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An Evaluation of Single-Featured EWMA-X (SFEWMA-X) Control Chart with Process Mean Shifts and Standard Deviation Changes

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Abstract: Quality control chart is an important tool for process monitoring and quality improvement. Several combined control charts have been proposed to enhance the detection ability in detecting small and large process parameter changes. However, two pairs of control limits and statistics on the control chart make the combined charts too complicated for the practitioners to use in practice. We have developed a single-featured EWMA-X (SFEWMA-X) control chart in previous study, which has the ability to monitor both large and small process shifts simultaneously using only one set of statistic and control limits. In this study, we presented an example to demonstrate the use of the SFEWMA-X chart, and performed the average run length (ARL) comparison between SFEWMA-X chart and the individual control charts to illustrate the detection ability of the SFEWMA-X chart. To facilitate the implementation of the proposed method in practice, we also introduced the algorithm for deriving optimal chart design tables.

Keywords: EWMA; combined EWMA-X; process shift detection; average run length; ARL.

1. Introduction

The statistical control-chart concept was first proposed by Shewhart [1], and control charts developed according to this concept are called Shewhart control charts. Shewhart control charts have been widely used for decades. However, since Shewhart control charts use only the information contained in the current sample observation, they are not efficient in detecting small process parameter changes. Exponentially weighted moving average (EWMA) is an alternative to Shewhart chart when small process shifts are of interest. Roberts [2] first proposed EWMA control chart in the name of geometrical moving average (GMA). Hunter [3] called GMA control chart the EWMA control chart. Several studies concluded that EWMA control chart provided greater sensitivity to small process

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mean shifts [3-6], but it is insensitive to large process mean changes.

In order to enhance the overall detection ability in detecting both small and large process parameter changes, Lucas and Saccucci [5] suggested a combined Shewhart EWMA control chart. Woodall and Maragah [7] emphasized that a Shewhart chart should always be used in conjunction with the EWMA chart. Klein [8] evaluated a group of combined Shewhart-EWMA statistical schemes with simulation, and showed that the Shewhart-EWMA schemes have better detection ability than Shewhart-Run Rulers schemes. Tolley and English [9] presented a comparison between the cost performance of EWMA and the combined EWMA-$\overline{X}$ control chart schemes. Zhang et al. [10] proposed a new charting procedure, which called a composite EWMA (CEWMA) control chart, combining two or more Exponentially Weighted Moving Averages (EWMAs). Serel and Moskowitz [11] presented an EWMA cost minimization model which assists in designing the joint control scheme based on pure economic or both economic and statistical performance criteria. Hawkins and Deng [12] proposed two new combined charts with the simplicity characteristic, exhibiting significant performance advantages over the traditional control chart ($\overline{\mathrm{X}}$ and S chart). Sheu et al. [13] designed a combined control chart scheme, consisting of a two-sided generally weighted moving average mean chart and a two-sided GWMA variance chart. Simões et al. [14] optimized the designs of the AEWMA and of the combined EWMA-Shewhart schemes with regard to pairs of shifts in the process mean.

In many applications, the control chart is based on individual observations (n = 1). The Shewhart control chart for monitoring process standard deviation using individual observations is moving average (MR) control chart. Albin [15] compared the performance of process control procedures for individual observations that include combinations of the X chart, MR chart, EWMA chart, and runs rules. The Shewhart-X and EWMA charts with no run rules for individual observation process control were recommended. Reynolds and Stoumbos [16] investigated control chart schemes for detecting drifts in the process mean μ and/or process standard deviation σ when individual observations are sampled. They showed that the combinations of the X chart and EWMA charts detect slow-rate and moderate-rate drifts much faster than the combined X and MR chart. Reynolds and Stoumbos [17] investigated control charts for monitoring a process to detect changes in the mean and/or variance of a normal quality variable when an individual observation is taken at each sampling point. Stoumbos and Reynolds [18] showed that the Shewhart-X chart detects shifts in standard deviation faster than the MR chart.

In this study, we extend our previous study [19], called a single featured EWMA-X (SFEWMA-X) control chart that combines the features of the EWMA and Shewhart-X control charts, to evaluate its performance for detecting process mean shifts and standard deviation changes. An example was presented to demonstrate the use of the SFEWMA-X chart. The average run length (*ARL*) comparison between SFEWMA-X chart and the individual control charts was performed to illustrate the detection ability of the SFEWMA-X chart. To facilitate the implementation of the SFEWMA-X chart, the algorithm for deriving the optimal chart design tables was introduced.

