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Multi-Objective Optimization of Crystallization Unit in a Fertilizer Plant Using Particle Swarm Optimization

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Abstract: In a real life situation, due to the complexity of the industrial systems and non-linearity of their behavior, it is very difficult to achieve optimum performance of system for desired industrial goals using uncertain, vague and imprecise data. Herein, an approach has been proposed through which the behavior of the system is analyzed in the form of well- known six reliability indices by using triangular fuzzy numbers which allow the consideration of expert opinions, linguistic variables, operating conditions in reliability information. Using their behavior analysis a fuzzy multi-objective optimization problem (FMOOP) has been formulated. Due to the conflicting nature of the multiple objectives, the decision making is difficult and it leads to the Pareto optimal solutions instead of single optimal solutions. Many evolutionary algorithms (EAs) already exist in the literature for solving a multi-objective optimization problem (MOOP), and are termed as multi-objective evolutionary algorithms (MOEAs). Particle swarm optimization (PSO) is one of such MOEA which demonstrates the ability to identity a Pareto-optimal front efficiently. Here, a crisp optimization problem is reformulated from FMOOP by taking into account the preference of decision maker (DM) and then PSO is applied to solve the resulting fuzzified MOOP. The presented approach is applied in order to solve the multi-objective series-parallel system reliability optimization problem for a crystallization unit of a urea plant.

Keywords: Multi-objective optimization; fuzzy optimization; PSO; Pareto-optimal solution, lambda-tau.

1. Introduction

In a production plant, to obtain maximum output it is necessary that each of its subsystem/unit should run failure free and furnish excellent performance to achieve desired goals. High performance of these units can be achieved with highly reliable subunits and perfect maintenance.

With advances in technology and growing complexity of technological systems, the job of reliability/system analyst has become more challenging as they have to study, characterize, measure and analyze the uncertain behavior of system using various techniques which requires the knowledge of precise numerical probabilities and component functional dependencies, the information which is rather difficult to obtain. Even if data is available, it is often inaccurate and thus subjected to uncertainty i. e. historical record can only represent the past behavior of the system but may be unable to predict the future behavior of the equipment.

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For failure analysis of these systems, various methods available in literature are reliability block diagrams (RBD's), Monte Carlo simulation (MCS), Markov modeling (MM), Failure mode effective analysis (FMEA), root cause analysis (RCA), Fault tree analysis (FTA), and Petri nets (PN) etc. These methods are recognized as a powerful tool for estimating the reliability of large scaled systems [27, 28], where system success or failure is described by the state of the top event. The probability of a top event is a function of the failure probability of a primary event. Variation in the primary event probabilities which contains the uncertainties may result variation in the top event probability. To resolve the uncertainties, fuzzy methodology [2, 4, 29] is one of the widely used technique applied to engineering systems e. g., in human reliability, software reliability, fault diagnosis and safety and risk engineering [3, 11, 17]. Also variety of methods and algorithms exists for optimization and applied in various technological fields, during the last three decades [6, 7, 19, 20].

The majority of the industrial systems are repairable one and data used for their behavior analysis were collected from historical records/logbooks/ experts opinions and taken as crisp data because most of the industrial systems exhibit constant failure and repair rates after initial burn-in period. Since records are either not properly updated or out of date, so did not reflect the actual behavior of the system. Thus, the used data were vague, imprecise, and uncertain. Also, the traditional analytical techniques need large amounts of data, which are difficult to obtain because of various practical constraints such as rare events of components, human errors, and economic considerations for the estimation of failure/repair characteristics of the system. In such circumstances, it is usually not easy to analyze the behavior and performance of these systems up to desired degree of accuracy by utilizing available resources, data, and information.

Thus, to analyze more closely the system's behavior, other reliability criteria should be included in the traditional analysis and involved uncertainties must be quantified. The inclusion of various reliability indices as criteria helps the management to understand the effect of increasing/ decreasing the failure and repair rates of a particular component or subsystem upon the overall performance of the system and quantification of uncertainties provide results closer to the real situational environment's results. These ideas were highlighted by Knezevic and Odoom [13] and analyzed the behavior of a general repairable system by introducing the concept of Fuzzy Lambda-Tau technique with Petri net (PN) in terms of various reliability indices utilizing quantified data. In their approach, PN is used to model the system while fuzzy set theory is used to quantify the uncertain, vague, and imprecise data. The use of fuzzy set theory and fuzzy arithmetic to determine components or system reliability can be found in literature [1, 2, 23, 30]. Behavior analysis of the press unit is analyzed by [14] using FTA instead of PN while [8, 9, 24, 25] analyzed by using PN and fuzzy approach of different industrial systems.

However, in many practical design situations, reliability apportionment is complicated because of presence of several conflicting objectives and imprecise cost of the components of the system. For instance, a designer is required to minimize the system cost while simultaneously maximizing the system reliability. Therefore, multi-objective functions become an important aspect in the reliability design of the engineering systems. In reliability problems, it is often required to maximize or minimize several objectives subject to several constraints. Such problem can be formulated as a multi-objective optimization problem (MOOP). Different methods and algorithms have been proposed to solve MOOPs can be found from [10, 16, 18, 22].

In the present paper, a conflicting multi-objective non-linear programming

(MONLP) problem is considered in which we maximize the reliability and minimize the cost of the given system. For this, firstly the problem is analyzed through its behavior analysis by well known six reliability indices and then viewed as fuzzy multi-objective mathematical programming (FMOMP) problem with corresponding generalized triangular fuzzy numbers to objectives which are applied as objective coefficients to a reliability problem for the system. Now with the choice of the DM/system expert regarding the priority amongst the objectives, the problem is converted to a single objective optimization problem and then solved by PSO. The technique is explained through an example of a crystallization unit of a fertilizer plant. Pareto optimal solution of this multi-objective mathematical programming is established.

2. Assumption

The basic assumptions used in this methodology are given below:

- components failure and repair rates are statistically independent, constant and obey exponential distribution functions;
- Separate maintenance facility is available for each component.
- After repairs, required component is considered as good as new.
- The standby units are of same nature and capacity as the active units.

