International Journal of Applied Science and Engineering
2011. 9, 3: 169-176

A new Method for Solving Sensitivity Analysis for Fuzzy Linear Programming Problems

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Abstract: The fuzzy set theory is being applied massively in many fields these days. One of these is linear programming problems. Kheirfam and Hasani (Sensitivity analysis for fuzzy linear programming problems with fuzzy variables, Advanced Modeling and Optimization, 12 (2010) 257-272), proposed a method for the sensitivity analysis for fuzzy linear programming (FLP) problems with fuzzy variables. There are some important cases that are not considered by Kheirfam and Hasani. In this paper, those cases are considered and numerical examples are solved.

Keywords: fuzzy linear programming problem; trapezoidal fuzzy numbers; ranking function; sensitivity analysis.

1. Introduction

Sensitivity analysis is well-explored area in classical linear programming. Sensitivity analysis is a basic tool for studying perturbations in optimization problems. There is considerable research on sensitivity analysis for some operations research and management science models such as linear programming and investment analysis.

In most practical applications of mathematical programming the possible values of the parameters required in the modeling of the problem are provided by either a decision maker subjectively or a statistical inference from the past data due to which there exists some uncertainty. In order to reflect this uncertainty, the model of the problem is often constructed with fuzzy data [19].

Fuzzy linear programming (FLP) provides the flexibility in values. But even after formulating the problem as FLP problem, one cannot stick to all the values for a long time or it is quite possible that the wrong values got entered. With time the factors like cost, required time or availability of product etc. changes widely. Sensitivity analysis for FLP problem needs to be applied in that case. Sensitivity analysis is one of the interesting researches in FLP problem.

Zimmermann [20] attempted to fuzzify a linear program for the first time, fuzzy numbers being the source of flexibility. Zimmermann also presented a fuzzy approach to multi-objective linear programming problem and its sensitivity analysis. Sensitivity analysis in FLP problem with crisp parameters and soft constraints was first considered by Hamacher et al. [6].

Tanaka and Asai [17] proposed a method of allocating the given investigation cost to each fuzzy coefficients by using sensitivity analysis. Tanaka et al. [16] formulated a FLP problem with fuzzy coefficients and the value of information was discussed via sensitivity analysis. Sakawa and Yano [15] presented a

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fuzzy approach for solving multi- objective linear fractional programming problem via sensitivity analysis.

Fuller [4] proposed that the solution to FLP problems with symmetrical triangular fuzzy numbers is stable with respect to small changes of centers of fuzzy numbers. Perturbations occur due to calculation errors or just to answer managerial questions "What if …". Such questions propose after the simplex method and the related research area refers to as basis invariance sensitivity analysis.

Dutta et al. [3] studied sensitivity analysis for fuzzy linear fractional programming problem. Verdegay and Aguado [18] proposed that in the case of FLP problems, whether or not a fuzzy optimal solution has been found by using linear membership functions modeling the constraints, possible further changes of those membership functions do not affect the former optimal solution. The sensitivity analysis performed for those membership functions and the corresponding solutions shows the convenience of using linear functions instead of other more complicated ones.

Gupta and Bhatia [5] studied the measurement of sensitivity for changes of violations in the aspiration level for the fuzzy multi-objective linear fractional programming problem. Precup and Preitl [14] performed the sensitivity analysis for some fuzzy control systems. Lotfi et al. [11] developed a sensitivity analysis approach for the additive model. Kheirfam and Hasani [8] studied the basis invariance sensitivity analysis for FLP problems. In this paper, some important cases that are not considered by Kheirfam and Hasani [8] are discussed and numerical examples are solved.

This paper is organized as follows: In Section 2, some basic definitions, arithmetic operations, defuzzification function and FLP problem are reviewed. Sensitivity analysis for the deletion of a fuzzy variable and a fuzzy constraint are discussed in Section 3. In Section 4, obtained results and sensitivity analysis for the deletion of a fuzzy variable and a fuzzy constraint are explained with the help of numerical examples. Conclusions are discussed in Section 5.

