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Approximately Optimal Testing Policy for Two-Unit Parallel Standby Systems

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Abstract: Due to the impact of time-dependent unit unavailability, the testing strategies (e.g., simultaneous or staggered testing) have significant effect on the system unavailability for complex standby systems. Several testing strategies have been studied to minimize average system unavailability for certain systems. For example, uniformly staggered testing has been shown to be the best for two identical units in parallel with and without common cause failure between units. However, this result may not be suitable for parallel systems with nonidentical units. This study provides a discussion to obtain the optimal testing strategy for parallel systems for the case in which units are not necessary to be identical. Moreover, a maintenance manager who is aware of the importance of each system in the plant should establish a testing schedule based on plant-level, not on system- or component-level. That is, the manager should set up testing schedule for each system based on its contribution to the plant. Therefore, a good testing schedule should consider the balance between the maintenance cost and risk (or unavailability) coming with the maintenance task for each system. This study provides a cost-effective model taking both cost and risk into account to establish a good testing policy (including testing strategy and test interval) for two-unit parallel systems.

Keywords: Surveillance testing policy; cost-effective model.

1. Introduction

Engineering safety systems are usually standby systems whose failures are hidden and can be discovered only by inspection or at the next activation. Therefore, most earlier research related to standby systems aims to find the optimal surveillance testing schedule minimizing average system unavailability [1-4]. However, the same type of systems may have different effects on plant safety. From reliability-centered maintenance point of view [5], a good preventive maintenance (PM) should be established based on plant-level, not on system- or component-level. In other words, two identical systems with different effects on plant safety should have different testing policy. Therefore, an optimal surveillance testing policy should be able to spend least expenditure to provide appropriate maintenance to find "hidden failure" before a demand and keep the whole plant away from accident or any risk [6-7].

Due to the complexity of the systems, it is not easy to obtain the optimal testing policy

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taken account of both testing strategy and test intervals from the analytical point of view. Most earlier research obtained the optimal testing policy for multi-unit systems by comparing the system unavailabilities over pre-specified policies [8-11]. Several computer codes have been provided to compute time-dependent system unavailability or safety [2, 12]. For two-unit parallel systems with identical units, uniformly staggered testing is shown to be the best testing strategy to minimize average system unavailability [1]. Uniformly staggered testing will also be shown to be the best testing strategy to minimize our objective function that balances system unavailability and maintenance cost in the cases in which two units in the system are identical in this study. It is believed that this result is also hold for systems with *n* identical units in parallel.

In order to improve the safety in the plant, the purpose of the preventive maintenance of the standby systems should aim to increase the system availability rather than the system reliability. Moreover, reliability-centered maintenance suggests to reduce unnecessary maintenance to reduce the maintenance cost and to avoid the risk provided by imperfect maintenance [5]. Unfortunately, the research studying system availability is not as much as that for system reliability. Nakagawa summarized the optimum testing policies for systems [13]. Most of paper studied the optimum number of inspect times that minimized the total cost in finite time span for single-unit systems or the redundant systems (one operating unit with a standby unit). Recently, due to the improvement of maintenance software and the quality of the system, less research focused on the testing policies or preventive maintenance for complexity standby systems. However, the maintenance software is just like a black box, the decision makers may not have any idea about the influence by the change of the system on the maintenance policy or the precision of the results obtained by the software. Recently, there are several catastrophes occurred (e.g., BP oil disaster in Gulf of Mexico and Fukushima Daiichi nuclear diaster), the importance for the safety systems should be studied to improve the safety in plant.

In this study, we inspect the optimal testing policies for two-unit standby systems not only to increase the system availability but also to reduce the maintenance cost. Although the uniformly staggered testing has been shown to be the optimal test policies for the systems with two identical unit in parallel. However, the it is not preferred for the case in which two units are not identical in the parallel systems. The optimal testing strategy for systems with two nonidentical units in parallel will be discussed in this study. Almost simultaneous testing becomes a better choice for the systems with significantly different units. These results suggest that the maintenance manager should be aware all the characteristics and property of the systems (e.g., importance of the system, failure rate, difference between units, etc.) before establishing the testing schedule for the systems. In this study, not only the optimal testing strategy but also an almost optimal test interval are for the two-unit parallel system.

