

Unsteady Magnetohydrodynamic Free Convective Flow Past a Vertical Porous Plate

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Abstract: A numerical solution to the problem of unsteady magnetohydrodynamic free convective flow past a vertical porous plate has been analyzed. The dimensionless governing equations are solved using finite element method. The velocity, temperature and concentration fields are studied for different physical parameters.

Keywords: Unsteady; MHD; free convection; porous plate; FEM.

1. Introduction

Convective heat transfer in a porous media is a topic of rapidly growing interest due to its application to geophysics, geothermal reservoirs, thermal insulation engineering, exploration of petroleum and gas fields, water movements in geothermal reservoirs, etc. The study of convective heat transfer mechanisms through porous media in relation to the applications to the above areas has been made by Nield and Bejan [1]. Hossain and Begum [2] have discussed unsteady free convective mass transfer flow past vertical porous plates. Recently, the study of free convective mass transfer flow has become the object of extensive research as the effects of heat transfer along with mass transfer effects are dominant features in many engineering applications such as rocket nozzles, cooling of nuclear reactors, high sinks in turbine blades, high speed aircrafts and their atmospheric reentry, chemical devices and process equipments. Unsteady effect on MHD free convective and mass transfer flow through porous medium with constant suction and constant heat flux in rotating system studied by Sharma [3]. But in all these papers thermal diffusion effects have been neglected, whereas in a convective fluid when the flow of mass is caused by a temperature difference, thermal diffusion effects cannot be neglected. In view of the importance of this diffusion-thermo effect, Jha and Singh [4] presented an analytical study for free convection and mass transfer flow past an infinite vertical plate moving impulsively in its own plane taking Soret effects into account. In all the above studies, the effect of the viscous dissipative heat was ignored in free-convection flow. However, Gebhart and Mollendorf [5] have shown that when the temperature difference is small or in high Prandtl number fluids or when the gravitational field is of high intensity, viscous dissipative heat should be taken into account in free convection flow past a semi-infinite vertical plate. The unsteady free convection flow of a viscous incompressible fluid past an infinite vertical plate with constant heat flux is considered on taking into account viscous dissipative heat, under the

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influence of a transverse magnetic field studied by Srihari. K et al [6]. The effect of temperature dependent viscosity and thermal conductivity on unsteady MHD convective heat transfer past a semi-infinite vertical porous plate has studied Seddek and Salama [7]. In recent years, progress has been considerably made in the study of heat and mass transfer in magneto hydrodynamic flows due to its application in many devices, like the MHD power generator and Hall accelerator. The influence of magnetic field on the flow of an electrically conducting viscous fluid with mass transfer and radiation absorption is also useful in planetary atmosphere research. Yih [8] numerically analyzed the effect of transpiration velocity on the heat and mass transfer characteristics of mixed convection about a permeable vertical plate embedded in a saturated porous medium under the coupled effects of thermal and mass diffusion. Elbashareshy [9] studied the effect of surface mass flux on mixed convection along a vertical plate embedded in porous medium. Chin et al. [10] obtained numerical results for the steady mixed convection boundary layer flow over a vertical impermeable surface embedded in a porous medium when the viscosity of the fluid varies inversely as a linear function of the temperature. Anand Rao and Shivaiah [11] studied the Chemical reaction effects on an unsteady MHD free convection flow past an infinite vertical porous plate with constant suction. The chemical reaction effects on an unsteady MHD flow past a semi-infinite vertical porous plate with viscous dissipation has been analyzed numerically by Anand Rao and Shivaiah [12].

The object of the present paper is to study the thermal radiation effect on unsteady magneto hydrodynamic flow past a vertical porous plate with variable suction. The problem is governed by the system of coupled non-linear partial differential equations whose exact solutions are difficult to obtain, if possible. So, Galerkin finite element method has been adopted for its solution, which is more economical from computational point of view.

2. Formulation of the problem

An unsteady two-dimensional laminar free convective boundary layer flow of a viscous, incompressible, electrically conducting and the chemical reaction effects on an unsteady magneto hydrodynamics free convection fluid flow past a semi-infinite vertical plate embedded in a porous medium with heat absorption is considered. The x' - axis is taken along the vertical plate and the y' - axis normal to the plate. It is assumed that there is no applied voltage, which implies the absence of an electric field. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field and the Hall Effect are negligible. The concentration of the diffusing species in the binary mixture is assumed to be very small in comparison with the other chemical species which are present, and hence the Soret and Dufour are negligible. Further due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y' and t' only. Now, under the usual Boussinesq's approximation, the governing boundary layer equations of the problem are:

$$\text{Continuity equation: } \frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\nu}{K'} \right) u' \tag{2}$$

$$\text{Energy equation: } \frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} \quad (3)$$

$$\text{Diffusion equation: } \frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (4)$$