2. Preliminaries

In this section, some basic definitions, notations and arithmetic operation of trapezoidal fuzzy numbers, are reviewed. In addition to that, a brief review of used ranking function and FLP problem is given.

2.1.Definitions

In this section, some basic definitions are presented.

Definition 2.1 [7] A fuzzy set $\tilde{A}$ in X (set of real number) is a set of ordered pairs:

$\tilde{A}=\{(x,\mu_{\tilde{A}}(x))|x\in X\}$

$\mu_{\tilde{A}}(x)$ is called the membership function of x in $\tilde{A}$ which maps X to [0,1].

Definition 2.2 [7] The support of a fuzzy set $\tilde{A}$ on X is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}}(x)>0$.

Definition 2.3 [7] The α - cut of a fuzzy set $\tilde{A}$ is the subset of X defined as:

$\tilde{A}_\alpha=\{x\in X \mid \mu_{\tilde{A}}(x)\geq\alpha\}$

2.2. Arithmetic operations

In this section, arithmetic operations between two trapezoidal fuzzy numbers, defined on universal set of real numbers X, are discussed [7].

Let $\tilde{A}_1$ = (a_1,b_1,c_1,d_1) and $\tilde{A}_2$ = (a_2,b_2,c_2,d_2) be two trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$

(ii) $\tilde{A}_1 \ominus \tilde{A}_2=(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2)$

(iii) $\tilde{A}_1 \otimes \tilde{A}_2 \approx (a',b',c',d')$ where

$a'=minimum\ (a_1a_2,a_1d_2,a_2d_1,d_1d_2)$,

$b'=minimum\ (b_1b_2,b_1c_2,b_2c_1,c_1c_2)$,

$c' = maximum \ (b_1 b_2, b_1 c_2, b_2 c_1, c_1 c_2)$,
$d' = maximum \ (a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2)$

(iv) $\lambda\tilde{A} = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1) & \lambda < 0 \end{cases}$

2.3. Defuzzification function

A convenient method for comparison of fuzzy number is by use of defuzzification function [10]. A defuzzification function *R*: F(R) $\rightarrow$ R, where F(R) is set of all fuzzy numbers defined on set of real numbers, maps each fuzzy number into a real line, where a natural order exists. Some orders on $\mathrm{F}(X)$ are defined as follows:

(i) $\tilde{A} \geq_R \tilde{B}$ if and only if $R(\tilde{A}) \geq R(\tilde{B})$

(ii) $\tilde{A} >_R \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$

(iii) $\tilde{A} =_R \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$

where, $\tilde{A}$ and $\tilde{B}$ belong to F(R), *R* is a defuzzification function and the symbol "$\geq_R$" represents the fuzzy order relation. For given trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$

$$R(\tilde{A}) = \frac{a+b+c+d}{4}$$

2.4. Fuzzy linear programming (FLP) problem

Linear programming is one of the most frequently applied operations research technique. In the conventional approach value of the parameters of linear programming models must be well defined and precise. However, in real world environment, this is not a realistic assumption. In the real life problems the following situation may occur:
Whenever a company wants to launch new product in the market there may exist uncertainty about the cost/profit of the product. In such a situation the cost/profit should be represented by fuzzy numbers. A FLP [15] problem is defined as follows:

Maximize $\tilde{z} =_R C^T \tilde{X}$

Subject to $A\tilde{X} =_R \tilde{b}$

$\tilde{X} \geq_R \tilde{0}.$

where
$\tilde{b} = [\tilde{b}_j]_{m\times 1}, \tilde{X} = [\tilde{x}_j]_{n\times 1}, A = [a_{ij}]_{m\times n}, C^T = [\tilde{c}_j]_{1\times n}$,
and *R* is a linear ranking function.

FLP provides the flexibility in values. But even after formulating the problem as FLP problem, one cannot stick to all the values for a long time or it is quite possible that the wrong values got entered. With time the factors like cost, required time or availability of product etc. changes widely. Sensitivity analysis for FLP problem needs to be applied in that case. Sensitivity analysis is one of the interesting researches in FLP problem.