2. Methodology

Consider to test units in a two-unit parallel system with the same test interval, say T. The maintenance schedule for two-units system is shown in Figure 1. Without loss of generality, assume that the i^{th} cycle begins with the i^{th} test of unit 1 at time O_i. After testing, unit 1 is restored at time A_i, and stays in the standby state until the next test at O_{i+1}. Similarly, unit 2 is tested at B_i and restored at C_i and stays in standby state until the next test at B_{i+1}. The cycle ends just before the $(i+1)^{st}$ test of unit

1; that is, at point O_{i+1}. The time lag between the tests of units 1 and 2 is assumed to be constant for all cycle i, and is denoted by L. Let τ_k denote the expected downtimes for testing, restorative maintenance, and repair of unit k. In order to keep the availability of the parallel systems, exactly simultaneous testing is not considered in this study. Therefore, the value of L in must be between τ_1 and T - τ_2.

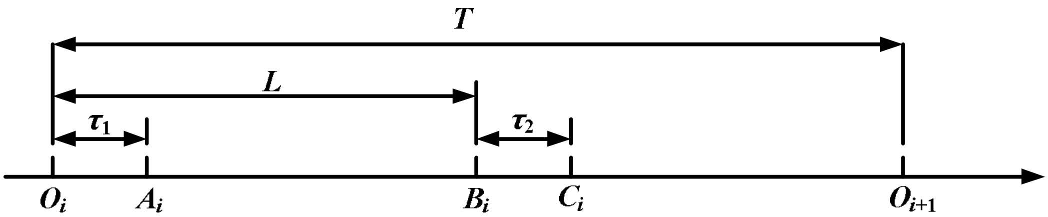


Figure 1. Maintenance schedule for two-unit systems

Unlike most of earlier research, the objective in this study is to find not only the optimal test interval for the system but also the optimal test strategy (simultaneous or staggered) for the units in the system. Moreover, the optimal testing policy takes both risk and maintenance cost into account. The objective function is given by

$$\text{Min} \quad M(T,L) = C_u\overline{U}(T,L) + \overline{C}(T) \qquad (1)$$

where C_u is the expected loss due to system unavailability per time unit, $\overline{U}(T,L)$ is the average system unavailability per cycle, and $\overline{C}(T)$ is the average cost for testing, maintenance and repair for the system per cycle. The average system unavailability is assessed by the time-dependent unavailabilities of units. The testing strategy, such as simultaneous or uniformly staggered testing, will affects the system unavailability. Therefore, both the objective function and average system unavailability are functions of the test interval (T) and the time lag between the test of units 1 and 2 (i.e., L). Since exactly simultaneous testing is not adopted in this study, the cost function is just a function of the test interval.

2.1. Assumptions

Assumptions adopted in this study are listed below,

(1) Surveillance tests of each unit are performed periodically.
(2) A unit is either operable or failed. There are no partially degraded states. Unit failure can occur either because of failure on demand. The state of each unit can be determined precisely during a test.
(3) Following surveillance testing, some minimal level of restorative maintenance (such as lubricating the unit and restoring it to standby status) is performed if the unit is found to be in the operable state during the test; otherwise, the unit is fully repaired or replaced.
(4) A unit is as good as new immediately after its restoration.
(5) A unit is unavailable during testing, restorative maintenance, and repair.
(6) Units are assumed to fail independently of each other during both standby and operation.