3. Methodology

The proposed methodology is divided into two phase. In the phase I, a behavior of the system is analyzed in the form of various reliability indices namely fuzzy failure rate, repair time, expected number of failure (ENOF), mean time between failures (MTBF), reliability and availability by using the uncertain, vague and imprecise data while in phase II, a non-linear multi-objective optimization problem is formulated along with the obtained results from phase I and by using the suggestions of DM/management personnel preferences/system expert and then solve it by using PSO. Both the phases are described in details as below.

3.1. Phase I: Behavior analysis of the system

The motive of this phase is to analyze the behavior of the system by utilizing quantified vague, imprecise and conflicting information/data. The procedural steps used for conducting the analysis of the system are given as follows:

a. In this step information is extracted from different sources. The data related to failure rates (λ_i) and repair times (τ_i) of the main components of the unit are collected from present/historical records and is integrated with expertise of maintenance personnel.
b. As the collected data are imprecise and vague due to various practical reasons as already given in the above discussion. To account for the imprecision and uncertainties in the data, crisp input data is converted into known triangular fuzzy number with specified spread.
c. In this step all the minimal cut sets of the considered systems are obtained using their Petri net model. Using these minimal cut sets, expressions for systems' λ_s and τ_s are obtained.
d. As most of the actions or decisions implemented by human or machines are binary or crisp, so it is necessary to convert the fuzzy output to a crisp value. The process of converting fuzzy output to a crisp value is said to be defuzzification. Out of the existence of the various defuzzification techniques in the literature,

center of area (COA) method is selected due to its property that it has the advantage to have taken the whole membership function into account for this transformation and therefore it is equivalent to mean of data [21, 30]. If the membership function $\mu_{\tilde{A}}(x)$ of the output fuzzy set $\tilde{A}$ is described on the interval $[x_1, x_2]$, then COA defuzzification $\bar{x}$ can be defined as

$$\bar{x} = \frac{\int_{x_1}^{x_2} x\,\mu_{\tilde{A}}(x)\,dx}{\int_{x_1}^{x_2} \mu_{\tilde{A}}(x)\,dx}$$

3.2. Phase II: Formulation of the MONLP problem

A general MONLP problem is to find the design variable set X that optimizes a vector of objective functions $f(X) = \{f_1(X), f_2(X), \ldots\ldots, f_n(X)\}$ over the feasible design space. The problem is modeled as follow

$$\begin{aligned} &Maximize &:\ & f(X) = \{f_1(X), f_2(X), \ldots, f_n(X)\} \\ &subject\ to &:\ & h_i(X) = 0, \qquad i = 1,2,\ldots, I \\ &&& g_j(X) \le 0, \qquad j = 1,2,\ldots, J \\ &&& X_k^l \le X_k \le X_k^u, \quad k = 1,2,\ldots, K \end{aligned} \tag{1}$$

where $f_1(X), f_2(X), \ldots\ldots, f_n(X)$ are the individual objective functions, $h_i(X)$ and $g_j(X)$ are equality and inequality constrained functions, respectively. X_k^l and X_k^u are the lower and upper bounds of decision vector X_k, respectively.

Basic definitions of the Pareto-optimal solutions are given below:

Definition: (Complete optimal solution): X^* is said to be complete optimal solution to the MONLP (1) if and only if there exists $X^* \in X$ such that $f_t(X^*) \ge f_t(X)$, for $t = 1,2,\ldots\ldots, n$.

However, when the objective functions of the MONLP conflict with each other, a complete optimal solution does not always exist and hence the Pareto optimality concept arises and it is defined as follows

Definition: (Pareto-optimal solution): A vector X^* is a Pareto optimal if there exists no feasible vector X which would decrease some objective function without causing a simultaneous increase in at least one objective function. Mathematically, the Pareto optimal solution is expressed as below:

A design vector X^* is a Pareto optimum if and only if, for any X and i, $f_j(X) \ge f_j(X^*)$, $j = 1,2,\ldots., n,\ j \ne i$ $\Rightarrow\ f_i(X) \le f_i(X^*)$

In general, there exist a number of Pareto optimal solutions to multi-objective optimization problems. Thus, the designer must select a compromise or satisfying solution from the Pareto optimal solution set according to his or her preference.

3.2.1. Methodology for solving the MONLP problem

In practical sense, the expression of the objective functions and constraints in the optimization problem (1) are not transparent. While determining these objectives and objective goal as well as goal of the constraints can be involved in many non-stochastic uncertain factors. Thus to make model more flexible and adoptable to human decision process, the optimization model (1) can be expressed as fuzzy non-linear programming problems with fuzzy numbers. Thus in fuzzy environment, the original problem (1) is converted into fuzzy multi-objective optimization problem (2) with fuzzy decision variables.

Therefore in fuzzy environment the optimization problem (1) becomes

$$\begin{aligned} \text{Maximize} \ &: \ \tilde{f}(X) = \{\tilde{f}_1(X), \tilde{f}_2(X), \ldots, \tilde{f}_n(X)\} \\ \text{subject to} \ &: \ \tilde{h}_i(X) = 0 \qquad ; \ i = 1,2,\ldots,I \\ & \quad \tilde{g}_j(X) \le 0 \qquad ; \ j = 1,2,\ldots,J \\ & \quad X_k^l \le X_k \le X_k^u \qquad ; \ k = 1,2,\ldots,K \end{aligned} \quad (2)$$

Here $\tilde{f}, \tilde{h}_i, \tilde{g}_j$ are taken as generalized fuzzy numbers.

In order to use the fuzzy set theory to solve the optimization problems, the fuzzy constraints have to be formed first. These constraints originated from the given crisp constraints by relaxing the bounds. A corresponding membership function is established to describe the fuzziness of each constraint. In detail, the following steps are used to solve the MONLP of problem (2).

