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Hydromagnetic Flow Near a Stagnation Point on a Stretching Sheet with Variable Thermal Conductivity and Heat Source/Sink

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Abstract: The hydro-magnetic flow of a viscous fluid near a stagnation point on a linearly stretching sheet with variable thermal conductivity and heat source is investigated. The governing non-linear partial differential equations of momentum and energy are transformed into non-linear ordinary differential equations using the usual similarity transformation. The resulting problem is then solved using perturbation technique. The effect of variable thermal conductivity, magnetic field and some other important parameters encountered in the problem are discussed. The result shows that the observed parameters have significance influence on the flow.

Keywords: Hydro-magnetic flow; stagnation point; source; variable thermal conductivity; steady; Nusselt number; skin-friction coefficient and Perturbation technique.

1. Introduction

The study of hydro-magnetic flow has become more important in the industry, due to the fact that many metallurgical processes involves the cooling of filaments by drawing them into an electrically conducting steady incompressible viscous fluid under the influence of magnetic fields, thereby controlling the rate of cooling. The quality of final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. The hydro-magnetic flow in electrically conducting fluid can control the rate of cooling and the study of heat transfer in boundary layer over stretching surface find applications in cooling of elastic sheets, spinning of fibres, extrusion of plastic sheets, e.t.c. Liquid metals have small prandthl number (e.g Pr=0.01 is for Bismuth, Pr=0.023 for mercury) and are generally used as coolants because of very large thermal conductivity. Flow in the neighbourhood of a stagnation point in a plane was initiated by Hiemenz (1911). Grubka and Bobba (1985) investigated fluid flow and heat transfer characteristics on stretching sheet with variable temperature condition. Bestman et al. (1992) looked at the effect of radiative heat transfer to a hydro-magnetic flow of a slightly rarefied binary gas in a vertical channel. Their results showed that increase in radiation parameter reduces the temperature. Chamkha (1997) investigated mixed convection stagnation flow with suction and blowing. Similarity equations were derived and solved numerically by an implicit and accurate finite difference method. The results illustrated the influence of wall mass transfer coefficient, heat absorption coefficient, Prandthl number and buoyancy or mixed