2.2. Unavailability and cost functions

Let t denote the time elapsed from the beginning of the given cycle, and $x_k(t)$ is the age of unit k at time t. The time-dependent unavailability of unit k is given by $u[x_k(t)]$ and defined as

$$u_k[x_k(t)] = \begin{cases} \rho_k + (1-\rho_k)F_k[x_k(t)] & \text{if } 0 \le x_k(t) < T - \tau_k \quad \text{(Standby period)} \\ 1 & \text{if } T - \tau_k \le x_k(t) < T \quad \text{(Testing period)} \end{cases} \tag{2}$$

where ρ_k is the constant probability that unit k fails on demand and $F_k(x_k)$ is the probability that the unit k fails at or before age x_k. By assuming that any given cycle begins with the test of unit 1, the age of unit 1 is equal to t-τ_1 for $\tau_1 \le t \le T$. The age of unit 2 is given by

$$x_2(t) = \begin{cases} t + (T - L - \tau_2) & \text{if } 0 \le t < L \\ t - (L + \tau_2) & \text{if } L + \tau_2 \le t < T \end{cases} \tag{3}$$

For one of the units is being tested, the availability of the whole systems depends on only the unit that is not being tested. For the period that none of the units are tested, by the independent failure assumption provided in Assumption 6, the time-dependent unavailability of the whole system can be assess by

$$u(t) = u_1[x_1(t)] \cdot u_2[x_2(t)] \tag{4}$$

The system unavailabilities during the four different periods (as shown in Figure 1) are given by Eqs. (5)-(10)

$$U_{OA} = \int_0^{\tau_1} u_2[x_2(t)]dt = \int_0^{\tau_1} [\rho_2 + (1-\rho_2)F_2(t + T - L - \tau_2)dt \tag{5}$$

$$\begin{aligned} U_{AB} &= \int_{\tau_1}^{L} u_1[x_1(t)] \cdot u_2[x_2(t)]dt \\ &= \int_{\tau_1}^{L} [\rho_1 + (1-\rho_1)F_1(t-\tau_1)] \cdot [\rho_2 + (1-\rho_2)F_2(t + T - L - \tau_2)]dt \end{aligned} \tag{6}$$

$$U_{BC} = \int_L^{L+\tau_2} u_1[x_1(t)]dt = \int_L^{L+\tau_2} [\rho_1 + (1-\rho_1)F_1(t-\tau_1)dt \tag{7}$$

$$\begin{aligned} U_{AB} &= \int_{L+\tau_2}^{T} u_1[x_1(t)] \cdot u_2[x_2(t)]dt \\ &= \int_{L+\tau_2}^{T} [\rho_1 + (1-\rho_1)F_1(t-\tau_1)] \cdot [\rho_2 + (1-\rho_2)F_2(t - L - \tau_2)]dt \end{aligned} \tag{8}$$

The average system unavailability per cycle can then be obtained by dividing the sum of the overall system unavailability by the length of the test interval, that is,

$$\overline{U}(T,L) = \frac{1}{T}(U_{OA} + U_{AB} + U_{BC} + U_{CO}) \tag{9}$$

By no exactly simultaneous testing assumption, there is no common maintenance cost between two units. Therefore, the unit average cost is equal to

$$\overline{C}(T) = \frac{1}{T}\sum_{k=1}^{2}[C_{T_k} + C_{F_k} \cdot u_k(T - \tau_k)] \tag{10}$$

where C_{T_k} and C_{F_k} are the testing (including maintenance) cost and expected additional repair cost for unit k, respectively.

After deriving the average system unavailability function and cost function, the objective function can be determined by plugging them into Eq. (1). The optimal testing policy (including optimal testing strategy, L^*, and test interval, T^*) can be obtained by solving the following equations:

$$\begin{cases} \dfrac{\partial}{\partial L} M(T,L) = 0 \\ \dfrac{\partial}{\partial T} M(T,L) = 0 \end{cases} \tag{11}$$

3. Example

In this section, we presented the case in which the failure rates of units are constant, i.e., the failure times of units are exponential distributed. Moreover, we presented two different cases as follows. Let λ_k denote the failure rate of unit k. Using Taylor's expansion, the probability that the unit i failed at or before age x_k can be approximated by $\lambda_k x_k$, i.e., $F_k(x_k) \approx \lambda_k x_k$. Then, the unit time-dependent unavailability is approximately equal to

$$u_k(x_k) = \rho_\kappa + (1-\rho_k)F_k(x_k) \approx \rho_k + (1-\rho_k)\lambda_k x_k \tag{12}$$

