Assignment and Travelling Salesman Problems with Coefficients as *LR* Fuzzy Parameters

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Abstract: Mukherjee and Basu (Application of fuzzy ranking method for solving assignment problems with fuzzy costs, International Journal of Computational and Applied Mathematics, 5, 2010, 359-368) proposed a new method for solving fuzzy assignment problems. In this paper, some fuzzy assignment problems and fuzzy travelling salesman problems are chosen which cannot be solved by using the fore-mentioned method. Two new methods are proposed for solving such type of fuzzy assignment problems and fuzzy travelling salesman problems. The fuzzy assignment problems and fuzzy travelling salesman problems which can be solved by using the existing method, can also be solved by using the proposed methods. But, there exist certain fuzzy assignment problems and fuzzy travelling salesman problems which can be solved only by using the proposed methods. To illustrate the proposed methods, a fuzzy assignment problem and a fuzzy travelling salesman problem is solved. The proposed methods are easy to understand and apply to find optimal solution of fuzzy assignment problems and fuzzy travelling salesman problems occurring in real life situations.

Keywords: Fuzzy assignment problem; fuzzy travelling salesman problem; Yager's ranking index; *LR* fuzzy number.

1. Introduction

The assignment problem is a special type of linear programming problem in which our objective is to assign n number of jobs to n number of persons at a minimum cost (time). The mathematical formulation of the problem suggests that this is a 0-1 programming problem and is highly degenerate. All the algorithms developed to find optimal solution of transportation problems are applicable to assignment problem. However, due to its highly degeneracy nature, a specially designed algorithm widely known as Hungarian method proposed by Kuhn [10] is used for its solution. Examples of these types of problems are assigning men to offices, crew (drivers and conductors) to buses, trucks to delivery routes etc. Over the past 50 years, many variations of the classical assignment problem are proposed e.g. bottleneck assignment problem, generalized assignment problem, quadratic assignment problem etc.

However, in real life situations, the parameters of assignment problem are imprecise numbers instead of fixed real numbers because time/cost for doing a job by a facility (machine/person) might vary due to different reasons. Zadeh [24] introduced the concept of fuzzy sets to deal with imprecision and vagueness in real life situations.

Since then, significant advances have been made in developing numerous methodologies and their applications to various decision problems. Fuzzy assignment problems have received great attention in recent years [5, 8, 11-13, 18, 21, 23].

Travelling salesman problem is a well-known NP-hard problem in combinatorial optimization. In the ordinary form of travelling salesman problem, a map of cities is given to the salesman and he has to visit all the cities only once and return to the starting point to complete the tour in such a way that the length of the tour is the shortest among all possible tours for this map. The data consists of weights assigned to the edges of a finite complete graph and the objective is to find a cycle passing through all the vertices of the graph while having the minimum total weight. There are different approaches for solving travelling salesman problem. Almost every new approach for solving engineering and optimization problems has been tried on travelling salesman problem. Many methods have been developed for solving travelling salesman problem. These methods consist of heuristic methods and population based optimization algorithms etc. Heuristic methods like cutting planes and branch and bound can optimally solve only small problems whereas the heuristic methods such as 2-opt, 3-opt, Markov chain, simulated annealing and tabu search are good for large problems. Population based optimization algorithms are a kind of nature based optimization algorithms. The natural systems and creatures which are working and developing in nature are one of the interesting and valuable sources of inspiration for designing and inventing new systems and algorithms in different fields of science and technology. Particle Swarm Optimization, Neural Networks, Evolutionary Computation, Ant Systems etc. are a few of the problem solving techniques inspired from observing nature. Travelling salesman problems in crisp and fuzzy environment have received great attention in recent years [1-4, 6, 9, 14, 16, 17, 19, 20].

With the use of LR fuzzy numbers, the computational efforts required to solve fuzzy assignment problems and fuzzy travelling salesman problem are considerably reduced [25]. Moreover, all types of crisp numbers, triangular fuzzy numbers and trapezoidal fuzzy numbers can be considered as particular cases of LR fuzzy numbers, thereby extending the scope of use of *LR* fuzzy numbers.

Mukherjee and Basu [15] proposed a new method for solving fuzzy assignment problems. In this paper, some fuzzy assignment problems and fuzzy travelling salesman problems are chosen which cannot be solved by using the fore-mentioned method. Two new methods are proposed for solving such type of fuzzy assignment problems and fuzzy travelling salesman problems. The fuzzy assignment problems and fuzzy travelling salesman problems which can be solved by using the existing method, can also be solved by using the proposed methods. But, there exist certain fuzzy assignment problems and fuzzy travelling salesman problems which can be solved only by using the proposed methods. To illustrate the proposed methods, a fuzzy assignment problem and a fuzzy travelling salesman problem is solved. The proposed methods are easy to understand and apply to find optimal solution of fuzzy assignment problems and fuzzy travelling salesman problems occurring in real life situations.

This paper is organized as follows: In Section 2, basic definitions and Yager's ranking approach for the ranking of fuzzy numbers are discussed. In Section 3, formulations of fuzzy assignment problems and fuzzy travelling salesman problems are presented. In Section 4, limitations of existing method [15] are discussed. In Section 5, to overcome the limitations discussed in Section 4, two new methods are proposed to find optimal solution of fuzzy assignment problems and fuzzy travelling salesman

problems. In Section 6, the advantages of proposed methods over existing method are discussed and illustrated by solving two examples. The results are discussed in Section 7 and the conclusions are discussed in Section 8.]

2. Preliminaries

In this section, some basic definitions and Yager's ranking approach for the ranking of fuzzy numbers are presented.

2.1. Basic definitions

In this section, some basic definitions are presented.

Definition 1. [7] A function $L:[0,\infty)\to[0,1]$ (or $R:[0,\infty)\to[0,1]$) is said to be reference function of fuzzy number if and only if
(i) $\quad L(x)=L(-x)$ (or $R(x)=R(-x)$)
(ii) $\quad L(0)=1$ (or $R(0)=1$)
(iii) $\quad L$ (or R) is non-increasing on $[0,\infty)$.

