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A New Method for Solving Fuzzy Sensitivity Analysis Problems

Amit Kumar and Neha Bhatia*

School of Mathematics and Computer Applications Thapar University, Patiala-147004, India

Abstract: Kheirfam and Hasani (Sensitivity analysis for fuzzy linear programming problems with fuzzy variables, Advanced Modeling and Optimization, 12 (2010) 257-272), proposed a new method to deal with the sensitivity analysis of such fuzzy linear programming (FLP) problems in which all the elements of coefficient matrix of the constraints and the coefficients of the decision variables in the objective function are represented by real numbers and remaining parameters are represented by trapezoidal fuzzy numbers. In this paper, it is shown that the existing method can't be used for solving sensitivity analysis of such FLP problems in which only the elements coefficient matrix of constraints are represented by real numbers and other parameters are represented by trapezoidal fuzzy numbers. To overcome this limitation of existing method, a new method, named as Mehar's method, is proposed to deal with the sensitivity analysis of FLP problems. The main advantage of Mehar's method over existing method is that Mehar's method is easy to apply as compare to existing method and can be used to deal with the sensitivity analysis of both type of fuzzy sensitivity analysis problem. To show the advantages of the Mehar's method over existing method some fuzzy sensitivity analysis problems, which may or may not be solved by using existing methods, are solved by using Mehar's method.

Keywords: fuzzy linear programming problems; ranking function; sensitivity analysis; trapezoidal fuzzy numbers.

1. Introduction

The fuzzy set theory is being applied massively in many fields these days. One of these is linear programming problems. Sensitivity analysis is well-explored area in classical linear programming. Sensitivity analysis is a basic tool for studying perturbations in optimization problems. There is considerable research on sensitivity analysis for some operations research and management science models such as linear programming and investment analysis.

In most practical applications of mathematical programming the possible values of the parameters required in the modeling of the problem are provided either by a decision maker subjectively or a statistical inference from the past data due to which there exists some uncertainty. In order to reflect this uncertainty, the model of the problem is often constructed with fuzzy data [14].

Fuzzy linear programming (FLP) provides the flexibility in values. But even after formulating the problem as FLP problem, one cannot stick to all the values for a long time or it is quite possible that the wrong values got entered. With time the factors like cost, required time or availability of product etc.

* Corresponding author; e-mail: neha26bhatia@gmail.com *Accepted for Publication: March 3, 2011*

changes widely. Sensitivity analysis for FLP problem needs to be applied in that case. Sensitivity analysis is one of the interesting researches in FLP problem.

Zimmermann [15] attempted to fuzzify a linear program for the first time, fuzzy numbers being the source of flexibility. Zimmermann also presented a fuzzy approach to multi-objective linear programming problem and its sensitivity analysis. Sensitivity analysis in FLP problem with crisp parameters and soft constraints was first considered by Hamacher et al. [4].

Tanaka and Asai [10] proposed a method of allocating the given investigation cost to each fuzzy coefficients by using sensitivity analysis. Tanaka et al. [11] formulated a FLP problem with fuzzy coefficients and the value of information was discussed via sensitivity analysis. Sakawa and Yano [9] presented a fuzzy approach for solving multi- objective linear fractional programming problem via sensitivity analysis.

Fuller [2] proposed that the solution to FLP problems with symmetrical triangular fuzzy numbers is stable with respect to small changes of centers of fuzzy numbers. Perturbations occur due to calculation errors or just to answer managerial questions "What if …". Such questions propose after the simplex method and the related research area refers to as basis invariance sensitivity analysis.

Dutta et al. [1] studied sensitivity analysis for fuzzy linear fractional programming problem. Verdegay and Aguado [12] proposed that in the case of FLP problems, whether or not a fuzzy optimal solution has been found by using linear membership functions modeling the constraints, possible further changes of those membership functions do not affect the former optimal solution. The sensitivity analysis performed for those membership functions and the corresponding solutions shows the convenience of using linear functions instead of other more complicated ones.

Gupta and Bhatia [3] studied the measurement of sensitivity for changes of violations in the aspiration level for the fuzzy multi-objective linear fractional programming problem. Precup and Preitl [8] performed the sensitivity analysis for some fuzzy control systems. Lotfi et al. [7] developed a sensitivity analysis approach for the additive model. Kheirfam and Hasani [6] studied the basis invariance sensitivity analysis for FLP problems.

In this paper, it is shown that the existing method can't be used for solving sensitivity analysis of such FLP problems in which only the elements coefficient matrix of constraints are represented by real numbers and other parameters are represented by trapezoidal fuzzy numbers. To overcome this limitation of existing method, a new method, named as Mehar's method, is proposed to deal with the sensitivity analysis of FLP problems. The main advantage of Mehar's method over existing method is that Mehar's method is easy to apply as compare to existing method and can be used to deal with the sensitivity analysis of both type of fuzzy sensitivity analysis problem. To show the advantages of the Mehar's method over existing method some fuzzy sensitivity analysis problems, which may or may not be solved by using existing methods, are solved by using Mehar's method.

This paper is organized as follows: In Section 2, some basic definitions, arithmetic operations and Yager's ranking approach for the ranking of fuzzy numbers are presented. In Section 3, the applicability of existing method is discussed. In Section 4, limitations of an existing method [6] for solving fuzzy sensitivity analysis problems are pointed out. In Section 5, to overcome the limitations of an existing method, a new method, named as Mehar's method, is proposed. In Section 6, advantages of Mehar's method over an existing method are discussed. Comparison of an existing method with the Mehar's method is discussed in Section 7. Conclusions and future work are discussed in Section 8.

2. Preliminaries

In this section, some basic definitions, arithmetic operations of trapezoidal fuzzy numbers and a ranking approach for the ranking of trapezoidal fuzzy numbers are presented.

2.1 Basic definitions

In this section, some basic definitions are presented [5].

