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Tuning of PID Controllers for Unstable Continuous Stirred Tank Reactors

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Abstract: Tuning Proportional Integral and Derivative (PID) controllers for unstable Second Order Plus Time Delay systems with a Zero (SOPTDZ), based on internal model control (IMC) and stability analysis (SA) principles is proposed. The controllers designed by the proposed methods are used to control unstable jacketed Continuous Stirred Tank Reactors (CSTR), carrying out irreversible first order reaction. The performance of proposed controllers is compared with the synthesis method. Simulation results on nonlinear unstable continuous stirred tank reactors and transfer function models are presented to the show the efficiency of the proposed controllers.

Keywords: Internal model control; stability analysis method; synthesis method; CSTR; PID controller; Unstable systems; Second Order Plus Time Delay system with a Zero (SOPTDZ).

1. Introduction

PID controllers give satisfactory performance for many of the control processes. Due to their simplicity and usefulness, PID controller has become a powerful solution to the control of a large number of industrial processes. The control systems performance is complicated by the numerator dynamics (presence of a zero) of the process. Several processes exhibit second order plus time delay system with a zero transfer function model. Examples for such processes are jacketed CSTR [1], distillation column [2], autocatalytic CSTR [3] and crystallizer [4]. Many recycle processes where energy and mass recycle takes place are represented by SOPTDZ transfer function model [2].

Very limited methods of designing PID controllers for unstable SOPTDZ systems are available in the literature. They are IMC method [5] and synthesis method [6]. In the synthesis method [6], closedloop transfer function is assumed. From the closedloop transfer function, controller transfer function is derived using process transfer function. Later controller transfer function is written as PID controller with a lead lag filter. In the IMC method [5], the PID controllers are designed for unstable FOPTD and SOPTD systems with and without a zero from IMC filter using Maclaurin series expansion.

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In the present work, a simple method for designing PID controllers based on internal model control principles and stability analysis is proposed. The PID tuning parameters are given as function of process model parameters. Simulation results for various transfer function models and nonlinear models are given to show the efficiency of the proposed controllers.

2. Proposed methods

2.1. Internal model control (IMC) method

The process transfer function for SOPTD with a negative zero is given by

$$G_p(s) = \frac{K_p(1+ps)e^{-Ls}}{(\tau_1 s+1)(\tau_2 s-1)} \quad (1)$$

Using Pade's approximation for the time delay $e^{-Ls} = \frac{(1-0.5Ls)}{(1+0.5Ls)}$, the process transfer function in Eq (1) is written as,

$$G_p(s) = \frac{K_p(1+ps)(1-0.5Ls)}{(\tau_1 s+1)(\tau_2 s-1)(1+0.5Ls)} \quad (2)$$

The process transfer function is written as

$$G_p(s) = G_p^{+}(s)G_P^{-}(s) \quad (3)$$

Where $G_P^{+}(s)$ is invertible part and $G_P^{-}(s)$ is noninvertible part. Therefore, the invertible part of the process transfer function in Eq (2) is $G_P^{+}(s) = \frac{K_p(1+ps)}{(\tau_1 s+1)(\tau_2 s-1)(1+0.5Ls)}$ (4)

The IMC controller for this system is given as, $Q = [G_P^{+}(s)]^{-1}.f$ (5)

Where f is IMC filter. It consists of both numerator and denominator dynamics. The order of the numerator is equal to the number of unstable poles of the process. The order of denominator is selected in such a way to make controller realizable (the controller numerator order should be less than or equal to that of the order of denominator). Therefore IMC filter for this transfer function model is given by $f = \frac{\eta s+1}{(\lambda s+1)^3}$ (6)

Using Eq (4) and Eq (6) in Eq (5), IMC controller is given by

$$Q = \frac{(1+0.5Ls)(\tau_1 s+1)(\tau_2 s-1)(\eta s+1)}{K_P(1+ps)(\lambda s+1)^3} \quad (7)$$

Where λ is IMC filter time constant, is a tuning parameter. The equivalent PID controller is obtained by using the following equation, $G_c(s) = \frac{Q}{(1-G_P(s)Q)}$ (8)

Substituting Eq (2) and Eq (7) in the above equation, the following transfer function model for the controller is obtained.

$$G_c(s) = \frac{(1+0.5Ls)(\tau_1 s+1)(\tau_2 s-1)(\eta s+1)}{K_p(1+ps)[(\lambda s+1)^3-(1-0.5Ls)(\eta s+1)]} \quad (9)$$

Eq (9) can be rearranged into the following form, $G_c(s) = K_c[1+\frac{1}{\tau_I s}+\tau_D s]\frac{(1+\alpha_0 s)}{(\alpha_1 s^2+\alpha_2 s+1)}$ (10)

This is a PID controller with a lead lag filter

Where, $K_c = \frac{\tau_1+\eta}{K_p(\eta-3\lambda-0.5L)}$ (11)

$$\tau_I = (\tau_1+\eta) \quad (12)$$

$$\tau_D = \frac{\tau_1\eta}{(\tau_1+\eta)} \quad (13)$$

$$\eta = \frac{\lambda^3+(3\lambda+0.5L)\tau_2^{\ 2}+3\lambda\tau_2}{(\tau_2^{\ 2}-0.5L\tau_2)} \quad (14)$$

$$\alpha_0 = 0.5L \quad (15)$$

$$\alpha_1 = \frac{\lambda^3 p}{(\eta-3\lambda-0.5L)\tau_2} \quad (16)$$

$$\alpha_2 = \frac{\lambda^3}{(\eta-3\lambda-0.5L)\tau_2}+p \quad (17)$$

λ is selected by trial and error procedure. If the value of λ is large, the response is sluggish but robust and if it is small the response is fast but less robust.