Step 1: Formulation of fuzzy region of satisfaction: - Solve the MONLP problem (2) as a single objective non-linear problem times for each problem by taking one of the objective at a time and ignoring the others. These solutions are known as ideal solutions. The solution to the above model is the ideal solution of each objective function, , and the corresponding objective function at the ideal solution may be given by

$$\tilde{f}_t^* = \tilde{f}_t(x_t^*) , \qquad t = 1,2,\ldots\ldots\ldots\ldots, n$$

The lower bound (m_t) and upper bound (M_t) corresponding to each objective $\tilde{f}_t$ is calculated as

$m_t = \min_{1\le p\le n} \tilde{f}_t(x_p^*)$ and $M_t = \max_{1\le p\le n} \tilde{f}_t(x_p^*)$

Hence the membership functions corresponding to the two constraints $\tilde{f}_t(x) \le m_t$ (i.e. for minimization) and $\tilde{f}_t(x) \ge M_t$ (i.e. for maximization) is defined as follows:

For $\tilde{f}_t(x) \le m_t \qquad (t = 1,2\ldots., n)$

$$\mu_{\tilde{f}_t}(x) = \begin{cases} 1 & ; \quad \tilde{f}_t(x) \le m_t \\ \dfrac{M_t - \tilde{f}_t(x)}{M_t - m_t} & ; \quad m_t \le \tilde{f}_t(x) \le M_t \\ 0 & ; \quad \tilde{f}_t(x) \ge M_t \end{cases} \quad (3)$$

Here, $\mu_{\tilde{f}_i}(x)$ is strictly monotonically decreasing function of $\tilde{f}_t(x)$.

For $\tilde{f}_t(x) \ge M_t \qquad (t = 1,2\ldots., n)$

$$\mu_{\tilde{f}_t}(x) = \begin{cases} 1 & ; \quad \tilde{f}_t(x) \ge M_t \\ \dfrac{\tilde{f}_t(x) - m_t}{M_t - m_t} & ; \quad m_t \le \tilde{f}_t(x) \le M_t \\ 0 & ; \quad \tilde{f}_t(x) \le m_t \end{cases} \quad (4)$$

Here, $\mu_{\tilde{f}_i}(x)$ is strictly monotonically increasing function of $\tilde{f}_t(x)$

Step 2: Formulation of Fuzzy multi- objective optimization problem (FMOOP): - Using the achieved objectives' membership functions and DM/system expert preferences in the form of weights, system performance optimization problem is formulated as a single objective optimization problem (by using Huang [10] approach) in the following form

$$\begin{aligned} &\mathit{Maximize} : \left(1 \wedge \frac{\alpha_1(X)}{w_1}\right) \wedge \left(1 \wedge \frac{\alpha_2(X)}{w_2}\right) \wedge \ldots \wedge \left(1 \wedge \frac{\alpha_n(X)}{w_n}\right) \\ &\mathit{subjectto} : \ \alpha_t(X) = \mu_{\tilde{f}_t(x)} \\ &\qquad \tilde{g}_j(X) \le 0, \qquad j = 1,2,\ldots, I + J \\ &\qquad w_i \in [0,1] \\ &\qquad X_k^l \le X_k \le X_k^u, \quad k = 1,2,\ldots,K \end{aligned} \quad (5)$$

where $\mu_{\tilde{f}}(X) = \{\mu_{\tilde{f}_1}(X), \mu_{\tilde{f}_2}(X), \ldots\ldots, \mu_{\tilde{f}_n}(X),$ is a set of n fuzzy regions of satisfaction corresponding to the objective functions, $\wedge$ indicates the intersection, w_t represents the t^{th} objective weight suggested by DM, α_t is the degree of satisfaction of the t^{th} objective, X is the vector of decision variables. Now the obtained problem is solved by using PSO technique, which is described in the next step.

Step 3: A survey of PSO algorithm: -Particle Swarm Optimization (PSO), first introduced by Kennedy and Eberhart [12], is a stochastic global optimization technique. The algorithm models the behavior of a group of particles whose initial values are specified by a group of proposed random solutions called particles. These particles repeatedly search the environment of the problem to reach new solutions. The position and velocity of i^{th} particle at iteration t are specified by $x_i(t)$ and $v_i(t)$ respectively in the searching space. Each particle conserves it best $pbest_i$ position and global best position $gbest$. Then velocity and position of particle i at iteration $t+1$ are updated as follow

$$v_i(t+1) = w * v_i(t) + c_1 * r_1 * (pbest_i(t) - x_i(t)) + c_2 * r_2 * (gbest(t) - x_i(t)) \tag{6}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{7}$$

where c_1, c_2 are the acceleration constants with positive values; r_1, r_2 are random numbers between 0 and 1. Also w is the inertia weight factor which is decreased and varied linearly from initial (w_1) to final (w_2) w.r.t. iteration number [26]. The particle velocity in Eq. (6) is an important parameter because it determines the resolution about the solution regions. Furthermore it was necessary to set a control parameter $v_{\max}$ for the velocity that is unable to exceed this value. The choice of a too small value for $v_{\max}$ can cause very small updating of velocities and positions of particles at each iteration. Hence, the algorithm may take a long time to converge and face a problem of getting struck to local minima. To overcome these situations, Clerc and Kennedy [5] have proposed improved velocity update rules employing a constriction factor χ and accordingly the velocity update equation is