Definition 2.1 A fuzzy number $\widetilde{A}=(a,b,c,d)$ is said to be a trapezoidal fuzzy number if its membership function is given by:

$$\mu_{\widetilde{A}}(x)=\begin{cases}\dfrac{(x-a)}{(b-a)}, & a\le x<b\\ 1\quad, & b\le x\le c\\ \dfrac{(x-d)}{(c-d)}, & c<x\le d\\ 0\quad, & \text{otherwise}\end{cases}$$

where, $a,b,c,d\in R$

Definition 2.2 A fuzzy number $\widetilde{A}=(a,b,c,d)$ is said to be a non-negative trapezoidal fuzzy number iff $a\ge 0$.

2.2. Arithmetic operations

In this section, arithmetic operations between two trapezoidal fuzzy numbers, defined on universal set of real numbers R, are presented [5].

Let $\widetilde{A}_1=(a_1,b_1,c_1,d_1)$ and $\widetilde{A}_2=(a_2,b_2,c_2,d_2)$ be two trapezoidal fuzzy numbers then

(i) $\widetilde{A}_1\oplus\widetilde{A}_2 = (a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$

(ii) $\widetilde{A}_1\Theta\,\widetilde{A}_2= (a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2)$

(iii) $\widetilde{A}_1\otimes\widetilde{A}_2\approx(a',b',c',d')$ where,

(iv) $a'=\text{minimum }(a_1a_2,a_1d_2,a_2d_1,d_1d_2)$, $b'=\text{minimum }(b_1b_2,b_1c_2,b_2c_1,c_1c_2)$,

(v) $c'=\text{maximum }(b_1b_2,b_1c_2,b_2c_1,c_1c_2)$, $d'=\text{maximum }(a_1a_2,a_1d_2,a_2d_1,d_1d_2)$]

(vi) $\lambda\widetilde{A}=\begin{cases}(\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1) & \lambda\ge 0\\ (\lambda d_1,\lambda c_1,\lambda b_1,\lambda a_1) & \lambda\le 0\end{cases}$

2.3. Yager's ranking approach

A number of approaches have been proposed for the ranking of fuzzy numbers. A relatively simple computational and easily understandable ranking method, proposed by Yager [13], is considered for the ranking of fuzzy numbers in this paper. Yager [13] proposed a procedure for ordering fuzzy sets in which a ranking index $\Re(\widetilde{A})$ is calculated for the fuzzy number $\widetilde{A}=(a,b,c,d)$ from its λ-cut $A_\lambda=[b-(b-a)\lambda,c+(d-c)\lambda]$ according to the following formula:

$$\Re(\widetilde{A})=\frac{1}{2}\left(\int_0^1(b-(b-a)\lambda)\,d\lambda+\int_0^1(c+(d-c)\lambda)\,d\lambda\right)=\frac{a+b+c+d}{4} \qquad (1)$$

Since $\Re(\lambda)$ is calculated from the extreme values of λ-cut of $\widetilde{A}$ i.e., $b-(b-a)\lambda$ and

$c+(d-c)\lambda$, rather than its membership function, it is not required knowing the explicit form of the membership functions of the fuzzy numbers to be ranked. That is, unlike most of the ranking methods that require the knowledge of the membership functions of all fuzzy numbers to be ranked, the Yager's ranking index is still applicable even if the explicit form of membership function of the fuzzy number is unknown.

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers then

$\tilde{A} \underset{\Re}{\geq} \tilde{B}$ if $\Re(\tilde{A}) \geq \Re(\tilde{B})$

$\tilde{A} \underset{\Re}{>} \tilde{B}$ if $\Re(\tilde{A}) > \Re(\tilde{B})$

$\tilde{A} \underset{\Re}{=} \tilde{B}$ if $\Re(\tilde{A}) = \Re(\tilde{B})$

3. Applicability of existing method

In this section, the applicability of existing method [6] for solving fuzzy sensitivity analysis problems is discussed:

The existing method [6] can be used only for solving such fuzzy sensitivity analysis problems in which the elements of coefficient matrix of constraints and coefficients of the decision variables in objective function are represented by real numbers and remaining parameters are represented by fuzzy numbers e.g., the fuzzy sensitivity analysis problem, chosen in Example 3.1, can be solved by using the existing method.

Example 3.1 [6] Consider the FLP problem,

Minimize $\ominus\, \tilde{x}_1 \ominus \tilde{x}_2 \ominus 2\tilde{x}_3$,

Subject to

$\tilde{x}_1 \oplus \tilde{x}_2 \oplus 2\tilde{x}_3 \leq_{\Re} (3,5,8,13)$,

$\tilde{x}_1 \ominus \tilde{x}_2 \leq_{\Re} (4,6,10,16)$,

$\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_{\Re} (-6,1,6,14)$,

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$ are non-negative trapezoidal fuzzy numbers.

(a) Discuss the effect of changing the requirement vector from $(3,5,8,13),(4,6,10,16),(-6,1,6,14)$ to $(8,10,12,15),(1,3,5,6),(2,4,8,11)$ on the fuzzy optimal solution of resulting FLP problem.

(b) Find the effect of addition of a new non-negative fuzzy variable $\tilde{x}_4$ with cost 8 and column vectors $(0,5,7)^T$ on the current fuzzy optimal solution.

(c) Find the effect of addition of a new fuzzy constraint $2\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_{\Re} (1,2,3,4)$ on the current fuzzy optimal solution.

(d) If $\tilde{x}_2$ is deleted from the given FLP problem then find the fuzzy optimal solution of resulting FLP problem.

(e) Discuss the effect of deletion of a fuzzy constraint $\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_{\Re} (-6,1,6,14)$ from given FLP problem on the fuzzy optimal solution of resulting FLP problem.

4. Limitations of existing method

In this section, on the basis of applicability of existing method [6], the limitations of the existing method are pointed out.

Since, for the ranking function, used in the existing method [6] the property $\Re(\tilde{A} \otimes \tilde{B}) = \Re(\tilde{A})\Re(\tilde{B})$ is not satisfied. So the existing method can not be used for solving such fuzzy sensitivity analysis problems in which both the coefficients of the decision variables in the objective function and decision variables are represented by fuzzy numbers. e.g., fuzzy sensitivity analysis problem, chosen in Example 4.1, can not be solved by using the existing method [6].