2. Proposed Method

2.1 Overview of SFEWMA-X

Let x_i represent quality characteristic of ith individual observation. The center line (CL) and control limits (UCL_x and LCL_x) of the Shewhart-X control chart (subgroup size n=1) are as follows:

$$\begin{aligned} UCL_x &= \mu_0 + L_x\sigma \\ CL &= \mu_0 \\ LCL_x &= \mu_0 - L_x\sigma \end{aligned} \quad (1)$$

where μ_0 is the mean of process, and L_X is a multiplier of the standard deviation σ. The ith statistic of EWMA control chart is defined as

$$z_i = \lambda x_i + (1 - \lambda) z_{i-1} \quad (2)$$

where λ is a weighted parameter ($0 < \lambda \le 1$) and the starting value z_0 is usually equal to process target μ_0. The variance of z_i is defined as

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2 - \lambda} \right) \left(1 - (1 - \lambda)^{2i} \right) \quad (3)$$

Thus, the center line (CL) and upper and lower control limits (UCL_z *and* LCL_z) of the EWMA are expressed as follows:

$$\begin{aligned} UCL_{z_i} &= \mu_0 + L_z\sigma_{z_i} \\ CL &= \mu_0 \\ LCL_{z_i} &= \mu_0 - L_z\sigma_{z_i} \end{aligned} \quad (4)$$

where L_z is a multiplier of EWMA standard deviation σ_{z_i}. When λ=1, the EWMA control chart is equivalent to Shewhart-X control chart. L_z and λ are two parameters need to be set carefully.

The proposed SFEWMA-X control chart is constructed by transforming the EWMA statistics into new statistics so that both Shewhart-X and EWMA control charts operate using the same magnitude of control limits. To transform the EWMA control limits into the same width as that in the Shewhart-X control chart, we multiply the time varying EWMA control width $L_z\sigma_{z_i}$ by M_i as

$$M_i L_z \sigma_{z_i} = L_x \sigma \quad (5)$$

M_i can be represented as

$$M_i = \frac{L_x}{L_z \sqrt{\frac{\lambda}{2-\lambda}\left(1 - (1-\lambda)^{2i}\right)}} \quad (6)$$

Thus, the EWMA statistic z_i is rescaled to the scale of the Shewhart-X chart and a new statistic m_i is derived for the rescaled EWMA as

$$\begin{aligned} m_i &= \mu_0 + M_i(z_i - \mu_0) \\ &= M_i z_i + (1 - M_i)\mu_0 \end{aligned} \quad (7)$$

Instead of tracking two different statistics, m_i and x_i, a single-featured metric y_i is determined by selecting x_i or m_i that has a relatively larger distance from the center line; y_i is defined as

$$y_i = \begin{cases} x_i & \text{if } d_{x_i} > d_{m_i} \\ m_i & \text{if } d_{x_i} < d_{m_i} \\ \text{either } x_i \text{ or } m_i & \text{if } d_{x_i} = d_{m_i} \end{cases} \quad (8)$$

where $d_{x_i} = |x_i - \mu_0|$ and $d_{m_i} = |m_i - \mu_0|$. When x_i and m_i have the same distance from the center line, either one of the value can be chosen. The center line (CL) and control limits (UCL_y and LCL_y) of the proposed SFEWMA-X chart are expressed as

$$\begin{aligned} UCL_y &= \mu_0 + L_y\sigma \\ CL &= \mu_0 \\ LCL_y &= \mu_0 - L_y\sigma \end{aligned} \quad (9)$$

where L_y is the same value as L_x. Thus, once parameters L_x, L_z, and λ are determined, the proposed SFEWMA-X control chart can be constructed. Because the transformed EWMA control width has the same magnitude as the Shewhart-X control width, the SFEWMA-X control chart alarms if either the statistic m_i or x_i is beyond the Shewhart-X control limits.

2.2 SFEWMA-X control chart design

Average run length (*ARL*) is a common performance evaluation metric of control charts. *ARL* is the number of plotted points on the control chart before an out-of-control signal occurs. An in-control *ARL* (noted as ARL_0) is the *ARL* when the process is in-control. An out-of-control *ARL* (noted as ARL_1) is the *ARL* when the process is out-of-control. The optimal control chart design is to compare various chart designs with a common ARL_0 and then select the chart with the smallest ARL_1. In general, the *ARL* for the Shewhart-X chart can be set using the cumulative standard normal distribution table. The *ARL* of EWMA control chart has been studied and proposed by many previous studies [4-6]. The *ARL* of SFEWMA-X control charts can be obtain from the *ARL* of EWMA and Shewhart-X control chart as

$$\frac{1}{ARL_{SFEWMA-X}} = \frac{1}{ARL_{EWMA}} + \frac{1}{ARL_{Shewhart-X}} - \left(\frac{1}{ARL_{EWMA}} \cap \frac{1}{ARL_{Shewhart-X}} \right) \tag{10}$$

where $ARL_{SFEWMA-X}$, ARL_{EWMA} and $ARL_{Shewhart-X}$ is the *ARL* of the SFEWMA-X, EWMA and Shewhart-X control charts respectively. Let $\alpha_{SFEWMA-X}=(1/ARL_{SFEWMA-X})$, $\alpha_{EWMA}= (1/ARL_{EWMA})$ and $\alpha_{Shewhart-X}=(1/ARL_{Shewhart-X})$, to show the changes of the intersection α_{EWMA} and $\alpha_{Shewhart-X}$, we simulate the ARL_0 of SFEWMA-X control chart designed by combining EWMA and Shewhart-X control chart with the same ARL_0=370. Table 1 and Figure 1 show that the intersection of α_{EWMA} and $\alpha_{Shewhart-X}$ exists increasingly as the value of λ increases. Due to lack the information of $(1/ARL_{EWMA})\cap(1/ARL_{Shewhart-X})$, the *ARL* of SFEWMA-X control chart cannot be determined with Eq. (10).