Where u', v' are the velocity components in x', y' directions respectively. t' - the time, ρ - the fluid density, ν - the kinematic viscosity, c_p - the specific heat at constant pressure, g - the acceleration due to gravity, β and β^* - the thermal and concentration expansion coefficient respectively, B_0 - the magnetic induction, α - the fluid thermal diffusivity, K' - the permeability of the porous medium, T' - the dimensional temperature, C' - the dimensional concentration, k - the thermal conductivity, μ - coefficient of viscosity, D - the mass diffusivity.

The boundary conditions for the velocity, temperature and concentration fields are:

$$\left. \begin{aligned} u' &= u'_p, \quad T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{n't'} \\ C' &= C'_w + \varepsilon(C'_w - C'_\infty)e^{n't'} \quad \text{at } y' = 0 \\ u' &\rightarrow 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty \end{aligned} \right\} \quad (5)$$

Where u'_p is the plate velocity, T'_w and C'_w are the wall dimensional temperature and concentration respectively, T'_∞ and C'_∞ are the free stream dimensional temperature and concentration respectively, n' - the constant.

Where A is a real positive constant, ε and εA are small values less than unity and V_0 is scale of suction velocity at the plate surface.

In order to write the governing equations and the boundary condition in dimension less form, the following non- dimensional quantities are introduced.

$$\left. \begin{aligned} u &= \frac{u'}{V_0}, \quad v = \frac{v'}{V_0}, \quad y = \frac{V_0 y'}{\nu}, \quad t = \frac{V_0^2 t'}{\nu}, \quad n = \frac{\nu n'}{V_0^2}, \\ T &= \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad Sc = \frac{\nu}{D}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \\ M &= \frac{\sigma B_0^2 \nu}{\rho V_0^2}, \quad K = \frac{K' V_0^2}{\nu^2}, \quad Pr = \frac{\nu \rho c_p}{k} = \frac{\nu}{\alpha}, \\ Gr &= \frac{g \beta \nu (T'_w - T'_\infty)}{V_0^3}, \quad Gm = \frac{g \beta^* \nu (C'_w - C'_\infty)}{V_0^3}, \end{aligned} \right\} \quad (6)$$

In view of equation (6), equations (2) - (4) reduced to the following dimensionless form.

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{iot}) \frac{\partial u}{\partial y} = GrT + GmC + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K} \right) u \quad (7)$$

$$\frac{\partial T}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} \tag{8}$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{i\omega t}) \frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} \tag{9}$$

And the corresponding boundary conditions are

$$t > 0 : u = 0, T = 1 + \varepsilon e^{i\omega t}, C = 1 \text{ at } y = 0, \quad u \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty. \tag{10}$$

3. Method of Solution

The Galerkin expansion for the differential equation (7) becomes

$$\int_{y_j}^{y_k} N^{(e)T} \left(\frac{\partial^2 u^{(e)}}{\partial y^2} + B \frac{\partial u^{(e)}}{\partial y} - \frac{1}{4} \frac{\partial u^{(e)}}{\partial t} - Nu^{(e)} + R_1 \right) dy = 0 \tag{11}$$

Where $R_1 = GrT + GmC$, $B = 1 + \varepsilon A e^{i\omega t}$, $N = M + \frac{1}{K}$.

Let the linear piecewise approximation solution $u^{(e)} = N_j(y)u_j(t) + N_k(y)u_k(t) = N_j u_j + N_k u_k$.

Where $N_j = \frac{y_k - y}{y_k - y_j}$, $N_k = \frac{y - y_j}{y_k - y_j}$, $N^{(e)T} = [N_j \quad N_k]^T = \begin{bmatrix} N_j \\ N_k \end{bmatrix}$.

The Galerkin expansion for the differential equation (11) becomes

$$N^{(e)T} \frac{\partial u^{(e)}}{\partial y} \Big|_{y_j}^{y_k} - \int_{y_j}^{y_k} \left\{ \begin{array}{l} \frac{\partial N^{(e)T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} \\ - N^{(e)T} \left(B \frac{\partial u^{(e)}}{\partial y} + \frac{1}{4} \frac{\partial u^{(e)}}{\partial t} + Nu^{(e)} - R_1 \right) \end{array} \right\} dy = 0 \tag{12}$$

Neglecting the first term in equation (12) we get

$$\int_{y_j}^{y_k} \left\{ \begin{array}{l} \frac{\partial N^{(e)T}}{\partial y} \frac{\partial u^{(e)}}{\partial y} \\ N^{(e)T} \left(B \frac{\partial u^{(e)}}{\partial y} - \frac{1}{4} \frac{\partial u^{(e)}}{\partial t} - Nu^{(e)} + R_1 \right) \end{array} \right\} dy = 0$$

$$\frac{1}{l^{(e)}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} - \frac{B}{2} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{l^{(e)}}{24} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} + \frac{Nl^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} = R_1 \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Where $l^{(e)} = y_k - y_j = h$ and dot denotes the differentiation with respect to t .