Example 4.1 Consider the FLP problem,

$$
\begin{aligned}
&\text{Minimize} \quad \Theta(1,2,4,7) \otimes \tilde{x}_1 \Theta (1,3,5,6) \otimes \tilde{x}_2 \Theta (2,4,7,9) \otimes \tilde{x}_3, \\
&\text{Subject to} \\
&\qquad \tilde{x}_1 \oplus \tilde{x}_2 \oplus 2\tilde{x}_3 \leq_{\Re} (3,5,8,13), \\
&\qquad \tilde{x}_1 \;\; \Theta \; \tilde{x}_2 \leq_{\Re} (4,6,10,16), \\
&\qquad \tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_{\Re} (-6,1,6,14), \qquad\qquad (\mathrm{E}_1) \\
&\qquad \tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \text{ are non-negative trapezoidal fuzzy numbers.}
\end{aligned}
$$

(i) Discuss the effect of changing the cost coefficients $(1,2,4,7)$, $(1,3,5,6)$ and $(2,4,7,9)$ of the decision variables $\tilde{x}_1, \tilde{x}_2$ and $\tilde{x}_3$ to $(1,3,5,7)$, $(1,4,7,9)$ and $(2,4,6,9)$ respectively on the fuzzy optimal solution of resulting FLP problem.

(ii) Discuss the effect of changing the requirement vector from $(3,5,8,13),(4,6,10,16),(-6,1,6,14)$ to $(8,10,12,15),(1,3,5,6),(2,4,8,11)$ on the fuzzy optimal solution of resulting FLP problem.

(iii) Find the effect of addition of a new non-negative fuzzy variable $\tilde{x}_4$ with cost $(0,3,8,10)$ and column vectors $(0,5,7)^T$ on the current fuzzy optimal solution.

(iv) Find the effect of addition of a new fuzzy constraint $2\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_{\Re} (1,2,3,4)$ on the current fuzzy optimal solution.

(v) If $\tilde{x}_2$ is deleted from the given FLP problem (E_1) then find the fuzzy optimal solution of resulting FLP problem.

(vi) Discuss the effect of deletion of a fuzzy constraint $\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_{\Re} (-6,1,6,14)$ from (E_1) on the fuzzy optimal solution of resulting FLP problem.

(vii) If the cost coefficients are assigned the values $(1,3,5,7)$, $(1,4,7,9)$, $(2,4,6,9)$ and right hand side vector is changed to $(3,5,10,12),(0,3,6,9),(1,4,7,10)$ then find the fuzzy optimal solution of resulting FLP problem.

5. Mehar's method

In this section, to overcome the limitations of the existing method [6], discussed in Section 4, a new method, named as Mehar's method, is proposed for solving such fuzzy sensitivity analysis problems. Any FLP problem in which the elements of coefficient matrix in the constraints are represented by real numbers and rest of the parameters are represented by fuzzy numbers can be formulated as follows:

$$
\begin{aligned}
&\text{Maximize (or Minimize)} \quad \tilde{C}^T \otimes \tilde{X}, \\
&\text{Subject to} \\
&\qquad A\tilde{X} \leq_{\Re} \text{ or } =_{\Re} \text{ or } \geq_{\Re} \tilde{b}, \qquad (\text{P}_1) \\
&\qquad \tilde{X} \text{ is non-negative trapezoidal fuzzy vector.}
\end{aligned}
$$

where, $\tilde{b}=[\tilde{b}_j]_{m\times 1}$, $\tilde{X}=[\tilde{x}_j]_{n\times 1}$, $A=[a_{ij}]_{m\times n}$, $\tilde{C}^T=[\tilde{c}_j]_{1\times n}$, $\Re$ is a linear ranking function, $\tilde{x}_j=(a_j,b_j,c_j,d_j)$, $\tilde{c}_j=(e_j,f_j,g_j,h_j)$ and $\tilde{b}_j=(p_j,q_j,r_j,s_j)$ are the trapezoidal fuzzy numbers.

The steps of proposed method are as follows:

Step 1 Convert the FLP problem (P_1) into the crisp linear programming (CLP) problem (P_2).

$$
\begin{aligned}
&\text{Maximize (or Minimize)} \quad \Re(\tilde{C}^T \otimes \tilde{X}), \\
&\text{Subject to} \\
&\qquad \Re(A\tilde{X}) \leq or = or \geq \Re(\tilde{b}), \\
&\qquad d_j - c_j \geq 0, \\
&\qquad b_j - a_j \geq 0, \\
&\qquad c_j - b_j \geq 0, \qquad (\text{P}_2) \\
&\qquad a_j \geq 0 \qquad j = 1,2,3,\ldots,n
\end{aligned}
$$

Step 2 Solve the CLP problem (P_2), obtained in Step 1, to find the optimal solution a_j, b_j, c_j and d_j.

Step 3 Find the fuzzy optimal solution by putting the values of a_j, b_j, c_j and d_j, obtained from Step 2, in $\tilde{x}_j=(a_j,b_j,c_j,d_j)$ and the fuzzy optimal value by putting the values of $\tilde{x}_j=(a_j,b_j,c_j,d_j)$ in $\sum_{j-1}^{n}\tilde{c}_j \otimes \tilde{x}_j$.

Step 4 Check that which of the following case is to be considered:

(i) Change in the cost vector.
(ii) Change in requirement vector.
(iii) Addition of a new fuzzy variable.
(iv) Addition of new fuzzy constraint.
(v) Deletion of a fuzzy variable.
(vi) Deletion of a fuzzy constraint.
(vii) Simultaneous change in cost and requirement vector.