If time delay is small, the time delay is approximated as $e^{-Ls} = 1-Ls$, the invertible part of the process transfer function of Eq (1) is $G_P^+(s) = \dfrac{K_p(1+ps)}{(\tau_1 s+1)(\tau_2 s-1)}$ (18)

The IMC controller for the given transfer function is given in Eq (5). Therefore, IMC filter f is given by $f = \dfrac{\eta s+1}{(\lambda s+1)^2}$ (19)

Using Eq (18) and Eq (19) in Eq (5), IMC controller is given by

$$Q = \frac{(\tau_1 s+1)(\tau_2 s-1)(\eta s+1)}{K_P(1+ps)(\lambda s+1)^2} \quad (20)$$

Substituting Eq (1) and Eq (20) in Eq (8), the following transfer function model for the controller is obtained.

$$G_c(s) = \frac{(\tau_1 s+1)(\tau_2 s-1)(\eta s+1)}{K_p(1+ps)[(\lambda s+1)^2-(1-Ls)(\eta s+1)]} \quad (21)$$

Eq (21) is rearranged into PID controller with a first order lag filter.

$$G_c(s) = K_c[1+\frac{1}{\tau_I s}+\tau_D s]\frac{1}{(\tau_f s+1)} \quad (22)$$

Where, $K_c = \dfrac{\tau_1+\eta}{K_p(\eta-2\lambda-L)}$ (23)

$$\tau_I = (\tau_1+\eta) \quad (24)$$

$$\tau_D = \frac{\tau_1\eta}{(\tau_1+\eta)} \quad (25)$$

$$\tau_f = p \quad (26)$$

$$\eta = \frac{\lambda^2+(2\lambda+L)\tau_2}{\tau_2-L} \quad (27)$$

2.2. Stability analysis method

The process transfer function for unstable SOPTD with a negative zero is given by Eq (1). The transfer function of the controller is given by

$$G_c(s) = K_c'(1+\frac{1}{\tau_I' s})(1+\tau_D' s)(\frac{1}{\alpha s+1}) \quad (28)$$

The combined transfer function model of the process and the controller is given by

$$G_c(s)G_p(s) = \frac{K_P(1+ps)e^{-Ls}}{(\tau_1 s+1)(\tau_2 s-1)}K_c'(1+\frac{1}{\tau_I' s})(1+\tau_D' s)(\frac{1}{\alpha s+1}) \quad (29)$$

Let $\tau_D' = \tau_1$ and $\alpha = p$, then Eq (29) is written as

$$G_c(s)G_p(s) = \frac{K_c' K_p(1+\frac{1}{\tau_I' s})e^{-Ls}}{\tau_2 s-1} \quad (30)$$

Where ω_c is crossover frequency and $\dfrac{5}{\omega_c}$ is given by τ_I'. Phase angle criteria for openloop transfer function [Eq (30)] is given by

$$\tan^{-1}(\frac{-1}{\omega_c\tau_I'}) - L\omega_c + \tan^{-1}(\omega_c\tau_2) = 0 \quad (31)$$

$$\tan^{-1}(-0.2) - L\omega_c + \tan^{-1}(\omega_c\tau_2) = 0 \quad (32)$$

Eq (32) is solved by using *fsolve* of MATLAB for ω_c. Using amplitude criteria, K_c' is given by

$$K_c' = \frac{\sqrt{1+\omega_c^2\tau_2^2}}{k_p\sqrt{1+(\frac{1}{\omega_c\tau_I'})^2}} = \frac{\sqrt{1+\omega_c^2\tau_2^2}}{k_p\sqrt{1+(0.2)^2}} \quad (33)$$

Eq (28) is compared with the conventional PID controller with a lag filter [Eq (22)] and the conventional PID settings are given by

$$K_c = \frac{K_c'(\tau_I'+\tau_D')}{\tau_I'} \quad (34)$$

$$\tau_I = \tau_I' + \tau_D' \quad (35)$$

$$\tau_D = \frac{\tau_I'\tau_D'}{\tau_I'+\tau_D'} \quad (36)$$

$$\alpha = p \quad (37)$$

3. Simulation results

In this section, proposed PID controller designed by IMC method (IMC-PID controller) and proposed PID controller designed by stability analysis method (SA-PID controller) are applied to various CSTR transfer function models and non-linear models to show the efficiency of the proposed controllers. The performance of the proposed controllers is compared with PID controller designed by synthesis method [3] (SM-PID controller).

Case study-1

Consider a jacketed CSTR carrying first order irreversible exothermic reaction. The heat of reaction is removed by a coolant in the jacket to maintain temperature of reaction. The model equations of jacketed CSTR without jacket dynamics [1] are,

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - K_0 \exp(-\frac{E_a}{RT})C_A \quad (38)$$

$$\frac{dT}{dt} = \frac{F}{V}(T_f - T) + (\frac{-\Delta H}{\rho C_p})K_0 \exp(\frac{-E_a}{RT})C_A - \frac{UA}{V\rho C_p}(T - T_j) \quad (39)$$

The above non-linear model equations are linearised around the unstable operating point C_A=0.0644 lb.mol/ft^3, T=560.77^{0}R and T_j=523.0122^{0}R. The values of the process parameters are given Table 1

Table 1. Parameter values of Jacketed CSTR for case study 1

Feed concentration, C_{Af}	0.132 lb.mol/ft^3
Feed temperature, T_f	519.67^0 R
Reactor volume(nominal)	668 ft^3
Heat transfer area	309 ft^2
Diameter	7.5 ft
Operating volume	500 ft^3
Operating flow rate	2000 ft^3/hr
Over all heat transfer coefficient, U	75 Btu/hr ft^2 ^{0}F
Activation energy, E_a	32400 Btu/lb.mol
Frequency factor,K_0	16.96×10^{12} hr^{-1}
Heat of reaction,-ΔH	39000 Btu/lb.mol
Product of density and specific heat, ρ C_P	53.25Btu/ft^3 ^{0}F
Ideal gas constant, R	1.987 Btu/lb.mol ^{0}R

The process transfer function relating the reactor temperature to the jacket temperature along with a measurement delay of 0.0317 hr is given by

$$\frac{T(s)}{T_J(s)} = \frac{0.82055s + 6.5565}{0.9416s^2 + 2..6977s - 1} e^{-0.0317s} \quad (40)$$

The IMC-PID controller, SA-PID controller and SM-PID controller parameters are given in Table 2. The servo and regulatory response with these controllers are shown in Figure1.