Definition 2. [7] A fuzzy number $\tilde{A}$ defined on the universal set of real numbers denoted as $\tilde{A}=(m,n,\alpha,\beta)_{LR}$, is said to be an *LR* flat fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by:

$$\mu_{\tilde{A}}(x)=\begin{cases} L\left(\dfrac{m-x}{\alpha}\right), & x\le m,\ \alpha>0 \\[2mm] R\left(\dfrac{x-n}{\beta}\right), & x\ge n,\ \beta>0 \\[2mm] 1, & m\le x\le n \end{cases}$$

Definition 3. [7] Let $\tilde{A}=(m,n,\alpha,\beta)_{LR}$ be an *LR* flat fuzzy number and λ be a real number in the interval [0,1] then the crisp set

$$A_{\lambda}=\{x\in X:\mu_{\tilde{A}}(x)\ge\lambda\}=[m-\alpha L^{-1}(\lambda),n+\beta R^{-1}(\lambda)]$$

is said to be λ-cut of $\tilde{A}$.

2.2. Yager's ranking approach

A number of ranking approaches have been proposed for comparing fuzzy numbers. In this paper, Yager's ranking approach [22] is used for ranking of fuzzy numbers. This approach involves relatively simple computational and is easily understandable. This approach involves a procedure for ordering fuzzy numbers in which a ranking index $\Re(\tilde{A})$ is calculated for an *LR* flat fuzzy number $\tilde{A}=(m,n,\alpha,\beta)_{LR}$ from its λ-cut $A_{\lambda}=[m-\alpha L^{-1}(\lambda),n+\beta R^{-1}(\lambda)]$ according to the following formula:

$$\Re(\tilde{A})=\frac{1}{2}\left(\int_0^1(m-\alpha L^{-1}(\lambda))\,d\lambda+\int_0^1(n+\beta R^{-1}(\lambda))\,d\lambda\right)$$

Let $\tilde{A}$ and $\tilde{B}$ be two *LR* flat fuzzy numbers then

(i) $\quad \tilde{A}\underset{\Re}{\ge}\tilde{B}$ if $\Re(\tilde{A})\ge\Re(\tilde{B})$

(ii) $\quad \tilde{A}\underset{\Re}{>}\tilde{B}$ if $\Re(\tilde{A})>\Re(\tilde{B})$

(iii) $\quad \tilde{A}\underset{\Re}{=}\tilde{B}$ if $\Re(\tilde{A})=\Re(\tilde{B})$

2. 2. 1. Linearity property of Yager's ranking index

Let $\tilde{A}=(m_1,n_1,\alpha_1,\beta_1)_{LR}$ and $\tilde{B}=(m_2,n_2,\alpha_2,\beta_2)_{LR}$ be two *LR* flat fuzzy numbers and k_1,k_2 be two non negative real numbers.

Using Definition 2, the λ-cut A_{λ} and B_{λ} corresponding to $\tilde{A}$ and $\tilde{B}$ are:

$A_{\lambda}=[m_1-\alpha_1 L^{-1}(\lambda),n_1+\beta_1 R^{-1}(\lambda)]$ and $B_{\lambda}=[m_2-\alpha_2 L^{-1}(\lambda),n_2+\beta_2 R^{-1}(\lambda)]$ Using the property, $(\delta_1 A_1+\delta_2 A_2)_{\lambda}=\delta_1(A_1)_{\lambda}+\delta_2(A_2)_{\lambda}\ \forall\ \delta_1,\delta_2\in$ (is set of real numbers), the $\lambda-$cut $(k_1 A+k_2 B)_{\lambda}$ corresponding to $k_1\tilde{A}\oplus k_2\tilde{B}$ is:

$$(k_1 A + k_2 B)_\lambda = [k_1 m_1 + k_2 m_2 - (k_1 \alpha_1 + k_2 \alpha_2) \\ L^{-1}(\lambda), k_1 n_1 + k_2 n_2 \\ + (k_1 \beta_1 + k_2 \beta_2) R^{-1}(\lambda)]$$

Using Section 2.2, the Yager's ranking index $\Re(k_1 \widetilde{A} \oplus k_2 \widetilde{B})$ corresponding to fuzzy number $(k_1 \widetilde{A} \oplus k_2 \widetilde{B})$ is:

$$\Re(k_1 \widetilde{A} \oplus k_2 \widetilde{B}) = \frac{1}{2} k_1 [\int_0^1 (m_1 - \alpha_1 L^{-1}(\lambda)) d\lambda$$

$$+ \int_0^1 (n_1 + \beta_1 R^{-1}(\lambda)) d\lambda$$

$$+ \frac{1}{2} k_2 [\int_0^1 (m_2 - \alpha_2 L^{-1}(\lambda)) d\lambda$$

$$+ \int_0^1 (n_2 + \beta_2 R^{-1}(\lambda)) d\lambda]$$

$$= k_1 \Re(\widetilde{A}) + k_2 \Re(\widetilde{B})$$

Similarly, it can be proved that $\Re(k_1 \widetilde{A} \oplus k_2 \widetilde{B}) = k_1 \Re(\widetilde{A}) + k_2 \Re(\widetilde{B}) \, \forall k_1, k_2 \in R$

3. Linear programming formulations of fuzzy assignment problems and fuzzy traveling salesman problems

In this section, linear programming formulations of fuzzy assignment problems and fuzzy travelling salesman problems are presented [15].

3.1. Linear programming formulation of fuzzy assignment problems

Suppose there are n jobs to be performed and n persons are available for doing these jobs. Assume that each person can do one job at a time and each job can be assigned to one person only. Let $\widetilde{c}_{ij}$ be the fuzzy cost (payment) if j^{th} job is assigned to i^{th} person. The problem is to find an assignment x_{ij} so that the total cost for performing all the jobs is minimum.

The chosen fuzzy assignment problem may

be formulated into the following fuzzy linear programming problem (FLPP):

Minimize $\qquad \sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} x_{ij}$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \qquad j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1, \qquad i = 1, 2, \ldots, n$$

$x_{ij} = 0 \text{ or } 1, \forall i, j \qquad\qquad (P_1)$

where,

$\widetilde{c}_{ij}$ is a trapezoidal fuzzy number.

$\sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} x_{ij} :$

Total fuzzy cost for performing all the jobs.

