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Design of PID Controllers for Pure Integrator Systems with Time Delay

C. V. Nageswara Rao [a], A. Seshagiri Rao[b], and R. Padma Sree [*,c]

[a] *Department of Chemical Engineering, G.V. P. College of Engineering (A), Madhurawada*
[b] *Department of Chemical Engineering, NIT, Trichy.*
[c] *Department of Chemical Engineering, AU College of Engineering (A), Visakhapatnam, India*

Abstract: In this paper, design of Proportional Integral and Derivative (PID) controllers based on internal model control (IMC) principles, direct synthesis method (DS), stability analysis (SA) method for pure integrating process with time delay is proposed. The performances of the proposed controllers are compared with the controllers designed by recently reported methods. The robustness of the proposed controllers for the uncertainty in model parameters is evaluated considering one parameter at a time using Kharitonov's theorem. The proposed controllers are applied to various transfer function models and to non linear model of isothermal continuous copolymerization of styrene-acrylonitrile in CSTR. An experimental set up of tank with the outlet connected to a pump is considered for implementation of the PID controllers designed by the three proposed methods to show the effectiveness of the methods.

Keywords: PID controller; integrating systems; internal model controller; stability analysis; direct synthesis method; robustness; Kharitonov's theorem; level control in cylindrical tank

1. Introduction

Integrating systems with time delay are found in the modeling of liquid level systems, liquid storage tanks, boilers, batch chemical reactors and the bottom level control of a distillation column [1]. Chien and Freuhauf [1] have suggested that many chemical processes can be modeled for the purpose of designing controllers by a transfer function $\frac{k_p e^{-Ls}}{s}$. Fuentes and Luyben [2] have reported that the composition control loop of a high purity distillation column has a large time constant and hence, the response resembles that of a pure integrator plus dead time model. An isothermal continuous copolymerization reactor can be modeled as an integrating system with dead time [3]. The model contains only two parameters (k_p *and* L) and the model is very simple for identification. The model is able to adequately represent the dynamics of many systems over the frequency range of interest for the PID controller design. In industries 95% of the controllers are of PID type [4, 5].

There are number of methods available in the literature for designing PID controllers for pure integrating systems with time delay. They are methods based on stability criteria [6-13], optimization method [14-16], IMC method [1, 17-19], Two Degree Of Freedom (2DOF) controller [20] and modified smith

[*] Corresponding author; e-mail: padvan@rediffmail.com *Accepted for Publication: September 29, 2011*

predictor method for large time delay [21, 22] and equating coefficient method [23] and synthesis method [24].

A simple IMC-PID controller design technique is given by [19] on the basis of the IMC principle for integrating processes with time delay. The method is mainly focused on the disturbance rejection, which causes the overshoot in the set point response, and a Two Degree Of Freedom (2DOF) control structure is used to eliminate this overshoot. PID controller is designed using Maclaurin series expansion. The controller is tuned for robustness by specifying the peak of maximum sensitivity function (Ms).

Recently Seshagiri Rao, V. S. R. Rao and Chidambaram [24] have designed PID controller for integrating systems with time delay by direct synthesis method. In the direct synthesis method, the closed loop transfer function is to be assumed, from which controller transfer function is derived using process transfer function.

The present work is intended to design PID controllers for pure integrating systems with time delay using three methods (i) IMC method (ii) direct synthesis method and (iii) stability analysis method. In the IMC method integrator is also treated as unstable pole for the design of IMC controller. From the IMC controller, conventional PID controller parameters are obtained [25]. In the direct synthesis method, conventional PID controller along with the first order filter is chosen and the resulting closed loop characteristic equation is compared with the desired characteristic equation to obtain the closed loop time constant. Using this closed loop time constant, the controller settings k_c, τ_I *and* τ_D are evaluated. The PID filter time constant, τ_f is chosen to be $\beta\tau_D$, β is selected in the range of 0.35 to 0.5. In the stability analysis method, ω_c the phase cross-over frequency is obtained by phase angle criteria for the process and The values of τ_I *and* τ_D are estimated [26] from phase cross-over frequency. The controller is chosen to be PID controller with a lag filter. The loop transfer function, which is the product of controller and the process transfer function, is written for the integrating system with time delay. The phase cross-over frequency ω_{cc} is obtained from the phase angle criteria for the loop transfer function using *fsolve* of MATLAB. The value of ultimate controller gain $k_{c,\max}$ is obtained from the gain margin criteria. By using a suitable value for gain margin between 1.5 and 4.0, the design value of controller gain (k_c) is obtained.

The performance of the proposed controllers based on IMC principles (IMC-PID), stability analysis method (SA-PID), direct synthesis method (DS-PID) for various transfer function models for both servo and regulatory problems is compared with the controllers designed by recently reported methods like direct synthesis method (DS-s) [24], equating coefficient method (EC) [23] and internal model control method with disturbance rejection (IMC-dr) [19]. An experiment is conducted to implement the proposed controllers to control level in the cylindrical tank for both set point tracking and load disturbance.

2. Design of the Controller

2.1. IMC Method

The process transfer function is given by

$$G_p = \frac{k_p e^{-Ls}}{s} \tag{1}$$

Using Pade's approximation for time delay, Eq (1) is rewritten as

$$G_p = \frac{k_p(1-0.5Ls)}{s(1+0.5Ls)} \tag{2}$$

IMC controller for the above system consists of two parts. First part is the inverse of the stable portion of the process and second part is IMC filter. The numerator order of the

IMC filter is equal to the number of unstable poles. If the process has an unstable pole, then IMC filter will have first order numerator dynamics. Here, integrator is considered as an unstable pole. The denominator order of the IMC filter is selected in such a way to make the IMC controller proper (order of the numerator is equal to or less than the order of denominator). The controller for the above process [Eq (2)] is given by

$$Q = \frac{s(1+0.5Ls)(\alpha s+1)}{k_p(\lambda s+1)^3} \tag{3}$$

From the IMC controller, conventional feedback controller is obtained by

$$G_c = \frac{Q}{1-G_p Q} \tag{4}$$

$$G_c = \frac{s(1+0.5Ls)(\alpha s+1)}{k_p[(\lambda s+1)^3-(1-0.5Ls)(\alpha s+1)]} \tag{5}$$

Equating coefficient of s in the denominator of the Eq (5) to zero, the α value is obtained as

$$\alpha = 3\lambda + 0.5L \tag{6}$$

The Eq (5) is simplified as,

$$G_c = \frac{s(1+0.5Ls)(\alpha s+1)}{s^2 k_p (3\lambda^2+0.5L\alpha)(\tau_f s+1)} \tag{7}$$

$$G_c = \left[\frac{\alpha+0.5L}{k_p(3\lambda^2+0.5L\alpha)}\right]\left[1+\frac{1}{(\alpha+0.5L)s}+\frac{0.5L\alpha}{(\alpha+0.5L)}s\right]\left[\frac{1}{\tau_f s+1}\right] \tag{8}$$

This is a PID controller with a lag filter. The parameters of PID controller are given by

$$k_c = \frac{\alpha+0.5L}{k_p(3\lambda^2+0.5L\alpha)}$$

$$\tau_I = \alpha + 0.5L$$

$$\tau_D = \frac{0.5L\alpha}{\alpha+0.5L}$$

$$\tau_f = \frac{\lambda^3}{3\lambda^2+0.5L\alpha}$$

The filter time constant λ is selected in such a way that the performance of the servo and regulatory problems is reasonably good. The value of λ is selected between 0.6L to 3L, to have good performance for both servo and regulatory problems.

