International Journal of Applied Science and Engineering
2011. 9, 4: 277-286

Efflux Time for Two Exit Pipe System

G. Santosh Kumar[a], Ch. V. Subbarao [*, a], and P. King[b]

[a] *Department of Chemical Engineering, MVGR College of Engineering, Chintalavalasa- 535005 Vizianagaram, Andhra Pradesh, India.*
[b] *Pollution Control Engineering laboratory, Department of Chemical Engineering, College of Engineering, Andhra University, Visakhapatnam.*

Abstract: Based on integral balances, mathematical equation for efflux time during gravity draining of a Newtonian liquid (below its bubble point) from a large open cylindrical storage tank (Where the flow in the storage tank is essentially laminar) through two exit pipes of same diameter located at the bottom of the vessel (The flow is assumed to be turbulent in each of the exit pipes) is developed. The equation is ultimately simplified and written in dimensionless form. The equation will be of use for arriving at the minimum time required for draining the contents of a cylindrical storage vessel through two exit pipe. The equation is verified with the experimental data. A maximum deviation of 19% and an average deviation of 16% are observed. The effect of initial height of liquid on Reynolds' number is also established. It is shown that while draining a liquid from the cylindrical storage tank through exit pipe system, Froude number remains constant.

Keywords: Efflux time; Newtonian liquid; open storage tank; minimum time

1. Introduction

In Chemical industry, processing and storage vessels appear in a large variety of geometries. The reasons for the choice of the typical shape or geometry may be attributed to convenience, insulation requirements, floor space, material costs, corrosion and safety considerations. The time required to drain these vessels off their liquid contents is known as efflux time as reported by Hart and Sommerfeld [1] and this is of crucial importance in many emergency situations besides productivity considerations.

They reported a general expression for efflux time (t) given by $t = \frac{1}{C_0 A_0 \sqrt{2g}} \int_0^H \frac{A}{\sqrt{h}} dh$, C_o is discharge coefficient, A_0 is area of orifice, A is area of tank, H is the height of liquid at t= 0 and h is the height of the liquid at any time, t.

Mathematical analysis and experimental work for gravity draining of a Newtonian liquid from a storage tank through restricted orifice of different diameters were reported [2]. It was shown that the acceleration of free surface of a liquid is less than the acceleration due to gravity.

Von Dogen and Roche. Jr [3] carried out efflux time analysis from tanks with exit pipes and fittings in the Reynolds number range of 40,000 - 60,000. The effect of pipes and fittings were expressed in terms of equivalent lengths.

Using computational tools, Morrison [4]

[*] Corresponding author; e-mail: subbaraochv@rediffmail.com *Accepted for Publication: October 9, 2011*

carried out efflux time analysis through an exit pipe at around a Reynolds number of 6,000. The maximum efflux time reported was only 35 seconds. The ratio of tank cross section to pipe cross section was only 228. This ratio was much less than the ratio reported by subbarao et al [5, 6].

Subbarao et al [5, 6] carried out efflux time analysis of draining a Newtonian liquid from a cylindrical tank through a single exit pipe based on macroscopic balances.

The efflux time equation was simplified and written as

$$t_{eff} = \sqrt{\frac{2}{g_m}}\left(\sqrt{H+L} - \sqrt{H'+L}\right),$$ where g_m is modified form of acceleration due to gravity and related to acceleration due to gravity g by

$$\frac{g_m}{g} = \frac{1}{\left(1+4f\frac{L}{d}\right)\left(\frac{A_t}{A_p}\right)^2},$$ f is the friction factor, L and d are length and diameter of the exit pipe, A_t and A_p is the cross sectional area of tank and pipe respectively. They used an exit pipe of $4X10^{-3}$ m dia.

Their data suggested that $\frac{V_2^{\ 2}}{2} << 4f\frac{LV_2^{\ 2}}{2d}$. They also mentioned that polymer solutions increase the Froude number. Froude number can also be increased either by changing the geometry of the vessel or by providing a large diameter exit pipe or providing two exit pipes.