3.1. Identical units case

In this subsection, two units in the systems are assumed to be identical, that is, the values of all parameters (e.g., failure rate, demand failure, cost, etc.) of two units are the same. Let λ denote the failure rate of the unit, the time-dependent unavailability of unit k at age x_k is approximated by

$$u_k(x_k) = \rho + (1-\rho)F_k(x_k) \approx \rho + (1-\rho)\lambda x_k \quad \text{for } k = 1, 2 \tag{13}$$

By plugging Eq. (13) into Eqs. (5)-(10) and solve Eq. (11), the uniformly staggered testing (i.e., $L^* = \frac{1}{2}T$) minimizing the objective function. In other words, the uniformly staggered testing is the best testing strategy for the system with two identical units in parallel. This result is the same the model minimizing average system unavailability as shown in previous research [1].

By adopting uniformly staggered testing, an optimal test interval is approximately by [14].

$$T^* = \begin{cases} \left[\left(R+\sqrt{D}\right)^{1/3} + \left(R-\sqrt{D}\right)^{1/3}\right] - \frac{1}{3}A_2 & \text{if } (A_0 < 0 \text{ and } Q \geq 0) \\ & \text{or } (A_0 = 0 \text{ and } A_2 < 0); \\ \frac{1}{3}A_2\left[2\cos\left(\frac{\theta}{3} - 1\right)\right] & \text{if } A_0 < 0 \text{ and } Q < 0; \\ 2\tau & \text{otherwise} \end{cases} \tag{14}$$

$$A_2 = \frac{12}{5}\left[\frac{\rho}{(1-\rho)\lambda} - \tau\right]$$

$$A_0 = \frac{8}{5}\left[\tau^3 + \frac{3}{2}\cdot\frac{(1-2\rho)\tau^2}{(1-\rho)\lambda} - \frac{3\rho\tau}{(1-\rho)\lambda^2}\right] - \frac{24}{5C_u(1-\rho)^2\lambda^2}\{C_T + C_F[\rho - (1-\rho)\lambda\tau]\}$$

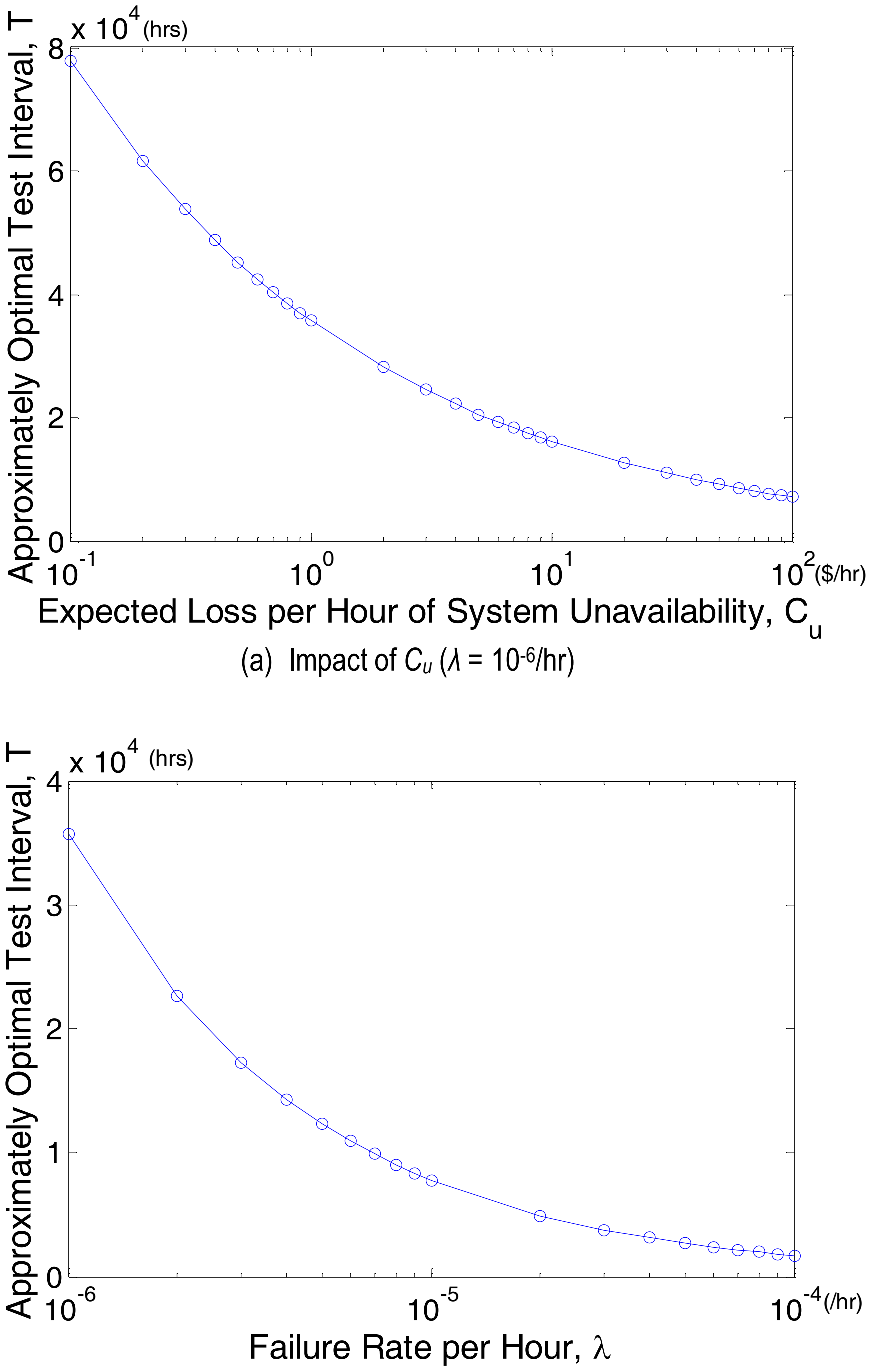
where $R = -\frac{1}{54}\left(27A_0 + 2A_2^3\right)$