2.2. Direct Synthesis method

The process transfer function is given by Eq (1). The controller transfer function is taken as

$$G_C = k_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1}{(1+\tau_f s)} \tag{9}$$

Where $\tau_f = \beta \tau_D$

The closed loop characteristic equation is given by

$$1 + G_P G_C = 0 \tag{10}$$

Substituting Eq (1) and Eq (9) in Eq (10), the following equation is obtained

$$1+\frac{k_p e^{-Ls}}{s} k_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\frac{1}{(1+\tau_f s)} = 0 \tag{11}$$

Rearranging the Eq (11) along with the first order Pades' approximation for time delay $\left(e^{-Ls} = \frac{1-0.5Ls}{1+0.5Ls}\right)$ gives the following equation

$$A s^2\ (1+\tau_f\, s)(1+0.5L s)+\ (1+\tau_I\, s+\tau_I\, \tau_D\, s^2)\ (1-0.5L s)=0 \tag{12}$$

Where $A=\dfrac{\tau_I}{k_p\, k_c}$ (12a)

On expanding and rearranging, the following polynomial in s is obtained for the characteristic equation

$$s^4\,(0.5L\tau_f\ A)+s^3\,(A\tau_f+0.5L A-0.5L\tau_I\,\tau_D)+s^2\,(A-0.5L\tau_I+\tau_I\,\tau_D)+s(-0.5L+\tau_I)+1=0 \tag{13}$$

The desired closed loop characteristic equation is given by

$$(\tau_{CL}^2\, s^2+2\,\tau_{CL}\, s+1)(\tau_{CL}^2\, s^2+1.414\,\tau_{CL}\, s+1)=0 \tag{14}$$

Rearranging the above equation, the following equation is obtained

$$s^4\,(\tau_{CL}^4)+s^3\,(3.414\,\tau_{CL}^3)+s^2\,(4.828\,\tau_{CL}^2)+s\,(3.414\ \tau_{CL})+1=0 \tag{15}$$

On comparing coefficients of s, s^2, s^3, s^4, in Eq (13) with Eq (15), the following equations are obtained

$$0.5L\tau_f\ A=\tau_{CL}^4 \tag{16}$$

$$A\ \tau_f+0.5L A-0.5L\tau_I\,\tau_D=3.414\,\tau_{CL}^3 \tag{17}$$

$$A-0.5L\tau_I+\tau_I\,\tau_D=4.828\,\tau_{CL}^2 \tag{18}$$

$$-0.5L+\tau_I=3.414\,\tau_{CL} \tag{19}$$

From Eq (19), equation for τ_I is obtained as

$$\tau_I=3.414\,\tau_{CL}+0.5L \tag{20}$$

Substitution for $A\tau_f$ and A from Eq (16) and Eq (18) respectively into Eq (17) the equation for τ_D is obtained.

$$\tau_D=\frac{1}{\tau_I}(f_1\,\tau_{CL}^4+f_2\,\tau_{CL}^3+f_3\,\tau_{CL}^2+f_4\,\tau_{CL}+f_5) \tag{21}$$

Where

$f_1=1/0.5L^2$; $f_2=(-3.414)/L$; $f_3=2.414/L$; $f_4=0.8535L$; $f_5=0.125L^3$

Rearranging Eq (16) on substituting for τ_I *and* τ_D from Eq (20) and Eq (21), a polynomial equation in τ_{CL} is obtained as

$$h_1\,\tau_{CL}^8+h_2\,\tau_{CL}^7+h_3\,\tau_{CL}^6+h_4\,\tau_{CL}^5+h_5\,\tau_{CL}^4+h_6\,\tau_{CL}^3+h_7\,\tau_{CL}^2+h_8\,\tau_{CL}+h_9=0 \tag{22}$$

Where

$$h_1=\frac{b_1\beta}{L};\ \ h_2=-0.1707b_1\beta+\frac{b_2\beta}{L};\ \ h_3=1.207b_1\beta-2.414Lb_1\beta-1.707\ b_2\beta+\frac{b_3\beta}{L}$$

$$h_4=g_1+0.4267b_1L^2\beta+1.207b_2\beta-2.414\ Lb_2\beta-1.707b_3\beta+0.8535\beta$$

$$h_5=g_2+0.0625b_1L^3\beta+0.426b_2\ L^2\beta+b_3(1.207\beta-2.414L\beta))-0.1707b_4\beta+\frac{b_5\beta}{L}$$

$$h_6=g_3+0.0625b_2L^3\beta+0.4267b_3L^2\beta+b_4(1.207\beta-2.414L\beta)-0.1707b_5\beta$$

$$h_7=g_4+0.0625b_3L^3\beta+0.4267b_4L^2\beta+b_5(1.207\beta-2.414L\beta)$$

$$h_8=0.0625b_4L^3\beta+0.4267b_5L^2\beta\ \ ;\ \ h_9=0.0625b_5L^3\beta$$

$$g_1=3.414\,[1-0.5\ \beta];\ \ g_2=2.663L\beta+0.5L;\ \ g_3=-2.0603L\beta+0.4267L^2\beta$$

$$g_4=0.7284L^3\beta-0.30175L^2\beta\ ;\ \ g_5=-0.015625L^5\beta$$

Using *roots* of MATLAB, Eq (22) is solved for closed loop time constant τ_{CL}. Selecting the large positive value from the available values for τ_{CL}, PID parameters are obtained from Eq (20), Eq (21) and Eq (16).

The first order filter time constant τ_f is obtained from $\beta\tau_D$ (β value varies between 0.35 and 0.5). By conducting simulations on various transfer function models, it has been observed that if the filter time constant is taken as 10% of the calculated value of τ_f, then the performance of the controller is good.

2.3. Stability analysis method

The process transfer function for pure integrator with time delay is given by equation Eq (1). The phase angle criterion for pure integrator with time delay is given by

$$\phi(\omega) = -\frac{\pi}{2} - L\omega \tag{23}$$

At the crossover frequency $\omega = \omega_c$, $\phi(\omega_c) = -\pi$. Hence, from Eq (23) the equation for ω_c is obtained as

$$\omega_c = \frac{\pi}{2L} \tag{24}$$

Then the values of $\tau_I \; and \; \tau_D$ are obtained from [26]. Typically the formulae chosen for $\tau_I \; and \; \tau_D$ in the literature [26] are $\frac{5}{\omega_c} \; and \; \frac{0.8}{\omega_c}$. For the pure integrator with time delay, by conducting simulations on various transfer function models it is observed that the choice of $\tau_I = \frac{4}{\omega_c}$ gives good performance in terms of faster response (less settling time) for both servo and regulatory problems. So the tuning rules are given by

$$\tau_I = \frac{4}{\omega_c} \tag{25}$$

$$\tau_D = \frac{0.8}{\omega_c} \tag{26}$$

The conventional PID controller along with a lag filter considered and the transfer function of controller is given by Eq (9). In the Eq (9), τ_f is equal to $0.1\tau_D$

The loop transfer function in s domain is

$$G_p(s)G_c(s) = k_p k_c \frac{e^{-Ls}}{s}\left[1 + \frac{1}{\tau_I s} + \tau_D s\right]\frac{1}{(1+\tau_f s)} \tag{27}$$

Eq (27) in frequency domain is written as

$$G_p(\omega)G_c(\omega) = k_p k_c \frac{e^{-Lj\omega}}{j\omega}\left[1 + j\left(\frac{\tau_I \tau_D \omega^2 - 1}{\tau_I \omega}\right)\right]\frac{1}{(1+j\tau_f \omega)} \tag{28}$$

The phase angle criteria for the loop transfer function is given by

$$\phi(\omega) = -\frac{\pi}{2} - L\omega + \tan^{-1}\left(\frac{\tau_I \tau_D \omega^2 - 1}{\tau_I \omega}\right) - \tan^{-1}(\tau_f \omega) \tag{29}$$

At the crossover frequency $\omega = \omega_{cc}$, $\phi(\omega_{cc}) = -\pi$. Hence, Eq (29) is written as

$$-\pi = -\frac{\pi}{2} - L\omega_{cc} + \tan^{-1}\left(\frac{\tau_I \tau_D \omega_{cc}^2 - 1}{\tau_I \omega_{cc}}\right) - \tan^{-1}(\tau_f \omega_{cc}) \tag{30}$$

The nonlinear equation [Eq (30)] is solved for ω_{cc} and the ultimate controller gain is obtained by using gain margin criteria. The gain margin criterion is given by

$$\left|G_p(\omega_{cc})\,G_c(\omega_{cc})\right| = 1 \tag{31}$$

From the above equation, ultimate controller gain is given by

$$k_{c,\max} = \frac{\omega_{cc}\,\sqrt{\tau_f^2\,\omega_{cc}^2+1}}{k_p\,\sqrt{\left[\dfrac{\tau_I\,\tau_D\,\omega_{cc}^2-1}{\tau_I\,\omega_{cc}}\right]^2+1}} \tag{32}$$

A gain margin of 1.5 to 4.0 is used to obtain the design value of controller gain k_c.

$$k_c = \frac{k_{c,\max}}{GM} \tag{33}$$

3. Robustness of the Controller

A control system is said to be robust, if the closed loop system is stable even when the model parameters of the actual process are different than that used for the controller design. To compare the robustness of the different controller design methods, the stability region in each of the model parameters for which the controller is stable is to be calculated. The robustness of the closed loop system for the perturbation separately in time delay and process gain is analyzed theoretically by Kharitonov's theorem [27, 28].