Subbarao [7] showed through mathematical analysis that efflux time for cone is less than that of cylinder through single exit pipe and hence Froude number for cone can be expected to be higher than that of a cylinder.

Subbarao and Sivakumar [8] also reported work for comparison of efflux times between cylindrical and spherical tanks. Their mathematical analysis suggested that the efflux time for sphere is less than that of cylinder.

Present work considers draining a Newtonian liquid through two exit pipes and the mathematical analysis is based on integral balances as reported by Bird et al [9]. Integral balances are useful for making preliminary estimate of an engineering problem. They are useful for deriving approximate relations.

The scope of work includes

(a) Development of mathematical equation for efflux time for two-exit pipe system
(b) Verification of efflux time equation developed with the experimental values
(c) Study of variation of efflux time with initial height of liquid in the tank
(d) Study of variation of Froude number with initial height of liquid in the tank.

2. Experimental Procedure

The Apparatus used for experimentation consisted of a stainless steel cylindrical tank of known diameter (Figure 1) provided with a level indicator (LI) and two mild steel pipes each of $4X10^{-3}$m I.D welded to the tank. One pipe is located at the centre of the bottom of the tank and the other at 2cm away from the centre. Two gate valves (GV_1 & GV_2) provided at the bottom most point served as the outlet for draining as shown in figure 1. Both the valves were closed and the tank was filled up to the mark and allowed to stabilize. The stopwatch was started immediately after the opening of the bottom gate valves. The drop in water level was read from the level indicator.

The time was recorded for a known drop in the liquid level. The measurements were continued till the water level reaches to a desired value just 0.02m above the tank bottom. The experimental efflux time was designated as t_{act} .

The experiments were repeated to check the consistency of data. The lists of variables covered in these studies are compiled in table 1.

3. Development of mathematical model for two-exit pipe system

As shown in Figure 1, the cylindrical tank along with two exit pipe system (21 & 22) is initially filled with a Newtonian liquid. The diameter of each exit pipe is $4X10^{-3}$m. At stations 21 and 22 the density and other physical properties are assumed to be uniform over the entire cross section. The liquid leaves both the pipes at station-2 under turbulent flow conditions. A level indicator (LI) shows the level in the tank. It is desired to find the time required to drain the liquid from the storage tank (not the pipes) through two-exit pipe system.

Table 1. List of variables covered with two-exit pipe system.

S.No	D, m	L, m	H, m
1	0.27	0.75	0.20, 0.18, 0.16, 0.14
2	0.32	0.75	0.32, 0.28, 0.24, 0.20
3	0.34	0.75	0.44, 0.40, 0.32, 0.20

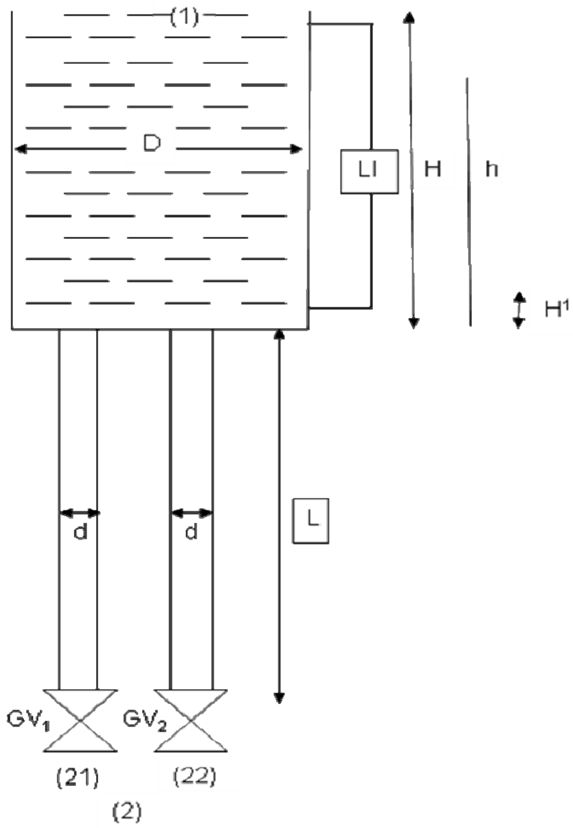

Figure 1. Cylindrical tank along with two exit pipes

The total mass flow rate is taken as sum of the individual mass flow rate from each of the pipes.