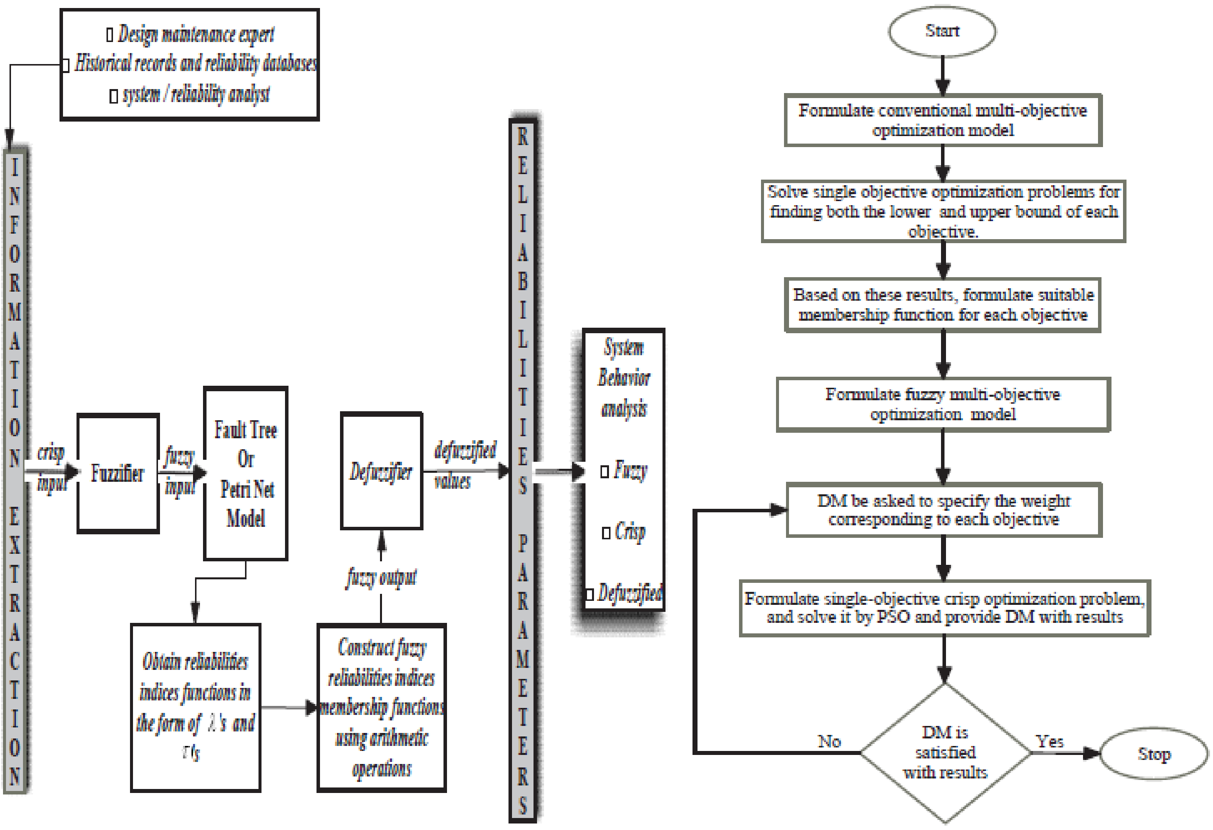
$$v_i(t+1) = \chi(v_i(t) + c_1 * r_1 * (pbest_i(t) - x_i(t)) + c_2 * r_2 * (gbest(t) - x_i(t)))$$

where

$$\chi = \frac{2}{\left|2 - \phi - \sqrt{\phi^2 - 4\phi}\right|} \text{ with } \phi = c_1 + c_2, \;\; \phi > 4$$

Clerc and Kennedy [5] found that the system behavior could be controlled to have the following features: (i) the system does not diverge in a real value region and finally can converge; and (ii) the system can search different regions efficiently by avoiding premature convergence.

The whole process of the Phase I and II methodologies may also be explained through the flowchart given in Figure 1(a) and 1(b) along with PSO algorithm flow chart in Figure 2



(a) Flow chart of Phase I (b) Flow chart of Phase II

Figure 1. Flow chart of Phase I and II methodology.

4. Illustrative Example

The above mentioned technique for solving MONLP problem is illustrated through the problem of optimization of reliability of a crystallization subunit of a urea plant. The brief description of the system (urea plant) is given below.

4.1. System Description

The urea plant considered here is a complex engineering system where the units are arranged in a random fashion and they run continuously for a longer period to produce the required quantity of urea [15]. The plant is a combination of two dependent systems namely ammonia production system and the urea production system. For the production of urea, liquid ammonia and Carbon dioxide are used as inputs which are obtained from ammonia plant. Further, processed in a reactor at controlled pressure and temperature the reactants (urea, ammonium carbonate, water and excess ammonia) are sent to decomposer for urea separation. In the crystallizer, the crystals of urea are separated by centrifuge and conveyed pneumatically to the prilling tower where they are melted, sprayed through distributors and finally fell down at the bottom of the tower, from where it is collected. Among the various functional units in the plant such as urea synthesis, urea decomposition, urea crystallization, urea prilling and urea recovery, urea crystallization is one of the most important and vital functional processes which is the subject of our discussion.