Table 2. PID settings for different methods

Case study	Controller	K_C	τ_I	τ_D	α_0	α_1	α_2
1	IMC-PID	21.6464	0.482	0.1098	0.0159	0.0015	0.1374
	SA-PID	49.4887	0.4288	0.084	---	---	0.12515
	SM-PID	0.5476	4.0146	0.0158	---	0.001	0.0801
2	IMC-PID	3.7102×10^5	2221.4	3.7987	---	---	766.075
	SA-PID	3.0152×10^5	2221.2	3.6363	---	---	766.075
	SM-PID	1.4728×10^5	4.5115	0.4446	---	0.0768	0.3842
3	IMC-PID	31.1037	1.1979	0.1908	0.03	0.0113	0.9636
	SA-PID	28.036	1.1891	0.1857	---	---	0.9518
	SM-PID	6.5786	0.3043	0.027	---	---	0.011
4	IMC-PID	-10.7689	1.7175	0.4288	0.15	0.0049	0.1831
	SA-PID	-6.274	2.043	0.5024	---	---	0.1507
	SM-PID	-0.7265	39.7268	0.1494	---	---	0.15

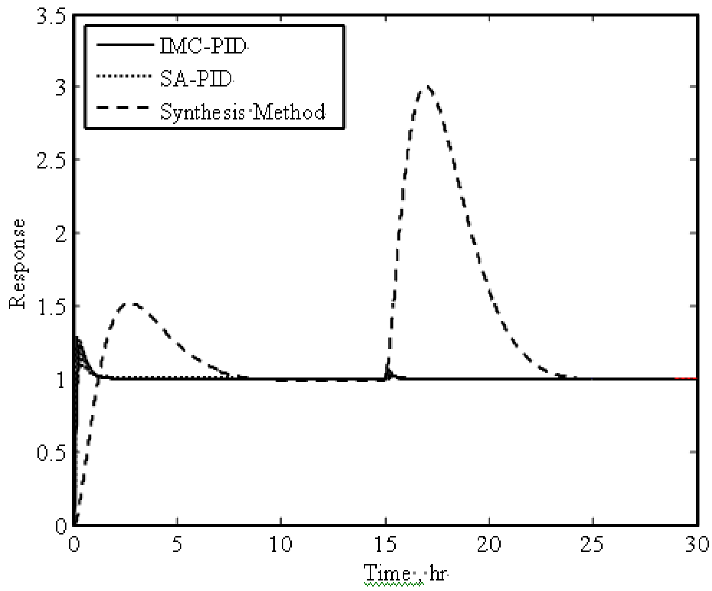

Figure1. Servo and regulatory response of jacketed CSTR (linear model) case study-1

The servo and regulatory performance of the proposed controllers is better than SM-PID controller. The performance comparison in terms ISE, IAE and ITAE for linear model for the servo and regulatory response is given in Table 3.

Table 3. Performance comparison for linear models

Case study	Controller	Servo problem			Regulatory problem		
		ISE	IAE	ITAE	ISE	IAE	ITAE
1	IMC-PID	0.1189	0.244	0.0816	0.000802	0.0223	0.0083
	SA-PID	0.0901	0.1775	0.0501	0.000221	0.0088	0.0027
	SM-PID	0.2626	0.6425	0.9397	10.4706	7.3547	20.9931
2	IMC-PID	2.7442	3.8358	11.641	1.0689×10^{-6}	0.0024	0.0120
	SA-PID	2.8934	3.7007	9.7226	1.5214×10^{-6}	0.0028	0.0133
	SM-PID	2.7084	4.1085	12.943	1.6999×10^{-6}	0.0031	0.0126
3	IMC-PID	0.1645	0.3747	0.287	0.0048	0.0385	0.0098
	SA-PID	0.1893	0.4063	0.2856	0.0083	0.0544	0.0162
	SM-PID	0.1506	0.2448	0.0506	0.0058	0.0463	0.0125
4	IMC-PID	0.6833	1.2014	1.3637	0.0107	0.1595	0.2580
	SA-PID	1.1263	1.6892	2.2689	0.0461	0.3256	0.5682
	SM-PID	37.216	30.248	415.06	128.447	54.683	720.176

The simulation of non-linear model equations [Eq (38) and Eq (39)] of the jacketed CSTR with the proposed IMC-PID controller, SA-PID controller and SM-PID controller for servo response (change in reactor temperature 560.77 to 563^0 R) is shown in Figure2.

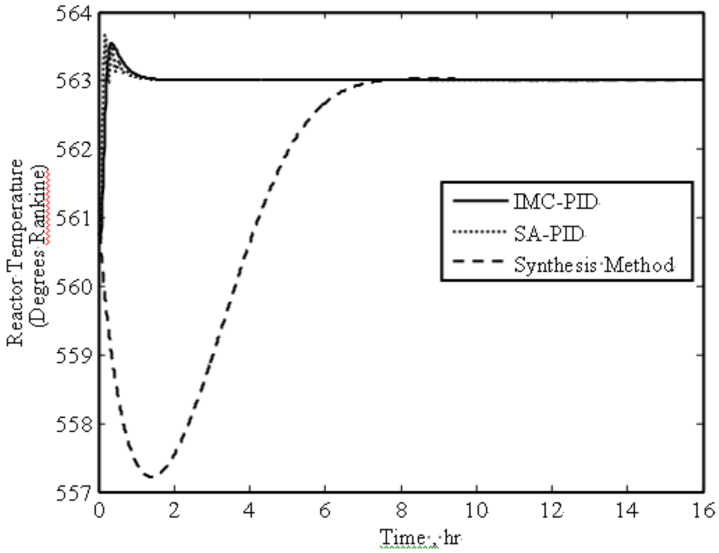

Figure 2. Servo response of jacketed CSTR (T=560.77 to 563^0 R)

The regulatory response of jacketed CSTR for the change in feed temperature from 519.67 to 523^0 R, change in feed concentration from 0.132 to 0.134 lb.mol/ft^3, change in jacket temperature from 524.67 to 520^0 R and change in ratio F/V from 4 to 4.2 hr^{-1} are shown in Figure 3, Figure 4, Figure 5, and Figure 6 respectively.