The process transfer function is given by Eq (1) and the controller transfer function is given by Eq (9). The characteristic equation of the system with PID controller using second order Pade's approximation for time delay $\left(e^{-Ls} = \dfrac{1-0.5\,L\,s+0.0833\,L^2\,s^2}{1+0.5\,L\,s+0.0833\,L^2\,s^2}\right)$, the following polynomial in s is obtained.

$$M(s) = c_1\,s^5 + c_2\,s^4 + c_3\,s^3 + c_4\,s^2 + c_5\,s + c_6 = 0 \tag{34}$$

Where

$c_1 = \tau_I\,\tau_f\,x_2$; $c_2 = \tau_I\,\tau_f\,x_1 + \tau_I\,x_2 + k\,\tau_I\,\tau_D\,x_2$

$c_3 = \tau_I\,\tau_f + \tau_I\,x_1 - k\,\tau_I\,\tau_D\,x_1 + k\,\tau_I\,x_2$

$c_4 = \tau_I + k\,\tau_I\,\tau_D - k\,\tau_I\,x_1 + k\,x_2$

$c_5 = k\,\tau_I - k\,x_1$; $c_6 = k$; $x_1 = 0.5\,L$

$x_2 = 0.0833\,L^2$; $k = k_c k_p$

Kharitonov's polynomials for $c_i^- < c_i < c_i^+$ (i = 1, 2, 3, 4, 5, 6) are given below where c_i^- and c_i^+ is the lower bound and upper bound for c_i respectively

$$c_1^-\,s^5 + c_2^-\,s^4 + c_3^+\,s^3 + c_4^+\,s^2 + c_5^-\,s + c_6^- = 0 \tag{35}$$

$$c_1^+\,s^5 + c_2^+\,s^4 + c_3^-\,s^3 + c_4^-\,s^2 + c_5^+\,s + c_6^+ = 0 \tag{36}$$

$$c_1^+\,s^5 + c_2^-\,s^4 + c_3^-\,s^3 + c_4^+\,s^2 + c_5^+\,s + c_6^- = 0 \tag{37}$$

$$c_1^-\,s^5 + c_2^+\,s^4 + c_3^+\,s^3 + c_4^-\,s^2 + c_5^-\,s + c_6^+ = 0 \tag{38}$$

For fixed value of k_p, a perturbation in time delay L i.e., when $(L-\Delta L) \le L \le (L+\Delta L)$ is substituted in the above coefficients and Kharitonov's polynomials are checked for stability using the Routh-Hurwitz method [28]. Similar procedure is repeated to find stability region for k_p.

4. Simulation Results

In this section, simulation results for various transfer function models are given for both servo and regulatory problems. The performance comparison of the proposed controllers (PID controller designed by IMC method [IMC-PID controller], PID controller designed by direct synthesis method [DS-PID controller] and PID controller designed by stability analysis method [SA-PID controller]) is made with the PID controllers designed by direct synthesis method [DS-s] [24], equating coefficient method (EC) [23] and IMC method with disturbance rejection (IMC-dr) [19] in terms of integral performance criteria like Integral Square Error (ISE), Integral Absolute Error (IAE) and Integral Time weighted Absolute Error (ITAE). The PID tuning rules for the proposed methods are given in Table 1. The gain margin and phase margin is evaluated for all the case studies and reported in Table 2. The gain margin and phase margin indicates the robustness of the controller. The larger values of gain margin and phase margin indicate that the controller is robust.

Table 1. PID Tuning Rules PID controller with lag filter : $G_c = k_c(1+\frac{1}{\tau_I s}+\tau_D s)\frac{1}{(\tau_f s+1)}$

Transfer function model	Method	k_c	τ_I	τ_D	τ_f	Tuning Parameter
$G_P = \frac{k_p e^{-Ls}}{s}$	IMC method	$\frac{\alpha+0.5L}{k_p(3\lambda^2+0.5L\alpha)}$ $\alpha = 3\lambda+0.5L$	$\alpha+0.5L$	$\frac{0.5L\alpha}{\alpha+0.5L}$	$\frac{\lambda^3}{3\lambda^2+0.5L}$	λ 0.4L to 1.7L
	Direct synthesis method	$\frac{\tau\tau_I\tau_f}{k_p\tau_{CL}^4}$	3.414 τ_{CL}+0.5 L	Eq (21)	$10\%(\beta\tau_D)$	β 0.35 to 0.5
	Stability analysis method	$k_{c,des} = \frac{k_{c,\max}}{GM}$ and $k_{c,\max} = \frac{\omega_{cc}\sqrt{\tau_f^2\omega_{cc}^2+1}}{k_p\sqrt{\left[\frac{\tau_I\tau_D\omega_{cc}^2-1}{\tau_I\omega_{cc}}\right]^2+1}}$	$4/\omega_c$	$0.8/\omega_c$	$0.1\tau_D$	GM=1.5 to 3.0

Table 2. Gain margin and phase margin for the pure integrator with time delay

Transfer Function	Phase margin(PM) and gain Margin (GM)	IMC-PID	DS-PID	SA-PID	DS-s	EC	IMC-dr
$\frac{e^{-5s}}{s}$	GM	1.49	1.37	1.49	1.71	1.86	--
	PM	28.14	27.26	34.87	39.37	38.20	--
$0.0506\frac{e^{-6s}}{s}$	GM	1.61	1.54	1.684	1.71	1.85	1.38
	PM	30.79	32.92	36.35	39.37	38.46	25.56
$0.2082\frac{e^{-50s}}{s}$	GM	1.62	1.56	1.73	1.88	1.86	--
	PM	31.41	33.63	37.17	43.72	38.98	--

4.1. Case Study-1: $G_p = \frac{e^{-5s}}{s}$

The transfer function model, $\frac{e^{-5s}}{s}$ [24] is considered. The PID parameters for direct synthesis method [24] and equating coefficient method [23] are given in Table 3. The response of the system for unit step change in the set point at t=0 s and for unit step change in load at t=70 s for all the proposed controllers is shown in Fig 1. The servo and regulatory performance of the system with the controller designed by direct synthesis method [24] and equating coefficient method [23] is also shown in Figure 1.