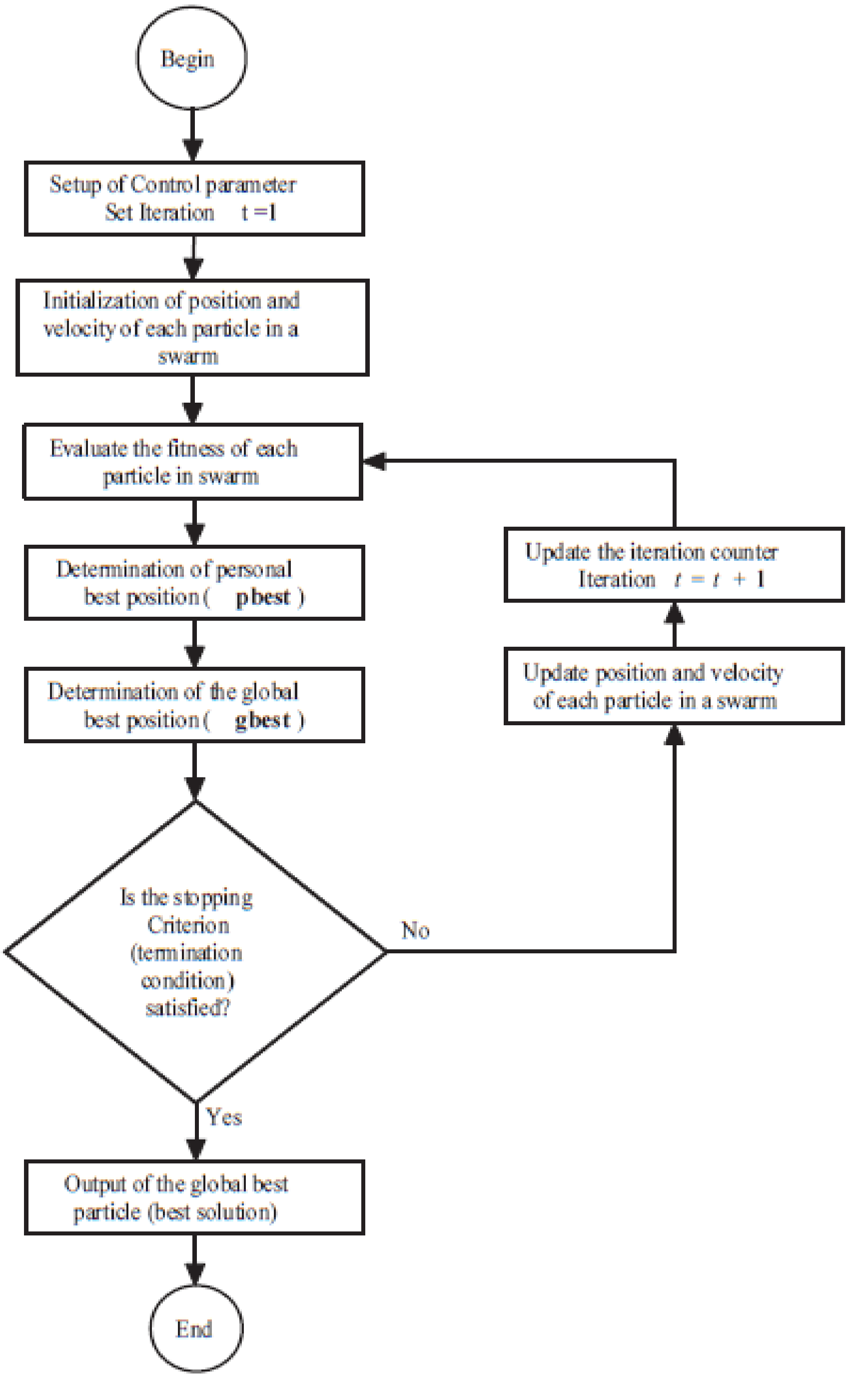


Figure 2. Flow chart of PSO algorithm.

In brief, this operating system comprises of five subsystems arranged in series defined as follows:

- **vacuum generator (A)**: It consists of two stage ejector, barometric condenser used to generate the pressure of 175mm of Hg.
- **crystallizer (B):** It consists of two units in series, concentrator and crystallizer. Failure of any one unit consid-

ered as the complete failure of the system.

- **centrifuge (D):** It consists of five centrifugal pumps arranged in series. Failure of any unit causes the complete failure of the system.
- **crystallizer pump (E):** It consists of two pumps one is operative and other in cold standby. Failure of both at a time will cause complete failure of the system.
- **slurry feed pump (F):** It consists of two pumps arranged in parallel. The urea slurry is removed from the crystallizer through slurry feed pumps and is sent to centrifuges which are arranged in parallel.

The systematic diagram of the system is given in Figure 3.

4.2. Behavior Analysis

Under the information extraction phase, the data related to failure rate (λ) and repair time (τ) of the components is collected from present/historical records of the urea plant and is integrated with expertise of maintenance personnel as presented in Table 1. The equivalent Petri net model of the system is shown in Figure 4. Based on that, the minimal cut sets obtained by using matrix method are $\{A\},\{B_i\}_{i=1,2}$, $\{D\}_{i=1,2,...,5}$,$\{E_1,E_2\}$ and $\{F_1,F_2\}$

For mission time t=10(hrs), the expressions of failure rate (λ_s) and repair time (τ_s) of the system are obtained using results given in Table 2. The remaining reliability indices' expressions of the system are evaluated (those given in Table 3) by using the expressions of λ_s and τ_s

Following the basic steps as described in section 3.1, the reliability indices for the system have been plotted in Figure 5 for $\pm 15\%$, $\pm 25\%$ and $\pm 50\%$ uncertainties whereas crisp and defuzzified values for the system is tabulated in Table 4. From Table 4, it is evident that defuzzified values change with change of spread but crisp values do not change. For instance failure rate first increases by 0.19%, when spread changes from $\pm 15\%$ to $\pm 25\%$, and further increases by 0.93%, when spread changes from $\pm 25\%$ to $\pm 50\%$. Similarly, for repair time and MTBF, the change in defuzzified values is observed with change of spread. On the other hand, reliability decreases by 0.03%, when spread changes from $\pm 15\%$ to $\pm 25\%$, and further decreases by 0.15%, when spread changes from $\pm 25\%$ to $\pm 50\%$. Thus, it is observed that the maintenance action should be based on defuzzified values will be more appropriate than the one based on crisp value, as a safe interval between maintenance action can be established and inspections can be conducted to monitor the condition or status of various equipments constituting the system before it reaches to crisp value. Maintenance/plant personnel using this approach may change their targeted goals to achieve higher profit.