Table 1. The ARL_0 of SFEWMA-X chart (ARL_0 of EWMA chart =ARL_0 of Shewhart-X chart=370)

λ	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
L_z	2.4907	2.7017	2.8013	2.8569	2.9006	2.9265	2.9448	2.9624	2.9683	2.9800
L_x	3	3	3	3	3	3	3	3	3	3
SFEWMA-X ARL_0	196.64	194.67	199.87	202.42	205.50	209.95	215.94	222.36	222.87	233.61
$\alpha_{SFEWMA-X}$	0.0051	0.0051	0.0050	0.0049	0.0049	0.0048	0.0046	0.0045	0.0045	0.0043
$\alpha_{EWMA} \cap \alpha_{Shewhart-X}$	0.0003	0.0003	0.0004	0.0005	0.0005	0.0006	0.0008	0.0009	0.0009	0.0011

λ	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
L_z	2.9811	2.9910	2.9917	2.9946	2.9949	2.9957	3.0034	3.0034	3.0034	3.0034
L_x	3	3	3	3	3	3	3	3	3	3
SFEWMA-X ARL_0	240.92	249.51	257.25	271.38	283.35	296.72	315.01	326.57	346.23	371.88
$\alpha_{SFEWMA-X}$	0.0042	0.0040	0.0039	0.0037	0.0035	0.0034	0.0032	0.0031	0.0029	0.0027
$\alpha_{EWMA} \cap \alpha_{Shewhart-X}$	0.0012	0.0014	0.0015	0.0017	0.0019	0.0020	0.0022	0.0023	0.0025	0.0027

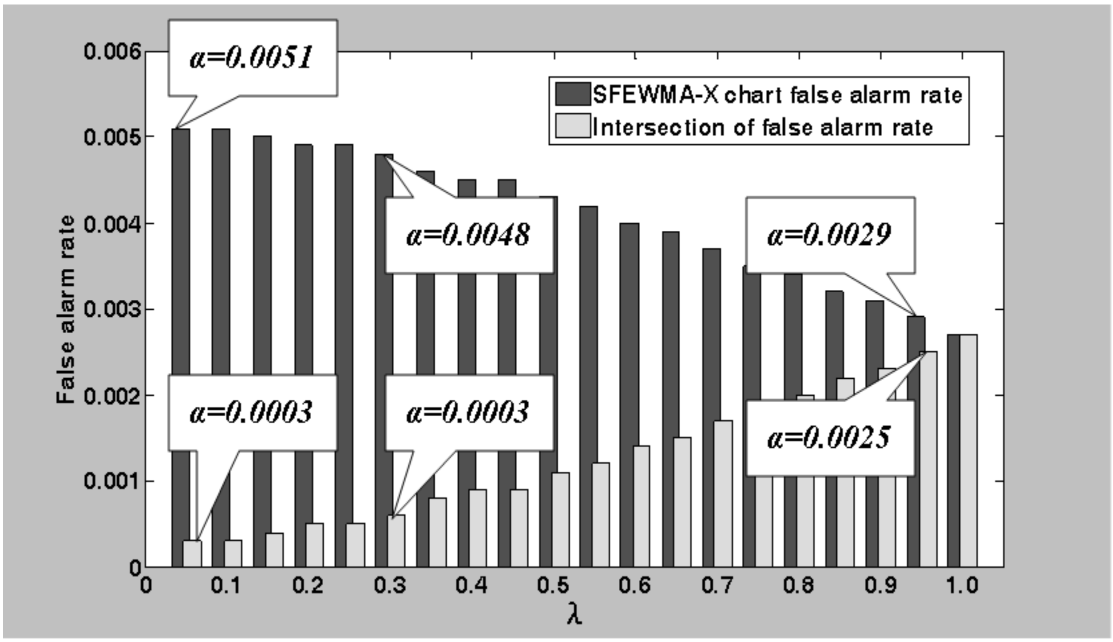


Figure 1. $\alpha_{SFEWMA\text{-}X}$ and intersection of α_{EWMA} and $\alpha_{Shewhart\text{-}X}$ as the value of λ increases. (α_{EWMA}=$\alpha_{Shewhart\text{-}X}$=0.0027)

Therefore, we can obtain the SFEWMA-X chart parameter combinations (λ, L_z and L_x) for a specified ARL_0 using simulation procedure as shown in Figure 2. Given a ARL_0 and λ, we select an initial L value (noted as L_0) within the range of (*Lmax*, *Lmin*), which are the upper and lower bounds, such that the value of SFEWMA-X statistics can be calculated using Eq. (6), (7) and (8). For a number of iterations (No_Of_Iteration), we draw a sample from standard normal distribution, and sequentially record the run length (*ARL_temp*) until a SFEWMA-X statistics y_i is plotted out of control width. When *ARL_temp* is greater or less than the specified ARL_0, we replace the *Lmax* or *Lmin* respectively, which reduces the width of searching bounds, and the search process restarts with a new initial value L_0=L_0+(*Lmax*-*Lmin*)/2. Meanwhile, if *ARL_temp* reaches the specified ARL_0 close enough, the L_z for the given ARL_0 is derived and the process stops. To facilitate the experiment in this study, we summarized the optimal designd parameters for ARL_0=250 into table, as shown in table 2.