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convection parameter. Chiam (1997) studied the heat transfer in fluid flow on stretching sheet at stagnation point in presence of internal dissipation, heat source and stress with constant fluid properties. Mahapatra and Gupta (2002) investigated the effect of viscous dissipation on heat transfer in stagnation point flow towards stretching sheet. Abdelkhalek (2005) studied the skin friction in the MHD mixed convection stagnation point with mass transfer. The results were obtained by solving the coupled non-linear partial differential equations describing the flow by perturbation technique. The results were presented to illustrates the influence of the mass transfer coefficient, heat absorption coefficient, Prandthl number and the mixed convection parameter. Seddeek and Salem (2005) looked at the effect of variable viscosity and thermal diffusivity on the heat and mass transfer distribution on a stretching surface. Abdelkhalek (2008) studied hydro-magnetic stagnation point flow. An analysis was performed to study the momentum, heat and mass transfer characteristics of MHD natural convection flow and heat generating/absorbing fluid at the stagnation point of an isothermal two-dimensional porous body immersed in a fluid with saturated porous medium. Sharma and Singh (2008) investigated the effects of heat source and variable thermal conductivity on flow of viscous incompressible electrically conducting fluid in the presence of transverse magnetic field. The results were obtained using Runge-Kuta fourth order technique along with shooting method. Norfifah and Anuar [2009] studied MHD stagnation-point flow of a micro-polar fluid with prescribed wall heat flux. The steady MHD mixed convection stagnation point flow towards a vertical immersed in an incompressible micro-polar fluid with prescribed wall heat flux was investigated. The governing partial differential equations were transformed into a system of ordinary differential equations by using similarity variables for transformation. The dimensionless equations were solved numerically by a finite difference method. Haquel et al. [2011] studied magnetohydrodynamics free convective heat generating unsteady micropolar fluid flow through a porous medium with constant heat and mass flux. Unsteady magnetohydrodynamics heat and mass transfer by free convective micropolar fluid flow over an infinite vertical porous medium under the action of transverse magnetic field with themal diffusion have been studied numerically in the presence of constant heat source. Muhammad and Anwar [2011] studied the boundary layer stagnation point flow and heat transfer characteristics of an clectrically conducting micropolar fluid impinging normally on a permeable horizontal surface in the presence of a uniform magnetic field. The governing continuity, momentum, angular momentum and heat equations together with the associated boundary conditions were reduced to a system of coupled non-linear differential equations. The reduced equations were then solved numerically by an algorithm based on finite difference discretization. The results were further refined by Richardson's extrapolation. Jashim [2011] studied convective flow of micro-polar fluid along an inclined flat plate with variable electric conductivity and uniform surface heat flux. MHD two-dimensional steady convective flow and heat transfer of micropolar fluid flow along an inclined flat plate with variable electric conductivity and uniform surface heat flux was analyzed numerically in the presence of heat generation. With appropriate transformations, the boundary layer partial differential equations were transformed into non-linear ordinary differential equations. The local similarity solutions of the transformed dimensionless equations for the velocity flow, microrotation and heat transfer characteristics were assessed using Nachtsheim-Swigert Shooting iteration technique along with the sixth order Runge-Kutta-Butcher initial value solver. Hamza et al. [2011] studied unsteady magnetohydrodynamics micropolar fluid flow and mass transfer past a vertical permeable plate with variable suction. The region of unsteady magnetohydrodynamics flow of viscous, incompressible, electrically-conducting fluid occupying a semi-infinite region of space bounded by an infinite vertical plate moving with constant velocity in the presence of a transverse magnetic field was considered for analysis. Govardhan and Kishan [2012] studied unsteady MHD boundary layer flow of an incompressible micro-polar fluid over a stretching sheet and the sheet was stretched in it's own plane. The governing non-linear equations and their associated boundary conditions were first cast into dimensionless form by similarity transformation. The resulted equations were solved numerically using the Adams-Predictor corrector method for the whole transient from the initial state to final steady-state flow. In this paper, We investigated the hydro-magnetic flow of a viscous fluid near a stagnation point on a linearly stretching sheet with variable thermal conductivity and heat source. The governing non-linear partial differential equations of momentum and energy are transformed into non-linear ordinary differential equations using the usual similarity transformation. The resulting problem is then solved using perturbation technique.

2. Formulation of the problem and method of solution

Consider two-dimensional steady viscous flow of an incompressible hydro-magnetic fluid with thermal conductivity near a stagnation point on non-conducting stretching sheet in the presence of transverse magnetic field and heat generation/absorption. It is assumed that external field is zero, the electric field owing to polarization of charges and hall effect are neglected. The stretching sheet has a uniform temperature T_w, linear velocity $u_w(x)$ and it is placed in the plane y=0. The x-axis is taken along the sheet as shown in the Figure 1 below;