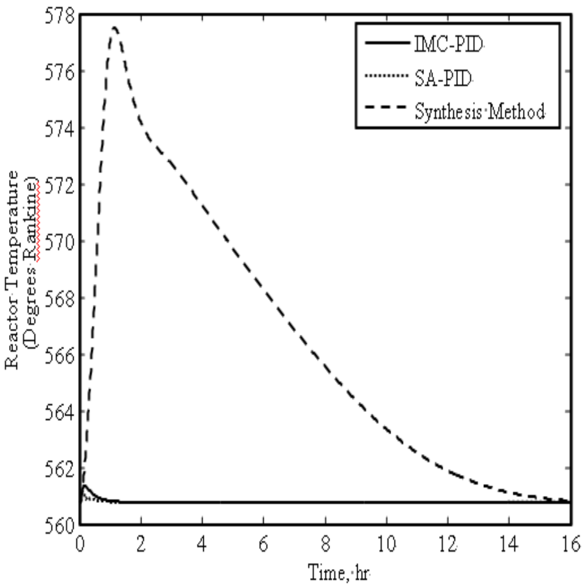

Figure 3. Regulatory response of jacketed CSTR for change in feed temperature (T_f=519.67 to 523^0 R) for non-linear model in case study-1

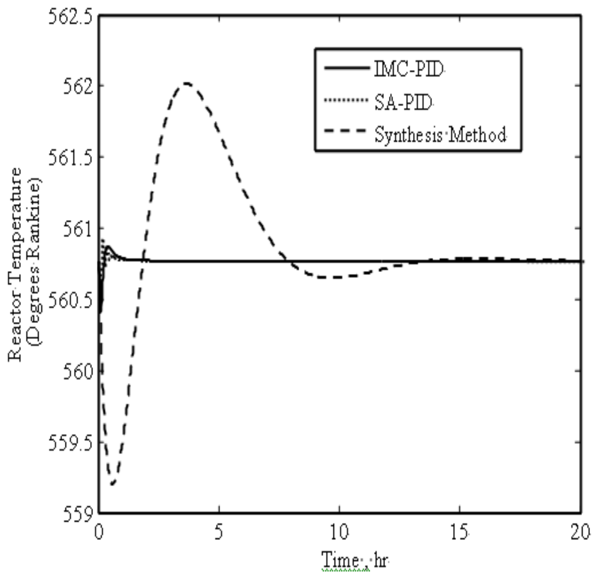

Figure 4. Regulatory response of jacketed CSTR for change in feed concentration (C_{Af}=0.132 to 0.134 lb.mol/ft^3) or non-linear model in case study-1

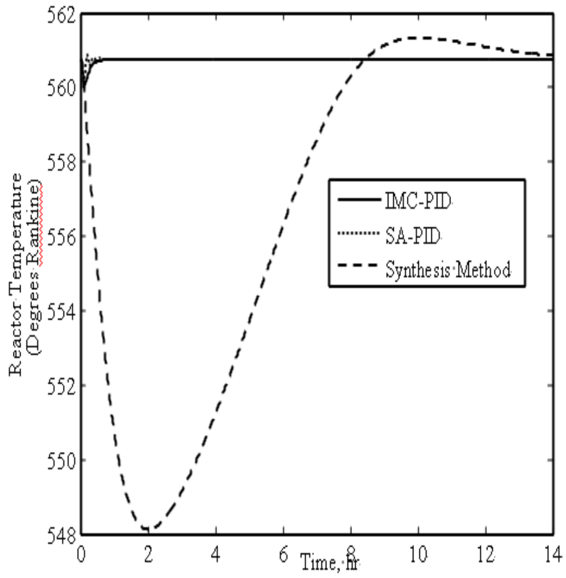

Figure 5. Regulatory problem of jacketed CSTR for change in jacket temperature(T_j=524.67 to 520^0 R) for non-linear model in case study-1

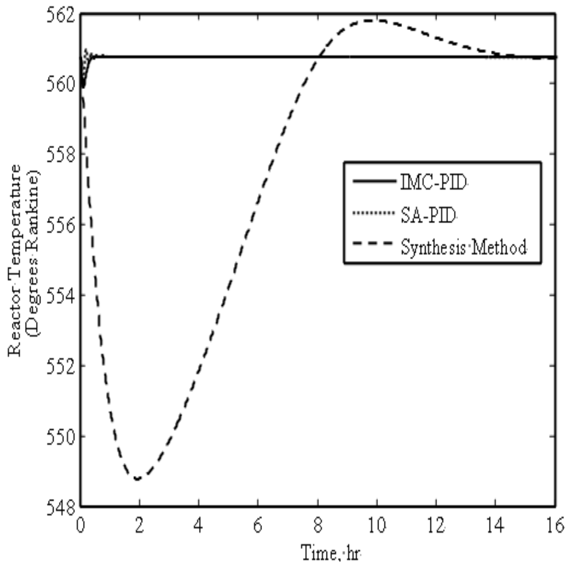

Figure 6. Regulatory problem of jacketed CSTR for change in feed rate to volume of reactor (F/V=4 to 4.2 min^{-1}) for non-linearmodel in case study-1

The performance of these three controllers for nonlinear models in terms of ISE, IAE and ITAE for both servo and regulatory problems is given in Table 4. The performance of proposed controllers is superior to the SM-PID controller. The robustness of the IMC-PID controller, SA-PID controller and SM-PID controller is studied for the model uncertainties considering $\pm$ 20% perturbation in parameters such as ΔH, K_0, and U considering one parameter at a time. The performance is reported in terms of ISE, IAE and ITAE values in Table 5.

The controllers are designed based on nominal model parameters and applied to the process with perturbed parameters. For $\pm$ 20% Δ H, the controller designed by synthesis method could not stabilize the jacketed CSTR. The robustness of proposed controllers is better for model uncertainty considering one parameter at a time when compared to SM-PID controller.