3.2. Linear programming formulation of fuzzy travelling salesman problems

Suppose a salesman has to visit n cities. He starts from a particular city, visits each city once and then returns to the starting point. The fuzzy travelling costs from i^{th} city to j^{th} city is given by $\widetilde{c}_{ij}$. The objective is to select the sequence (tour) in which the cities are visited in such a way that the total travelling cost is minimum. The chosen fuzzy travelling salesman problem may be formulated into the following FLPP:

Minimize $\qquad \sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} x_{ij}$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n \text{ and } j \neq i \quad (1)$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n \text{ and } i \neq j \quad (2) \quad (P_2)$$

$$x_{ij} + x_{ji} \leq 1, \quad 1 \leq i \neq j \leq n \qquad (3)$$

$$x_{ij} + x_{jk} + x_{ki} \leq 2, \quad 1 \leq i \neq j \neq k \leq n \qquad (4)$$

$$\vdots$$

$$x_{ip_1} + x_{p_1 p_2} + x_{p_2 p_3} + \ldots + x_{p_{(n-2)} i} \leq$$

$$(n-2), 1 \leq i \neq p_1 \neq \ldots \neq p_{(n-2)} \leq n, \qquad (5)$$

where,

$\widetilde{c}_{ij}$ is a trapezoidal fuzzy number.

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} \, x_{ij} :$$

Total fuzzy travelling cost of completing the tour.

$x_{ij} = 1$ if the salesman visits city j immediately after visiting city i and $x_{ij} = 0$ otherwise. Constraints (1) and (2) ensure that each city is visited only once. Constraint (3) is known as subtour elimination constraint and eliminates all 2-city subtours. Constraint (4) eliminates all 3-city subtours. Constraint (5) eliminates all $(n-1)$-city subtours. For a feasible solution of travelling salesman problem, the solution should not contain subtours. So, for a 5-city travelling salesman problem, we should not have subtours of length 2, 3 and 4. For a 6-city travelling salesman problem, we should not have subtours of length 2 3, 4 and 5. Similarly, for a n-city travelling salesman problem, we should not have subtours of length 2 to $(n-1)$.

3.3. Optimal solution of fuzzy assignment problems

The optimal solution of the fuzzy assignment problem (P_1) is the set of non-negative integers $\{x_{ij}\}$ which satisfies the following characteristics:

(i) $\sum_{i=1}^{n} x_{ij} = 1, j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, n.$

(ii) If there exist any set of non-negative integers $\{x'_{ij}\}$ such that

$\sum_{i=1}^{n} x'_{ij} = 1, j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} x'_{ij} = 1, i = 1, 2, \ldots, n,$

then

$$\Re(\sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} \, x_{ij}) \leq \Re(\sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} \, x'_{ij}).$$

3.4. Optimal solution of fuzzy travelling salesman problems

The optimal solution of the fuzzy assignment problem (P_2) is the set of non-negative integers $\{x_{ij}\}$ which satisfies the following characteristics:

(i) $\sum_{i=1}^{n} x_{ij} = 1, j = 1, 2, \ldots, n, \quad j \neq i$ and

$\sum_{j=1}^{n} x_{ij} = 1, i = 1, 2, \ldots, n, \quad i \neq j$ and also

satisfies subtour elimination constraints.

(ii) If there exist any set of non-negative integers $\{x'_{ij}\}$ such that

$\sum_{i=1}^{n} x'_{ij} = 1, j = 1, 2, \ldots, n, \quad j \neq i$ and

$\sum_{j=1}^{n} x'_{ij} = 1, i = 1, 2, \ldots, n, \quad i \neq j$ and also

satisfies subtour elimination constraints,

then $\Re(\sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} \, x_{ij}) \leq \Re(\sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} \, x'_{ij}).$

4. Limitations of existing method

In this section, the limitations of existing method are discussed.

(i) The existing method [15] can be applied only to solve the following type of fuzzy assignment problems:

Example 4.1.

The fuzzy assignment problem, solved by Mukherjee and Basu [15], may be formulated into the following FLPP:

Minimize $(3,5,6,7)x_{11} \oplus (5,8,11,12)x_{12} \oplus (9,10,11,15)x_{13}$
$\oplus (5,8,10,11)x_{14} \oplus (7,8,10,11)x_{21} \oplus (3,5,6,7)x_{22}$
$\oplus (6,8,10,12)x_{23} \oplus (5,8,9,10)x_{24} \oplus (2,4,5,6)x_{31}$
$\oplus (5,7,10,11)x_{32} \oplus (8,11,13,15)x_{33} \oplus (4,6,7,10)x_{34}$
$\oplus (6,8,10,12)x_{41} \oplus (2,5,6,7) x_{42} \oplus (5,7,10,11)x_{43}$
$\oplus (2,4,5,7) x_{44}$

subject to

$x_{11} + x_{12} + x_{13} + x_{14} = 1, \quad x_{11} + x_{21} + x_{31} + x_{41} = 1,$

$x_{21} + x_{22} + x_{23} + x_{24} = 1, \quad x_{12} + x_{22} + x_{32} + x_{42} = 1,$

$x_{31} + x_{32} + x_{33} + x_{34} = 1, \quad x_{13} + x_{23} + x_{33} + x_{43} = 1,$

$x_{41} + x_{42} + x_{43} + x_{44} = 1, \quad x_{14} + x_{24} + x_{34} + x_{44} = 1$

$x_{ij} = 0$ or $1, \forall \ i = 1,2,3,4$ and $j = 1,2,3,4$.

The existing method [15] cannot be used for solving the following type of fuzzy assignment problems:

Minimize $\qquad \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$

subject to

$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n$

$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n \qquad (P_3)$

$x_{ij} = 0$ or $1, \forall i, j$

where,

$\tilde{c}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$:

Fuzzy payment to i^{th} person for doing j^{th} job.

$\sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$:

Total fuzzy cost for performing all the jobs.

Example 4.2.