We write the element equations for the elements $y_{i-1} \leq y \leq y_i$ and $y_j \leq y \leq y_k$ assemble three element equations, we obtain

$$\frac{1}{l^{(e)^2}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} - \frac{B}{2l^{(e)}} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{1}{24} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}^* \\ u_i^* \\ u_{i+1}^* \end{bmatrix} + \frac{N}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} = \frac{R_1}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad (13)$$

Now put row corresponding to the node i to zero, from equation (13) the difference schemes is

$$\frac{1}{l^{(e)^2}} [-u_{i-1} + 2u_i - u_{i+1}] - \frac{B}{2l^{(e)}} [-u_{i-1} + u_{i+1}] + \frac{1}{24} [u_{i-1}^* + 4u_i^* + u_{i+1}^*] + \frac{N}{6} [u_{i-1} + 4u_i + u_{i+1}] = R_1$$

Applying Crank-Nicholson method to the above equation then we gets

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} = A_4 u_{i-1}^j + A_5 u_i^j + A_6 u_{i+1}^j + R^* \quad (14)$$

Where $A_1 = 1 - 12r + 6Brh + 2Nk$, $A_2 = 4 + 24r + 8Nk$, $A_3 = 1 - 12r - 6Brh + 2Nk$,
 $A_4 = 1 + 12r - 6Brh - 2Nk$, $A_5 = 4 - 24r - 8Nk$, $A_6 = 1 + 12r + 6Brh - 2Nk$,
 $R^* = 24(Gr)kT_i^j + 24(Gm)kC_i^j$

Applying similar procedure to equation (8) and (9) then we gets

$$B_1 T_{i-1}^{j+1} + B_2 T_i^{j+1} + B_3 T_{i+1}^{j+1} = B_4 T_{i-1}^j + B_5 T_i^j + B_6 T_{i+1}^j \quad (15)$$

$$C_1 C_{i-1}^{j+1} + C_2 C_i^{j+1} + C_3 C_{i+1}^{j+1} = C_4 C_{i-1}^j + C_5 C_i^j + C_6 C_{i+1}^j \quad (16)$$

$B_1 = 1 - 12r Pr + 6Brh$, $B_2 = 4 + 24r Pr$, $B_3 = 1 - 12r Pr - 6Brh$,
 $B_4 = 1 + 12r Pr - 6Brh$, $B_5 = 4 - 24r Pr$, $B_6 = 1 + 12r Pr + 6Brh$
 $C_1 = S_c - 12r + 6BS_c rh$, $C_2 = 4S_c + 24r$, $C_3 = S_c - 12r - 6BS_c rh$,
 $C_4 = S_c + 12r - 6BS_c rh$, $C_5 = 4S_c - 24r$, $C_6 = S_c + 12r + 6BS_c rh$

Here $r = \frac{k}{h^2}$ and h, k are the mesh sizes along y -direction and time t -direction respectively.

Index i refers to the space and j refers to the time. In Equations (14)-(16), taking $i=1(1)n$ and using initial and boundary conditions (10), the following system of equations are obtained:

$$A_i X_i = B_i, \quad i = 1(1)3$$

Where A_i 's are matrices of order n and X_i, B_i 's column matrices having n - components. The solutions of above system of equations are obtained by using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by C-program. In order to prove the convergence and stability of finite element method, the same C-program was run with slightly changed values of h and k and no significant change was observed in the values of u, T and C . Hence, the finite element method is stable and convergent.

4. Skin Friction

The skin-friction, Nusselt number and Sherwood number are important physical parameters for this type of boundary layer flow. The skin friction, rate of heat and mass transfer are Skin

$$\text{friction coefficient } (C_f) \text{ is given by } C_f = \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad (17)$$

$$\text{Nusselt number } (Nu) \text{ at the plate is } Nu = \left(\frac{\partial T}{\partial y} \right)_{y=0} \quad (18)$$

$$\text{Sherwood number } (Sh) \text{ at the plate is } Sh = \left(\frac{\partial C}{\partial y} \right)_{y=0} \quad (19)$$

5. Results and discussion

As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behaviour have been discussed for variations in the governing parameters viz., the thermal Grashof number G_r , modified Grashof number G_c , magnetic field parameter M , permeability parameter K , Prandtl number Pr , and Schmidt number Sc . In the present study we adopted the following default parameter values of finite element computations:

$$Gr = 5.0, Gm = 5.0, M = 1.0, K = 5.0, Pr = 0.71, Sc = 0.6, A = 0.01, \varepsilon = 0.002, \omega = 1.0, t = 1.0.$$

All graphs therefore correspond to these values unless specifically indicated on the appropriate graph.