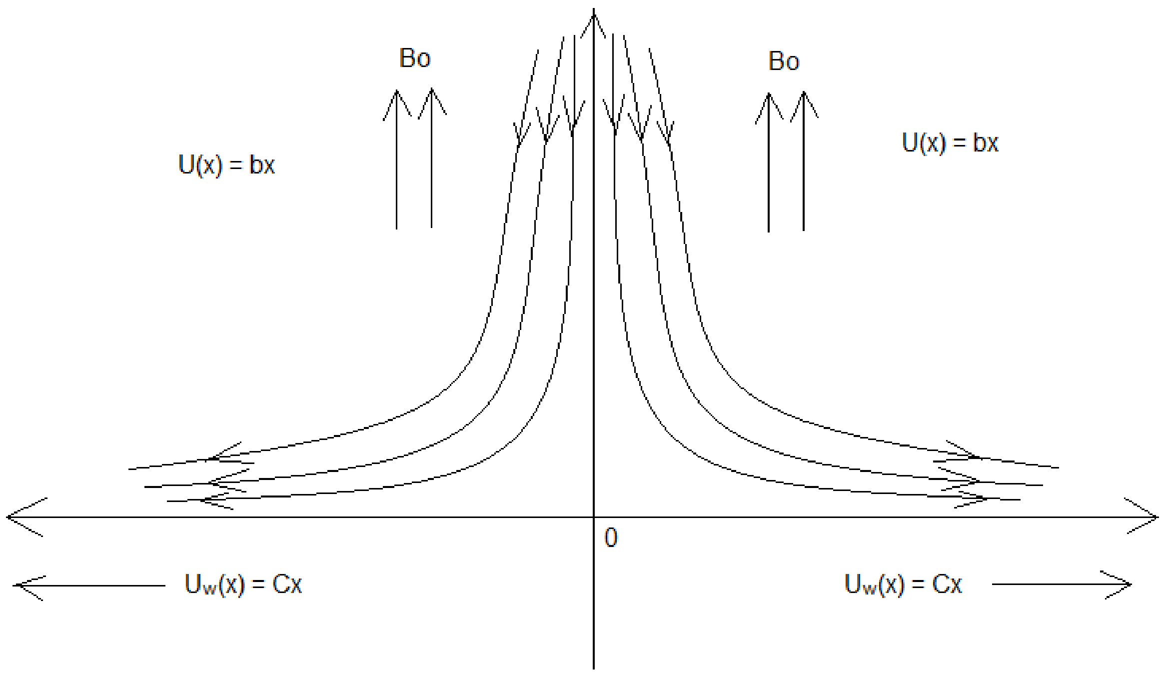

Figure 1. Physical model

The fluid occupies the upper half plane y>0. The continuity, momentum and energy equations governing the flow under the influence of externally imposed transverse magnetic field in the boundary layer are:

$$\frac{\partial_u}{\partial_x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + V\frac{\partial^2 u}{\partial y^2} + \sigma B^2{}_0 u/\rho \tag{2}$$

$$\rho C_p\left[u\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y}\right] = \frac{\partial}{\partial y}\left(K^*\frac{\partial T}{\partial y}\right) + Q(T + T_\infty) \tag{3}$$

Where u and v are the velocity components along x and y axes, p, σ, B_0, ρ, C_p, K*, Q, T, and T_∞ are pressure of the fluid, electrical conductivity, magnetic field intensity, density of fluid, specific heat at constant pressure, variable thermal conductivity, volumetric rate of heat, fluid temperature and free stream temperature respectively.

The second derivatives of u and T with respect to x have been eliminated on the basis of magnitude analysis considering that Reynolds number is high. Hence the Navier-Stoke's equation modifies into Prandthl's boundary layer equation. In the free stream, $u + U(x) = bx$.

Then equation (2) reduces to

$$U\frac{dU}{dx} = \frac{-1}{\rho}\frac{\partial p}{\partial x} - \sigma\frac{B^2{}_0}{\rho}U \tag{4}$$

$\frac{\partial p}{\partial x}$ can be eliminated by subtracting equation(4) from equation(2) to have

$$u\frac{\partial u}{\partial x} + V\frac{\partial U}{\partial y} = U\frac{dU}{dx} + V\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B^2{}_0}{\rho}(u - U) \tag{5}$$

Subject to:

$$y = 0;\quad u = u_{x(x)},\quad V = 0, T = T_w$$

$$y \to \infty;\quad u = U(x) = bx, T = T_\infty \tag{6}$$

Following Chiam(1998), the thermal conductivity is

$$K^* = K(1 + \in\theta) \tag{7}$$

In equation (3)-(6), we have used the following similarity variables and stream function for transformation;

$$u = \frac{\partial\psi}{\partial y} \qquad v = -\frac{\partial\psi}{\partial x} \tag{8}$$

$$\eta = \left(\frac{c}{v}\right)^{1/2} y \text{ and } \psi(x, y) = (cv)^{\frac{1}{2}} x f(\eta) \tag{9}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

$$f''' + ff'' - f'^2 - M^2(f' - \lambda) + \lambda^2 = 0 \tag{11}$$

$$(1+\in\theta)\theta'''+\in\theta''^2+\text{prf}\theta''+\text{prS}\theta=0 \quad (12)$$

$$f'(0)=1,\quad f(0)=0,\quad \theta(0)=1,\quad f'(\infty)=\lambda,\quad \theta(\infty)=0 \quad (13)$$

$$M^2=\frac{\sigma B^2{}_0}{\rho C},\ \lambda=\frac{b}{c},\ \Pr=\frac{\mu C_p}{K},\ S=\frac{Q}{\rho C_{pC}} \quad (14)$$