The fuzzy assignment problem, for which the fuzzy costs are represented by *LR* fuzzy numbers, may be formulated into the following FLPP:

Minimize $\qquad ((9,10,1,3)_{LR})x_{11} \oplus ((6,8,3,5)_{LR})x_{12}$

$\oplus ((8,9,1,3)_{LR})x_{13} \oplus ((9,10,2,4)_{LR})x_{21}$

$\oplus ((10,11,3,1)_{LR})x_{22} \oplus ((4,5,1,3)_{LR})x_{23}$

$\oplus ((7,8,1,3)_{LR})x_{31} \oplus ((10,11,3,4)_{LR})x_{32}$

$\oplus ((7,8,2,3)_{LR})x_{33}$

subject to

$x_{11} + x_{12} + x_{13} = 1 \quad x_{11} + x_{21} + x_{31} = 1,$

$x_{21} + x_{22} + x_{23} = 1, \quad x_{12} + x_{22} + x_{32} = 1,$

$x_{31} + x_{32} + x_{33} = 1, \quad x_{13} + x_{23} + x_{33} = 1$

$x_{ij} = 0$ or $1, \forall \ i = 1,2,3$ and $j = 1,2,3$.

where, $L(x) = $ maximum $\{0, 1 - x^2\}$ and $R(x)$

$\qquad = $ maximum $\{0, 1 - x\}$

(i) The existing method [15] can be used for solving following type of fuzzy travelling salesman problems:

Example 4.3.

The fuzzy travelling salesman problem, solved by existing method [15], may be formulated into the following FLPP:

Minimize $\qquad (5,8,11,12) x_{12} \oplus (9,10,11,15) x_{13}$

$\oplus (5,8,10,11) x_{14} \oplus (7,8,10,11) x_{21}$

$\oplus (6,8,10,12) x_{23} \oplus (5,8,9,10) x_{24}$

$\oplus (2,4,5,6) x_{31} \oplus (5,7,10,11) x_{32}$

$\oplus (4,6,7,10) x_{34} \oplus (6,8,10,12) x_{41}$

$\oplus (2,5,6,7) x_{42} \oplus (5,7,10,11) x_{43}$

subject to

$x_{12} + x_{13} + x_{14} = 1, \quad x_{21} + x_{31} + x_{41} = 1,$

$x_{21} + x_{23} + x_{24} = 1, \quad x_{12} + x_{32} + x_{42} = 1,$

$x_{31} + x_{32} + x_{34} = 1, \quad x_{13} + x_{23} + x_{43} = 1,$

$x_{41} + x_{42} + x_{43} = 1, \quad x_{14} + x_{24} + x_{34} = 1,$

$x_{12} + x_{21} \leq 1, \quad x_{13} + x_{31} \leq 1, \quad x_{14} + x_{41} \leq 1,$

$x_{23} + x_{32} \leq 1, \quad x_{24} + x_{42} \leq 1, \quad x_{34} + x_{43} \leq 1,$

$x_{ij} = 0$ or $1, \forall \ i = 1,2,3,4$ and $j = 1,2,3,4$.

The existing method [15] cannot be used for solving the following type of fuzzy travelling salesman problems:

Minimize $\qquad \sum_{i=1}^{n} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij}$

subject to

$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n$ and $j \neq i$

$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n$ and $i \neq j \qquad (P_4)$

$x_{ij} + x_{ji} \leq 1, \quad 1 \leq i \neq j \leq n$

$x_{ij} + x_{jk} + x_{ki} \leq 2, \quad 1 \leq i \neq j \neq k \leq n$

$\vdots$

$$x_{ip_1} + x_{p_1p_2} + x_{p_2p_3} + \ldots + x_{p_{(n-2)^i}} \leq (n-2), 1$$
$$\leq i \neq p_1 \neq \ldots \neq p_{(n-2)} \leq n,$$

where,

$$\widetilde{c}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}:$$

Fuzzy travelling cost from i^{th} city to j^{th} city.

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \widetilde{c}_{ij} \, x_{ij}:$$

Total fuzzy travelling cost of completing the tour.
$x_{ij} = 1$ if the salesman visits city j immediately after visiting city i and $x_{ij} = 0$ otherwise.

Example 4.4.

The fuzzy travelling salesman problem, for which the fuzzy costs are represented by *LR* fuzzy numbers, may be formulated into the following FLPP:

Minimize $((9,10,1,3)_{LR})x_{12} \oplus ((6,8,3,5)_{LR})x_{13}$
$\oplus ((8,9,1,3\ )_{LR})x_{14} \oplus ((9,10,2,4\ )_{LR})x_{21}$
$\oplus ((10,11,3,1\ )_{LR})x_{23} \oplus ((4,5,1,3\ )_{LR})x_{24}$
$\oplus ((7,8,1,3\ )_{LR})x_{31} \oplus ((10,11,3,4\ )_{LR})x_{32}$
$\oplus ((7,8,2,3\ )_{LR})x_{34} \oplus ((9,10,3,5\ )_{LR})x_{41}$
$\oplus ((9,11,3,4\ )_{LR})x_{42} \oplus ((6,8,1,5)_{LR})x_{43}$

subject to

$x_{12} + x_{13} + x_{14} = 1, \quad x_{21} + x_{31} + x_{41} = 1,$

$x_{21} + x_{23} + x_{24} = 1, \quad x_{12} + x_{32} + x_{42} = 1,$

$x_{31} + x_{32} + x_{34} = 1, \quad x_{13} + x_{23} + x_{43} = 1,$

$x_{41} + x_{42} + x_{43} = 1, \quad x_{14} + x_{24} + x_{34} = 1,$

$x_{12} + x_{21} \leq 1, \quad x_{13} + x_{31} \leq 1, \quad x_{14} + x_{41} \leq 1,$

$x_{23} + x_{32} \leq 1, \quad x_{24} + x_{42} \leq 1, \quad x_{34} + x_{43} \leq 1,$

$x_{ij} = 0$ or $1, \forall\ i = 1,2,3,4$ and $j = 1,2,3,4.$

where,

$L(x) = \text{maximum} \ \{0, 1 - x^2\}$ and
$R(x) = \text{maximum} \ \{0, 1 - x\}$

5. Proposed methods to find the optimal solution of fuzzy assignment problems and fuzzy travelling salesman problems

In this section, two new methods (based on fuzzy linear programming formulation and classical methods) are proposed to find the optimal solution of fuzzy assignment problems and fuzzy travelling salesman problems.

5.1. Method based on fuzzy linear programming formulation (FLPF)

In this section, a new method (based on FLPF) is proposed to find the optimal solution of fuzzy assignment problems and fuzzy travelling salesman problems occurring in real life situations.
The steps of proposed method are as follows:

Step 1 Check that the chosen problem is fuzzy assignment problem or fuzzy travelling salesman problem.
Case (i) If the chosen problem is fuzzy assignment problem, then formulate it as (P_3).
Case (ii) If the chosen problem is fuzzy travelling salesman problem, then formulate it as (P_4).