Figure 1 presents typical velocity profiles in the boundary layer for various values of the Grashof number Gr , while all other parameters are kept at some fixed values. The Grashof number Gr defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value.

The influence of the modified Grashof number Gm on the velocity is presented in Figure 2. The modified Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of Gm correspond to cooling of the plate. Also, as Gm increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.

For various values of the magnetic parameter M , the velocity profiles are plotted in Figure 3. It can be seen that as M increases, the velocity decreases. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the flow. The effect of the permeability parameter K on the velocity field is shown in Figure 4. An increase the resistance of the porous medium which will tend to increase the velocity. This behavior is evident from Figure 4.

Figure 5(a) and Figure 5(b) illustrate the velocity and temperature profiles for different values of the Prandtl number Pr . The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Figure 5(a)). From Figure 5(b), it is observed that an

increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr . Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

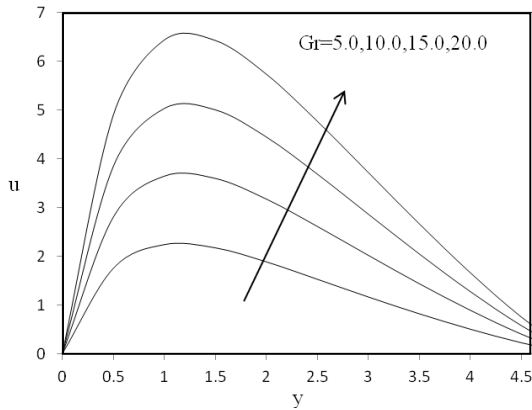


Figure 1. Velocity profile for different values of Gr

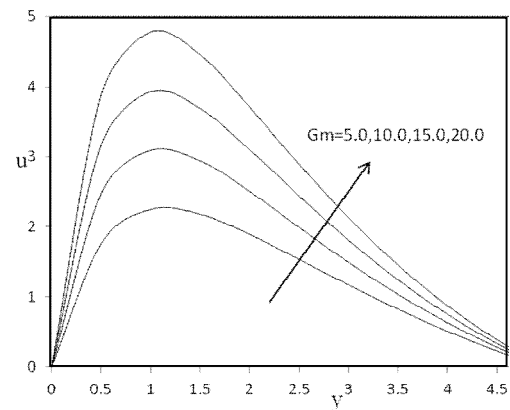


Figure 2. Velocity profile for different values of Gm

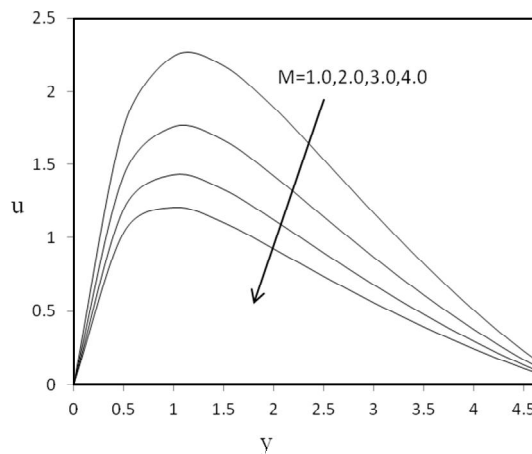


Figure 3. Velocity profile for different values of M