Case study-2

Consider a jacketed CSTR carrying first order irreversible exothermic reaction. The model equations of jacketed CSTR without jacket dynamics [7] are given by

$$\frac{dC_A}{dt} = \frac{F}{V}(C_{A0} - C_A) - K_0 C_A \exp(-\frac{E_a}{RT}) \quad (41)$$

Table 4. Performance comparison for non-linear model (case study-1)

Performance	Controller	Change in reactor temperature	Change in feed temperature	Change in feed concentration	Change in jacket temperature	Change in F/V
		Servo problem	Regulatory problem			
ISE	IMC-PID	0.7264	0.0993	0.0148	0.0714	0.0922
	SA-PID	0.5103	0.0198	0.0080	0.0248	0.0387
	SM-PID	89.2459	882.5869	6.0840	560.8599	509.029
IAE	IMC-PID	0.5945	0.272	0.0935	0.1728	0.1587
	SA-PID	0.4202	0.1058	0.0539	0.0790	0.0965
	SM-PID	20.5838	89.5717	6.3562	60.8603	59.2889
ITAE	IMC-PID	0.1882	0.116	0.0382	0.0487	0.0299
	SA-PID	0.1136	0.0392	0.0165	0.0183	0.0200
	SM-PID	46.837	399.4815	23.6052	211.7889	214.049

Table 5. Performance comparison under model uncertainty for non-linear model in case study-1

performance	Controller	+20%ΔH	-20% ΔH	+20% K_0	-20% K_0	+20% U	-20% U
ISE	IMC-PID	1.4545	1.694	0.4377	0.9449	0.0780	0.0179
	SA-PID	0.3466	0.449	0.177	0.3214	0.0384	0.0031
	SM-PID	unstable	unstable	1305.5	3118.6	478.44	403.85
IAE	IMC-PID	0.9194	1.057	0.3809	0.6183	0.1913	0.1174
	SA-PID	0.3602	0.411	0.2236	0.2635	0.1404	0.0471
	Synthesis	unstable	unstable	125.42	203.68	62.987	79.63
ITAE	IMC-PID	0.3320	0.407	0.0945	0.1616	0.0592	0.0611
	SA-PID	0.1160	0.128	0.0534	0.0536	0.0578	0.0217
	Synthesis	unstable	unstable	707.09	1260.1	266.23	847.49

$$\frac{dT}{dt} = \frac{F}{V}(T_0 - T) + \frac{(-\Delta H)K_0 C_A}{\rho C_p}\exp\left(-\frac{E_a}{RT}\right) + \frac{UA}{V\rho C_p}(T_j - T) \tag{42}$$

The above non-linear model equations are linearised around unstable operating point C_A =3.734 Kmol/m^3, T=344^0 K and T_j=317.4^0 K. The values of the parameters are given in Table 6.

Table 6. Parameter values of Jacketed CSTR for case study 2

Feed concentration, C_{A0}	7.5Kmol/m^3
Feed temperature, T_0	300 ^{0}K
Density	850Kg/m^3
Volume	1m^3
Inlet flow rate	0.00065m^3/sec
Over all heat transfer coefficient, UA	1.45J/s ^{0}K
Activation energy, E_a	69000KJ/Kmol
Frequency factor,K_0	1.87×10^7s^{-1}
Heat of reaction,- ΔH	50000KJ/Kmol
specific heat, C_P	3.5KJ/kg ^{0}K
Ideal gas constant, R	8.345KJ/Kmol ^{0}K

The transfer function relating the reactor temperature to the jacket temperature along with a measurement time delay of one second is given by

$$\frac{T(s)}{T_j(s)}=\frac{0.0004706s+6.143\times10^{-7}}{s^2-0.0004483s-4.055\times10^{-7}}e^{-1s} \quad (43)$$

The IMC-PID controller, SA-PID controller, and SM-PID controller parameters are shown in Table 2. The servo and regulatory response of IMC-PID controller, SA-PID controller and SM-PID controller for linear model are shown in Figure 7 and Figure 8 respectively.The performance of the proposed controllers is better than the SM-PID controller for both servo and regulatory problem. The performance comparison in terms ISE, IAE and ITAE for linear model for the servo and regulatory problem is given Table 3.

The simulation of non-linear model equations [Eq (41), Eq (42)] of jacketed CSTR with IMC-PID controller, SA-PID controller and SM-PID controller for servo problem (change in reactor temperature from 344 to 346^0 K) is shown in Figure 9. The regulatory response of jacketed CSTR for change in feed temperature from 300 to 305^0 K is shown in Figure 10. The regulatory response of jacketed CSTR for change in initial concentration from 7.5 to 7.7 kmol/m^3, change in jacket temperature from 317.4 to 319^0 K and change in F/V from 0.00065 to 0.0007 s^{-1} is also studied.