Step 2 Convert FLPP ((P_3) or (P_4)), obtained in Step 1, into the following crisp linear programming problem:

Minimize $\sum_{i=1}^{n} \sum_{j=1}^{n} \Re(\widetilde{c}_{ij}) x_{ij}$

subject to respective constraints.

Step 3 Solve crisp linear programming problem, obtained in Step 2, to find the optimal solution $\{x_{ij}\}$ and Yager's ranking index $\sum_{i=1}^{n} \sum_{j=1}^{n} \Re(\widetilde{c}_{ij}) x_{ij}$ corresponding to minimum total fuzzy cost.

5.2. Method based on classical methods

In this section, a new method (based on classical methods) is proposed to find the

optimal solution of fuzzy assignment problems or fuzzy travelling salesman problems occurring in real life situations.

The steps of the proposed method are as follow:

Step 1 Check that the chosen problem is fuzzy assignment problem or fuzzy travelling salesman problem.

Case (i) If the chosen problem is fuzzy assignment problem, then formulate it as (P_3). Represent (P_3) into tabular form as shown in Table 1.

Case (ii) If the chosen problem is fuzzy travelling salesman problem, then formulate it as (P_4). Represent (P_4) into tabular form as shown in Table 2.

Step 2 Convert the fuzzy assignment problem or fuzzy travelling salesman problem obtained in Step 1 into crisp problem as follows:

Case (i) For (P_3) construct a new Table 3 as shown below:

Case (ii) For (P_4) construct a new Table 4 as shown below:

Step 3 Solve crisp linear programming problem, obtained in Step 2, to find the optimal solution $\{x_{ij}\}$ and Yager's ranking index $\sum_{i=1}^{n}\sum_{j=1}^{n}\Re(\tilde{c}_{ij})x_{ij}$ corresponding to minimum total fuzzy cost.

Table 1. Fuzzy assignment costs

Job→ Person↓	J_1	J_2	...	J_j	...	J_n
P_1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	...	$\tilde{c}_{1j}$	...	$\tilde{c}_{1n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
P_j	$\tilde{c}_{j1}$	$\tilde{c}_{j2}$	...	$\tilde{c}_{jj}$	...	$\tilde{c}_{jn}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
P_n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	...	$\tilde{c}_{nj}$	...	$\tilde{c}_{nn}$

where, $\tilde{c}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$

Table 2. Fuzzy travelling costs

City→ ↓	1	2	...	j	...	n
1	-	$\tilde{c}_{12}$	...	$\tilde{c}_{1j}$	...	$\tilde{c}_{1n}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
j	$\tilde{c}_{j1}$	$\tilde{c}_{j2}$	...	-	...	$\tilde{c}_{jn}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	$\tilde{c}_{n1}$	$\tilde{c}_{n2}$	...	$\tilde{c}_{nj}$	...	-

where, $\tilde{c}_{ij} = (m_{ij}, n_{ij}, \alpha_{ij}, \beta_{ij})_{LR}$

Table 3. Crisp assignment costs

Job→ Person↓	J_1	J_2	…	J_j	…	J_n
P_1	$\Re(c_{11})$	$\Re(\widetilde{c}_{12})$	…	$\Re(\widetilde{c}_{1j})$	…	$\Re(\widetilde{c}_{1n})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
P_j	$\Re(\widetilde{c}_{j1})$	$\Re(\widetilde{c}_{j2})$	…	$\Re(\widetilde{c}_{jj})$	…	$\Re(\widetilde{c}_{jn})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
P_n	$\Re(\widetilde{c}_{n1})$	$\Re(\widetilde{c}_{n2})$	…	$\Re(\widetilde{c}_{nj})$	…	$\Re(\widetilde{c}_{nn})$

where, $\quad \Re(\widetilde{c}_{ij}) = \dfrac{1}{2}(\int_0^1 (m_{ij} - \alpha_{ij} L^{-1}(\lambda))\, d\lambda + \int_0^1 (n_{ij} + \beta_{ij} R^{-1}(\lambda))\, d\lambda)$

Table 4. Crisp travelling costs

City→ ↓	1	2	…	j	…	n
1	-	$\Re(\widetilde{c}_{12})$	…	$\Re(\widetilde{c}_{1j})$	…	$\Re(\widetilde{c}_{1n})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
j	$\Re(\widetilde{c}_{j1})$	$\Re(\widetilde{c}_{j2})$	…	-	…	$\Re(\widetilde{c}_{jn})$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	$\Re(\widetilde{c}_{n1})$	$\Re(\widetilde{c}_{n2})$	…	$\Re(\widetilde{c}_{nj})$	…	-

where, $\quad \Re(\widetilde{c}_{ij}) = \dfrac{1}{2}(\int_0^1 (m_{ij} - \alpha_{ij} L^{-1}(\lambda))\, d\lambda + \int_0^1 (n_{ij} + \beta_{ij} R^{-1}(\lambda))\, d\lambda)$

6. Advantage of the proposed methods over existing method

In this section, the advantage of the proposed methods over existing method are discussed:

The existing method [15] can be used for solving only such fuzzy assignment problems wherein all the cost parameters are represented by trapezoidal or triangular fuzzy numbers. However, in real life situations, it is not possible to represent all the parameters by trapezoidal or triangular fuzzy numbers i.e., there may exist certain fuzzy assignment problems in which some uncertain parameters are represented by *LR* fuzzy numbers given by model (P_3). The existing method [15] can not be used for solving such fuzzy assignment problems (P_3). The main advantage of both the proposed methods is that these can be used for solving both type of fuzzy assignment problems and in addition, both types of fuzzy travelling salesman problems. To show the advantage of proposed methods over existing methods [15], fuzzy assignment and fuzzy travelling salesman problems chosen in Example 4.2 and Example 4.4, which cannot be solved by using the existing method [15], are solved by using the proposed methods.

6.1. Optimal solution of fuzzy assignment problem

In this section, fuzzy assignment problem chosen in Example 4.2, which cannot be solved by using the existing method, is solved by using the proposed methods.