$$Q = \frac{A_0}{108}\left(27A_0 + 4A_2^3\right)$$

$$\theta = \cos^{-1}\left(\frac{27R}{A_2^3}\right) \in (0, \pi)$$

Figure 2 shows the behavior of the approximately optimal surveillance test interval as a function of the expected loss due the system unavailability, C_u, and the units' failure rate, λ. This figure suggests that (1) The approximately optimal test interval T is decreasing in the cost per time unit of system unavailability, C_u, and the failure rate λ. That is, when the system are more important (larger value of C_i) or less reliable (larger value of λ), the units are suggested to be tested more frequently.

(2) The approximately optimal test interval T appears to be roughly inversely proportional to $\lambda^{2/3}$; for example, T decrease by a factor of about four if the failure rates of both component increase by a factor of eight.



(a) Impact of C_u ($\lambda = 10^{-6}$/hr)

(b) Impact of λ (C_u = \$1/hr)

Figure 2. Approximately optimal test interval for a system with identical units in parallel, as a function of C_u and λ (for the case in which $\rho = 10^{-3}$/demand, τ = 1 hr, C_T = \$10/test, C_F = \$100/repair)

3.2. Nonidentical units case

In this subsection, units in the system are different. Let λ_k denote the failure rate of unit k. By Eq. (12), the time-dependent unit unavailability is approximately equal to

$$u_k(x_k) = \rho_k + (1-\rho_k)F_k(x_k) \approx \rho_k + (1-\rho_k)\lambda_k x_k \quad \text{for } k = 1,2 \tag{15}$$