Table 3. PID Parameters for Case Study – 1

Method	k_c	τ_I	τ_D	τ_f
IMC-PID $(\lambda = 0.72L)$	0.219	15.8	2.1044	0.6468
SA-PID (Gain Margin=1.5)	0.2156	12.7324	2.5465	0.25465
DS-PID $(\beta = 0.35)$	0.2531	13.0687	2.0324	0.07113
DS-s	0.2	17.5	2.143	0.0714
EC	0.1791	12.5	2.45	--

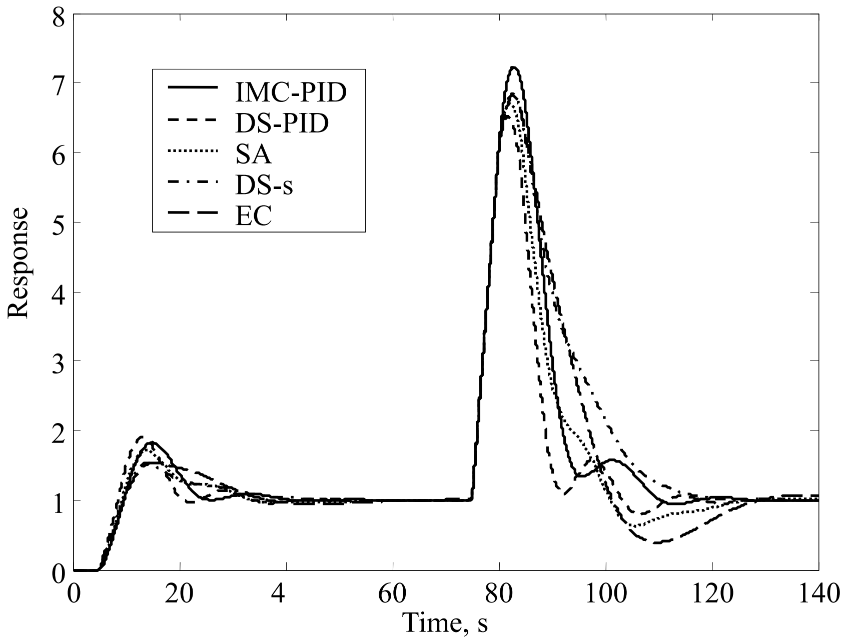

Figure 1. The response of the pure integrating process with time delay (Case study 1)

The performance of the controllers is given in terms of ISE, IAE and ITAE. The servo and regulatory performance of the proposed DS-PID controller is superior when compared with the proposed IMC-PID, SA-PID controllers and reported DS-s and EC controllers. The servo performance of the IMC-PID controller is better when compared with SA-PID, DS-s and EC and gives better performance for the regulatory problem than the reported DS-s and EC controllers. The proposed SA-PID controller gives superior regulatory performance when compared to IMC-PID, DS-s and EC controllers (refer to Table 4)

The stability regions of the model parameters for the PID controller designed are calculated by Kharitonov's theorem considering uncertainty in one parameter at a time and are tabulated in Table 5. The proposed controllers give a lower region of stability for the model parameters when compared with the controllers designed by direct synthesis method [24] and equating coefficient method [23], which can also be observed from the gain margin and phase margin calculations given in Table 2.

4.2. Case Study-2: $G_p = 0.0506\dfrac{e^{-6s}}{s}$

The transfer function model $0.0506\dfrac{e^{-6s}}{s}$ [19, 14] is considered The PID controller parameters for various methods are reported in Table 6.

The PID parameters for direct synthesis method [24], equating coefficient method [23 and IMC method with disturbance rejection (IMC-dr) [19] are given in Table 6. The response of the system for unit step change in the set point at t=0 s and for unit step change in load at t=150 s for all the proposed controllers is shown in Fig 2. For IMC method with disturbance rejection [19], the PID is implemented without a set point filter. For fair comparison with other methods, set point filter is not used for IMC-dr method. The servo and regulatory performance of the system with the controller designed by direct synthesis method [24], equating coefficient method [23] and IMC method with disturbance rejection [19] is also shown in Figure 2.

The performance of the controllers is given in terms of ISE, IAE and ITAE. The regulatory performance of reported IMC-dr [19] controller is superior to all the proposed controllers, DS-s controller and EC controller. The proposed DS-PID controller gives superior performance when compared to the proposed IMC-PID controller, SA-PID controller, reported DS-s, IMC-dr and EC controllers. The servo performance of reported IMC-dr controller is better than proposed controllers (IMC-PID, SA-PID), DS-s controller and EC controller (refer to Table 4). The proposed IMC-PID controller gives almost similar performance for both servo and regulatory problems as compared to DS-s controller and shows better performance than EC controller. The servo and regulatory performance of the proposed SA-PID controller gives better performance than EC controller. The servo performance of reported DS-s controller is better when compared with the proposed SA-PID controller and EC controller.

The stability regions of the model parameters for the PID controller designed are calculated by Kharitonov's theorem considering uncertainty in one parameter at a time and are tabulated in Table 5. The proposed IMC-PID controller gives a lower region of stability for the uncertainty in model parameters k_p *and* L when compared with the controller designed by direct synthesis method [24]. The proposed controllers (IMC-PID, DS-PID) gives almost similar region of stability for the uncertainty in the model parameter k_p and gives a lower region of stability for the uncertainty in the model parameter L (time delay) compared to the controller designed by equating coefficient method [23].

Table 4. Performance comparison

Transfer function	Method	Servo Problem			Regulatory Problem		
		ISE	IAE	ITAE	ISE	IAE	ITAE
$\frac{e^{-5s}}{s}$	IMC-PID	11.36	15.53	172.94	308.087	72.81	1.13×10^{3}
	DS-PID	10.74	14.42	151.30	202.84	54.31	779.72
	SA-PID	10.28	15.67	191.23	248.36	68.56	1.12×10^{3}
	DS-s	9.01	14.77	182.26	329.37	87.78	1.54×10^{3}
	EC	10.1	16.720	241.12	329.87	88.70	1.67×10^{3}
$0.0506\frac{e^{-6s}}{s}$	IMC-PID	13.29	19.13	268.27	1.62	6.14	119.98
	DS-PID	11.72	17.28	221.46	1.11	4.85	88.278
	SA-PID	12.54	20.08	334.22	1.37	6.02	131.20
	DS-s	10.805	17.73	263.25	1.46	6.40	135.38
	EC	12.062	20.20	361.98	1.45	6.51	151.57
	IMC-dr	13.25	17.54	223.26	0.85	3.76	63.93
$0.2082\frac{e^{-50s}}{s}$	IMC-PID	0.098	4.62	532.73	2.57×10^{-5}	0.069	11.82
	DS-PID	0.088	4.18	434.48	1.77×10^{-5}	0.055	8.65
	SA-PID	0.089	4.71	595.51	2.15×10^{-5}	0.064	10.39
	DS-s	0.076	4.35	579.82	2.82×10^{-5}	0.089	17.40
	EC	0.085	4.65	616.09	2.14×10^{-5}	0.067	11.31

Table 5. Comparison of Stability regions

Transfer Function	Method	Stability region	
		k_p	L
$\frac{e^{-5s}}{s}$	IMC-PID	± 52.4%	± 21.94%
	DS-PID	± 42.0%	± 17.7%
	SA-PID	± 49.05%	± 23.25%
	DS-s	± 68.2%	± 29.15%
	EC	± 61.1%	± 34.81%
$0.0506\frac{e^{-6s}}{s}$	IMC-PID	± 59.1%	± 26.1%
	DS-PID	± 60.9%	± 24.6%
	SA-PID	± 58.7%	± 30%
	DS-s	± 68.2%	± 29.16%
	EC	± 61.1%	± 34.63%
	IMC-dr	± 41.2%	± 17.15%
$0.2082\frac{e^{-50s}}{s}$	IMC-PID	± 60.0%	± 26.5%
	DS-PID	± 59.2%	± 24%
	SA-PID	± 59.5%	± 31.4%
	DS-s	± 72.0%	± 34.8%
	EC	± 60.5%	± 34.7%

IMC-PID: Internal Model Control; DS-s: Direct Synthesis method [24]
DS-PID: Direct Synthesis method; EC: Equating Coefficient method [23]
SA-PID: Stability analysis method; IMC-dr: IMC method with disturbance rejection

Table 6. PID Parameters for Case Study-2

Method	k_c	τ_I	τ_D	τ_f
IMC-PID $(\lambda = 0.8L)$	3.3231	20.40	2.5588	0.9116
SA-PID (Gain margin=1.7)	3.1324	15.2789	3.0558	0.3056
DS-PID $(\beta = 0.45)$	3.6805	17.4309	2.5266	0.11370
DS-s $(\lambda = L)$	3.2938	21	2.5714	0.08571
EC	2.9499	15	3	--
IMC-dr	4.282	14.91	2.34	--