3. Sensitivity analysis for the deletion of a fuzzy variable and a fuzzy constraint

Kheirfam and Hasani [8] proposed a method for the sensitivity analysis for the FLP problems with fuzzy variables. In this section cases, not considered by Kheirfam and Hasani [8], are discussed.

(i) Deletion of a fuzzy variable.
(ii) Deletion of a fuzzy constraint.

Case 1: Deletion of a fuzzy variable

There arise two cases under this category:

(i) If a non basic fuzzy variable or a basic fuzzy variable at zero level (i.e. its rank is zero) is deleted, then there will be no change in the fuzzy optimal solution.
(ii) However, deletion of positive basic fuzzy variable (i.e. its rank is positive) will affect the optimal solution.

Considering (ii), note that deleting a fuzzy variable at positive level is equivalent to convert it into non basic fuzzy variable. For the same, first remove the entire column associated with the basic fuzzy variable to be deleted from the optimal table and then multiply the entire row corresponding to this variable by -1 so that feasibility gets disturbed. Now, applying fuzzy dual simplex method for FVLP, restore feasibility, which will include the removal of variable to be deleted.

Case 2: Deletion of a fuzzy constraint

While deleting a fuzzy constraint we observe two situations:

(i) If any fuzzy constraint is satisfied on the boundary, i.e., rank of slack or surplus variable corresponding to this fuzzy constraint is at zero level then deletion of such a constraint may cause change in the fuzzy optimal solution.

(ii) If any fuzzy constraint is satisfied in the interior of feasible region P_F i.e., rank of slack or surplus variable corresponding to this fuzzy constraint are positive then deletion of such a constraint will not affect the fuzzy optimal solution.

In other words, situation (i) is a binding on the fuzzy optimal solution, while situation (ii) is a nonbinding on the fuzzy optimal solution.

4. Numerical examples

In this section proposed method is illustrated with the help of numerical examples:

Example 1 Considered the following FLP model:

Minimize $\tilde{z} =_R \ominus \tilde{x}_1 \ominus \tilde{x}_2 \ominus 2\tilde{x}_3$

Subject to

$\tilde{x}_1 \oplus \tilde{x}_2 \oplus 2\tilde{x}_3 \leq_R (3,5,8,13)$

$\tilde{x}_1 \ominus \tilde{x}_2 \leq_R (4,6,10,16)$

$\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_R (-6,1,6,14)$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq_R \tilde{0}.$

(i) Discuss the effect of deletion of fuzzy variable $\tilde{x}_3$ and $\tilde{x}_1$ from the FLP problem on the fuzzy optimal solution.

(ii) If a fuzzy constraint $\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_R (-6,1,6,14)$ deleted from the above FLP problem discuss its affect on the fuzzy optimal solution.

Solution:

Minimize $\tilde{z} =_R \ominus \tilde{x}_1 \ominus \tilde{x}_2 \ominus 2\tilde{x}_3$

Subject to

$\tilde{x}_1 \oplus \tilde{x}_2 \oplus 2\tilde{x}_3 \leq_R (3,5,8,13)$

$\tilde{x}_1 \ominus \tilde{x}_2 \leq_R (4,6,10,16)$

$\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_R (-6,1,6,14)$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq_R \tilde{0}.$

Adding fuzzy slack variables, above FLP problem can be written as:

Minimize $\tilde{z} =_R \ominus \tilde{x}_1 \ominus \tilde{x}_2 \ominus 2\tilde{x}_3$

Subject to

$\tilde{x}_1 \oplus \tilde{x}_2 \oplus 2\tilde{x}_3 \oplus \tilde{s}_1 =_R (3,5,8,13)$

$\tilde{x}_1 \ominus \tilde{x}_2 \oplus \tilde{s}_2 =_R (4,6,10,16)$

$\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \oplus \tilde{s}_3 =_R (-6,1,6,14)$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{s}_1, \tilde{s}_2, \tilde{s}_3 \geq_R \tilde{0}.$

The fuzzy optimal solution is $\tilde{x}_1 =_R (0,0,0,0)$, $\tilde{x}_2 =_R (0,0,0,0)$, $\tilde{x}_3 =_R \left(\frac{3}{2},\frac{5}{2},4,\frac{13}{2}\right)$ and fuzzy optimal value is $\tilde{z} =_R (-13,-8,-5,-3)$.