$W_2 = W_{21} + W_{22}$ (Since the liquid is drained through two exit pipes) (1)

$$\therefore W_2 = \rho V_{21} A_p + \rho V_{22} A_p \qquad (2)$$

Since the pipes are of equal diameter, it is reasonable to assume that the average velocity in both the pipes is same. ($V_{21} = V_{22}$)

$$\therefore W_2 = 2\rho V_{21} A_p \qquad (3)$$

W_2 can also be written as

$$W_2 = \rho V_2 S_2 \qquad (4)$$

but $S_2 = 2A_p$ (5)

Hence $V_2 = V_{21}$ (6)

The mechanical energy balance equation between station-1 and station-2 can be written as

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + gZ_2 + \frac{V_2^2}{2} + \sum h_{fs} \qquad (7)$$

For two exit pipe system $\Sigma h_{fs} = \frac{4f(L/d)V_{21}^2}{2} + \frac{4f(L/d)V_{22}^2}{2}$ Hence equation 7 becomes

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + gZ_2 + \frac{V_2^2}{2} + \frac{4f(L/d)V_{21}^2}{2} + \frac{4f(L/d)V_{22}^2}{2} \qquad (8)$$

Since the inlet and outlets are open to atmosphere and liquid drains very slowly, $P_1 = P_2$, $V_1 = 0$ & $V_{21} = V_{22} = V_2$ noting that $Z_1 = Z_2 + h + L$ and at low Reynolds numbers, it was established that

$$\frac{V_2^2}{2} << \frac{4f(L/d)V_{21}^2}{2} + \frac{4f(L/d)V_{22}^2}{2}$$

$f = \frac{0.0791}{\mathrm{Re}^{0.25}}$ as reported by Bird et al [9] and is valid for $2100 < \mathrm{Re} < 10^5$

$\mathrm{Re} = \frac{dV_2\rho}{\mu}$ (Since $V_2 = V_{21} = V_{22}$)

Substituting the value of f and Re gives the following expression for V_2

$$V_2 = \frac{1.93 * g^{4/7}(h+L)^{4/7} d^{5/7} \rho^{1/7}}{\mu^{1/7} L^{4/7}} \qquad (9)$$

The mass balance equation between station-1 and station-2 can be written as

$$\frac{d}{dt}\left(\frac{\pi}{4} D^2 h\right) = -2V_2 S \qquad (10)$$

Substituting the value of V_2 and S in eq.10, eq.10 upon integration between the limits (at t=0, h=H and at $t = t_{eff}$, h=0) gives the following equation for efflux time

$$t_{eff} = \frac{0.6044\left[(H+L)^{3/7} - (L)^{3/7}\right]L^{4/7}}{g^{4/7}\left(\frac{\rho}{\mu}\right)^{1/7} d^{5/7} d^{2}} D^{2} \quad (11)$$

Eqn.11 suggests that the plot of $\left[(H + L)^{3/7} - L^{3/7}\right]$ Vs t_{eff} is a straight line.

Eqn.11 can also be written in the following dimensionless form as

$$\frac{t_{eff}}{\left(\frac{\mu d^{2}}{\rho g^{4}}\right)^{1/7}} = 0.6044 * \left[\left(1+\frac{H}{L}\right)^{3/7} - 1\right]\frac{A_t}{A_p}\frac{L}{d} \quad (12)$$

$$\theta = 0.6044 * \left[\left(1+\frac{H}{L}\right)^{3/7} - 1\right]\frac{A_t}{A_p}\frac{L}{d} \quad (13)$$

4. Results and discussions

4.1. Verification of efflux time for two-exit pipe system:

Even though Eqn.13 suggests that complete draining is possible, experimentally complete draining could not be achieved. This is suspected to be due to surface tension forces. Hence, Eqn.13 is modified as

$$\theta = 0.6044\left[\left(1+\frac{H}{L}\right)^{3/7} - \left(1+\frac{H'}{L}\right)^{3/7}\right]\frac{A_t}{A_p}\frac{L}{d} \quad (14)$$

It can be seen from the equation that dimension less time is influenced only by the dimensions of the tank and exit pipe.