Where M, λ, Pr, S and ε are magnetic field parameter, ratio of free stream velocity to stretching sheet parameter, Prandth number, heat source/sink and thermal conductivity parameter respectively.

Equations (11) and (12) are highly couple non-linear ordinary differential equation and since $\in << 1$ we assumed a perturbation of this form:

$$f(\eta)=f_0+\in f_1+\in^2 f_2+-----$$

$$\theta(\eta)=\theta_0+\in\theta_1+\in^2\theta_2+----- \quad (15)$$

Where $\in$ is the perturbation parameter. We used it to transform equation (11) and (12) to system of linear non- homogenous and homogenous differential equation by neglecting terms in $\in^3$ and higher. We then solved the system of equations with the boundary conditions to obtain solutions for the flow velocity and temperature respectively as follows:
f_0 and θ_0 are taken as the given conditions θ_0= and $f_0=1+\lambda$ [Adomian(2001)].

Substituting equation (15) into equations (11)-(12) and neglecting the higher order terms of $O(\in^3)$, we obtain the following set of equations

$$f_1'''+(1+\lambda)f_1''-M^2f_1'=-\lambda(M^2+\lambda) \quad (16)$$

$$\theta_1''+\Pr(1+\lambda)\theta_1'+\Pr S\theta_1=0 \quad (17)$$

$$f_2'''+\Pr(1+\lambda)f_2''-M^2f_2'=-\lambda(M^2+\lambda)-f_1f_1''+f_1'^2 \quad (18)$$

$$\theta_2''+\Pr(1+\lambda)\theta_2'+\Pr S\theta_2=-\theta_1'-\Pr\theta_1'f_1 \quad (19)$$

Solving equations (16)-(19) under the boundary conditions equation(13), we obtain the expressions for velocity flow and temperature as follows

$$f(\eta)=1+\lambda+\in\left(A_1+A_2e^{-K_{2\eta}}\right)+\in^2\left(A_3+A_4e^{-K_{2\eta}}+A_{6\eta}e^{-K_{2\eta}}\right) \quad (20)$$

$$\theta(\eta)=1+\in B_1e^{-q\eta}+\in^2\left(B_2e^{-q\eta}+B_{3\eta}e^{-q\eta}+B_4e^{-(K_{2+q})\eta}\right) \quad (21)$$

Where

$$K_2=\frac{(1+\lambda)+\sqrt{(1+\lambda)^2+4M^2}}{2},\ A_1=\frac{1}{K_2},\ A_2=-\frac{1}{K_2},\ A_3=\frac{1-A_6}{K_2},\ A_4=\frac{A_6-1}{K_2},$$

$$A_6=\frac{-A_1A_2K_2^2}{3K_2^2-2(1+\lambda)K-M^2},\ q=\frac{\Pr(1+\lambda)+\sqrt{\Pr^2(1+\lambda)-4\Pr S}}{2},\ B_1=1,$$

$$B_3=\frac{-(B_1q^2-\Pr B_1qA_1)}{-q(2+q)+\Pr(1+\lambda)-\Pr(1+\lambda)q+\Pr S},\ B_4=\frac{A_2\Pr B_1q}{(q+K_2)-\Pr(1+\lambda)(q+K_2)+\Pr S},$$

$$B_2=1-B_4$$

3. Skin-friction

The skin-friction co-efficient at the sheet is given by

$$C_f = \frac{\tau_w}{\rho C (CV)^{1/2}} = x f''(0) \tag{22}$$

where $\tau_w = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0}$ is the shear stress at the sheet.