(i) The effect of deletion of a fuzzy variable $\tilde{x}_3$ and $\tilde{x}_1$ from the given FLP problem on the fuzzy optimal solution are discussed below:

Case1. When a fuzzy variable $\tilde{x}_3$ is deleted:

Since $\tilde{x}_3$ is a basic fuzzy variable with positive value $\left(\frac{3}{2},\frac{5}{2},4,\frac{13}{2}\right)$ (i.e. its rank is positive), its deletion will affect the fuzzy optimal solution. Remove the column associated to $\tilde{x}_3$ and multiply all the entries in $\tilde{x}_3$-row by -1. Note that feasibility has been disturbed as now, basic variable $\tilde{x}_3$ is at negative level. Now apply fuzzy dual simplex method [14] to restore the feasibility.

Table 1. Fuzzy optimal table of the chosen FLP problem

basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	RHS	R(RHS)
$\tilde{z}$ row	0	0	0	-1	0	0	$(-13,-8,-5,-3)$	$-\frac{29}{4}$
$\tilde{x}_3$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	$\left(\frac{3}{2},\frac{5}{2},4,\frac{13}{2}\right)$	$\frac{29}{8}$
$\tilde{s}_2$	1	-1	0	0	1	0	$(4,6,10,16)$	9
$\tilde{s}_3$	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	1	$\left(-\frac{25}{2},-3,\frac{7}{2},\frac{25}{2}\right)$	$\frac{1}{8}$

Table 2. Table of resulting FLP problem

basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	RHS	R(RHS)
$\tilde{z}$ row	0	0	-1	0	0	$(-13,-8,-5,-3)$	$-\frac{29}{4}$
$\tilde{x}_3$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\left(-\frac{13}{2},-4,-\frac{5}{2},-\frac{3}{2}\right)$	$-\frac{29}{8}$
$\tilde{s}_2$	1	-1	0	1	0	$(4,6,10,16)$	9
$\tilde{s}_3$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	1	$\left(-\frac{25}{2},-3,\frac{7}{2},\frac{25}{2}\right)$	$\frac{1}{8}$

Table 3. Fuzzy optimal table of resulting FLP problem after deletion of fuzzy variable $\tilde{x}_3$

basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	RHS	R(RHS)
$\tilde{z}$ row	1	1	0	0	0	$(-10,-3,3,10)$	0
$\tilde{s}_1$	1	1	1	0	0	$(3,5,8,13)$	$\frac{29}{4}$
$\tilde{s}_2$	1	-1	0	1	0	$(4,6,10,16)$	9
$\tilde{s}_3$	1	1	0	0	1	$(-11,-.5,7.5,19)$	$\frac{15}{4}$

So, the fuzzy optimal solution is $\tilde{x}_1 =_R (0,0,0,0)$, $\tilde{x}_2 =_R (0,0,0,0)$, $\tilde{x}_3 =_R (0,0,0,0)$ and fuzzy optimal value is $\tilde{z} =_R (0,0,0,0)$.

Case 2. When a fuzzy variable $\tilde{x}_1$ is deleted:

Since $\tilde{x}_1$ is a non basic fuzzy variable so its deletion will not affect the fuzzy optimal solution.

(ii) Since the fuzzy constraint $\tilde{x}_1 \oplus \tilde{x}_2 \oplus \tilde{x}_3 \leq_R (-6,1,6,14)$ is satisfied as strict inequality, because slack variable $\tilde{s}_3 =_R \left(-\frac{25}{2}, -3, \frac{7}{2}, \frac{25}{2}\right)$ i.e. inside the feasible region P_F or rank of slack variable corresponding to this fuzzy constraint are positive level, hence its deletion causes no changes in fuzzy optimal solution.