3. Experiment

In this section, an example was proposed to demonstrate the use of the SFEWMA-X chart, and a comparison between SFEWMA-X chart and the individual control charts was performed to illustrate the detection ability of SFEWMA-X chart.

3.1 Example

We adopted a set of simulation data [20] to illustrate the proposed SFEWMA-X chart in detection of process shift. As shown in Table 3, x_i represents the *i*th observation and z_i is the statistic of EWMA. The calculations of EWMA and SFEWMA-X control chart are shown from Column 3 to 8 in Table 3, in which UCL_{z_i} and LCL_{z_i} are the upper and lower control limits of EWMA; y_i is the statistic of proposed SFEWMA-X statistic; UCL_y and LCL_y are the upper and lower control limits of y_i respectively. Among the data, the first 10 data, drawn from the standard normal distribution, represent an in-control process, while the last 20 data, drawn from

$N(0.5, 1)$, represent the process mean shifts $\delta=0.5$. Given the minimal acceptable $ARL_0=250$, and the process changes to be detected quickly at $\delta=0.5$ and $\sigma_1/\sigma_0=1$, the SFEWMA-X control chart is designed as following the proposed procedures.

Step 1. Set the desired overall acceptable minimal $ARL_0=250$.

Step 2. Set the process mean and standard deviation changes to be detected quickly are $\delta=0.5$ and $\sigma_1/\sigma_0=1$.

Step 3. From Table 2, select the optimal control chart parameter combination $L_x=3$, $L_z=2.7311$ and $\lambda=0.05$.

Step 4. Implement the SFEWMA-X control chart.

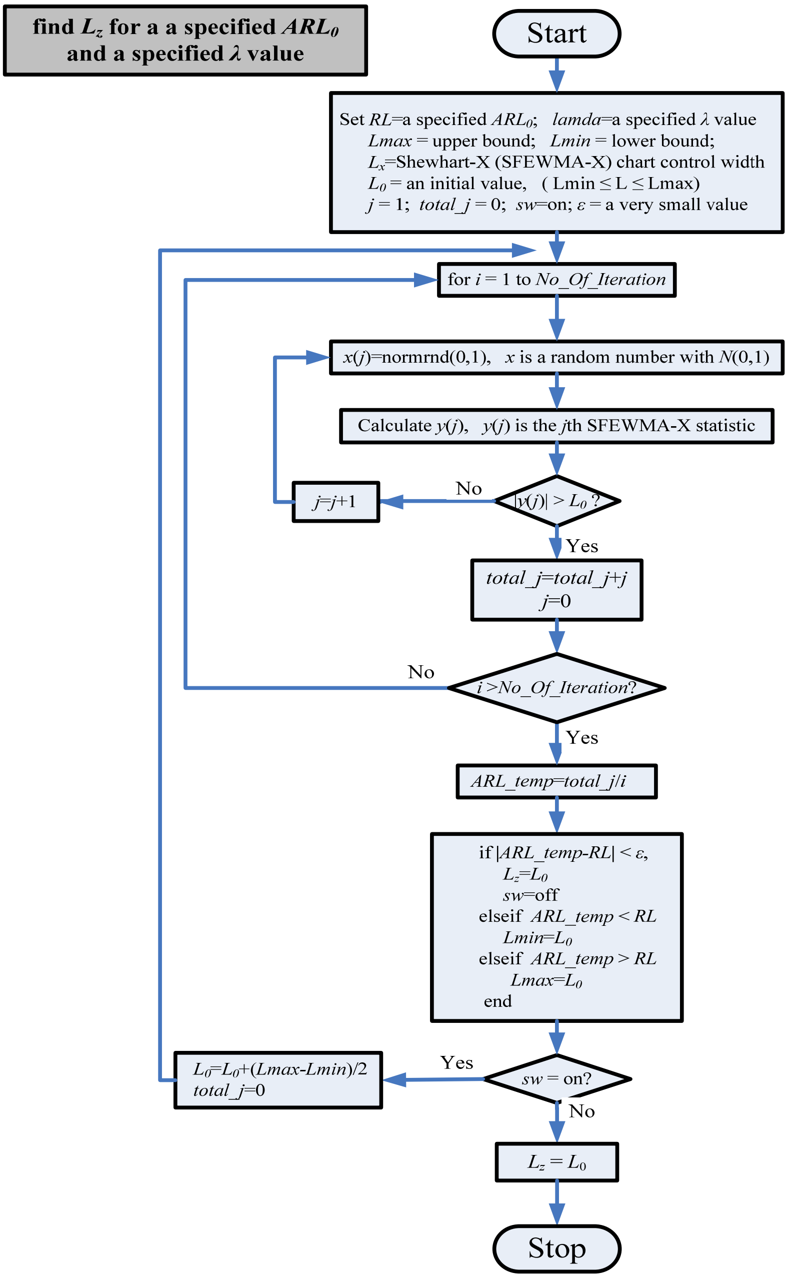


Figure 2. Simulation procedures for obtaining the SFEWMA-X chart combinations

Table 2. The optimal parameter design for SFEWMA-X control chart (ARL_0=250, and L_x=3)