Sine there are two exit pipes, the Reynolds number in each of the exit pipes is calculated as

$$\mathrm{Re} = \frac{d V_{2\exp}\rho}{\mu} \quad (15)$$

$$V_{2\exp} = \frac{\pi D^{2}(H - H')}{2\pi d^{2} t_{act}} \quad (16)$$

Since the fluid is water, its density and viscosity are assumed to be 1000 kg/m^3 and 1X10^{-3} kg/m.sec cp respectively. The flow in each of the pipes is calculated using eq. 15 and found to be turbulent only.

In order to verify the validity of eq.14, a plot of $\left[\left(1+\frac{H}{L}\right)^{3/7} - \left(1+\frac{H'}{L}\right)^{3/7}\right]$ vs

t_{eff} (t_{eq} and t_{act}) is shown in figure 2. Even though the maximum deviation was 19%, the graph being a straight line suggests the validity of eq.15. This is further confirmed by the plots (figure 3-4) drawn for two different diameters of tanks.

Since, there are two-exit pipes, there is an increased cross section for flow, the flow possibly transformed into pipe flow and there by reducing the formation of vortices and reduction in contraction coefficient and hence leading to reduction in percentage error.

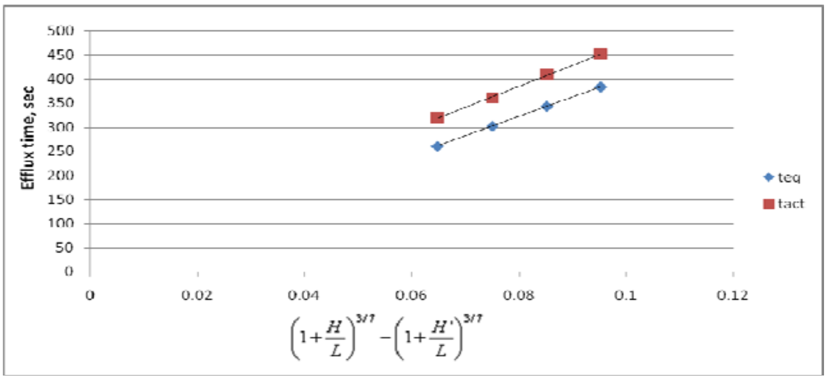

Figure 2. Comparison of efflux time for two-exit pipe system (D=0.27m, L=0.75m)

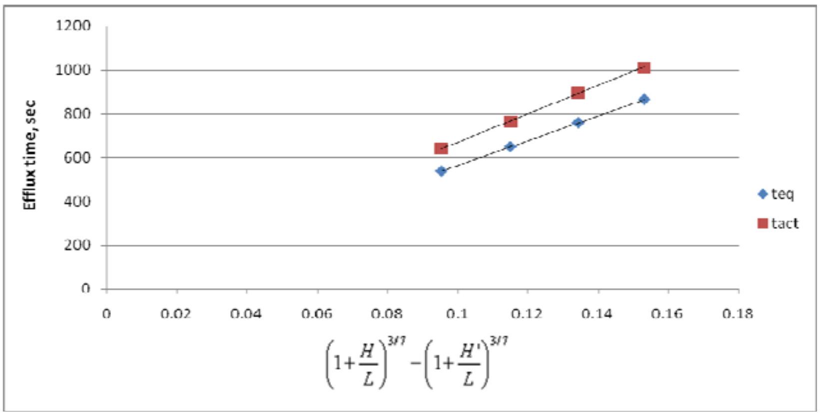

Figure 3. Comparison of efflux time for two-exit pipe system (D=0.32m, L=0.75m)