Case 1: Change in the cost vector

If the cost vector $\tilde{C}^T$ changes to $\tilde{C}'^T$ in the given FLP problem (P_1) then replace $\Re(\tilde{C}^T \otimes \tilde{X})$ by $\Re(\tilde{C}'^T \otimes \tilde{X})$ in CLP problem (P_2) to obtain (P_3):

$$
\begin{array}{ll}
\text{Maximize (or Minimize)} & \Re(\tilde{C}'^T \otimes \tilde{X}), \\
\text{Subject to} & \\
 & \Re(A\tilde{X}) \le or = or \ge \Re(\tilde{b}), \\
 & d_j - c_j \ge 0, \\
 & b_j - a_j \ge 0, \\
 & c_j - b_j \ge 0, \qquad\qquad\qquad\qquad (\mathrm{P}_3) \\
 & a_j \ge 0 \qquad j = 1,2,3,\ldots,n
\end{array}
$$

Now apply the existing sensitivity analysis technique to find the optimal solution of (P_3) with the help of optimal solution of (P_2) and use Step 3 of the Mehar's method to find the fuzzy optimal solution and fuzzy optimal value of the resulting FLP problem.

Case 2: Change in requirement vector $\tilde{b}$

If the change in right hand side (RHS) or requirement vector is made i.e., $\tilde{b}$ is changed to $\tilde{b}'$ in (P_1) then, replace $\Re(\tilde{b})$ by $\Re(\tilde{b}')$ in CLP problem (P_2) to obtain (P_4):

$$
\begin{array}{ll}
\text{Maximize (or Minimize)} & \Re(\tilde{C}^T \otimes \tilde{X}), \\
\text{Subject to} & \\
 & \Re(A\tilde{X}) \le or = or \ge \Re(\tilde{b}'), \\
 & d_j - c_j \ge 0, \\
 & b_j - a_j \ge 0, \\
 & c_j - b_j \ge 0, \qquad\qquad\qquad\qquad (\mathrm{P}_4) \\
 & a_j \ge 0 \qquad j = 1,2,3\ldots n
\end{array}
$$

Now apply the existing sensitivity analysis technique to find the optimal solution of (P_4) with the help of optimal solution of (P_2) and use Step 3 of the Mehar's method to find the fuzzy optimal solution and fuzzy optimal value of the resulting FLP problem.

Case 3: Addition of a new fuzzy variable

Suppose a new fuzzy variable, say $\tilde{x}_{n+1}$ is added in (P_1). Assume that $\tilde{c}_{n+1}$ is the cost and A_{n+1} is the column associated with $\tilde{x}_{n+1}$ then replace $\Re(A\tilde{X})$ by $\Re(A\tilde{X} \oplus A_{n+1}\tilde{x}_{n+1})$ and $\Re(\tilde{C}^T \otimes \tilde{X})$ by $\Re(\tilde{C}^T \otimes \tilde{X} \oplus \tilde{c}_{n+1} \otimes \tilde{x}_{n+1})$ in (P_2) to obtain the new CLP problem (P_5).

$$
\begin{array}{ll}
\text{Maximize (or Minimize)} & \Re(\tilde{C}'^T \otimes \tilde{X}'), \\
\text{Subject to} &
\end{array}
$$

$$\begin{aligned}
&\Re(A'\tilde{X}') \leq or = or \geq \Re(\tilde{b}), \\
&d_j - c_j \geq 0, \\
&b_j - a_j \geq 0, \\
&c_j - b_j \geq 0, \qquad\qquad (P_5) \\
&a_j \geq 0 \qquad j = 1,2,3,\ldots,n+1
\end{aligned}$$

Now apply the existing sensitivity analysis technique to find the optimal solution of (P_5) with the help of optimal solution of (P_2) and use Step 3 of the Mehar's method to find the fuzzy optimal solution and fuzzy optimal value of the resulting FLP problem.

Case 4: Addition of a new fuzzy constraint

Suppose a new fuzzy constraint is added in the original FLP problem (P_1) then, replace $\Re(A\tilde{X}) \leq or = or \geq \Re(\tilde{b})$ by $\Re(A'\tilde{X}) \leq or = or \geq \Re(\tilde{b}')$ in (P_2) to obtain new CLP problem (P_6):

$$\begin{aligned}
&\text{Maximize (or Minimize)} \quad \Re(\tilde{C}^T \otimes \tilde{X}), \\
&\text{Subject to} \\
&\Re(A'\tilde{X}) \leq or = or \geq \Re(\tilde{b}'), \\
&d_j - c_j \geq 0, \\
&b_j - a_j \geq 0, \\
&c_j - b_j \geq 0, \qquad\qquad (P_6) \\
&a_j \geq 0 \qquad j = 1,2,3,\ldots,n
\end{aligned}$$

Now apply the existing sensitivity analysis technique to find the optimal solution of (P_6) with the help of optimal solution of (P_2) and use Step 3 of the Mehar's method to find the fuzzy optimal solution and fuzzy optimal value of the resulting FLP problem.

Case 5: Deletion of a fuzzy variable

Suppose a fuzzy variable $\tilde{x}_n$ is deleted from the original FLP problem (P_1) then, replace $\Re(A\tilde{X})$ by $\Re(A'\tilde{X}')$ and $\Re(\tilde{C}^T \otimes \tilde{X})$ by $\Re(\tilde{C}^T \otimes \tilde{X}')$ in (P_2) to obtain new CLP problem (P_7):

$$\begin{aligned}
&\text{Maximize (or Minimize)} \quad \Re(\tilde{C}^T \otimes \tilde{X}') \\
&\text{Subject to} \\
&\Re(A'\tilde{X}') \leq or = or \geq \Re(\tilde{b}), \\
&d_j - c_j \geq 0, \\
&b_j - a_j \geq 0, \\
&c_j - b_j \geq 0, \qquad\qquad (P_7) \\
&a_j \geq 0 \qquad j = 1,2,3,\ldots,n-1
\end{aligned}$$

Now apply the existing sensitivity analysis technique to find the optimal solution of (P_7) with the help of optimal solution of (P_2) and use Step 3 of the Mehar's method to find the fuzzy optimal solution and fuzzy optimal value of the resulting FLP problem.