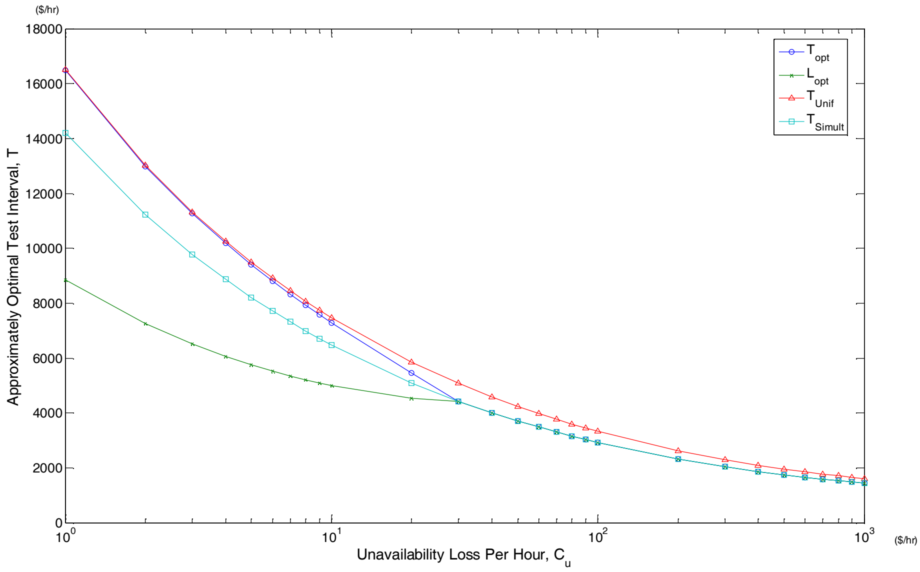
Plugging Eq. (15) into Eqs. (5)-(10) and solving Eq. (11), the optimal time lag between the tests of units 1 and 2 can be approximated by [14]

$$L_{12} = \begin{cases} T-\tau_2 \quad \text{(almost simultaneous testing)} \quad \text{if } \dfrac{1}{T}\left[\dfrac{\tau_1^2-\tau_2^2}{2}+\dfrac{\tau_1}{\lambda_1}-\dfrac{\tau_2}{\lambda_2}\right] \geq \dfrac{1}{2}T-\tau_2 \\ \tau_1 \quad \text{(almost simultaneous testing)} \quad \text{if } \dfrac{1}{T}\left[\dfrac{\tau_1^2-\tau_2^2}{2}+\dfrac{\tau_1}{\lambda_1}-\dfrac{\tau_2}{\lambda_2}\right] \leq -\dfrac{1}{2}T+\tau \\ \dfrac{1}{2}T+\dfrac{1}{T}\left[\dfrac{\tau_1^2-\tau_2^2}{2}+\dfrac{\tau_1}{\lambda_1}-\dfrac{\tau_2}{\lambda_2}\right] \text{(staggered testing)} \\ \qquad \text{if } -\dfrac{1}{2}T+\tau_1 < \dfrac{1}{T}\left[\dfrac{\tau_1^2-\tau_2^2}{2}+\dfrac{\tau_1}{\lambda_1}-\dfrac{\tau_2}{\lambda_2}\right] < \dfrac{1}{2}T-\tau_2 \end{cases} \tag{16}$$