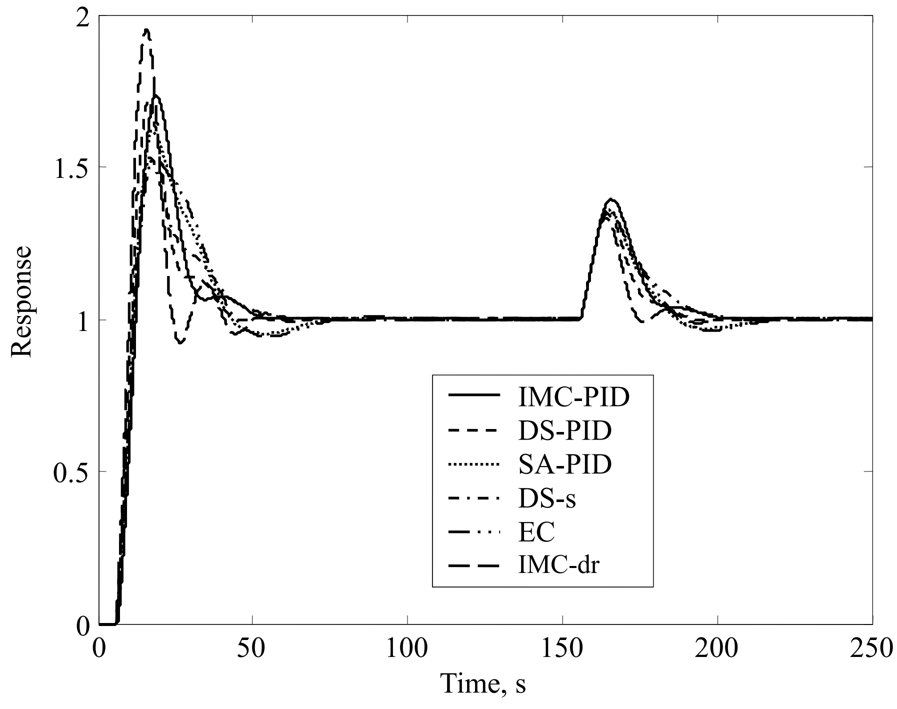

Figure 2. The response of the pure integrating process with time delay (Case study 2)

The proposed SA-PID controller gives lower region of stability for uncertainty in the model parameter k_p and almost similar region of stability for the uncertainty in the model parameter L compared to the controller designed by direct synthesis method [24]. All the proposed controllers give superior region of stability for the uncertainty in model parameters k_p *and* L when compared with the controller designed by IMC-dr [19]. Gain margin and phase margins in Table 2 also indicate the same stability criteria as explained above.

4.3. Case Study-3: Isothermal continuous copolymerization of styrene- acrylonitrile In CSTR [29]

Copolymerization is characterized by the presence of two or more distinct monomers. The model equations for CSTR carrying out this polymerization reaction are given by

$$V\left(\frac{dA}{dt}\right)=Q_1\,A_f-Q_3\,A-\frac{\left[V\,R_i^{0.5}\left(r_1\,A^2+A\,B\right)\right]}{f\left(A,B\right)} \quad (39)$$

$$V\left(\frac{dB}{dt}\right)=Q_2\,B_f-Q_3\,B-\frac{\left[V\,R_i^{0.5}\left(r_1\,B^2+A\,B\right)\right]}{f\left(A,B\right)} \quad (40)$$

With the initial conditions (at t = 0) A = A_0, B = B_0, where

$$f\left(A,B\right)=\left(r_1^2\,\xi_1^2\,A^2+2\,\Phi\,\xi_1\,\xi_2\,A\,B+r_2^2\,\xi_2^2\,B^2\right)^{0.5}$$

$$\xi_1=\left(\frac{2\,k_{t11}}{k_{p11}^2}\right)^{0.5} r_1\,;\quad \xi_2=\left(\frac{2\,k_{t22}}{k_{p22}^2}\right)^{0.5} r_2$$

$$r_1=\frac{k_{p11}}{k_{p12}}\,;\quad r_2=\frac{k_{p22}}{k_{p21}}\,;\quad \Phi=\frac{k_{t12}}{2\left(k_{t11}\,k_{t22}\right)^{0.5}}$$

R_i is rate of free radical initiation
A_f and B_f are the feed compositions of A and B
Q_1 and Q_2 are the flow rates of A and B entering the reactor
Q_3 is the flow out of the reactor
V is the volume of the reactor
The objective is to control the copolymer composition by controlling the mole fraction, F of monomer (styrene) in the copolymer

$$F=\frac{\left(r_1\,f_1^2+f_1\,f_2\right)}{\left(r_1\,f_1^2+2\,f_1\,f_2+r_2\,f_2^2\right)} \quad (41)$$

Where $f_1=1-f_2=\dfrac{A}{\left(A+B\right)}$ (42)

The nominal parameter values chosen for the simulation study [30] of copolymerization of acrylonitrile and styrene in CSTR are given in Table 7.

Padma Sree and Chidambaram [23] have identified the transfer function model of copolymerization of styrene-acrylonitrile in CSTR along with measurement delay of 50 seconds using relay identification method as an integrating system with dead-time (k_p=0.2082 and L = 50). The PID controller parameters for various methods are reported in Table 8.

Table 7. Parameter values used in the present simulation study

k_{t11}	4214.14 l mol^{-1}s^{-1}
k_{t12}	185.06 l mol^{-1}s^{-1}
k_{p11}	1.2458 × 10^{8} l mol^{-1}s^{-1}
k_{p12}	9.0433 ×10^{7} l mol^{-1}s^{-1}
r_1	0.455
r_2	0.033
R_i	0.9927 × 10^{-6}
Φ	15
A_f	2 mol/l
B_f	2 mol/l
A_0	0.827 mol/l
B_0	0.563 mol/l
Q_1/V	1.9 × 10^{-4} s^{-1}
Q_2/V	1.25 × 10^{-4} s^{-1}

Table 8. PID Parameters for Case Study-3

Method	k_c	τ_I	τ_D	τ_f
IMC-PID ($\lambda = 0.8L$)	0.0959	171.87	21.5581	7.68
SA-PID (Gain Margin=2.9)	0.0886	128.7245	25.7449	2.5745
DS-PID ($\beta = 0.45$)	0.1062	146.8557	21.2862	0.95788
DS-s ($\lambda = 1.2L$)	0.0854	197.1450	22.0346	0.83862
EC	0.0851	126.4	25.23	--

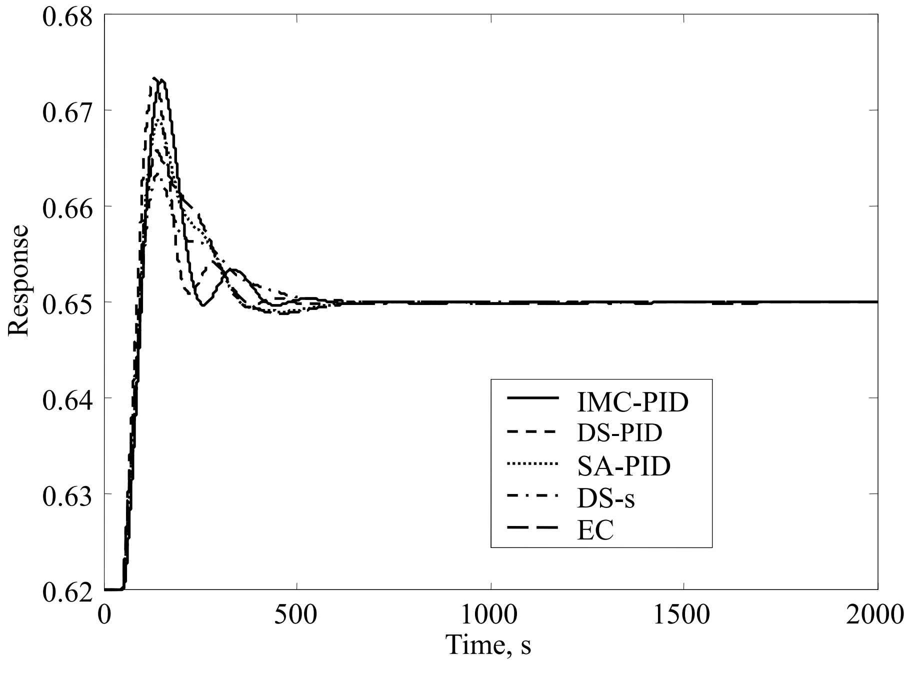

Figure 3. Servo response of copolymerization reactor (Nonlinear equations) (Case study 3)