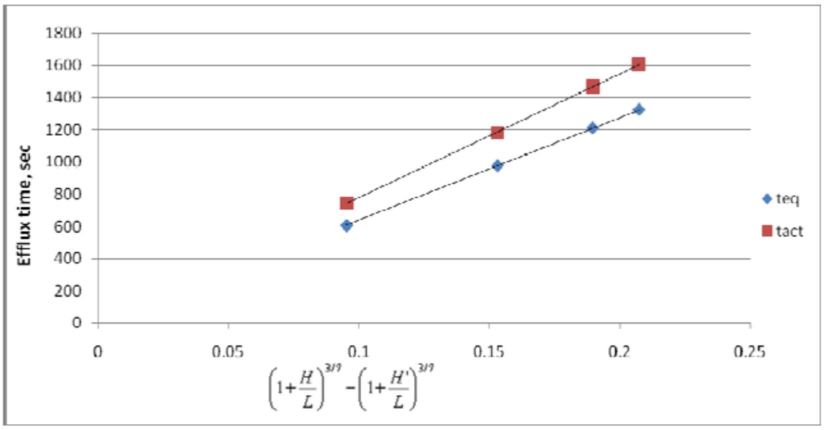

Figure 4. Comparison of efflux time for two-exit pipe system (D=0.34m, L=0.75m).

4.2. Effect of initial height of liquid on average Reynolds Number

The plot of initial height of liquid on Reynolds number for 0.27m dia. Tank is shown in figure5. The plot suggests that, as the height of the liquid in the tank decreases, average Reynolds number decreases. The reduction in Reynolds number is marginal and the flow in the exit pipe still remains turbulent

(Re>2100).

This is further confirmed by the plots (figure6-7) of intial height vs Reynolds number for the two diameters of tanks (D=0.32m and D=0.34m).

4.3. Varation of $\frac{V_2^{\ 2}}{2}$ with $4f\frac{L}{d}\frac{V_2^{\ 2}}{2}$

While deriving eq.13, it has been stated that

$$\frac{V_2^{\ 2}}{2} << \frac{4f(L/d)V_{21}^{\ 2}}{2} + \frac{4f(L/d)V_{22}^{\ 2}}{2}$$

The plot of $\frac{V_2^{\ 2}}{2}$ vs $4f\frac{L}{d}\frac{V_2^{\ 2}}{2}$ for 0.27m dia. tank shown in figure 8 justifies that statement. The trend was similar for other tank diameters as well.

4.4. Variation of Froude number with initial height of liquid in the tank

Froude number is defined as

$$Fr = \sqrt{\frac{V_1^2}{2g(H+L)}} = \sqrt{\frac{V_2^2 2A_p^2}{g(H+L)A_t^2}} = \sqrt{\frac{2V_2^{\ 2}}{g(H+L)}}\frac{A_p}{A_t}$$

Figure 9 illustrates the variation of Initial height of liquid in the tank with Froude number for 0.27m dia. tank.

It can be concluded from the figure that Froude number remains constant and is independent of initial height of liquid in the tank. Similar trend was observed for 0.32m dia as well as 0.34m dia. tanks.

5. Conclusions

Some of the conclusions of the above study are

(e) The average deviation between the theoretical values and experimental values was 16%.

(f) As the height of the liquid in the tank decreases, the efflux time decreases.

(g) The Reynolds number decreases with a decrease in initial height of liquid in the tank. The decrease in Reynolds number with initial height of liquid in the tank was marginal.

(h) While draining a liquid from the storage vessel through two-exit piping system, Froude number remains constant and is not influenced by the initial height of liquid in the tank.

Acknowledgement

The authors gratefully acknowledges the Principal, Dr K. V. L. Raju and the Management of MVGR College of Engineering –Vizianagaram for providing the necessary infrastructural facilities for carrying out experiments. The authors also would like to thank Prof Ch.Durgaprasada Rao, Retd. Professor of Chemical Engineering, IIT, Chennai, India and Prof C.Bhaskara Sarma, Professor of Chemical Engineering, Gayatri Vidyaparishad College of Engineering, Visakhapatnam, India for their useful discussions.