6.1.1. Optimal solution using the method based on FLPF

The optimal solution of fuzzy assignment problem, chosen in Example 4.2, may be obtained by using the following steps of the proposed method:

Step 1 The FLPF of fuzzy assignment problem, chosen in Example 4.2 is:

Minimize $((9,10,1,3)_{LR})x_{11}$
$\oplus ((6,8,3,5)_{LR})x_{12} \oplus ((8,9,1,3)_{LR})x_{13}$
$\oplus ((9,10,2,4)_{LR})x_{21} \oplus ((10,11,3,1)_{LR})x_{22}$
$\oplus ((4,5,1,3)_{LR})x_{23} \oplus ((7,8,1,3)_{LR})x_{31}$
$\oplus ((10,11,3,4)_{LR})x_{32} \oplus ((7,8,2,3)_{LR})x_{33}$

$x_{11} + x_{12} + x_{13} = 1 \quad x_{11} + x_{21} + x_{31} = 1,$
$x_{21} + x_{22} + x_{23} = 1, \quad x_{12} + x_{22} + x_{32} = 1,$
$x_{31} + x_{32} + x_{33} = 1, \quad x_{13} + x_{23} + x_{33} = 1$
$x_{ij} = 0$ or $1, \forall\ i = 1,2,3$ and $j = 1,2,3.$

Step 2 Using Step 2 of proposed method, the formulated FLPP is converted into the following crisp linear programming problem:

Minimize $(\Re(9,10,1,3)_{LR})x_{11}$
$\oplus (\Re(6,8,3,5)_{LR})x_{12} \oplus (\Re(8,9,1,3)_{LR})x_{13}$
$\oplus (\Re(9,10,2,4)_{LR})x_{21} \oplus (\Re(10,11,3,1)_{LR})x_{22}$
$\oplus (\Re(4,5,1,3)_{LR})x_{23} \oplus (\Re(7,8,1,3)_{LR})x_{31}$
$\oplus (\Re(10,11,3,4)_{LR})x_{32} \oplus (\Re(7,8,2,3)_{LR})x_{33}$

subject to

$x_{11} + x_{12} + x_{13} = 1, \quad x_{11} + x_{21} + x_{31} = 1,$
$x_{21} + x_{22} + x_{23} = 1, \quad x_{12} + x_{22} + x_{32} = 1,$
$x_{31} + x_{32} + x_{33} = 1, \quad x_{13} + x_{23} + x_{33} = 1$
$x_{ij} = 0$ or $1, \forall\ i = 1,2,3$ and $j = 1,2,3.$

Step 3 Using Definition 2 and Section 2.2, the values of $\Re(\tilde{c}_{ij}), \forall i, j$ are

$\Re(\tilde{c}_{11}) = 9.91667, \Re(\tilde{c}_{12}) = 7.25, \Re(\tilde{c}_{13})$
$= 8.91667, \Re(\tilde{c}_{21}) = 9.8333, \Re(\tilde{c}_{22})$
$= 9.75, \Re(\tilde{c}_{23}) = 4.91667, \Re(\tilde{c}_{31})$
$= 7.91667, \Re(\tilde{c}_{32}) = 10.5, \Re(\tilde{c}_{33})$
$= 7.5833.$

Using the values of $\Re(\tilde{c}_{ij})$ the crisp linear programming problem obtained in Step 2, may be written as:

Minimize $(9.91667\ x_{11} + 7.25\ x_{12}$
$+ 8.91667\ x_{13} + 9.8333\ x_{21} + 9.75\ x_{22}$
$+ 4.91667\ x_{23} + 7.91667\ x_{31} + 10.5\ x_{32}$
$+ 7.5833\ x_{33})$

subject to

$x_{11} + x_{12} + x_{13} = 1, \quad x_{11} + x_{21} + x_{31} = 1,$
$x_{21} + x_{22} + x_{23} = 1, \quad x_{12} + x_{22} + x_{32} = 1,$
$x_{31} + x_{32} + x_{33} = 1, \quad x_{13} + x_{23} + x_{33} = 1$
$x_{ij} = 0$ or $1, \forall\ i = 1,2,3$ and $j = 1,2,3.$

Step 4 Solving the crisp linear programming problem, obtained in Step 3, the optimal solution is $x_{12} = 1, x_{23} = 1, x_{31} = 1$ and minimum total fuzzy cost is $(17,21,5,11)_{LR}$. Yager's ranking index corresponding to minimum total fuzzy cost is 20.08334.

6.1.2. Optimal solution using method based on classical assignment method

The optimal solution of fuzzy assignment problem, chosen in Example 4.2, by using the method based on classical assignment method, proposed in Section 5.2, may be obtained as follows:

Step 1 The tabular representation of fuzzy assignment problem chosen in Example 4.2 is:

Step 2 Using Step 2 of proposed method the

fuzzy assignment problem shown in Table 5 may be converted into crisp assignment problem as shown in Table 6.

Step 3 The optimal solution of the crisp linear programming problem, obtained in Step 2, is $x_{12} = 1, x_{23} = 1, x_{31} = 1$ and minimum total fuzzy cost is

$(17,21,5,11)_{LR}$. Yager's ranking index corresponding to minimum total fuzzy cost is 20.08334.

The membership function of the *LR* type fuzzy number representing the minimum total fuzzy cost of the fuzzy assignment problem, chosen in Example 4.2, is shown in Figure 1.

Table 5. Fuzzy costs as *LR* fuzzy numbers

Job→ Person↓	J_1	J_2	J_3
P_1	$(9,10,1,3)_{LR}$	$(6,8,3,5)_{LR}$	$(8,9,1,3)_{LR}$
P_2	$(9,10,2,4)_{LR}$	$(10,11,3,1)_{LR}$	$(4,5,1,3)_{LR}$
P_3	$(7,8,1,3)_{LR}$	$(10,11,3,4)_{LR}$	$(7,8,2,3)_{LR}$

Table 6. Crisp costs

Job→ Person↓	J_1	J_2	J_3
P_1	9.91667	7.25	8.91667
P_2	9.8333	9.75	4.91667
P_3	7.91667	10.5	7.5833
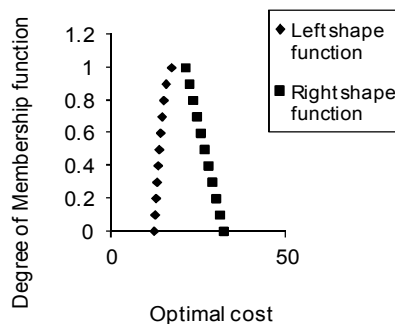
Figure 1. Membership function of *L-R* fuzzy number representing the minimum total fuzzy assignment cost

6.2. Optimal solution of fuzzy travelling salesman problems

In this section, fuzzy travelling salesman problem chosen in Example 4.4, which cannot be solved by using the existing method, is solved by using the proposed methods.