Case 6: Deletion of a fuzzy constraint

Suppose a fuzzy constraint is deleted from original FLP problem (P_1) then, replace $\Re(A\tilde{X}) \leq or = or \geq \Re(\tilde{b})$ by $\Re(A'\tilde{X}) \leq or = or \geq \Re(\tilde{b}')$, in (P_2) to obtain new CLP problem (P_8):

$$
\begin{array}{ll}
\text{Maximize (or Minimize)} & \Re(\tilde{C}^T \otimes \tilde{X}), \\
\text{Subject to} & \\
& \Re(A'\tilde{X}) \leq or = or \geq \Re(\tilde{b}'), \\
& d_j - c_j \geq 0, \\
& b_j - a_j \geq 0, \\
& c_j - b_j \geq 0, \\
& a_j \geq 0 \qquad j = 1,2,3,\ldots,n
\end{array}
\qquad (\mathrm{P}_8)
$$

Now apply the existing sensitivity analysis technique to find the optimal solution of (P_8) with the help of optimal solution of (P_2) and use Step 3 of the Mehar's method to find the fuzzy optimal solution and fuzzy optimal value of the resulting FLP problem.

Case 7: Simultaneous change in cost and requirement vector

If the cost $\tilde{c}_j$ corresponding to variable $\tilde{x}_j$ is changed to $\tilde{c}_j'$ and requirement vector $\tilde{b}$ is changed to $\tilde{b}'$ in (P_1) then, replace $\Re(\tilde{C}^T \otimes \tilde{X})$ by $\Re(\tilde{C}'^T \otimes \tilde{X})$ and $\Re(\tilde{b})$ by $\Re(\tilde{b}')$ in CLP problem (P_2) to obtain (P_9):

$$
\begin{array}{ll}
\text{Maximize (or Minimize)} & \Re(\tilde{C}'^T \otimes \tilde{X}), \\
\text{Subject to} & \\
& \Re(A\tilde{X}) \leq or = or \geq \Re(\tilde{b}'), \\
& d_j - c_j \geq 0, \\
& b_j - a_j \geq 0, \\
& c_j - b_j \geq 0, \\
& a_j \geq 0 \qquad j = 1,2,3,\ldots,n
\end{array}
\qquad (\mathrm{P}_9)
$$

Now apply the existing sensitivity analysis technique to find the optimal solution of (P_9) with the help of optimal solution of (P_2) and use Step 3 of the Mehar's method to find the fuzzy optimal solution and fuzzy optimal value of the resulting FLP problem.

6. Advantages of Mehar's method over the existing method

The main advantage of the Mehar's method over the existing method [6] is that the fuzzy sensitivity analysis problems which can be solved by the existing methods can also be solved by the Mehar's method. But there may exist several fuzzy sensitivity analysis problems, as discussed in Section 4, which can not be solved by the existing method. To show the advantage of Mehar's method over existing method [6] the fuzzy sensitivity analysis problem, chosen in Example 4.1, is solved by the Mehar's method.

6.1. Solution of the chosen problem

The solution of the fuzzy sensitivity analysis problem, chosen in Example 4, by using the Mehar's method can be obtained as follows:

Assuming $\tilde{x}_1 = (a_1, b_1, c_1, d_1)$, $\tilde{x}_2 = (a_2, b_2, c_2, d_2)$ and $\tilde{x}_3 = (a_3, b_3, c_3, d_3)$ the FLP problem chosen in Example 3.2, can be written as:

Minimize $\Theta(1,2,4,7) \otimes (a_1, b_1, c_1, d_1) \Theta (1,3,5,6) \otimes (a_2, b_2, c_2, d_2) \Theta (2,4,7,9) \otimes (a_3, b_3, c_3, d_3)$,

Subject to

$$(a_1, b_1, c_1, d_1) \oplus (a_2, b_2, c_2, d_2) \oplus (2a_3, 2b_3, 2c_3, 2d_3) \leq_{\Re} (3,5,8,13),$$
$$(a_1, b_1, c_1, d_1) \Theta (a_2, b_2, c_2, d_2) \leq_{\Re} (4,6,10,16),$$
$$(a_1, b_1, c_1, d_1) \oplus (a_2, b_2, c_2, d_2) \oplus (a_3, b_3, c_3, d_3) \leq_{\Re} (-6,1,6,14), \qquad (\mathrm{E}_2)$$

(a_1, b_1, c_1, d_1), (a_2, b_2, c_2, d_2) and (a_3, b_3, c_3, d_3) are non-negative trapezoidal fuzzy numbers.

Using Step 1 of the Mehar's method the FLP problem (E_2) is converted into the following CLP problem:

Minimize $\frac{1}{4}(-a_1 - a_2 - 2a_3 - 2b_1 - 3b_2 - 4b_3 - 4c_1 - 5c_2 - 7c_3 - 7d_1 - 6d_2 - 9d_3)$,

Subject to

$$a_1 + a_2 + 2a_3 + b_1 + b_2 + 2b_3 + c_1 + c_2 + 2c_3 + d_1 + d_2 + 2d_3 \leq 29,$$
$$a_1 - a_2 + b_1 - b_2 + c_1 - c_2 + d_1 - d_2 \leq 36,$$
$$a_1 + a_2 + a_3 + b_1 + b_2 + b_3 + c_1 + c_2 + c_3 + d_1 + d_2 + d_3 \leq 15,$$
$$a_1 \geq 0,\ a_2 \geq 0,\ a_3 \geq 0$$
$$b_1 - a_1 \geq 0,\ c_1 - b_1 \geq 0, d_1 - c_1 \geq 0,$$
$$b_2 - a_2 \geq 0,\ c_2 - b_2 \geq 0, d_2 - c_2 \geq 0, \qquad (\mathrm{E}_3)$$
$$b_3 - a_3 \geq 0,\ c_3 - b_3 \geq 0, d_3 - c_3 \geq 0.$$

The optimal solution of the CLP problem (E_3) is:

$a_1 = 0, a_2 = 0, a_3 = 0, b_1 = 0, b_2 = 0, b_3 = 0, c_1 = 0, c_2 = 0, c_3 = 0, d_1 = 1, d_2 = 0, d_3 = 14$ and the optimal value is -33.25.