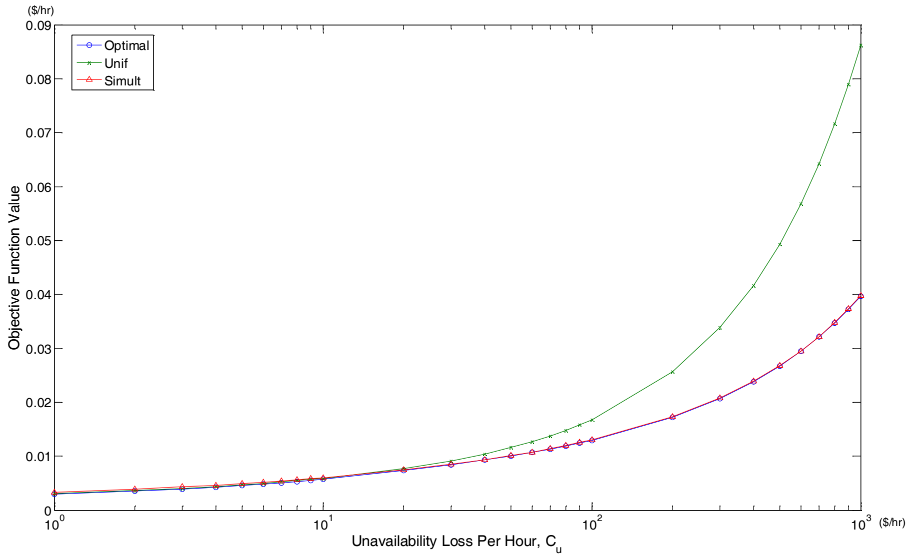
To summarize, if $\frac{1}{T}\left[\frac{\tau_1^2-\tau_2^2}{2}+\frac{\tau_1}{\lambda_1}-\frac{\tau_2}{\lambda_2}\right] \geq \frac{1}{2}T-\tau_2$, then $L_{12} = T-\tau_2$. This is, almost simultaneous testing is preferred, and unit 2 should be tested before unit 1. Similarly, if $\frac{1}{T}\left[\frac{\tau_1^2-\tau_2^2}{2}+\frac{\tau_1}{\lambda_1}-\frac{\tau_2}{\lambda_2}\right] \leq -\frac{1}{2}T+\tau_1$, then $L_{12} = \tau_1$. In this case, almost simultaneous is again preferred, but unit 1 now should be tested first. Otherwise, when $-\frac{1}{2}T+\tau_1 < \frac{1}{T}\left[\frac{\tau_1^2-\tau_2^2}{2}+\frac{\tau_1}{\lambda_1}-\frac{\tau_2}{\lambda_2}\right] < \frac{1}{2}T-\tau_2$, the two units should be tested in a staggered manner, but not necessarily uniformly staggered.

In the real world, it may not be practical to implement to optimal time lag shown in Eq. (16), and either (almost) simultaneous or uniformly staggered testing may be more convenient. Therefore, we compare the results of using the optimal time lag in Eq. (16) with the results of using uniformly staggered testing and also almost simultaneous testing (where the unit with the smaller value of $\frac{1}{2}\tau_k^2 + \frac{\tau_k}{\lambda_k} \approx \frac{\tau_k}{\lambda_k}$ is tested first.)

Figure 3 illustrates the performance achieved by these three testing policies, as a function of the expected loss per time unit of unavailability (C_u). As expected, the approximately optimal test interval decreases when the expected unavailability loss C_u increases. Figure 3 (a) shows that when C_u is small, the approximately optimal test interval is large. In this case, the optimal testing strategy is staggered (not necessarily uniformly staggered); however, there is no significant difference between the approximately optimal test interval corresponding to the optimal time lag (T_{opt}) and that corresponding to uniformly staggered testing (T_{Unif}). The objective function values achieved by these three testing policies do not differ significantly when C_u is small, as shown in Figure 3 (b). However, when C_u is large, the approximately optimal test interval becomes smaller, and the optimal testing strategy changes to almost simultaneous testing. In this case, the difference in the objective function values achieved by the optimal testing strategy and almost simultaneous testing is insignificant. In order to avoid the large loss caused by using uniformly staggered testing when C_u is large, almost simultaneous testing is suggested for all values of C_u for two-unit parallel systems in which the units are significantly different.



(a) Approximately optimal test interval and optimal time Lag



(b) Objective function value

Figure 3. Approximately optimal test interval, and approximately optimal time lag, and objective functions as functions of the expected unavailability loss per hour C_u, provided that $\left|\frac{1}{2}\left(\tau_1^2-\tau_2^2\right)+\frac{\tau_1}{\lambda_1}-\frac{\tau_2}{\lambda_2}\right|$ is not small relative to the test interval (for the case in which $\lambda_1 = 10^{-6}$/hr, $\lambda_2 = 10^{-5}$/hr, $\tau_1 = 10$ hrs, $\tau_2 = 1$ hr, $\rho_k = 10^{-3}$/demand, $C_{T_k} = \$10$/test, $C_{F_k} = \$100$/repair, for $k = 1, 2$)