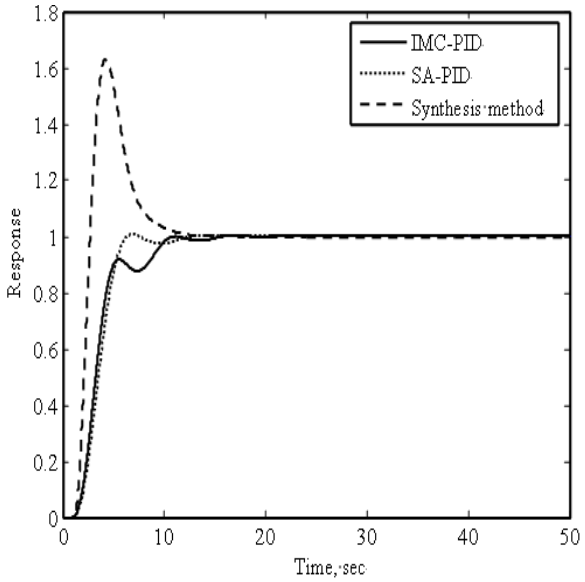

Figure 7. Servo response of jacketed CSTR for linear model in case study-2

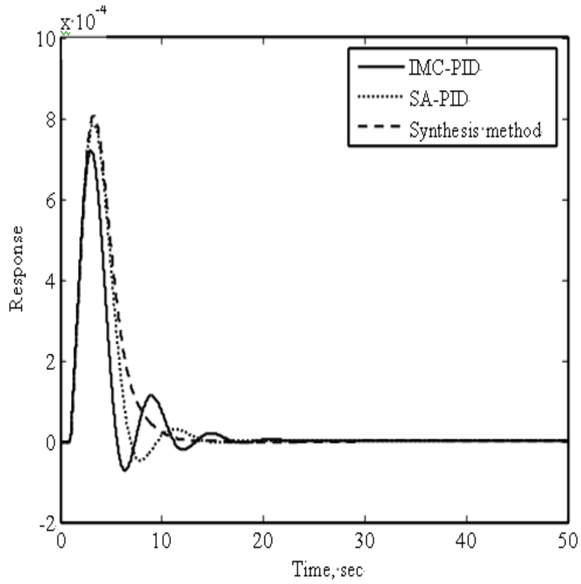

Figure 8. Regulatory response of jacketed CSTR for linear model in case study-2

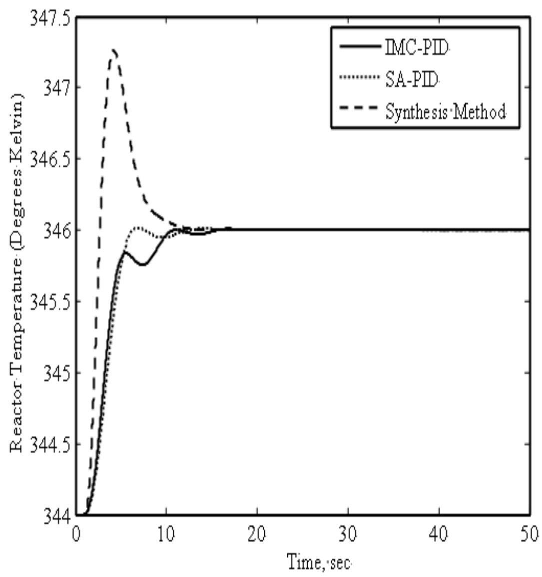

Figure 9. Servo response of jacketed CSTR ($T=344$ to 346^0 K) for non-linear model in case study-2

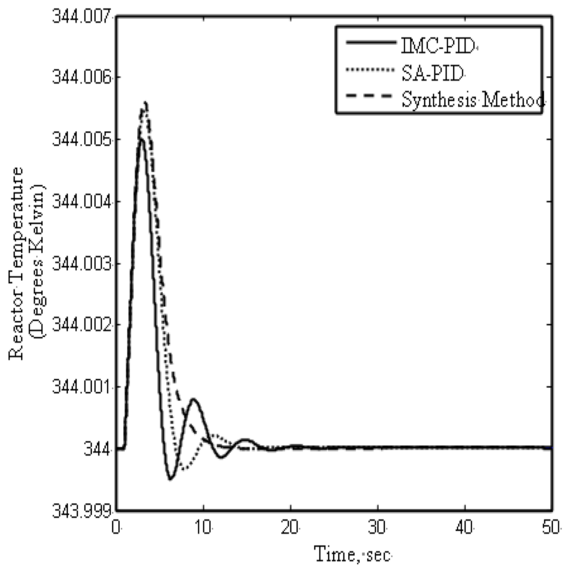

Figure 10. Regulatory problem of jacketed CSTR for change in feed temperature ($T_f=300$ to 305^0 K) for non-linear model in case study-2

Table 7. Performance comparison for non-linear model in case study-2

Performance	Controller	Change in temperature	Change in feed temperature	Change in feed concentration	Change in jacket temperature	Change in F/V
		Servo problem	Regulatory problem			
ISE	IMC-PID	10.9761	5.1421×10^{-5}	3.1342×10^{-9}	2.8393×10^{-6}	2.297×10^{-5}
	SA-PID	11.573	7.3194×10^{-5}	4.688×10^{-9}	4.0415×10^{-6}	3.268×10^{-5}
	SM-PID	10.8417	8.1778×10^{-5}	6.0684×10^{-9}	4.5156×10^{-6}	3.649×10^{-5}
IAE	IMC-PID	7.6701	0.0165	3.53×10^{-4}	0.0039	0.0108
	SA-PID	7.4008	0.0195	4.356×10^{-4}	0.0046	0.0127
	SM-PID	8.2196	0.0213	5.082×10^{-4}	0.0050	0.0146
ITAE	IMC-PID	23.2699	0.0831	0.0080	0.0196	0.0481
	SA-PID	19.4397	0.0921	0.0099	0.0217	0.0517
	SM-PID	25.8931	0.0878	0.0115	0.0207	0.0727

Table 8. Performance under model uncertainty for non-linear model in case study-2

Performance	Controller	+20% ΔH	-20% ΔH	+20% E_a	-10% E_a	+20% K_0	-20% K_0	+20% U	-20% U
ISE	IMC-PID	10.901	11.05	11.358	8.131	10.901	11.052	10.599	11.673
	SA-PID	11.482	11.66	12.039	8.273	11.482	11.665	10.959	12.535
	SM-PID	10.909	10.78	10.529	15.969	10.909	10.776	11.114	11.301
IAE	IMC-PID	7.6299	7.711	7.8708	6.092	7.6292	7.711	7.919	7.663
	SA-PID	7.355	7.447	7.632	6.096	7.354	7.448	7.385	8.342
	SM-PID	8.258	8.181	8.031	10.969	8.259	8.181	7.826	9.256
ITAE	IMC-PID	23.138	23.41	23.946	23.413	23.103	23.427	28.354	21.192
	SA-PID	19.311	19.57	20.109	24.433	19.261	19.604	20.976	25.054
	SM-PID	26.096	25.69	24.894	57.739	26.147	25.681	23.003	36.71

The performance of the proposed controllers for nonlinear models in terms of ISE, IAE, and ITAE for both servo and regulatory problems is given in Table 7. For both servo and regulatory problem, IMC-PID controller and SA-PID controller gives better response compared to SM-PID controller. The robustness of the IMC-PID controller, SA-PID controller and SM-PID controller is studied for the model uncertainties considering ± 20% perturbation in parameters such as ΔH, E_a, K_0, and U considering one parameter at a time. The performance is reported in terms of ISE, IAE and ITAE values in Table 8. The controllers are designed based on nominal model parameters and applied to the process with perturbed parameters. The proposed IMC-PID, SA-PID and SM-PID controllers are robust under model uncertainty considering one parameter at a time.