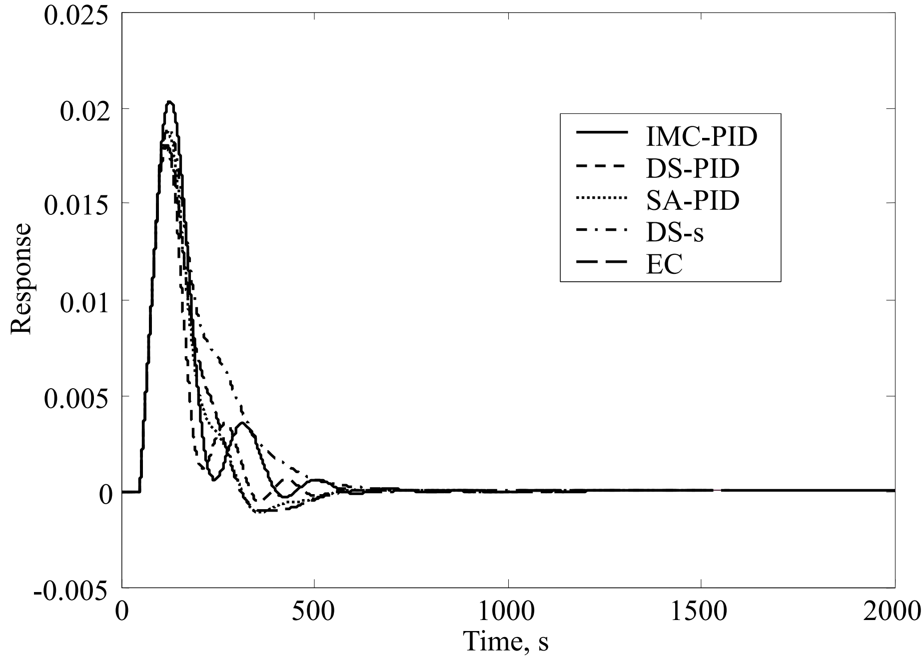

Figure 4. Fractional regulatory response of copolymerization reactor (Case study 3)

The closed loop response (in F) of the non linear system for a set point change from 0.62 to 0.65 is shown in Figure 3. The performance for the regulatory problem (in F) by giving a step disturbance in Q_2 from 1.25×10^{-4} to 10^{-4} is also evaluated and the fractional response is shown in Figure 4. Without any controller the response of the system goes from 0.62 to 0.648. The servo and regulatory performance of the system with the controller designed by direct synthesis method [24] and equating coefficient method [23] is also shown in Figure 3 and Figure 4.

The performance of the controllers is given in terms of ISE, IAE and ITAE (refer to Table 4). The proposed DS-PID controller gives superior performance when compared to the other proposed controllers and DS-s, EC controllers for both servo and regulatory problems. The proposed IMC-PID controller gives better performance when compared to DS-s and EC controllers for servo problem and similar performance when compared to EC controller. The proposed SA-PID controller gives almost similar servo performance when compared to DS-s controller and shows better performance than EC controller for both servo and regulatory problems.

The stability regions of the model parameters for the PID controller designed are calculated by Kharitonov's theorem considering uncertainty in one parameter at a time and are tabulated in Table 5. DS-s method gives better region of stability when compared to all the controllers. The proposed controllers gives almost gives almost similar region of stability for the uncertainty in the model parameter k_p and gives a lower region of stability for the uncertainty in the model parameter L compared to the controller designed by equating coefficient method [23]. The proposed controllers have lower region of stability for the model parameters when compared with the controller designed by direct synthesis method [24]. Gain margin and phase margins in Table 2 also indicate the same stability criteria as explained above.

4.4. Case Syudy-4: Level control in a cylindrical tank

The process has a liquid stream feeding the top of the cylindrical tank and a single exit stream pumped out at the bottom. The measured process variable (PV) is liquid level in the tank. To maintain level, the controller output (CO) signal adjusts a throttling valve at the discharge of a constant pressure pump to manipulate flow rate out of the bottom of the tank. This approximates the behavior of a centrifugal pump operating at relatively low throughput. The schematic diagram of the process is shown in Figure 5a. The experimental set up is shown in Figure 5b.

By pumping out the water from the cylindrical tank at a constant rate, the process is developed as an integrating process. The water to the tank from the storage tank is pumped through pump1. Pump2 is connected in the outlet of the tank to maintain a constant outflow. Inorder to provide a constant out flow irrespective of the valve position, a bypass with a non- returnable valve is connected.

The graphical model-fitting technique is used for the identification of level control in cylindrical tank. The integrating model is obtained [30] as in time domain form as

$$\frac{Y(s)}{U(s)} = \frac{k_p e^{-Ls}}{s} = \frac{-0.05688}{s} e^{-0.0333s} \qquad (43)$$

The PID tuning parameters by the proposed IMC method, direct synthesis method, stability analysis for the control of level in the tank based on the identified model are given in the Table 9.

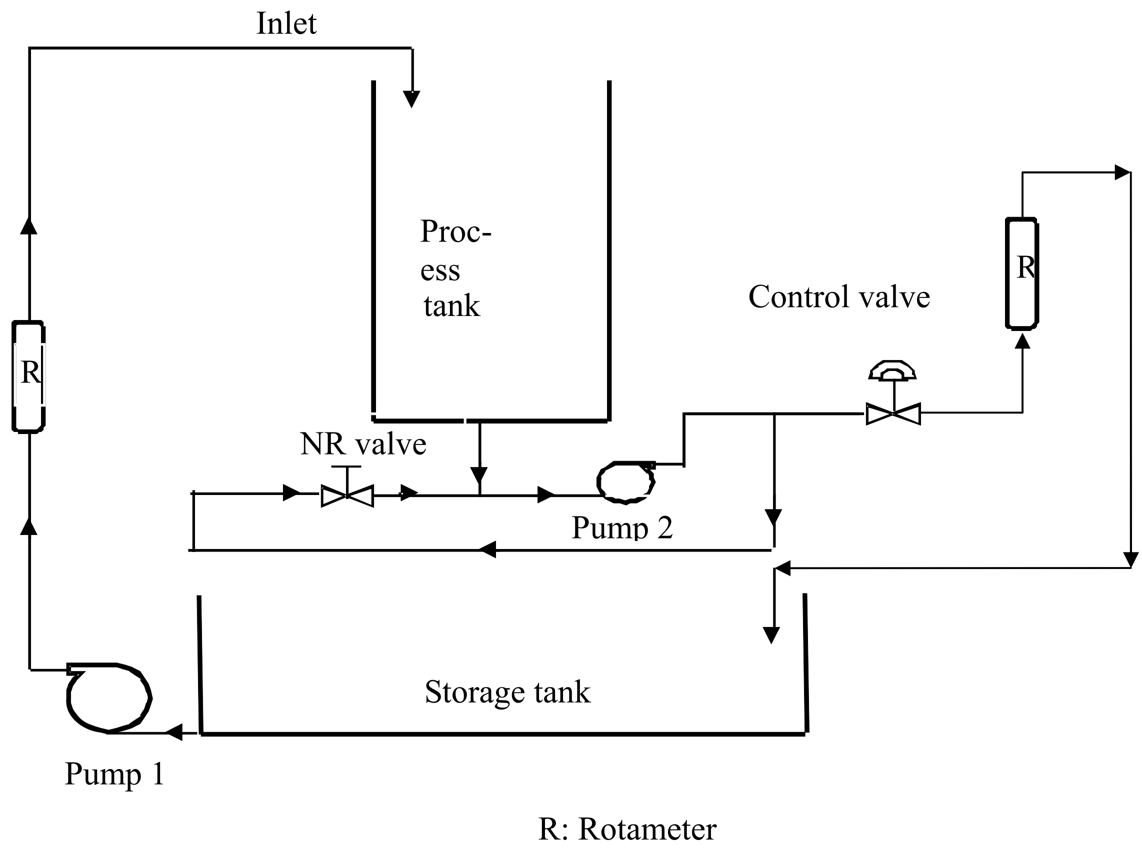

Figure 5a. Schematic diagram of the Integrating process

Figure 5b. Photographic view of the integrating tank experimental setup.