6.2.1. Optimal solution using the method based on FLPF

The optimal solution of fuzzy travelling salesman problem, chosen in Example 4.4, may be obtained by using the following steps of the proposed method:

Step 1 The FLPF of the fuzzy travelling salesman problem, chosen in Example 4.4, is:

Minimize $\qquad ((9,10,1,3)_{LR})x_{12}$
$\oplus ((6,8,3,5)_{LR})x_{13} \oplus ((8,9,1,3)_{LR})x_{14}$
$\oplus ((9,10,2,4)_{LR})x_{21} \oplus ((10,11,3,1)_{LR})x_{23}$
$\oplus ((4,5,1,3)_{LR})x_{24} \oplus ((7,8,1,3)_{LR})x_{31}$
$\oplus ((10,11,3,4)_{LR})x_{32} \oplus ((7,8,2,3)_{LR})x_{34}$
$\oplus ((9,10,3,5)_{LR})x_{41} \oplus ((9,11,3,4)_{LR})x_{42}$
$\oplus ((6,8,1,5)_{LR})x_{43}$

subject to

$x_{12}+x_{13}+x_{14}=1, \quad x_{21}+x_{31}+x_{41}=1,$
$x_{21}+x_{23}+x_{24}=1, \quad x_{12}+x_{32}+x_{42}=1,$
$x_{31}+x_{32}+x_{34}=1, \quad x_{13}+x_{23}+x_{43}=1,$
$x_{41}+x_{42}+x_{43}=1, \quad x_{14}+x_{24}+x_{34}=1,$
$x_{12}+x_{21} \leq 1, \quad x_{13}+x_{31} \leq 1,$
$x_{14}+x_{41} \leq 1, \quad x_{23}+x_{32} \leq 1,$
$x_{24}+x_{42} \leq 1, \quad x_{34}+x_{43} \leq 1,$
$x_{ij}=0$ or $1, \forall \ i=1,2,3,4$ and $j=1,2,3,4.$

Step 2 Using Step 2 of proposed method, the formulated FLPP is converted into the following crisp linear programming problem:

Minimize $\qquad (\Re(9,10,1,3)_{LR})x_{12}$
$+(\Re(6,8,3,5)_{LR})x_{13}+(\Re(8,9,1,3)_{LR})x_{14}$
$+(\Re(9,10,2,4)_{LR})x_{21}+(\Re(10,11,3,1)_{LR})x_{23}$
$+(\Re(4,5,1,3)_{LR})x_{24}+(\Re(7,8,1,3)_{LR})x_{31}$
$+(\Re(10,11,3,4)_{LR})x_{32}+(\Re(7,8,2,3)_{LR})x_{34}$
$+(\Re(9,10,3,5)_{LR})x_{41}+(\Re(9,11,3,4)_{LR})x_{42}$
$+(\Re(6,8,1,5)_{LR})x_{43})$

subject to

$x_{12}+x_{13}+x_{14}=1, \quad x_{21}+x_{31}+x_{41}=1,$
$x_{21}+x_{23}+x_{24}=1, \quad x_{12}+x_{32}+x_{42}=1,$
$x_{31}+x_{32}+x_{34}=1, \quad x_{13}+x_{23}+x_{43}=1,$
$x_{41}+x_{42}+x_{43}=1, \quad x_{14}+x_{24}+x_{34}=1,$
$x_{12}+x_{21} \leq 1, \quad x_{13}+x_{31} \leq 1,$
$x_{14}+x_{41} \leq 1, \quad x_{23}+x_{32} \leq 1,$
$x_{24}+x_{42} \leq 1, \quad x_{34}+x_{43} \leq 1,$
$x_{ij}=0$ or $1, \forall \ i=1,2,3,4$ and $j=1,2,3,4.$

Step 3 Using Definition 2 and Section 2.2, the values of $\Re(\tilde{c}_{ij}), \forall i,j$ are
$\Re(\tilde{c}_{12})=9.91667, \Re(\tilde{c}_{13})=7.25, \Re(\tilde{c}_{14})$
$=8.91667, \Re(\tilde{c}_{21})=9.8333, \Re(\tilde{c}_{23})$
$=9.75, \Re(\tilde{c}_{24})=4.91667, \ \Re(\tilde{c}_{31})$
$=7.91667, \Re(\tilde{c}_{32})=10.5, \Re(\tilde{c}_{34})$
$=7.5833,, \Re(\tilde{c}_{41})=9.75, \Re(\tilde{c}_{42})$
$=10, \Re(\tilde{c}_{43})=7.91667$

Using the values of $\Re(\tilde{c}_{ij})$, the crisp linear programming problem obtained in Step 2, may be written as:

Minimize $\qquad (9.91667 \, x_{12}+7.25 \, x_{13}$
$+8.91667 \, x_{14}+9.8333 \, x_{21}+9.75 \, x_{23}$
$+4.91667 x_{24}+7.91667 \, x_{31}+10.5 \, x_{32}$
$+7.5833 x_{34}+9.75 x_{41}+10 x_{42}+7.91667 x_{43})$

subject to

$x_{12}+x_{13}+x_{14}=1, \quad x_{21}+x_{31}+x_{41}=1,$
$x_{21}+x_{23}+x_{24}=1, \quad x_{12}+x_{32}+x_{42}=1,$
$x_{31}+x_{32}+x_{34}=1, \quad x_{13}+x_{23}+x_{43}=1,$
$x_{41}+x_{42}+x_{43}=1, \quad x_{14}+x_{24}+x_{34}=1,$
$x_{12}+x_{21} \leq 1, \quad x_{13}+x_{31} \leq 1,$
$x_{14}+x_{41} \leq 1, \quad x_{23}+x_{32} \leq 1,$
$x_{24}+x_{42} \leq 1, \quad x_{34}+x_{43} \leq 1,$

$x_{ij}= 0$ or 1, $\forall$ $i=1,2,3,4$ and $j=1,2,3,4$.