Case study-3

Consider a constant volume jacketed CSTR carrying first order irreversible exothermic reaction. The model equations of jacketed CSTR without jacket dynamics [8] are given by

$$\frac{dC_A}{dt} = \frac{q}{V}(C_{A0} - C_A) - K_0 C_A \exp(-\frac{E_a}{RT}) \quad (44)$$

$$\frac{dT}{dt} = \frac{q}{V}(T_0 - T) + \frac{(-\Delta H)K_0 C_A}{\rho C_p}\exp(-\frac{E_a}{RT}) + \frac{UA}{V\rho C_p}(T_j - T) \quad (45)$$

The above non-linear model equations are linearised around unstable operating point C_A =0.4758mol/l, T=312.7316^0 K and T_j=300.0^0 K. The values of the parameters are given in Table 9.

Table 9. Parameter values of jacketed CSTR for case study -3

Volume (V)	100 l
Inlet flow rate (q)	100 l/min
Feed temperature (T_0)	350^0 K
Feed concentration (C_{A0})	0.5 mol/l
Overall heat transfer coefficient (UA)	5×10^4
Specific heat (C_p)	0.239 J/(g ^{0}K)
Heat of reaction (-ΔH)	-50000 J/mol
Activation energy (E/R)	8750^0 K
Frequency factor (K_0)	7.2×10^{10}
Density (ρ)	1000 g/l

The transfer function relating the reactor temperature to the jacket temperature along with a measurement time delay of 0.06 min is given by

$$\frac{T(s)}{T_j(s)} = \frac{2.092s + 2.198}{s^2 - 0.4939s - 1.6} e^{-0.06s} \tag{46}$$

The IMC-PID controller, SA-PID controller, and SM-PID controller parameters are shown in Table 2. The servo and regulatory response of IMC-PID controller, SA-PID controller and SM-PID controller for linear model are shown in Figure 11. The servo performance of SM-PID controller shows a large overshoot compared to the proposed controllers. For regulatory problem, proposed controllers perform better than SM-PID controller. The performance comparison in terms ISE, IAE and ITAE for linear model for the servo and regulatory problem is given Table 3. The simulation of non-linear model equations [Eq (44), Eq (45)] of jacketed CSTR with IMC-PID controller, SA-PID controller and SM-PID controller for servo problem (change in reactor temperature from 312.73 to 314^0 K) is shown in Figure 12.

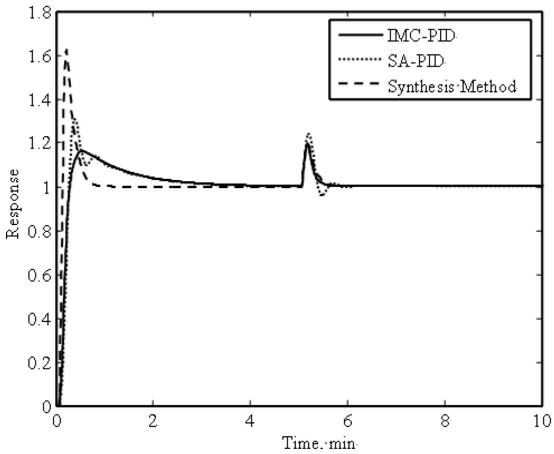

Figure 11. Servo and regulatory response of jacketed CSTR for linear model in case study-3

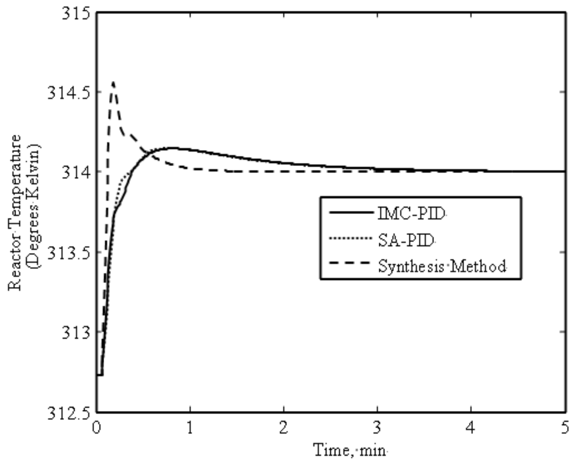

Figure 12. Servo response of jacketed CSTR (T=312.73 to 314^0 K) for non-linear model in case study-3

The regulatory response of jacketed CSTR for change in feed concentration from 0.4758 to 0.6 mol/l is shown in Figure 13. The regulatory response of jacketed CSTR for change in jacket temperature from 300 to 290^0K, change in coolant flow rate from 100 to 110 l/min, and change in feed temperature from 350 to 340^0 K is also studied.

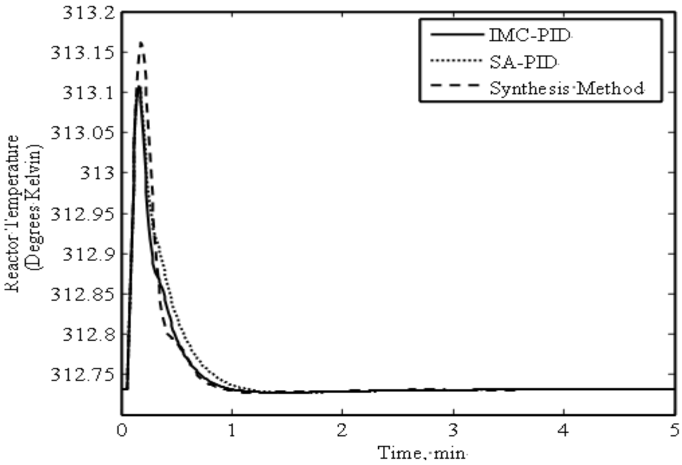

Figure 13. Regulatory response of jacketed CSTR for change in feed concentration (C_{Af}=0.4758 to 0.6 mol/l) for non-linear model in case study-3

The performance of the proposed controllers for nonlinear models in terms of ISE, IAE, and ITAE for both servo and regulatory problems is given in Table 10. For servo problem, PID controller designed by synthesis method gives best performance in terms of ISE, IAE and ITAE. However, the response shows large overshoot (refer to Figure 11 and Figure 12). The controllers designed by proposed methods give less overshoot for servo response.