Process mean shifts (δ)	σ_1/σ_0=1	σ_1/σ_0=1.5	σ_1/σ_0=2	σ_1/σ_0=3
0.00	-----	λ=0.50, L_z=3.0254	λ=0.50, L_z=3.0254	λ=0.50, L_z=3.0254
0.25	λ=0.05, L_z=2.7311	λ=0.45, L_z=3.0355	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
0.50	λ=0.05, L_z=2.7311	λ=0.30, L_z=3.0512	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
0.75	λ=0.10, L_z=2.9010	λ=0.30, L_z=3.0512	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
1.00	λ=0.15, L_z=2.9793	λ=0.30, L_z=3.0512	λ=0.50, L_z=3.0254	λ=0.50, L_z=3.0254
1.25	λ=0.15, L_z=2.9793	λ=0.30, L_z=3.0512	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
1.50	λ=0.25, L_z=3.0504	λ=0.35, L_z=3.0535	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
1.75	λ=0.30, L_z=3.0512	λ=0.35, L_z=3.0535	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
2.00	λ=0.35, L_z=3.0535	λ=0.35, L_z=3.0535	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
2.25	λ=0.35, L_z=3.0535	λ=0.45, L_z=3.0355	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
2.50	λ=0.45, L_z=3.0355	λ=0.45, L_z=3.0355	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
2.75	λ=0.45, L_z=3.0355	λ=0.45, L_z=3.0355	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254
3.00	λ=0.45, L_z=3.0355	λ=0.45, L_z=3.0355	λ=0.45, L_z=3.0355	λ=0.50, L_z=3.0254

Figure 3(a) is the SFEWMA-X control chart and an out-of-control signal occurs at observation 27. Note that the SFEWMA-X control chart uses only one pair of control limits and one statistic, y_i, while the original combined EWMA-X control chart, as shown in Figure 3(b), has two pairs of control limits and two statistics, x_i and z_i. The proposed method is obviously much clearer and neater in visual than the original EWMA-X control chart.

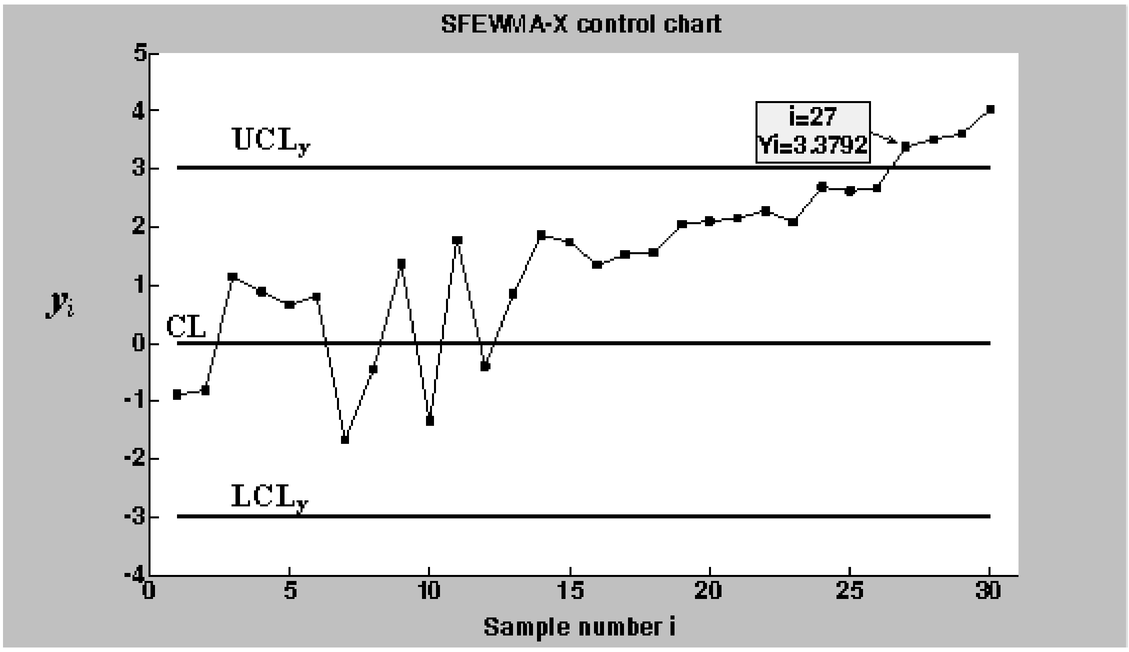
3.2 Evaluation of SFEWMA-X control charts

The purpose of this experiment is to compare the performance of SFEWMA-X control chart, EWMA control chart and Shewhart-X control chart at desired minimal ARL_0=250. For a fair comparison, designing the control charts quickly detect δ=0.25 and σ_1/σ_0=1.5, we obtained the designed parameters of SFEWMA-X control chart are λ=0.45 and control width L_z=3.0355 as presented in Table 2. Following the procedure, shown in Figure 2, we obtained the designed parameters of individual EWMA control chart with λ=0.5 and L_z=2.8511. Furthermore, since EWMA control chart is generally designed for the purposed of detecting only process mean shift, we had the design parameters of EWMA chart with λ=0.05 and L_z=2.3149 if taking no consideration of process standard deviation change. Meanwhile, the design parameter of Shewhart-X chart L_x=2.88 and relative ARL_1 were derived from cumulative standard normal distribution table.