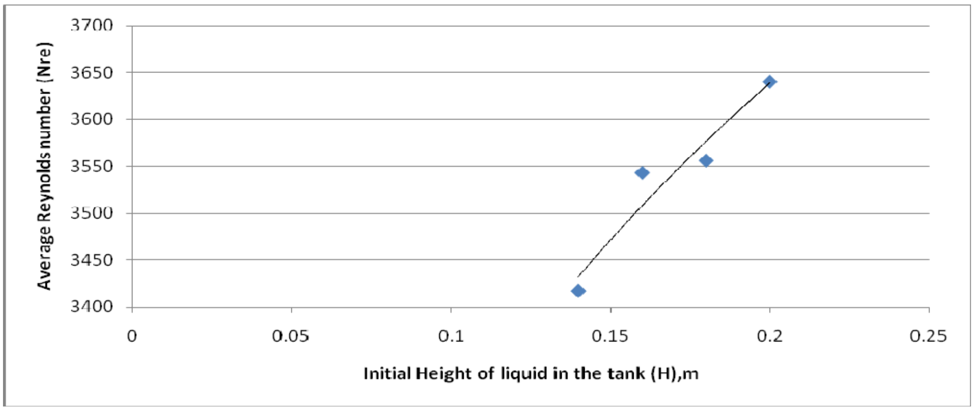

Figure 5. Variation of average Reynolds number with initial height of liquid in the tank, m (D=0.27m)

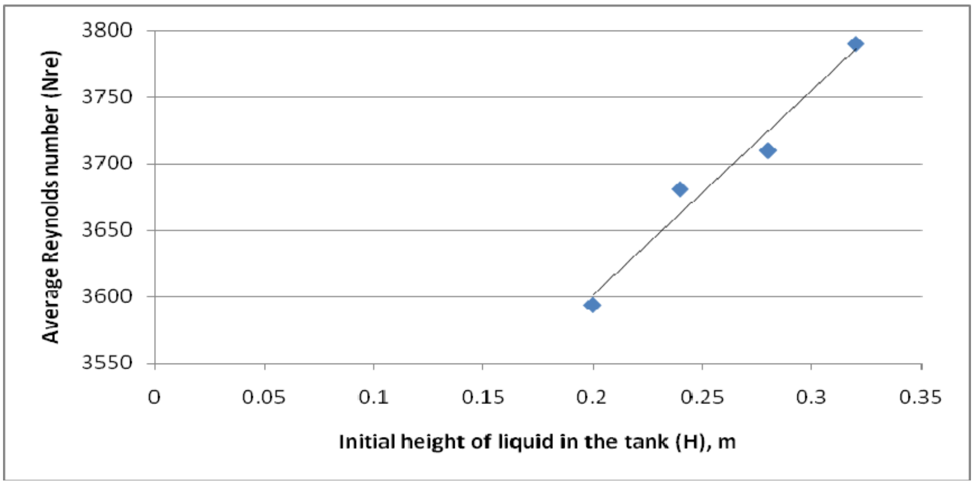

Figure 6. Effect of initial height of liquid in the tank (D=0.32m)