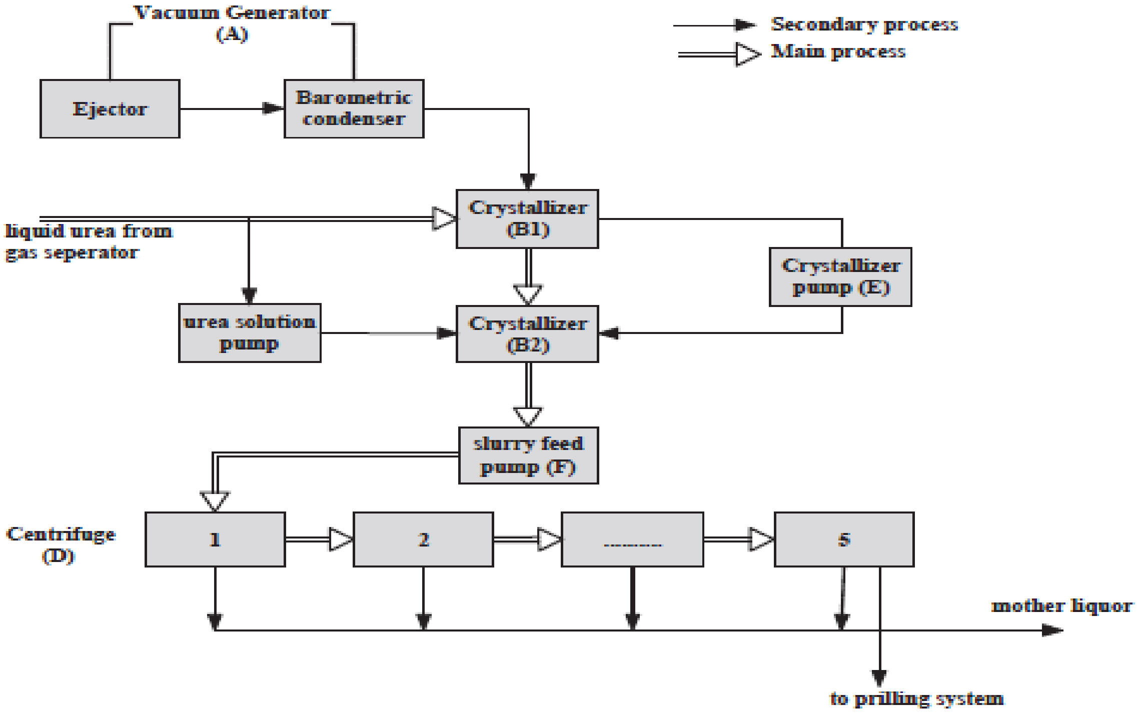


Figure 3. Systematic diagram of the Crystallization unit.

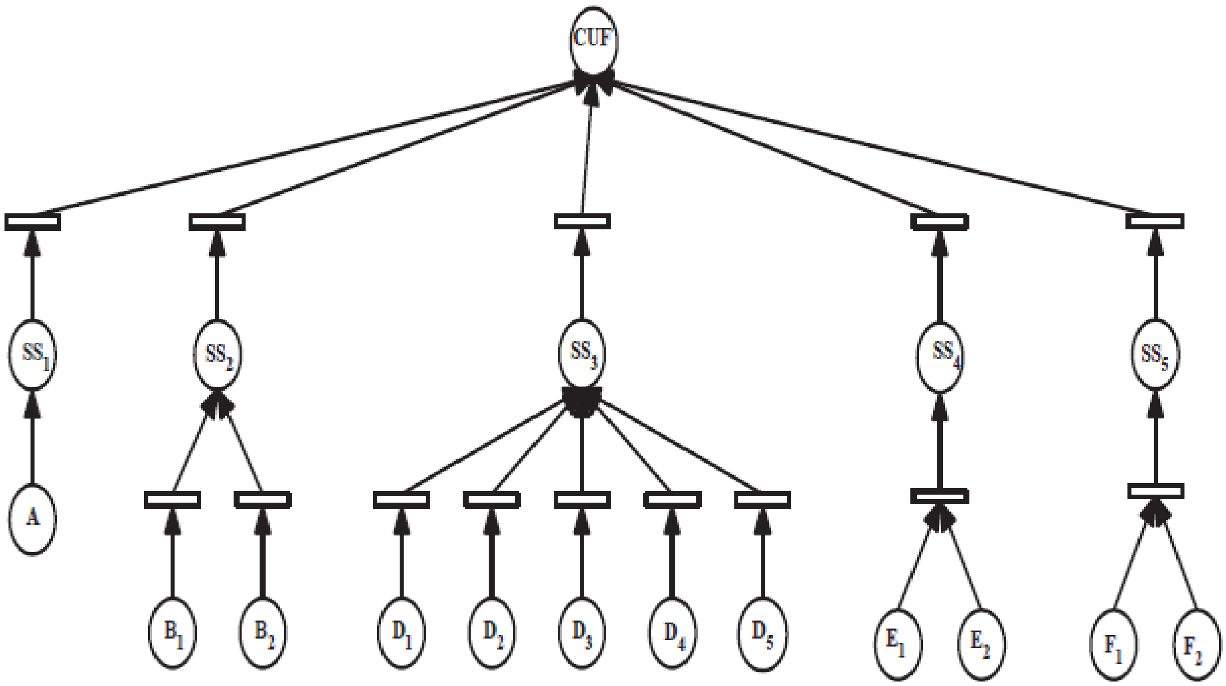


Figure 4. Petri-Net model of the crystallization unit.

Table 1. Data for failure rate (λ_i in per hrs) and repair time (τ_i in hrs)

Subsystems	**A (i = 1)**	**B (i = 2,3)**	**D (i = 4,...,8)**	**E (i = 9,10)**	**F (i = 11,12)**
$\lambda_i{}'s$	0.001	0.003	0.002	0.005	0.005
$\tau_i{}'s$	2	2	3	5	5

Table 2. Basic Expressions of Lambda-Tau Methodology

Gate $\rightarrow$	λ_{AND}	τ_{AND}	λ_{OR}	τ_{OR}
Expressions	$\prod_{j=1}^{n}\lambda_j\left[\sum_{i=1}^{n}\prod_{\substack{i=1\\i\neq j}}^{n}\tau_j\right]$	$\dfrac{\prod_{i=1}^{n}\tau_i}{\sum_{j=1}^{n}\left[\prod_{\substack{i=1\\i\neq j}}^{n}\tau_i\right]}$	$\sum_{i=1}^{n}\lambda_i$	$\dfrac{\sum_{i=1}^{n}\lambda_i\tau_i}{\sum_{i=1}^{n}\lambda_i}$

Table 3. Some Reliability parameters

Parameters	**Expressions**
Mean Time to Failure	$MTTF_s = \dfrac{1}{\lambda_s}$
Mean Time to Repair	$MTTR_s = \dfrac{1}{\mu_s} = \tau_s$
Mean Time Between Failures	$MTBF_s = MTTF_s + MTTR_s$
Availability	$A_s = \dfrac{\mu_s}{\mu_s+\lambda_s} + \dfrac{\lambda_s}{\mu_s+\lambda_s}e^{-(\lambda_s+\mu_s)t}$
Reliability	$R_s = \exp(-\lambda_s t)$
ENOF	$W_s(0,t) = \dfrac{\lambda_s\mu_s t}{(\lambda_s+\mu_s)} + \dfrac{\lambda_s^2}{(\lambda_s+\mu_s)^2}\left[1 - e^{-(\lambda_s+\mu_s)t}\right]$