Using Step 3 of the Mehar's method the fuzzy optimal solution is given by $\tilde{x}_1 = (0,0,0,1), \tilde{x}_2 = (0,0,0,0), \tilde{x}_3 = (0,0,0,14)$ and the fuzzy optimal value is $(-133,0,0,0)$.

(a) Since the cost coefficients corresponding to the variables $\tilde{x}_1, \tilde{x}_2$ and $\tilde{x}_3$ changes to $(1,3,5,7)$, $(1,4,7,9)$ and $(2,4,6,9)$ respectively in the original problem so replacing CLP problem (E_3) by (E_4):

Minimize $\frac{1}{4}(-a_1 - a_2 - 2a_3 - 3b_1 - 4b_2 - 4b_3 - 5c_1 - 7c_2 - 6c_3 - 7d_1 - 9d_2 - 9d_3)$,

Subject to

$$
\begin{aligned}
& a_1 + a_2 + 2a_3 + b_1 + b_2 + 2b_3 + c_1 + c_2 + 2c_3 + d_1 + d_2 + 2d_3 \le 29, \\
& a_1 - a_2 + b_1 - b_2 + c_1 - c_2 + d_1 - d_2 \le 36, \\
& a_1 + a_2 + a_3 + b_1 + b_2 + b_3 + c_1 + c_2 + c_3 + d_1 + d_2 + d_3 \le 15, \\
& a_1 \ge 0,\ a_2 \ge 0,\ a_3 \ge 0, \\
& b_1 - a_1 \ge 0,\ c_1 - b_1 \ge 0, d_1 - c_1 \ge 0, \\
& b_2 - a_2 \ge 0,\ c_2 - b_2 \ge 0, d_2 - c_2 \ge 0, \qquad (\mathrm{E}_4) \\
& b_3 - a_3 \ge 0,\ c_3 - b_3 \ge 0, d_3 - c_3 \ge 0.
\end{aligned}
$$

Now applying existing sensitivity analysis techniques, the optimal solution of resulting CLP problem (E_4) is:

$a_1 = 0, a_2 = 0, a_3 = 0, b_1 = 0, b_2 = 0, b_3 = 0, c_1 = 0, c_2 = 0, c_3 = 0, d_1 = 0, d_2 = 1, d_3 = 14$ and the optimal value is -33.75.

Using Step 3 of the Mehar's method the fuzzy optimal solution is $\tilde{x}_1 = (0,0,0,0)$, $\tilde{x}_2 = (0,0,0,1)$, $\tilde{x}_3 = (0,0,0,14)$ and the fuzzy optimal value is $(-135,0,0,0)$.

(b) Since the requirement vector is changed from $(3,5,8,13), (4,6,10,16)$ and $(-6,1,6,14)$ to $(8,10,12,15), (1,3,5,6)$ and $(2,4,8,11)$ in the original FLP problem (E_1) so replacing $\Re(3,5,8,13)$, $\Re(4,6,10,16)$ and $\Re(-6,1,6,14)$ by $\Re(8,10,12,15)$, $\Re(1,3,5,6)$ and $\Re(2,4,8,11)$ respectively i.e., $29, 36$ and 15 by $45, 15$ and 25 respectively in (E_3)

Minimize $\frac{1}{4}(-a_1 - a_2 - 2a_3 - 3b_1 - 4b_2 - 4b_3 - 5c_1 - 7c_2 - 6c_3 - 7d_1 - 9d_2 - 9d_3)$,

Subject to

$$
\begin{aligned}
& a_1 + a_2 + 2a_3 + b_1 + b_2 + 2b_3 + c_1 + c_2 + 2c_3 + d_1 + d_2 + 2d_3 \le 45, \\
& a_1 - a_2 + b_1 - b_2 + c_1 - c_2 + d_1 - d_2 \le 15, \\
& a_1 + a_2 + a_3 + b_1 + b_2 + b_3 + c_1 + c_2 + c_3 + d_1 + d_2 + d_3 \le 25, \\
& a_1 \ge 0,\ a_2 \ge 0,\ a_3 \ge 0, \\
& b_1 - a_1 \ge 0,\ c_1 - b_1 \ge 0, d_1 - c_1 \ge 0, \\
& b_2 - a_2 \ge 0,\ c_2 - b_2 \ge 0, d_2 - c_2 \ge 0, \qquad (\mathrm{E}_5) \\
& b_3 - a_3 \ge 0,\ c_3 - b_3 \ge 0, d_3 - c_3 \ge 0.
\end{aligned}
$$

Applying existing sensitivity analysis technique for the case of change in RHS vector, the optimal solution of CLP problem (E_5) is:

$a_1 = 0, a_2 = 0, a_3 = 0, b_1 = 0, b_2 = 0, b_3 = 0, c_1 = 0, c_2 = 0, c_3 = 0, d_1 = 5, d_2 = 0, d_3 = 20$ and the optimal value is -53.75.

Using Step 3 of the Mehar's method the fuzzy optimal solution is

$\tilde{x}_1 = (0,0,0,5)$, $\tilde{x}_2 = (0,0,0,0)$, $\tilde{x}_3 = (0,0,0,20)$ and the fuzzy optimal value is $(-215,0,0,0)$.