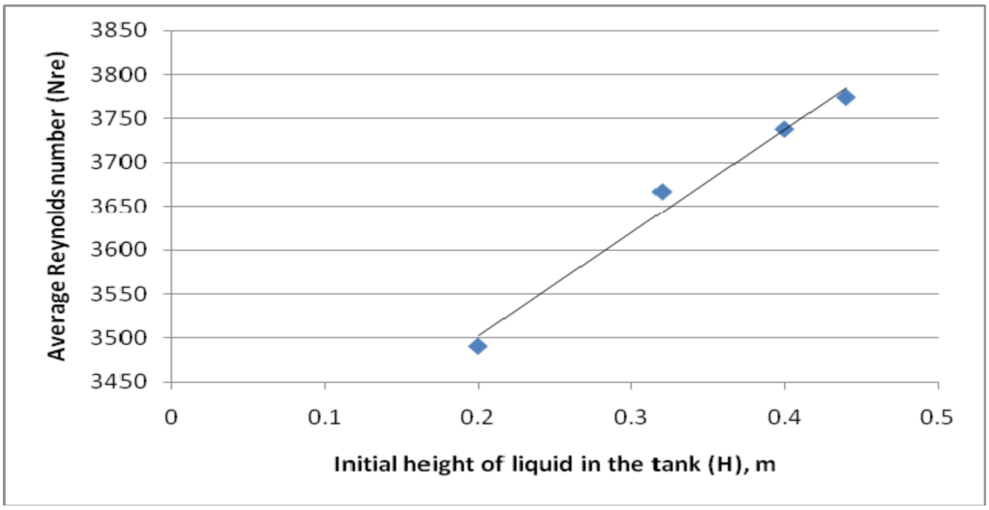

Figure 7. Effect of initial height of liquid in the tank (D=0.34m)

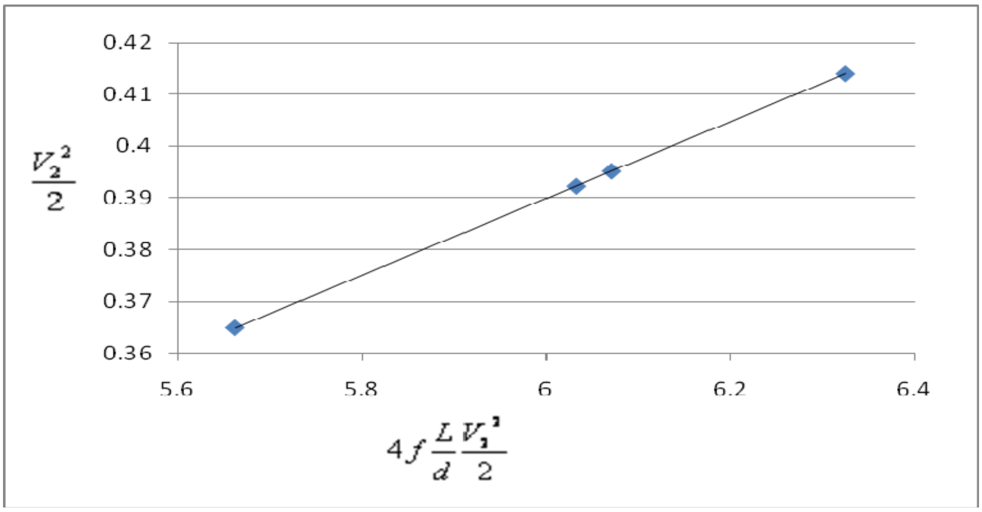

Figure 8. Plot of $\frac{V_2^2}{2}$ vs $4f\frac{L}{d}\frac{V_2^2}{2}$ for 0.27m dia. tank