Table 9. PID Settings by various methods for the model [$-0.05688\ e^{-0.0333s}/s$]

Method	k_c	τ_I	τ_D	τ_f
IMC-PID $(\lambda = 2L)$	-242.3356	0.2331	0.0155	0.0175
SA-PID (Gain margin=1.5)	-568.6338	0.0848	0.0170	0.0017
DS-PID $(\beta = 0.5)$	-559.5157	0.1013	0.0143	0.00071

First the experimental setup is operated for 50 runs with a constant 75% outflow such that a steady state level of 16 cm is maintained. In the computer, PID parameters are tuned based on the IMC method and then a step change of 2 cm is given and the servo response is recorded. It took around 15 min for level to reach the new steady state value (18 cm). Then a pulse input of 0.4 l/s for duration of 5 s is given to observe the capability of the controller to reject load disturbance. The level comes back to 18 cm after 25 min. The dynamic behavior of the process variable (liquid level in the tank) is recorded against the number of runs and represented against the time in the Fig 6 and the control action is also represented against the time in the Figure 7.

The procedure is repeated for other controllers to compare the performance of the controllers. For DS-PID controller, load disturbance is given at 25min as it took a long to settle to the given setpoint change (18 cm). For DS-PID and SA-PID controllers, the performance for both servo and regulatory problems is also given in Fig 6 and the corresponding control action response for these controllers is given in Fig 8 and Fig 9 respectively. The proposed IMC-PID controller performs better than SA-PID and DS-PID controllers. The proposed SA-PID controller performs better than DS-PID controller for both set point tracking and disturbance rejection. The performance of the proposed controllers are evaluated in terms of ITAE and reported in Table 10.

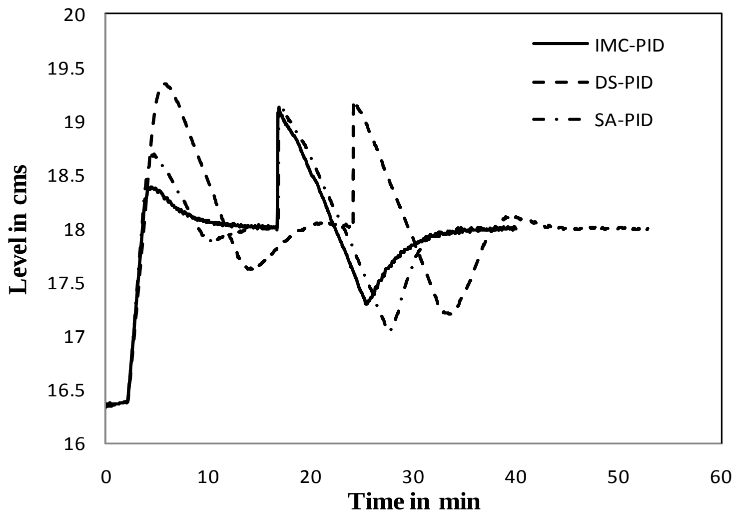

Figure 6. The closed loop response of level in cylindrical tank with the proposed controllers

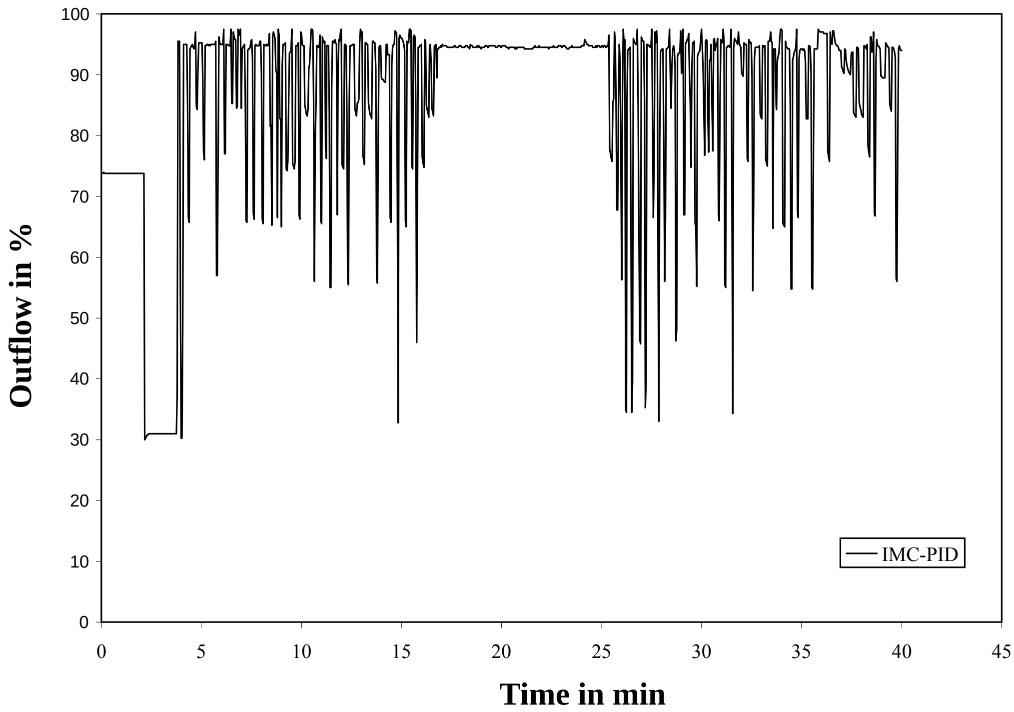

Figure 7. The manipulated variable (% outflow) behaviour (IMC-PID controller)

Table 10. Performance comparison in terms of ITAE

Method	Servo Problem	Regulatory Problem
IMC-PID	17.0744	12360.74
DS-PID	80.3816	20709.92
SA-PID	23.60223	11452.85

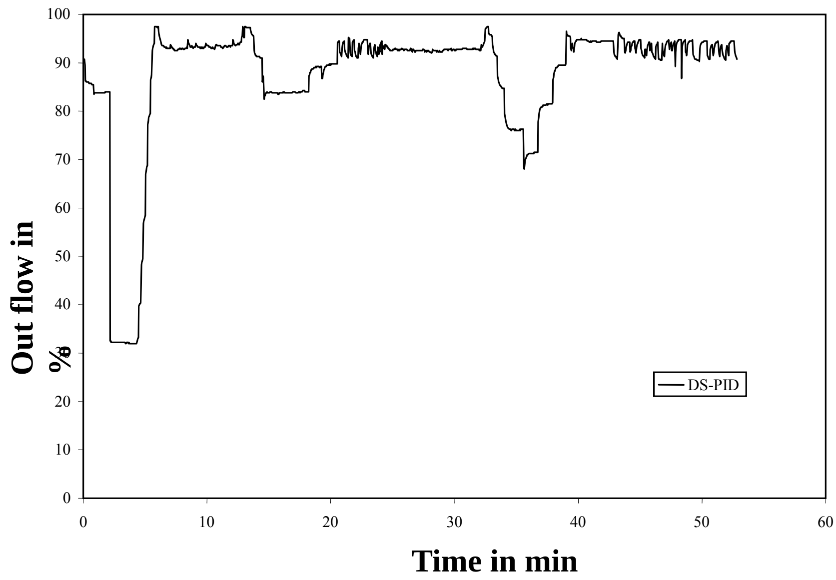

Figure 8. The manipulated variable (% outflow) behaviour (DS-PID controller)