Example 2 Considered the FLP problem

Maximize $\tilde{z} =_R 6\tilde{x}_1 \oplus 2\tilde{x}_2 \oplus 10\tilde{x}_3$

Subject to

$3\tilde{x}_1 \ \Theta\ \tilde{x}_2 \oplus \tilde{x}_3 \leq_R (7,9,10,12)$

$\tilde{x}_1 \oplus \tilde{x}_3 \leq_R (7,8,9,10)$

$\tilde{x}_2 \oplus 2\tilde{x}_3 \leq_R (2,4,5,7)$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq_R \tilde{0}$.

Table 4. Fuzzy optimal table of the chosen FLP problem

basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	RHS	R(RHS)
$\tilde{z}$ row	0	0	0	2	0	4	$(20,30,50,60)$	40
$\tilde{x}_3$	1	$-\frac{1}{2}$	0	$\frac{1}{3}$	0	$\frac{1}{6}$	$(1,2,3,4)$	$\frac{5}{2}$
$\tilde{s}_2$	0	0	0	$-\frac{1}{3}$	1	$-\frac{1}{3}$	$(0,2,4,6)$	3
$\tilde{s}_3$	0	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$(1,2,3,4)$	$\frac{5}{2}$

Show that (a) deletion of first fuzzy constraint affects the fuzzy optimal solution; (b) deletion of third fuzzy constraint does not change the fuzzy optimal solution.

Solution

Let us work out the deletion of first fuzzy constraint $3\tilde{x}_1 \ \Theta\ \tilde{x}_2 \oplus \tilde{x}_3 \leq_R (7,9,10,12)$. Since this fuzzy constraint is of the type $3\tilde{x}_1 \ \Theta\ \tilde{x}_2 \oplus \tilde{x}_3 \oplus \tilde{s}_1 =_R (7,9,10,12)$ and is satisfied on the boundary of the feasible region because $\tilde{s}_1 =_R \tilde{0}$. thus its deletion will impact the fuzzy optimal solution. Add a variable $-\tilde{s}_1'$ to this fuzzy constraint. Now, treat the problem as addition of a fuzzy variable. Note that, $\tilde{z}_j - \tilde{c}_j$ for $\tilde{s}_1' =-$ relative cost of $\tilde{s}_1 = -2$, and column below $\tilde{s}_1' =-$ column below $\tilde{s}_1 =_R \left[\frac{1}{3}, -\frac{1}{3}, 0\right]^T$. Applying existing sensitivity analysis technique for the addition of a fuzzy variable the fuzzy optimal table is

Table 5. Fuzzy optimal table of the resulting FLP problem after deletion of fuzzy constraint

basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{s}_1'$	$\tilde{s}_1$	$\tilde{s}_2$	$\tilde{s}_3$	RHS	R(RHS)
$\tilde{z}$ row	0	0	0	0	0	6	2	$(30,40,72,90)$	58
$\tilde{x}_3$	1	$-\frac{1}{2}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{6}$	$(0,6,7,9)$	$\frac{11}{2}$
$\tilde{s}_2$	0	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	$-\frac{1}{3}$	$(2,8,10,16)$	9
$\tilde{s}_3$	0	$\frac{1}{2}$	1	0	0	0	$\frac{1}{2}$	$(1,2,3,4)$	$\frac{5}{2}$

Fuzzy optimal solution is $\tilde{x}_1 =_R (0,0,0,0)$, $\tilde{x}_2 =_R (0,0,0,0)$ $\tilde{x}_3 =_R (0,6,7,9)$ and fuzzy optimal value is $\tilde{z} = (30,40,72,90)$.

For the case (b), the second fuzzy constraint is satisfied as a strict inequality, because $R(\tilde{s}_2) = 3$, i.e., inside the feasible region P_F, hence, its deletion causes no change in the fuzzy optimal solution.

5. Conclusions and future work

In this paper, some important cases for the sensitivity analysis for fuzzy variable linear programming problems that are not discussed in the literature are discussed and some numerical examples are solved. We will try to solve sensitivity analysis problems in which all the parameters are represented by fuzzy numbers.

Since there is no existing rule for the product of two arbitrary fuzzy numbers so we left it as a future work. This will be an interesting research work in the future.

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