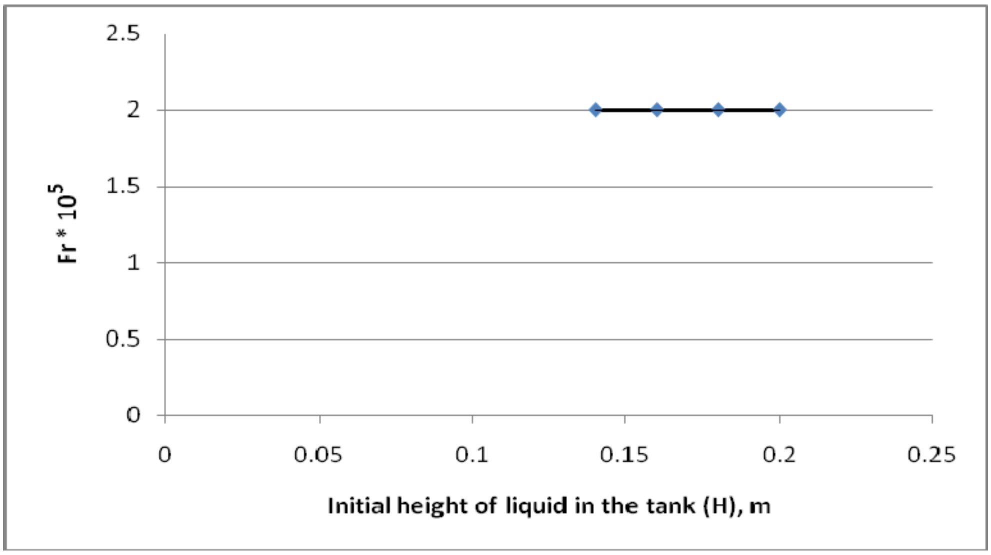

Figure 9. Variation of Froude number with initial height of liquid in the tank for 0.27m dia. Tank

Nomenclature

A_p	=	Area of each pipe , m^2
A_t	=	Area of tank, m^2
D	=	Diameter of tank, m
d	=	Diameter of each exit pipe, m
f	=	Friction factor, dimensionless
Fr	=	Froude number, dimensionless
GV_1 & GV_2	=	Gate Valves
H	=	Initial height of liquid in the tank at t=0, m

H'	=	Final height of liquid in the tank on draining, m
h	=	Height of liquid in the tank at any time, m
h_{fs}	=	Skin friction losses, j/kg
L	=	Length of the exit pipe, m
LI	=	Level indicator
S_2	=	Combined area of both pipes, m^2
P_1 & P_2	=	Pressures at station 1 and station 2 respectively, N/m^2
t_{eff}	=	Efflux time, sec
t_{act}	=	Actual efflux time, sec
t_{eqt}	=	Efflux time based on equation, sec
V_{21} & V_{22}	=	Velocities at station 21 and 22 respectively, m/sec
V_2	=	Velocity at station 2 , m/sec
V_{2exp}	=	Experimental average velocity, m/sec
W_{21} & W_{22}	=	Mass flow rate at station 21 and 22 respectively, kg/sec
Z_1 & Z_2	=	Elevation at station 1 and 2 respectively, m $Z_1 = Z_2 + h + L$
ρ	=	Density of liquid, kg/m^3
μ	=	Viscosity of liquid, kg/m. sec
θ	=	Dimensionless time

References

[1] Hart, W. P. and Sommerfeld, T. 1995. Expression for gravity drainage of annular and Toroidal containers. *Process Safety Progress*, 14, 4: 238-243.

[2] Libii, J. N. 2003. Mechanics of slow draining of a large tank under gravity. *American Journal of Physics*, 71, 11: 1204-1207.

[3] Von Dogen, D. B. and Roche, JR. E. C. 1999. Efflux time from tanks with exit pipes and fittings. *International Journal of Engineering Education,* 15, 3: 206-212.

[4] Morrison. K. R. 2001. Modeling and computation techniques for fluid mechanics experiments. *International Journal of Engineering Education*, 17, 3: 288-293.

[5] Subbaraoch, Ch. V., King. P., and Prasad, V. S. R. K. 2008. Effect of polymer additions on the mechanics of slow draining of large tank under gravity. *ARPN Journal of Engineering and Applied Sciences*, 3, 1: 68-83.

[6] Subbarao, Ch. V., King. P., and Prasad, V. S. R. K. 2008. Effect of polymer additives on the Dynamics of a Fluid for once through system. *International Journal of Fluid Mechanics Research,* 35, 4: 374-393.

[7] Subbarao, Ch. V. 2011. Comparison of Efflux Time between Cylindrical and Conical Tanks Through an Exit Pipe. *International Journal of Applied Science and Engineering*, 9, 1: 33-41.

[8] Reddy, G. V. S. K. and Subbarao, Ch.V. 2011. Comparison of Efflux Times between Cylindrical and Spherical Tank through an Exit Pipe. *International Journal of Applied Science and Engineering*, 3, 2: 61-68.

[9] Bird, R. B., Stewart, W. E., and Lightfoot, E. N. 2006. “*Transport Phenomena*”. 2nd edition. John Wiley & sons (Asia) Pvt. Ltd.