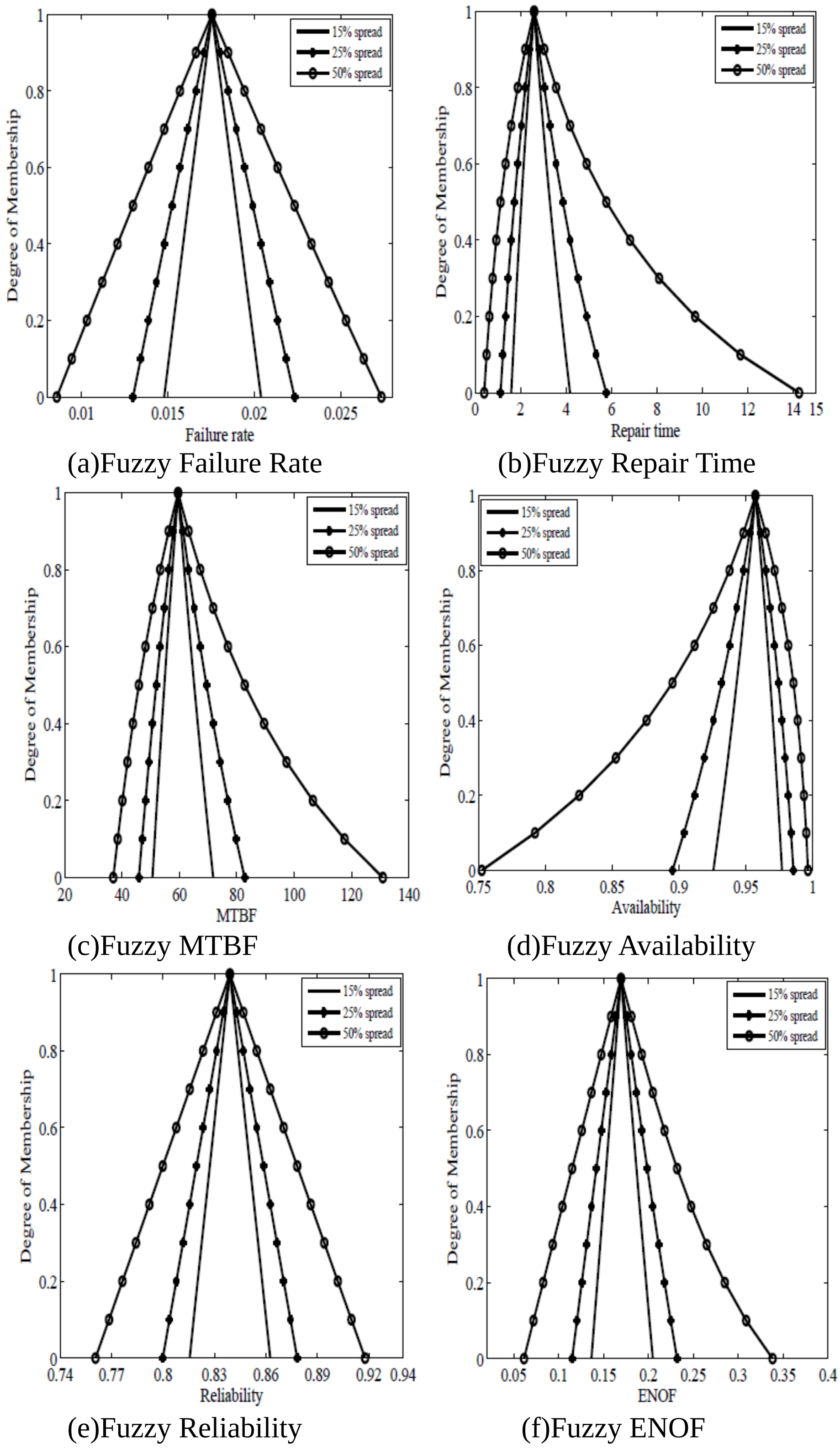


Figure 5. Various reliability indices at $\pm 15\% \pm 25\%$ and $\pm 50\%$ spreads.

Table 4. Crisp and defuzzified values at different spreads for crystallization unit

Reliability parameters	Crisp values	Defuzzified values at (spread)		
		±15%	± 25%	± 50%
Failure rate	0.017550	0.017568	0.017602	0.017757
Repair time	2.594017	2.738773	3.017784	4.857207
MTBF	59.574074	59.658419	59.830316	61.172424
Reliability	0.839037	0.838881	0.838603	0.837301
Availability	0.957231	0.955092	0.951099	0.929082
ENOF	0.169721	0.169682	0.169605	0.169103

4.3. Mathematical Model of the multi-objective optimization problem of the system

Let R_j and C_j be the reliability and cost of j^{th} component of the system while R_s and C_s denote the total reliability and cost of the system. Then in addition to maximization of system reliability, it is also oftenly required that the cost be minimized simultaneously. Incorporating both these requirements the MOOP of the system may be expressed mathematically as

$$\begin{aligned}
&\textit{Maximize} \quad R_s = \exp(-\lambda_s t)\\
&\textit{Minimize} \quad C_s = \sum_{j=1}^{12}\left\{a_j \log\left(\frac{1}{1-\exp(-\lambda_j t)}\right)+b_j\right\}\\
&\textit{subject to} \quad (1-s)X_k \le X_k \le (1+s)X_k\\
&\qquad \lambda_s^l \le \lambda_s \le \lambda_s^u\\
&\qquad X = [\lambda_1, \lambda_2, \ldots, \lambda_{12}, \tau_1, \tau_2, \ldots, \tau_{12}]^T\\
&\qquad \lambda_2 = \lambda_3;\ \lambda_4 = \lambda_5 = \ldots. = \lambda_8;\\
&\qquad \lambda_9 = \lambda_{10}\ ;\quad \lambda_{11} = \lambda_{12};\\
&\qquad \tau_2 = \tau_3;\ \tau_4 = \tau_5 = \ldots = \tau_8;\\
&\qquad \tau_9 = \tau_{10}\ ;\quad \tau_{11} = \tau_{12};\\
&\qquad k = 1,2,\ldots.,12\\
&\qquad s = 0.15\ (\textit{considered uncertainity level})
\end{aligned} \tag{8}$$