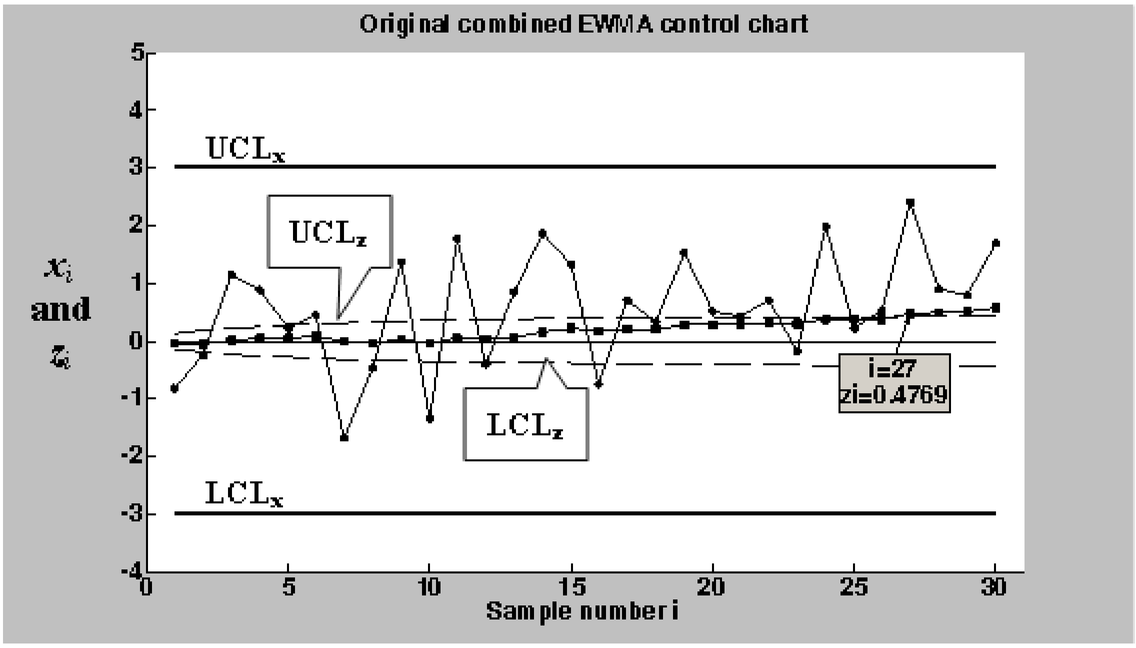
The comparative results of these four charts are shown in Table 4. When the change of process standard deviation were σ_1/σ_0 =1.5 and 2, the SFEWMA-X chart had better detection ability for the cases of process mean shifts from δ=0 to δ=3. When σ_1/σ_0 =1, the SFEWMA-X control chart also had better detection ability in detecting process mean shifts for the case in which δ>2.25. When σ_1/σ_0=3, the ARL_1s of SFEWMA-X and Shewhart-X chart had small difference in all the cases of process mean shifts, although Shewhart-X control chart had better ability in detecting most of the process mean shifts. In order to improve the detection ability for the cases of large process variation such as σ_1/σ_0=3, we may slightly decrease the value of L_z of SFEWMA-X control chart but this cause a little higher false alarm. Table 4 also showed that the detection ability of these four different control charts decreases as the ratio of process standard deviation (σ_1/σ_0) increases, while the detection ability increases as the process mean shift increases. For example, in Table 4, when the process mean shifted to δ=3, the detection abilities of these four control charts in the case of σ_1/σ_0>1 were smaller than that in the case of σ_1/σ_0=1.

