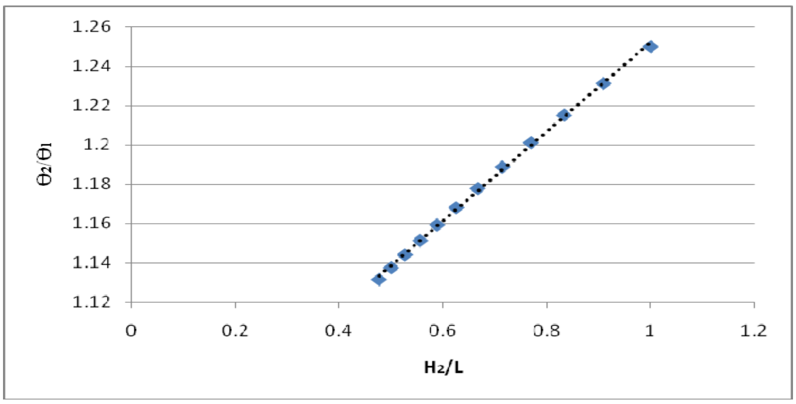

Figure 5. Efflux time ratio for different values of (H_2/L)>0, H_2 fixed)) ($D_1=D_2$)

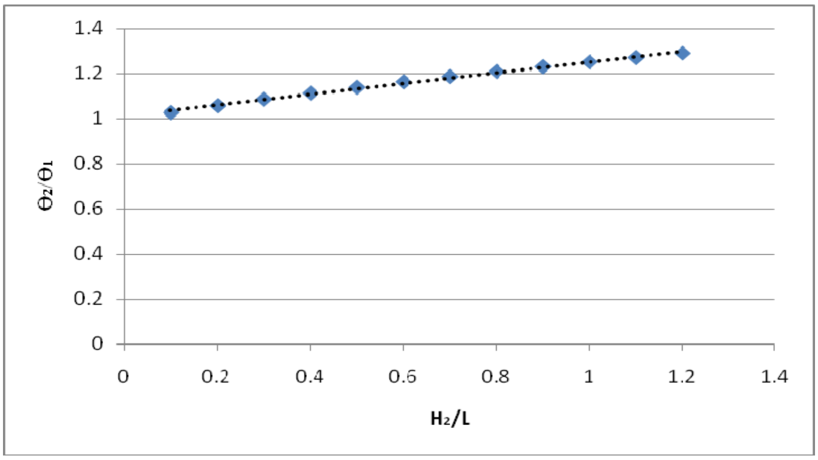

Figure 6. Efflux time ratio for different values of (H_2/L)>0)) ($H_1=H_2$)

(b) When the height of liquid in both the tanks is same i.e $H_1=H_2$

In this case, Eq.20 becomes $D_1^{\,2} = \frac{D_2^{\,2}}{3}$ (21)

Eq.18 can be written as

$$\frac{\theta_1}{\theta_2} = \frac{0.78*\left(\frac{H_2}{L}\right)^2\left[\left(1+\frac{H_2}{L}\right)^{3/7}-1\right]}{\left\{\left[\frac{7}{17}*(1+\frac{H_2}{L})^{17/7}-1\right]+\left[\frac{7}{3}*(1+\frac{H_2}{L})^{3/7}-1\right]-\left[\frac{7}{5}*(1+\frac{H_2}{L})^{10/7}-1\right]\right\}} \quad (22)$$

A plot of $\frac{H_2}{L}$ vs $\frac{\theta_2}{\theta_1}$ is shown in figure 6. From the plot, it can be observed that the ratio $\frac{\theta_1}{\theta_2}$ is > 1 suggesting draining time for cylinder is greater than that of a cone under turbulent flow conditions in the exit pipe.

(c) While deriving the above equations for efflux time, (eq.7 and 14), the contraction coefficient term is neglected.

The value of contraction coefficient for cylinder is given as 1.5 by Joye et al (2003) where as Mott [9] reported a contraction coefficient of 0 for cone. This will further increase the efflux time ratio. Hence cone is expected to drain much rapidly than cylinder as predicted by the above equations.

4. Conclusions

Some of the conclusions of the present work are

(a) For draining the same volume of liquid through exit pipe of same diameter, efflux time for cone < efflux time for cylinder. The efflux time ratio is influenced only by H_2/L.

(b) The efflux time ratio is influenced by both diameter as well as height of liquid in the storage vessels.

(c) The plots (Fig.3 & 6) also suggest that efflux time ratio is more for the case of equal diameters of tanks than equal heights of liquid in the tanks.

The theoretical values obtained are to be verified experimentally to find out the exact ratio of efflux times for the geometries considered.

Nomenclature

D_1	Diameter of cylinder, m
D_2	Diameter of cone corresponding to height H_2
d	Diameter of exit pipe, m
g	Acceleration due to gravity, m/sec^2
h	Height of liquid in the tank at any time, m
h_1,h_2	Height of liquid at any time t for cylinder and cone , m
H_1,H_2	Height of liquid at t=0 for cylinder and cone , m
f	Friction factor, dimensionless
L	Length of exit pipe, m
Re	Reynolds number, dimensionless
t_1,t_2	Efflux time for cylinder and cone , sec
V	Volume of liquid, m^3
V_1&V_2	Velocities at station-1 and 2 , m/sec
P_1 & P_2	Pressures at station 1 and station 2 , N/m^2
R_1	Radius of cylinder, m
R_2	Radius of cylinder corresponding to height H_2, m
r_2	Radius of cone corresponding to height h_2, m
W_1 & W_2	Mass flow rate at station 1 and 2 , kg/sec
Z_1 & Z_2	Elevations at 1 &2, Z_1=Z_2+h, m
ρ	Density of liquid, kg/m$_3$
µ	Viscosity of liquid, kg/m.sec
θ_1	Dimensionless time for cylindrical tank, sec
θ_2	Dimensionless time for conical tank , sec

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