(c) Suppose a new fuzzy variable $\tilde{x}_4$ having $\Re(\tilde{x}_4) \geq 0$ with cost (0,3,8,10) and column $(0,5,7)^T$ is added in the original FLP problem (E_1) then, replace (E_3) by (E_6):

Minimize $\frac{1}{4}(-a_1 - a_2 - 2a_3 - 2b_1 - 3b_2 - 4b_3 + 3b_4 - 4c_1 - 5c_2 - 7c_3 + 8c_4 - 7d_1 - 6d_2 - 9d_3 + 10d_4),$

Subject to

$$
\begin{aligned}
& a_1 + a_2 + 2a_3 + b_1 + b_2 + 2b_3 + c_1 + c_2 + 2c_3 + d_1 + d_2 + 2d_3 \leq 29, \\
& a_1 - a_2 + 5a_4 + b_1 - b_2 + 5b_4 + c_1 - c_2 + 5c_4 + d_1 - d_2 + 5d_4 \leq 36, \\
& a_1 + a_2 + a_3 + 7a_4 + b_1 + b_2 + b_3 + 7b_4 + c_1 + c_2 + c_3 + 7c_4 + d_1 + d_2 + d_3 + 7d_4 \leq 15, \quad a_1 \geq 0, \\
& a_2 \geq 0,\ a_3 \geq 0,\ a_4 \geq 0, \\
& b_1 - a_1 \geq 0,\ c_1 - b_1 \geq 0, d_1 - c_1 \geq 0, \\
& b_2 - a_2 \geq 0,\ c_2 - b_2 \geq 0, d_2 - c_2 \geq 0, \\
& b_3 - a_3 \geq 0,\ c_3 - b_3 \geq 0, d_3 - c_3 \geq 0, \qquad (\text{E}_6) \\
& b_4 - a_4 \geq 0,\ c_4 - b_4 \geq 0, d_4 - c_4 \geq 0.
\end{aligned}
$$

By applying existing sensitivity analysis technique for addition of variable the optimal solution of the above CLP problem (E_6) is:

$a_1 = 0, a_2 = 0, a_3 = 0, b_1 = 0, b_2 = 0, b_3 = 0, c_1 = 0, c_2 = 0, c_3 = 0, d_1 = 1, d_2 = 0, d_3 = 14, a_4 = 0, b_4 = 0,$ $c_4 = 0, d_4 = 0$ and the optimal value is -33.25.

Using Step 3 of the Mehar's method the fuzzy optimal solution is given by $\tilde{x}_1 = (0,0,0,1)$, $\tilde{x}_2 = (0,0,0,0)$, $\tilde{x}_3 = (0,0,0,14)$, $\tilde{x}_4 = (0,0,0,0)$ and the fuzzy optimal value is $(-133,0,0,0)$.

(d) Suppose a new fuzzy constraint $2\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_{\Re} (1,2,3,4)$ is added to the original FLP problem (E_1) then, add the constraint:

$2a_1 + a_2 + a_3 + 2b_1 + b_2 + b_3 + 2c_1 + c_2 + c_3 + 2d_1 + d_2 + d_3 \leq 10$ to (E_3) and the resulting CLP problem become

Minimize $\frac{1}{4}(-a_1 - a_2 - 2a_3 - 2b_1 - 3b_2 - 4b_3 - 4c_1 - 5c_2 - 7c_3 - 7d_1 - 6d_2 - 9d_3),$

Subject to

$$
\begin{aligned}
& a_1 + a_2 + 2a_3 + b_1 + b_2 + 2b_3 + c_1 + c_2 + 2c_3 + d_1 + d_2 + 2d_3 \leq 29, \\
& a_1 - a_2 + b_1 - b_2 + c_1 - c_2 + d_1 - d_2 \leq 36, \\
& a_1 + a_2 + a_3 + b_1 + b_2 + b_3 + c_1 + c_2 + c_3 + d_1 + d_2 + d_3 \leq 15, \\
& 2a_1 + a_2 + a_3 + 2b_1 + b_2 + b_3 + 2c_1 + c_2 + c_3 + 2d_1 + d_2 + d_3 \leq 10, \\
& a_1 \geq 0,\ a_2 \geq 0,\ a_3 \geq 0, \\
& b_1 - a_1 \geq 0,\ c_1 - b_1 \geq 0, d_1 - c_1 \geq 0, \\
& b_2 - a_2 \geq 0,\ c_2 - b_2 \geq 0, d_2 - c_2 \geq 0, \qquad (\text{E}_7) \\
& b_3 - a_3 \geq 0,\ c_3 - b_3 \geq 0, d_3 - c_3 \geq 0.
\end{aligned}
$$

Applying existing sensitivity analysis technique, the optimal solution of resulting CLP problem

(E_7) is:

$a_1 = 0, a_2 = 0, a_3 = 0, b_1 = 0, b_2 = 0, b_3 = 0, c_1 = 0, c_2 = 0, c_3 = 0, d_1 = 0, d_2 = 0, d_3 = 10$ and the optimal value is -22.50.

Using Step 3, the fuzzy optimal solution is $\tilde{x}_1 = (0,0,0,0)$, $\tilde{x}_2 = (0,0,0,0)$, $\tilde{x}_3 = (0,0,0,10)$ and the fuzzy optimal value is $(-90,0,0,0)$.

(e) Suppose a non-negative fuzzy variable $\tilde{x}_2$ is deleted from the given FLP problem (E_1) then, replace FLP problem (E_3) by (E_8).

Minimize $\quad \frac{1}{4}(-a_1 - 2a_3 - 2b_1 - 4b_3 - 4c_1 - 7c_3 - 7d_1 - 9d_3),$

Subject to

$$
\begin{aligned}
& a_1 + 2a_3 + b_1 + 2b_3 + c_1 + 2c_3 + d_1 + 2d_3 \le 29, \\
& a_1 + b_1 + c_1 + d_1 \le 36, \\
& a_1 + a_3 + b_1 + b_3 + c_1 + c_3 + d_1 + d_3 \le 15, \\
& a_1 \ge 0, a_3 \ge 0, \\
& b_1 - a_1 \ge 0,\ c_1 - b_1 \ge 0, d_1 - c_1 \ge 0, \\
& b_3 - a_3 \ge 0,\ c_3 - b_3 \ge 0, d_3 - c_3 \ge 0.
\end{aligned}
\tag{E_8}
$$

Applying existing sensitivity analysis technique, the optimal solution of resulting CLP problem (E_8) is:

$a_1 = 0, a_3 = 0, b_1 = 0, b_3 = 0,\ c_1 = 0, c_3 = 0,\ d_1 = 1, d_3 = 14$ and the optimal value is -33.25. Using Step 3, the fuzzy optimal solution is

$\tilde{x}_1 = (0,0,0,1)$, $\tilde{x}_3 = (0,0,0,14)$ and the fuzzy optimal value is $(-133,0,0,0)$.