Table 3. Examples of an EWMA-X control scheme and a SFEWMA-X control scheme

i	x_i	z_i	UCL_{z_i}	LCL_{z_i}	y_i	UCL_y	LCL_y
1	-0.812	-0.0406	0.1366	-0.1366	-0.8919	3	-3
2	-0.245	-0.0508	0.1884	-0.1884	-0.8094	3	-3
3	1.134	0.0084	0.2251	-0.2251	1.1340	3	-3
4	0.878	0.0519	0.2537	-0.2537	0.8780	3	-3
5	0.221	0.0604	0.2770	-0.2770	0.6536	3	-3
6	0.449	0.0798	0.2965	-0.2965	0.8073	3	-3
7	-1.673	-0.0079	0.3130	-0.3130	-1.6730	3	-3
8	-0.456	-0.0303	0.3272	-0.3272	-0.4560	3	-3
9	1.363	0.0394	0.3395	-0.3395	1.3630	3	-3
10	-1.343	-0.0297	0.3503	-0.3503	-1.3430	3	-3
11	1.766	0.0601	0.3597	-0.3597	1.7660	3	-3
12	-0.401	0.0370	0.3680	-0.3680	-0.4010	3	-3
13	0.848	0.0776	0.3753	-0.3753	0.8480	3	-3
14	1.863	0.1668	0.3818	-0.3818	1.8630	3	-3
15	1.320	0.2245	0.3876	-0.3876	1.7378	3	-3
16	-0.741	0.1762	0.3927	-0.3927	1.3463	3	-3
17	0.707	0.2028	0.3973	-0.3973	1.5312	3	-3
18	0.334	0.2093	0.4013	-0.4013	1.5646	3	-3
19	1.525	0.2751	0.4050	-0.4050	2.0378	3	-3
20	0.503	0.2865	0.4083	-0.4083	2.1053	3	-3
21	0.430	0.2937	0.4112	-0.4112	2.1427	3	-3
22	0.705	0.3142	0.4138	-0.4138	2.2782	3	-3
23	-0.173	0.2899	0.4162	-0.4162	2.0897	3	-3
24	1.986	0.3747	0.4183	-0.4183	2.6874	3	-3
25	0.226	0.3673	0.4202	-0.4202	2.6222	3	-3
26	0.528	0.3753	0.4219	-0.4219	2.6688	3	-3
27	2.408	0.4769	0.4234	-0.4234	3.3792	3	-3
28	0.905	0.4983	0.4248	-0.4248	3.5194	3	-3
29	0.794	0.5131	0.4260	-0.4260	3.6133	3	-3
30	1.704	0.5727	0.4271	-0.4271	4.0221	3	-3



(a) The example of SFEWMA-X scheme



(b) The example of combined EWMA-X scheme

Figure 3. Demonstration of proposed SFEWMA-X control chart

Table 4. A comparison of the *ARL* among SFEWMA-X, EWMA and Shewhart-X control chart (ARL_0 of SFEWMA-X, EWMA control chart and Shewhart-X control chart are 250)

	shift δ=	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00.	2.25	2.50	2.75	3.00
σ_1/σ_0 =1	**SFEWMA-X control chart (λ=0.45, L_z=3.0355)**	251.60	153.59	65.08	28.32	14.56	8.65	5.73	4.16	3.28	2.67	**2.22**	**1.91**	**1.67**
	Individual EWMA control chart (λ=0.5, L_z=2.8511)	248.96	138.80	54.11	24.20	12.78	7.81	**5.33**	**3.97**	**3.16**	**2.62**	2.24	1.96	1.76
	Individual EWMA control chart (λ=0.05, L_z=2.3149)	249.17	**60.85**	**23.52**	**13.94**	**9.86**	**7.64**	6.26	5.31	4.63	4.12	3.72	3.39	3.14
	Sewhart-X control chart (L_x=2.88)	250	193.35	110.3	59.51	33.08	19.32	11.89	7.71	5.27	3.77	2.84	2.23	1.82
σ_1/σ_0 =1.5	**SFEWMA-X control chart (λ=0.45, L_y=3.0355)**	**17.50**	**16.04**	**12.81**	**9.69**	**7.26**	**5.61**	**4.42**	**3.61**	**3.03**	**2.58**	**2.25**	**1.99**	**1.79**
	Individual EWMA control chart (λ=0.5, L_z=2.8511)	21.06	18.77	14.26	10.18	7.39	5.64	4.44	3.63	3.06	2.63	2.29	2.05	1.85
	Individual EWMA control chart (λ=0.05, L_z=2.3149)	53.81	36.18	20.54	13.49	9.84	7.72	6.37	5.45	4.75	4.23	3.81	3.48	3.20
	Shewhart-X control chart (L_x=2.88)	18.18	17.16	14.58	11.66	9.08	7.06	5.53	4.41	3.58	2.96	2.50	2.15	1.88
σ_1/σ_0 =2	**SFEWMA-X control chart (λ=0.45, L_z=3.0355)**	**6.48**	**6.28**	**5.91**	**5.34**	**4.72**	**4.15**	**3.61**	**3.16**	**2.78**	**2.48**	**2.23**	**2.01**	**1.85**
	Individual EWMA control chart (λ=0.5, L_z=2.8511)	8.02	7.67	7.08	6.26	3.32	4.59	3.96	3.40	2.95	2.60	2.33	2.11	1.93
	Individual EWMA control chart (λ=0.05, L_z=2.3149)	24.63	22.80	16.91	12.60	9.77	7.85	6.54	5.63	4.92	4.38	3.94	3.59	3.30
	Shewhart-X control chart (L_x=2.88)	6.66	6.52	6.14	5.60	5.00	4.40	3.85	3.37	2.96	2.62	2.33	2.1	1.9
σ_1/σ_0 =3	**SFEWMA-X control chart (λ=0.45, L_z=3.0355)**	2.98	2.97	2.93	2.85	**2.75**	**2.64**	2.55	2.42	**2.29**	**2.17**	**2.05**	**1.95**	**1.85**
	Individual EWMA control chart (λ=0.5, L_z=2.8511)	3.48	3.46	3.38	2.25	3.11	2.98	2.81	2.65	2.48	2.32	2.17	2.04	1.93
	Individual EWMA control chart (λ=0.05, L_z=2.3149)	10.82	10.60	9.92	8.86	7.79	6.97	6.18	5.46	4.88	4.39	4.00	3.68	3.40
	Shewhart-X control chart (L_x=2.88)	**2.96**	**2.95**	**2.91**	**2.84**	**2.75**	2.65	**2.53**	**2.41**	**2.29**	**2.17**	2.06	**1.95**	**1.85**