Step 4 The optimal solution of the crisp linear programming problem, obtained in Step 3, is

$x_{12} = 1, x_{24} = 1, x_{43} = 1, x_{31} = 1$ and

minimum total fuzzy cost is

$(26,31,4,14)_{LR}$. Yager's ranking index corresponding to minimum total fuzzy cost is 30.6667.

6.2.2. Optimal solution using the method based on classical travelling salesman method

The optimal solution of the fuzzy travelling salesman problem chosen in Example 4.4 by using the method based on classical travelling salesman method, proposed in Section 5.2, can be obtained as follows:

Step 1 The tabular representation of fuzzy travelling salesman problem, chosen in Example 4.4, is:

Step 2 Using Step 2 of proposed method, the travelling salesman problem shown in Table 7 may be converted into crisp travelling salesman problem as shown in Table 8.

Step 3 The optimal solution of the crisp linear programming problem, obtained in Step 2, is

$x_{12} = 1, x_{24} = 1, x_{43} = 1, x_{31} = 1$ and

minimum total fuzzy cost is

$(26,31,4,14)_{LR}$. Yager's ranking index corresponding to minimum total fuzzy cost is 30.6667.

The membership function of the *LR* type fuzzy number representing the minimum total fuzzy cost of the fuzzy travelling salesman problem, chosen in Example 4.4 is shown in Figure 2.

Table 7. Fuzzy costs as *LR* fuzzy numbers

City → ↓	1	2	3	4
1	-	$(9,10,1,3)_{LR}$	$(6,8,3,5)_{LR}$	$(8,9,1,3)_{LR}$
2	$(9,10,2,4)_{LR}$	-	$(10,11,3,1)_{LR}$	$(4,5,1,3)_{LR}$
3	$(7,8,1,3)_{LR}$	$(10,11,3,4)_{LR}$	-	$(7,8,2,3)_{LR}$
4	$(9,10,3,5)_{LR}$	$(9,11,3,4)_{LR}$	$(6,8,1,5)_{LR}$	-

Table 8. Crisp costs

City → ↓	1	2	3	4
1	-	9.91667	7.25	8.91667
2	9.8333	-	9.75	4.91667
3	7.91667	10.5	-	7.5833
4	9.75	10	7.91667	-
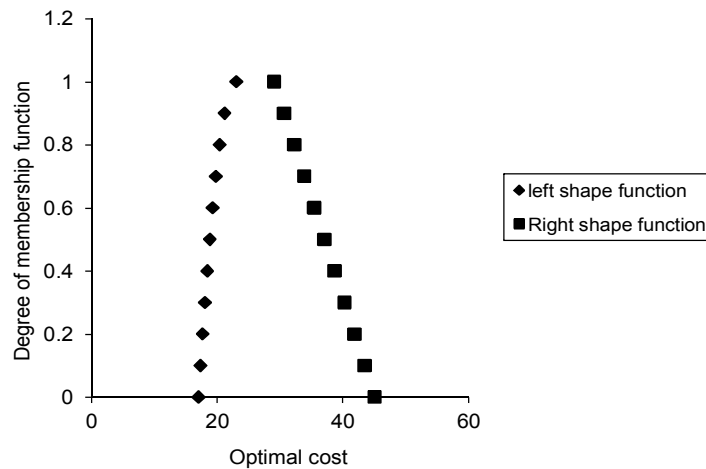
Figure 2. Membership function of *L-R* fuzzy number representing the minimum total fuzzy travelling cost

7. Results and discussion

To compare the existing [15] and the proposed methods, the results of fuzzy assigment problems and fuzzy travelling salesman problems chosen in Example 4.1, Example 4.2, Example 4.3 and Example 4.4 obtained by using the existing and the proposed methods are shown in Table 9.

It is obvious from the results shown in Table 9 that irrespective of whether we use existing or proposed methods, same results are obtained for Example 4.1 and Example 4.3, while, Example 4.2 and Example 4.4 can be solved only by using the proposed methods. On the basis of above results, it can be suggested that it is better to use the proposed methods instead of existing method [15] to solve fuzzy assignment problems and fuzzy travelling salesman problems.

8. Conclusions

In this paper, limitation of an existing method [15] for solving fuzzy assignment problems and fuzzy travelling salesman problems is discussed and to overcome this limitation, two new methods are proposed. By comparing the results of the proposed methods and existing method, it is shown that it is better to use the proposed methods instead of existing method.

In future, the proposed method may be modified to find fuzzy optimal solution of fuzzy assignment problems, fuzzy travelling salesman problems and generalized assignment problems with intuitionistic fuzzy numbers.

Table 9. Comparison of results obtained by using existing and proposed methods

Examples	Existing method [16]	Proposed method based on FLPF	Proposed method based on classical methods
4.1	$J_1 \to P_3, J_2 \to P_2,$ $J_3 \to P_1, J_4 \to P_4$ and minimum total fuzzy cost = (16,23,27,35)	$J_1 \to P_3, J_2 \to P_2,$ $J_3 \to P_1, J_4 \to P_4$ and minimum total fuzzy cost = (16,23,27,35)	$J_1 \to P_3, J_2 \to P_2,$ $J_3 \to P_1, J_4 \to P_4$ and minimum total fuzzy cost = (16,23,27,35)
4.2	Not applicable	$J_1 \to P_3$ $J_2 \to P_1,$ $J_3 \to P_2$ and minimum total fuzzy cost = $(17,21,5,11)_{LR}$	$J_1 \to P_3$ $J_2 \to P_1,$ $J_3 \to P_2$ and minimum total fuzzy cost = $(17,21,5,11)_{LR}$
4.3	$1 \to 4 \to 2 \to 3 \to 1$ and minimum total fuzzy cost = (15,25,31,36)	$1 \to 4 \to 2 \to 3 \to 1$ and minimum total fuzzy cost = (15,25,31,36)	$1 \to 4 \to 2 \to 3 \to 1$ and minimum total fuzzy cost = (15,25,31,36)
4.4	Not applicable	$1 \to 2 \to 4 \to 3 \to 1$ and minimum total fuzzy cost = $(26,31,4,14)_{LR}$	$1 \to 2 \to 4 \to 3 \to 1$ and minimum total fuzzy cost = $(26,31,4,14)_{LR}$

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