(f) Suppose a fuzzy constraint $\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \le_{\Re} (-6,1,6,14)$ is deleted from the original FLP problem (E_1) then, delete the constraint

$$a_1 + a_2 + a_3 + b_1 + b_2 + b_3 + c_1 + c_2 + c_3 + d_1 + d_2 + d_3 \le 15$$

from (E_3) resulting CLP problem become:

Minimize $\quad \frac{1}{4}(-a_1 - a_2 - 2a_3 - 2b_1 - 3b_2 - 4b_3 - 4c_1 - 5c_2 - 7c_3 - 7d_1 - 6d_2 - 9d_3),$

Subject to

$$
\begin{aligned}
& a_1 + a_2 + 2a_3 + b_1 + b_2 + 2b_3 + c_1 + c_2 + 2c_3 + d_1 + d_2 + 2d_3 \le 29, \\
& a_1 - a_2 + b_1 - b_2 + c_1 - c_2 + d_1 - d_2 \le 36, \\
& a_1 \ge 0,\ a_2 \ge 0,\ a_3 \ge 0, \\
& b_1 - a_1 \ge 0,\ c_1 - b_1 \ge 0, d_1 - c_1 \ge 0, \\
& b_2 - a_2 \ge 0,\ c_2 - b_2 \ge 0, d_2 - c_2 \ge 0, \\
& b_3 - a_3 \ge 0,\ c_3 - b_3 \ge 0, d_3 - c_3 \ge 0.
\end{aligned}
\tag{E_9}
$$

By applying existing sensitivity analysis technique, the optimal solution of resulting CLP problem (E_9) is:

$a_1 = 0, a_2 = 0, a_3 = 0, b_1 = 0, b_2 = 0, b_3 = 0, c_1 = 0, c_2 = 0, c_3 = 0, d_1 = 29, d_2 = 0, d_3 = 0$ and the optimal

value is -50.75.
Using Step 3, the fuzzy optimal solution is $\tilde{x}_1=(0,0,0,29)$, $\tilde{x}_2=(0,0,0,0)$, $\tilde{x}_3=(0,0,0,0)$ and the fuzzy optimal value is $(-203,0,0,0)$.

(g) Since the cost coefficients corresponding to variables $\tilde{x}_1, \tilde{x}_2$ and $\tilde{x}_3$ changes to (1,3,5,7), (1,4,7,9) and (2,4,6,9) respectively and right hand side vector is changed to (3,5,10,12), (0,3,6,9) and (1,4,7,10) simultaneously in the original FLP problem (E_1) so replace (E_3) by (E_{10}):

$$\text{Minimize } \frac{1}{4}(-a_1-a_2-2a_3-3b_1-4b_2-4b_3-5c_1-7c_2-6c_3-7d_1-9d_2-9d_3),$$

Subject to

$$\begin{aligned}
&a_1+a_2+2a_3+b_1+b_2+2b_3+c_1+c_2+2c_3+d_1+d_2+2d_3\le 30,\\
&a_1-a_2+b_1-b_2+c_1-c_2+d_1-d_2\le 18,\\
&a_1+a_2+a_3+b_1+b_2+b_3+c_1+c_2+c_3+d_1+d_2+d_3\le 22,\\
&a_1+b_1+c_1+d_1\ge 0,\\
&a_2+b_2+c_2+d_2\ge 0,\\
&a_3+b_3+c_3+d_3\ge 0,\\
&b_1-a_1\ge 0,\ c_1-b_1\ge 0, d_1-c_1\ge 0,\\
&b_2-a_2\ge 0,\ c_2-b_2\ge 0, d_2-c_2\ge 0,\\
&b_3-a_3\ge 0,\ c_3-b_3\ge 0, d_3-c_3\ge 0.
\end{aligned}\tag{E_{10}}$$

By applying existing sensitivity analysis technique, the optimal solution of resulting CLP problem (E_{10}) is:

$a_1=0, a_2=0, a_3=0, b_1=0, b_2=0, b_3=0, c_1=0, c_2=0, c_3=0, d_1=0, d_2=14, d_3=8$ and the optimal value is -49.5. Using Step 3, the fuzzy optimal solution is given by $\tilde{x}_1=(0,0,0,0)$, $\tilde{x}_2=(0,0,0,14)$, $\tilde{x}_3=(0,0,0,8)$ and the fuzzy optimal value is $(-198,0,0,0)$.

7. Comparison of Mehar's method with the existing method

To compare Mehar's method with the existing method [6] the results of fuzzy sensitivity analysis problems, obtained by using existing method as well as Mehar's method, are shown in Table 1

Table 1. Comparison of existing method and Mehar's method

Example	Existing method [6]	Mehar's method
3.1	Applicable	Applicable
4.1	Not applicable	Applicable

.

From Table 1 it can be concluded that:

(a) The fuzzy sensitivity analysis problems in which the decisions variables and the requirements are represented by fuzzy numbers and rest of the parameters are represented by real numbers e.g., the fuzzy sensitivity analysis problem, chosen in Example 3.1, can be solved by using both the existing method as well as Mehar's method also the obtained results are same.

(b) As explained in the Section 4, the fuzzy sensitivity analysis problems in which only the elements of coefficient matrix in the constraints are represented by real numbers and rest of the parameters are represented by fuzzy numbers e.g., the fuzzy sensitivity analysis problem, chosen in Example 4.1, can be solved only by using Mehar's method but can't be solved by using the existing method [6].

8. Conclusions and future work

In this paper, limitations of an existing method [6] for solving fuzzy sensitivity analysis problems are pointed out and to overcome these limitations a new method, named as Mehar's method, is proposed for solving fuzzy sensitivity analysis problems. To show the advantage of the Mehar's method over existing methods some fuzzy sensitivity analysis problem, which may or may not be solved by using the existing methods, are solved by using the Mehar's method.

Since there is no existing rule for the product of two arbitrary fuzzy numbers so to solve sensitivity analysis problems in which all the parameters are represented by fuzzy numbers is left as a future work.

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