4. Conclusions

Quality control chart is a useful process monitoring technique which can discover unusual variation in the process, and prevent producing defective products. Combined control charts have been proposed for monitoring both small and large process parameter changes; however, two pairs of control limits and statistics on the control chart make it too complicated to be broadly used in practice. In our previous study, we developed the SFEWMA-X control chart that incorporates Shewhart-X with EWMA control chart, such that it not only retains the detection power for small and large process parameter changes but also operates neatly and easily for practical use. An *ARL* comparison is presented to illustrate that the proposed method has excellent detection ability. The design and implementing procedures of SFEWMA-X control chart were also introduced. To facilitate the use of the proposed method, we introduced the algorithm for deriving optimal chart design tables, and illustrated the design procedures of the SFEWMA-X chart with a simple example. Since a large value of the SFEWMA-X parameter λ increases the probability that Shewhart-X and EWMA chart signal simultaneously. In order to avoid Shewhart-X and EWMA chart behave similarly, we suggest that the value of λ is set no more than 0.5 in practice. Since EWMA chart is sensitive to small process mean shifts and Shewhart-X chart is sensitive to both standard deviation changes and large process mean shifts, the proposed SFEWMA-X chart can help QC engineers to analyze whether a small process mean shift or standard deviation change occurs. However, what type of process parameter variation is crucial information to improve quality; it is worth a further study on this analysis of parameter changes.

References

[1] Shewhart, W. A. 1931. "*Economic Control of Quality of Manufactured Product*". Van Nostrand, New York.

[2] Roberts, S. W. 1959. Control chart tests based on geometric moving average. *Technomertics*, 1: 239-250.

[3] Hunter, J. S. 1986. The exponentially weighted moving average. *Journal of Quality Technology*, 18: 203-210.

[4] Crowder, S. V. 1987. A simple method for studying run-Length distribution of exponentially weighted moving average charts. *Technometrics*, 29: 401-407.

[5] Lucas, J. M. and Saccucci, M. S. 1990. Exponentially weighted moving average control Scheme: properties and enhancement. *Technometrics*, 32: 1-12.

[6] Neubauer, A. S. 1997. The EWMA control chart: properties and comparison with other quality-control procedures by computer simulation. *Clinical Chemistry*, 43: 594-601.

[7] Woodall, W. H. and Maragah, H. D. 1990. Discussion. Exponentially weighted moving average control schemes: properties and enhancements. *Technometrics*, 32: 1-29.

[8] Klein, M. 1996. Composite Shewhart-EWMA statistical control schemes. *IIE Transactions*, 28: 475-481.

[9] Tolley, G. O. and English, J. R. 1999. Economic design of constrained EWMA and combined EWMA- $\overline{X}$ control scheme. *IIE Transactions*, 33: 429-436.

[10] Zhang, L., Bebbington, M. S., Govindaraju, K. and Lai, C. D. 2004. Composite EWMA control charts. *Communication in Statistics: Simulation and Computation*, 33: 1133-1158.

[11] Serel, D. A. and Moskowitz, H. 2008. Joint economic design of EWMA control charts for mean and variance. *European Journal of Operational Research*, 184: 157-168.

[12] Hawkins, D. M. and Deng, Q. 2009. Combined charts for mean and variance Information. *Journal of Quality Technology*, 41: 415-425.

[13] Sheu, S. H. and Lu, S. L. 2009. The effect of auto-correlated observations on a GWMA control chart performance. *International Journal of Quality and Reliability Management*, 26: 112-128.

[14] Simões, B. F. T., Epprecht, E. K,. and Costa, A. F. B. 2010. Performance comparisons of EWMA control chart schemes. *Quality Technology & Quantitative Management*, 3: 249-261.

[15] Albin, S. L., Kang, L., and Shea, G. 1997. An X and EWMA chart for individual observations. *Journal of Quality Technology*, 29: 41-48.

[16] Reynolds, M. R. and Stoumbos, Z. G. 2001. Individuals control schemes for monitoring the mean and variance of process subject to drifts. *Stochastic Analysis and Applications*, 19: 863-892.

[17] Reynolds, M. R. and Stoumbos, Z. G. 2001. Monitoring the process mean and variances using individual observations and variable sampling intervals. *Journal of Quality Technology*, 33: 181-205.

[18] Stoumbos, Z. G. and Reynolds, M. R. 2005. Economic statistical design of adaptive control schemes for monitoring the mean and variance: an application to analyzers. *Nonlinear Analysis*, 6: 817-844.

[19] Liu, C. S. and Tien, F. C. 2011. A single-featured EWMA-X control chart for detecting shifts in process mean and standard deviation. *Journal of Applied Statistics*. (accepted January 2011)

[20] Sheu, S. H., and Lin, T. C. 2003. The generally weighted moving average control chart for detecting small shifts in the process mean. *Quality Engineering*, 16: 209-231.