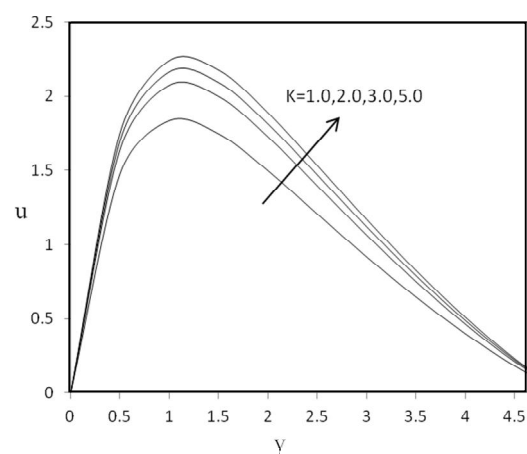


Figure 4. Velocity profile for different values of K

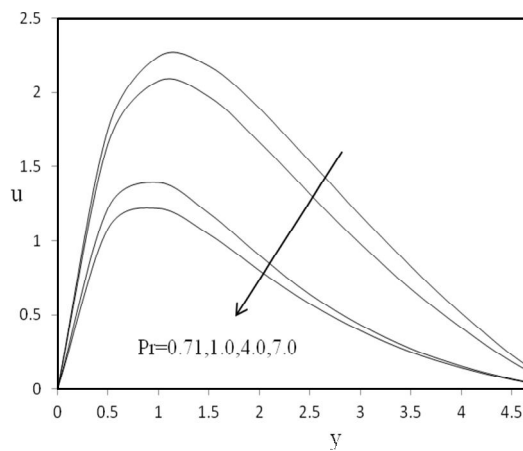


Figure 5. (a) Velocity profile for different values of Pr

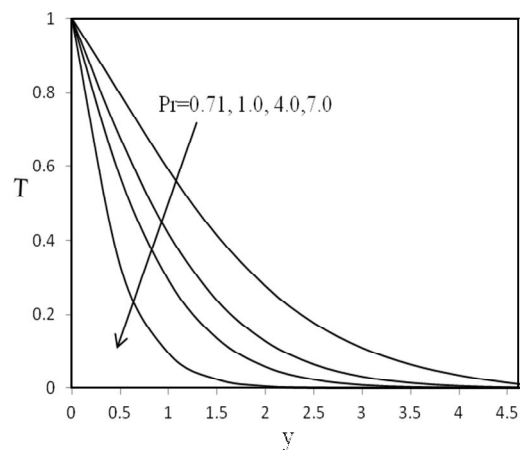


Figure 5. (b) Temperature profile for different values of Pr

The influence of the Schmidt number S_c on the velocity and concentration profiles are plotted in Figure 6(a) and Figure 6(b) respectively. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figure 6(a) and Figure 6(b).

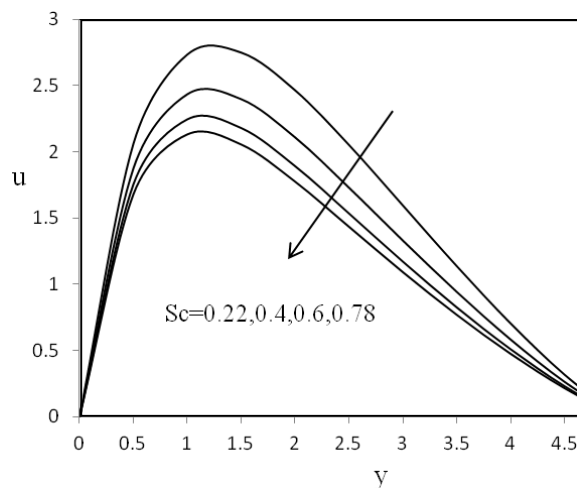


Figure 6. (a) Velocity profile for different values of S_c

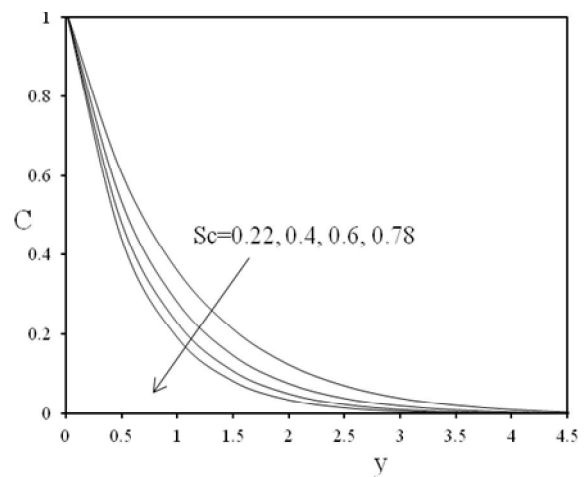


Figure 6. (b) Concentration profile for different values of S_c

6. Conclusion

In this article a mathematical model has been presented for the thermal radiation effect on unsteady magneto hydrodynamic flow past a vertical porous plate with variable suction. The non-dimensional governing equations are solved with the help of finite element method. The conclusions of the study are as follows:

- a) The velocity increases with the increase Grashof number and modified Grashof number.
- b) The velocity decreases with an increase in the magnetic parameter.
- c) The velocity increases with an increase in the permeability of the porous medium parameter.
- d) Increasing the Prandtl number substantially decreases the velocity and the temperature function.
- e) The velocity as well as concentration decreases with an increase in the Schmidt number.

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