where λ_s is the system failure rate whose expression is given by

$$\lambda_s = \sum_{i=1}^{8}\lambda_i + \lambda_9\lambda_{10}(\tau_9 + _{10}) + \lambda_{11}\lambda_{12}(\tau_{11} + \tau_{12})$$

and λ_s^l, λ_s^u are the lower and upper bound of the system failure rate taken from the Figure 5(a) at the considered uncertainty level corresponding to $\alpha = 0$.

The different values for the parameters $a_j\ (j = 1,2,\ldots,12)$ are 8, 24, 24, 8.75, 8.75, 8.75, 8.75, 8.75, 7.14, 7.14, 3.33, 3.33 respectively and for $b_j\ (j = 1,2,\ldots,12)$ are 80, 60, 60, 70, 70, 70, 70, 70, 50, 50, 30, 30 respectively.

5. Computational Results

5.1. Parameter setting

The optimization method is implemented in Matlab (MathWorks) and the program has been run on a T6400 @ 2GHz Intel Core(TM) 2 Duo processor with 2GB of Random Access Memory (RAM). In order to eliminate stochastic discrepancy, 30 independent runs were made involving 30 different initial trial solutions with population size 100 and maximum number of generation as 150 along with $c_1 = c_2 = 2.05$.

5.2. Results and Discussion

The FMOOP as given by optimization problem (2) is formulated for the system and the obtained FMOOP is converted into equivalent crisp optimization problem by using [10] approach. Corresponding to each computed fuzzy objective function; a fuzzy region of satisfaction is to be constructed. Using the constructed membership functions for fuzzy region of satisfactions (as objectives) and their weight vector as suggested by DM/system expert corresponding to the two objective functions, the following equivalent crisp optimization problem is formulated.

For the solution of the optimization problem (9), PSO is used with the parameter given in section 5.1. The results corresponding to weight sets W1=[1 1], W2 =[1 0.5], W3 =[0.8 0.2], W4 = [0.2 0.8], W5 = [0.5 1] which are suggested by DMs / system experts are given in Table 5.

$$
\begin{aligned}
\textit{Maximize} \quad &: \left(1 \wedge \frac{\alpha_1(X)}{w_1}\right) \wedge \left(1 \wedge \frac{\alpha_2(X)}{w_2}\right) \\
\textit{subject to} \quad &: \alpha_t(X) = \mu_{\tilde{f}_t}(X), \quad t = 1,2 \\
&(1-s)X_k \le X_k \le (1+s)X_k \\
&\lambda_s^l \le \lambda_s \le \lambda_s^u \\
&X = [\lambda_1, \lambda_2, ..., \lambda_{12}, \tau_1, \tau_2, ..., \tau_{12}]^T \\
&\lambda_2 = \lambda_3;\ \lambda_4 = \lambda_5 = ... = \lambda_8; \\
&\lambda_9 = \lambda_{10}\ ; \quad \lambda_{11} = \lambda_{12}; \\
&\tau_2 = \tau_3;\ \tau_4 = \tau_5 = ... = \tau_8; \\
&\tau_9 = \tau_{10}\ ; \quad \tau_{11} = \tau_{12}\ ; \\
&k = 1,2,....,12\ ; \quad w_t \in [0,1] \\
&s = 0.15\ (\textit{considered uncertainity level})
\end{aligned} \tag{9}
$$

Table 5. Pareto results for different weight sets suggested by decision maker

Sub components	Reliability	W1	W2	W3	W4	W5
A (i = 1)	R_i	0.988715	0.990147	0.989751	0.989080	0.988565
B (i = 2, 3)	R_i	0.966881	0.971042	0.974103	0.966948	0.966088
D (i = 4, ...,8)	R_i	0.983144	0.983144	0.983108	0.978984	0.981145
E (i = 9, 10)	R_i	0.944122	0.944122	0.947004	0.944321	0.944122
F (i = 11, 12)	R_i	0.944122	0.944122	0.461822	0.947707	0.944122
System R_s		0.844229	0.852746	0857839	0.826835	0.834190
System C_s		1168.475639	1176.007236	1181.966702	1159.674539	1162.334966
α_1		0.601520	0.784256	0.892526	0.228321	0.386121
α_2		0.601520	0.392128	0.226444	0.846207	0.772242

6. Conclusion

A real life reliability optimization problem of a crystallization system in a fertilizer plant (situated in the Northern part of India) has been discussed here. A behavior analysis as well as multi-objective reliability optimization (maximize reliability and minimize cost) of the considered plant has been analyzed. Based on the behavior analysis of the system, a multi - objective optimization problem is constructed. A mutual conflicting nature of the objectives are resolved with the help of fuzzy after constructing the fuzzy region of satisfication by taking linear membership functions. A very interactive fuzzy satisficing method for deriving a biased Pareto-optimal solution preferred by the DM is presented in this paper. Resulting FMOOP has been solved using one of the meta-heuristic technique namely as PSO. In a practical point of view, PSO enables the trade - off between the system reliability and desigining cost while the widely used single - objective approaches tend to optimize the system reliability without saving any designing cost. Since reliability decision is usually made in the earliest stage of system design and the information at this stage is incomplete and imprecise, it is necessary to rely on the experience of decision-makers and experts. The proposed approach can efficiently deal with the vagueness and subjectivity of expert information. Through this approach, a decision support system has been developed which helps the plant maintenance personnel in deciding his/her future strategy to gain optimum performance of the system.

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