Table 10. Performance comparison for non-linear model in case study-3

Performance	Controller	Change in temperature	Change in feed temperature	Change in feed concentration	Change in jacket temperature	Change in q
		Servo problem	Regulatory problem			
ISE	IMC-PID	0.2128	0.0133	0.0195	0.1591	0.058
	SA-PID	0.2198	0.0184	0.0273	0.2213	0.0802
	SM-PID	0.1844	0.0161	0.0235	0.194	0.0705
IAE	IMC-PID	0.4232	0.0815	0.0967	0.2824	0.1707
	SA-PID	0.4146	0.0897	0.1074	0.3111	0.1881
	SM-PID	0.2728	0.0979	0.1159	0.3393	0.2052
ITAE	IMC-PID	0.3557	0.0285	0.0395	0.0993	0.0598
	SA-PID	0.3438	0.0296	0.0424	0.1026	0.062
	SM-PID	0.0687	0.0374	0.0521	0.1295	0.0786

The regulatory response of the IMC-PID controller is the best when compared with the controller designed by synthesis method. The robustness of the IMC-PID controller, SA-PID controller and SM-PID controller is studied for the model uncertainties considering $\pm 20\%$ perturbation in parameters such as ΔH, K_0, E_a and U considering one parameter at a time.

The performance is reported in terms of ISE, IAE and ITAE values in Table 11. The controllers are designed based on nominal model parameters and applied to the process with perturbed parameters. The proposed controllers are robust for model uncertainty considering one parameter at a time compared to SM-PID controller.

Table 11. Performance under model uncertainty for non-linear model in case study-3

Performance	Controller	+20% ΔH	-20% ΔH	+20% E_a	-10% E_a	+20% K_0	-20% K_0	+20% U	-20% U
ISE	IMC-PID	0.2229	0.205	0.1873	3.3782	0.2228	0.2043	0.2653	0.1991
	SA-PID	0.1754	0.196	0.2643	2.8519	0.1756	0.1956	0.1596	0.2943
	SM-PID	0.1754	0.196	0.2643	2.8475	0.1756	0.1956	0.1596	0.2994
IAE	IMC-PID	0.432	0.416	0.4056	1.6118	0.4330	0.4149	0.4622	0.4466
	SA-PID	0.252	0.294	0.3800	1.4749	0.2535	0.2927	0.1788	0.4301
	SM-PID	0.252	0.294	0.38	1.4687	0.2583	0.2928	0.1788	0.4302
ITAE	IMC-PID	0.3559	0.356	0.366	0.9644	0.3574	0.3553	0.3513	0.3891
	SA-PID	0.0599	0.077	0.1113	0.7745	0.0619	0.0763	0.0241	0.1411
	SM-PID	0.0599	0.077	0.1113	0.7478	0.0621	0.0764	0.0242	0.1413

Case study-4

The following transfer function model is considered[6].

$$G_P(s) = \frac{(0.3119s + 2.07)e^{-0.3s}}{-2.85s^2 - 2.31s + 1} \tag{47}$$

The PID parameters of the IMC-PID controller, SA-PID controller, and SM-PID controller are given in Table 2. The servo and regulatory response of the process transfer function model with IMC-PID controller, SA-PID controller and SM-PID controller are shown in Figure 14. The performance of IMC-PID controller and SA-PID controller for the servo and regulatory problem are better than the SM-PID controller. The performance comparison of IMC-PID controller, SA-PID controller, and SM-PID controller in terms of ISE, IAE, and ITAE is shown in Table 3.

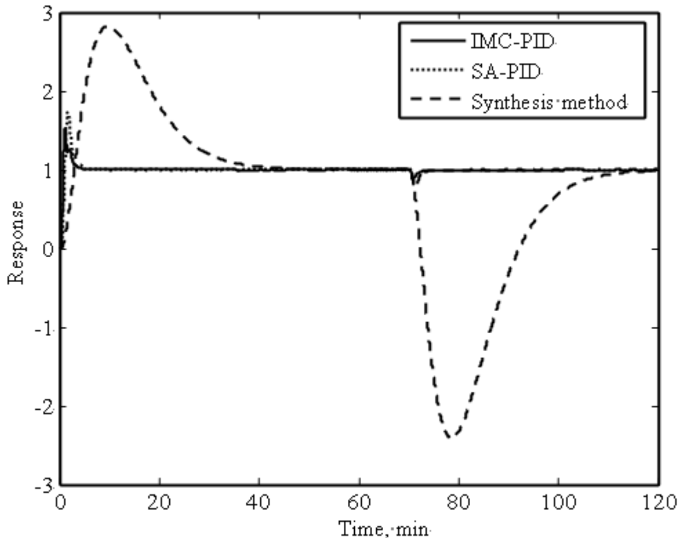

Figure 14. Servo and regulatory response for case study - 4

4. Conclusions

Two PID controller design methods based on internal control principles and stability analysis method for unstable SOPTDZ system is proposed. The controllers designed by IMC method and stability analysis method are used to control the various unstable jacketed stirred tank reactors carrying out first order irreversible reaction. Simple equations for the PID parameters are given in terms of process model parameters. The performance of the controllers designed by IMC method and stability analysis method is compared with the controller designed by synthesis method. The proposed controller performs better for both servo and regulatory problems than the controller designed by synthesis method. The performance of controllers designed by IMC method and stability analysis method for model uncertainty considering one parameter at a time is studied. When there is ±20% uncertainty in ΔH, the controller designed by synthesis method could not stabilize the reactor (Case study 1). The controllers designed by proposed methods give robust performance. The performance comparison for model uncertainty is given in terms of ISE, IAE and ITAE for both servo and regulatory problems. Simulation results on non-linear model equations of jacketed CSTR carrying out irreversible first order chemical reaction show that the controllers designed by proposed methods perform better than the controllers designed by synthesis method.

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