By adopting the simultaneous testing, the optimal test interval for systems can also be obtained by

$$T^* = \begin{cases} \left[\left(R+\sqrt{D}\right)^{1/3} + \left(R-\sqrt{D}\right)^{1/3}\right] - \frac{1}{3}A_2 & \text{if } (A_0 < 0 \text{ and } Q \ge 0) \\ & \text{or } (A_0 = 0 \text{ and } A_2 < 0); \\ \frac{1}{3}A_2\left[2\cos\left(\frac{\theta}{3} - 1\right)\right] & \text{if } A_0 < 0 \text{ and } Q < 0; \\ \tau_1 + \tau_2 & \text{otherwise} \end{cases} \tag{17}$$

This is almost the same as the approximately optimal test interval for two-unit parallel systems with identical units given in Eq. (14), expect the approximately optimal test interval corresponding to the continuous testing is now equal to $\tau_1+\tau_2$ rather than 2τ, and we now have

$$A_2 = \frac{3}{4}\left[\frac{\rho_2}{(1-\rho_2)\lambda_2} + \frac{\rho_1}{(1-\rho_1)\lambda_1} - (\tau_1+\tau_2+S_1)\right] \tag{18}$$

$$\begin{aligned} A_0 = \frac{1}{4}\Bigg[&(S_2^3 - 2S_z^3 - 3S_2S_1^2) + \frac{3\rho_2\tau_1^2}{(1-\rho_2)\lambda_2} + \frac{3\rho_1\tau_2^2}{(1-\rho_1)\lambda_1} - 3\left(\frac{\tau_1^2}{\lambda_1} - \frac{\tau_2^2}{\lambda_2}\right)S_3 - \frac{6\tau_1\tau_2}{S_4} \\ &+ \frac{6\rho_2\tau_1}{(1-\rho_2)\lambda_1\lambda_2} + \frac{6\rho_1\tau_2}{(1-\rho_1)\lambda_1\lambda_2}\Bigg] + \frac{3\sum_{i=1}^{2}\left\{C_{T_k} + C_{F_k}\left[\rho_k - (1-\rho_k)\lambda_k\tau_k\right]\right\}}{2C_u(1-\rho_1)(1-\rho_2)\lambda_1\lambda_2} \end{aligned} \tag{19}$$

where for the case in which the value of $\frac{1}{2}\tau_1^2 + \frac{\tau_1}{\lambda_1}$ for component 1 is not greater than that value of component 2, then $S_1 = \tau_1$, $S_2 = \tau_2$, $S_3 = 1$ and $S_4 = \lambda_1$; otherwise, $S_1 = \tau_2$, $S_2 = \tau_1$, $S_3 = -1$ and $S_4 = \lambda_2$. (Note that for the convenience of decision makers, it may be adequate to replace $\frac{1}{2}\tau_k^2 + \frac{\tau_k}{\lambda_k}$ by $\frac{\tau_k}{\lambda_k}$, since the τ_k^2 term will generally have relative little impact.)

4. Conclusion

Uniformly staggered testing has been shown to be the best choice to test two identical units in parallel. The optimal test interval for system decreases when the system is more important (larger value of expected lose due to system unavailability, C_u) or when the units are less reliable (larger value of failure rate, λ). In particular, the optimal test interval is inversely proportional to $\lambda^{2/3}$. That is, if the failures rates of the units increase by a factor of eight, the optimal test interval will decrease by a factor of four.

The uniformly staggered testing is not suitable for parallel systems with significantly different units. The results presented in this study indicate the importance of the system (i.e., the expected unavailability loss per hour) and the difference between units have significantly effect on the choice of the testing strategies. For parallel systems with nonidentical units, almost simultaneous with smaller value of the product of the testing duration and expected time to failure tested first has better results than uniformly staggered testing. This study provides an analytic perspective for the optimal testing strategy and test interval for two-unit parallel systems. The maintenance manager is suggested to be aware of all the characteristics and properties of the system (such as the importance of the system and the difference between the units) in order to establish the most cost-effective for the plant.

After the tsunami followed by the earthquake in Fukushima, in March, 2011, caused the catastrophe in Japan, the common cause failure has been recognized as one of the major reason for the failure of the emergence systems. Therefore, the impacts of the common cause failure on the redundant systems should be studied in the future research in order to improve the availability for the standby redundant systems and reduce the degree of the possible damage.

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