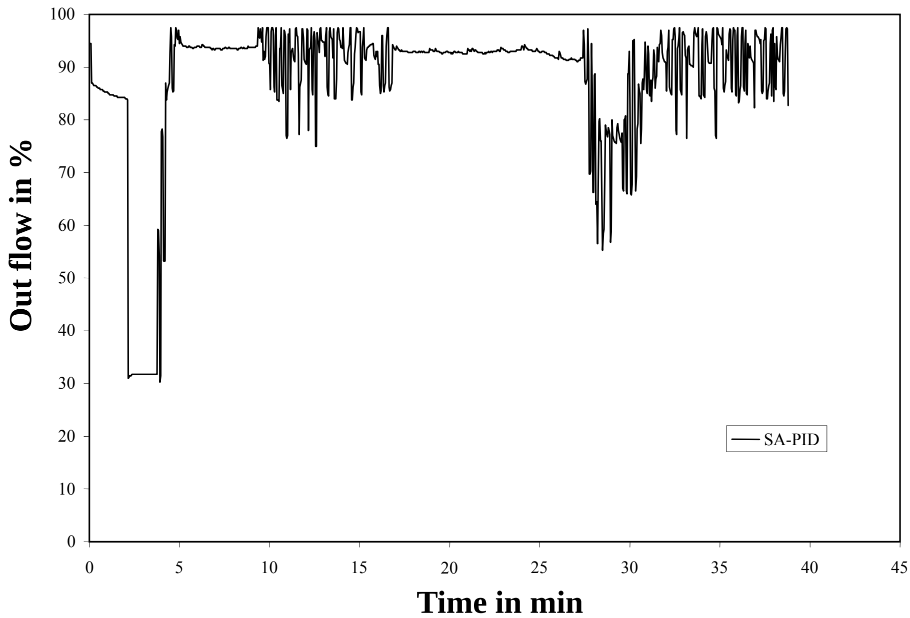

Figure 9. The manipulated variable (% outflow) behaviour (SA-PID)

5. Conclusions

Three methods of designing PID controllers for pure integrating system with time delay are proposed based on IMC method, stability analysis method and direct synthesis method. The performance of the proposed controllers is better than the recently reported methods. Stability region for various model parameters considering uncertainty in one parameter at a time is obtained using Kharitonov's theorem and compared with that of the literature reported methods. The stability region for all the model parameters is comparable with that of the literature reported methods. The advantage of these methods is that the controller is PID and simple conventional feed back control structure is used. Simulation results on various transfer function models, the nonlinear models of isothermal continuous copolymerization of styrene-acrylonitrile in CSTR are given to show the effectiveness of the proposed controllers. For level control in the cylindrical tank experimental verification is carried out to show the efficiency of the proposed controllers for both servo and regulatory problems.

References

[1] Chien, L. and Fruehauf, P. S. 1990. Consider IMC tuning to improve performance. *Chem Eng Progr,* 10: 33-41.

[2] Fuentes, C. and Luyben, W. L. 1983. Control of high purity distillation columns. *Ind Eng Chem Process Des Dev*, 22: 362.

[3] Srividya, R. and Chidambaram, M. 1997. On line controller tuning for integrator plus delay processes. *Process Contr Qual*, 9: 59-66.

[4] Astrom, K. J., and Haggland, T. 1995. "*PID controllers: Theory, Design and Tuning",* Instr Soc. America, North Carolina.

[5] Yu, C. C. 1999. "*Auto Tuning of PID controllers",* Springer-Verlag, Berlin

[6] Ziegler, J. G. and Nichols, N. B. 1942. Optimum settings for automatic controllers. *Trans ASME*, 64: 759-765.

[7] Tyreus, B.D. and Luyben, W.L. 1992. Tuning PI controllers for integrator/ dead-time processes. *Ind Eng Chem Res*, 31: 2625.

[8] Chidambaram, M. 1994. Design of PI controller for integrator/dead-time processes. *Hungar J Ind Chem*, 22: 37.

[9] Poulin, E. and Pomerleau, A. 1999. PI settings for integrating processes based on ultimate cycle information. *IEEE Trans Contr Syst Tech*, 7: 509.

[10] Luyben, W. L. 1996. Design of Proportional Integral and Derivative controllers for integrating dead-time processes. *Ind Eng Chem Res*, 35: 3480.

[11] Wang, L. and Cluette, W.R. 1997. Tuning PID controllers for integrating processes. *IEE Proc Contr Theor Appl,* 144: 385.

[12] Kookos, I. K., Lygros, A. I., and Arvanitis,K. G. 1999. On-line PI controller tuning for integrator / dead time processes. *Eur J Contr*, 5: 19.

[13] Wang, Y.G. and Cai, W.J. 2002. Advanced proportional integral derivative tuning for integrating and unstable processes with gain and phase margin specifications. *Ind Eng Chem Res*, 41: 2910-2914.

[14] Visioli, A. 2001. Optimal tuning of PID controllers for integral and unstable processes. *IEE Proc Contr Theor Appl,* 148: 180.

[15] Zhang, W., Xu, X., and Sun, Y. 1999. Quantitative performance design for integrating processes with time delay. *Automatica,* 35: 719-723.

[16] Ali, A. and Majhi, S. 2010. PID controller tuning for integrating processes. *ISA Trans,* 49: 70-78.

[17] Rivera, D.E., Morari, M., and Skogestad, S. 1986. Internal Model Control – 4, PID controller design. *Ind Eng Chem Process Des Dev,* 25: 1684.

[18] Skogestad, S. 2003. Simple analytical

rules for model reduction and PID controller tuning. *J Process Contr*, 13: 291-309.
[19] Shamsuzzoha, Md. and Lee, M. 2008. PID controller design for integrating processes with time delay. *Kor J Chem Eng*, 25, 4: 637-645.
[20] Sung, S.W. and Lee, I.B. 1996. Limitations and counter measures of PID controllers. *Ind Eng Chem Res*, 35: 2596-2610.
[21] Matausek, M. R., and Micic, A. D. 1999. On the modified Smith predictor for controlling a process with an integrator and long time. *IEEE Trans. Automat. Contr*, 44, 8: 1603-1606.
[22] Majhi, S. and Atherton, D. P. 1999. Modified Smith predictor and controller for process with time delay. *IEE Proc. Part D*, 146, 5: 359-366.
[23] Padma Sree, R. and Chidambaram, M. 2005. A Simple method of tuning PID controllers for integrator/dead-time processes. *Journal* of *Chemical Engineering* of *Japan*, 38: 113-119.
[24] Seshagiri Rao, A., Rao, V. S. R. and Chidambaram, M. 2009. Direct synthesis based controller design for integrating processes with time delay. *J. Franklin Inst*, 346: 38-56.
[25] Ogunnaike, B. A., and Ray, W. H. 1994. "*Processes dynamics modeling and control*", Oxford university press, New York. 666-667.
[26] Douglas, J. M. 1972. "*Process dynamics and control*", Prentice Hall, Englewood Cliffs. NJ. 85-148.
[27] Kharitonov, V. L. 1978. Asymptotic stability of an equilibrium position of a family of systems of linear differential equations. *Differentsial'nye Uraveniya*, 14: 1483-1485.
[28] Sinha, N. K. 2002. "*Control Systems*", New year international publishers. 142-144.
[29] Kravaris, C., Wright, R. A. and Carrier, J. F. 1989. Nonlinear controllers for Trajectory tracking in batch processes. *Computers* and *Chemical Engineering*, 13: 73-82.
[30] Kravaris, C., Wright, R. A., and Carrier, J. F. 1989. Nonlinear controllers for Trajectory tracking in batch processes. *Comput Chem Eng*, 13: 73-82.
[31] Friman, M. and Waller, K. V. 1994. Auto tuning of multi loop control system, *Ind Eng Chem Res*, 33: 1708.
[32] Rice, R. and Cooper, D. J. 2008. Improve control of liquid level loops. *Chem Eng Progr*, www.aiche.org/cep: 54-61.