4. Nusselt number

The rate of heat transfer in terms of the Nusselt number at the sheet is given by

$$Nu = \left(\frac{v}{c}\right)^{1/2} \frac{q_w}{K*(T_{w-T_x})} = -\theta''(0) \tag{23}$$

5. Results and discussion

We have formulated and solve the problem of hydro-magnetic flow near a stagnation point on a stretching sheet with variable thermal conductivity and heat source by perturbation technique. Using the software "Mathematica", the numerically computed and graphical results are presented.

From Table 3 it can be seen that $f''(0)$ for different values of λ when M=0 is in good agreement with the results obtained by Mahapatra and Gupta (2002) and Sharma and Singh (2008). It is noted from Table 4 that -θ'(0) for different values of λ are in good agreement with the results obtained by Mahapatra and Gupta (2002) and Sharma and Singh (2008). It is observed from Table 5 and Table 6 that the result obtained in the present paper for -θ'(0) when Pr=0.01, 0.023 and ε=0.0, 0.1 are in good agreement with that of Sharma and Singh (2008).

The Skin-friction co-efficient and Nusselt number are presented by equation (22) and (23). The effects of M, S, ε, λ and S on $f''(0)$ and -θ(0) has been presented through Table 1 and Table 2 respectively.

Table 1. Values of $f''(0)$ for different values of λ and M

λ	M	$f''(0)$	M	λ	$f''(0)$
0.1	0.0	-0.986868	0.0	0.1	-0.941954
0.2		-0.947548			
0.5		-0.686197	0.5		-0.963018
2.0		2.082568			
3.0		4.806621	1.0		-0.974906

Table 2. Values of -θ'(0) for different values of λ, M, S, Pr and ε

λ	M	S	Pr	ε	-θ'(0)
0.1	0.0	0.0	0.01	0.1	0.081496
0.2					0.096990
0.5					0.129719
2.0					0.235392
3.0					0.313076
	M				
	0.1				0.101064
	0.5				0.108786
	1.0				0.108788
		S			
		0.1			0.011978
		0.2			0.010878
			Pr		
0.0			0.01		0.006616
0.0			0.023		0.015217
			0.05	0.0	0.050558
			0.05	0.1	0.045562

Table 3. $f''(0)$ for different values of λ compared with the results obtained by Mahapatra and Gupta (2002) and Sharma and Singh (2008)

λ	Mahapatra and Gupta (2002)	Sharma and Singh (2008)	Present paper
0.1	-0.9694	-0.969386	-0.986868
0.2	-0.9181	-0.918106	-0.947548
0.5	-0.6673	-0-667263	-0.686197

Table 4.-θ'(0) for different values of λ compared with the results obtained by Mahapatra and Gupta (2002) and Sharma and Singh (2008)

λ	Mahapatra and Gupta (2002)	Sharma and Singh (2008)	Present paper
0.1	0.081	0.081245	0.081497
0.5	0.136	0.13557	0.129719
2-0	0.241	0.241025	0.235392

Table 5. -θ'(0) for different values Pr compared with results obtained by Sharma and Singh (2008)

Pr	Sharma and Singh (2008)	Present paper
0.01	0.006570	0.006616
0.023	0.015002	0.015217

Table 6. -θ'(0) for different values of ε compared with results obtained by Sharma and Singh (2008)

ε	Sharma and Singh (2008)	Present paper
0.0	0.050281	0.0505585
0.1	0.047069	0.0455562

Figure 2 shows that the velocity profile decreases with increase in magnetic field parameter. This is because of the presence of transverse magnetic field which sets in Lorentz force, and this result in retarding force on the velocity field and therefore as magnetic field parameter increases, so does the retarding force.

It is observed from Table 2 that the rate of heat transfer at the sheet increases due to increase in the Prandtl number in the absence of magnetic field when thermal conductivity of the fluid is constant or variable in the presence/absence of the source/sink parameter. This is so, because the thermal boundary layer for low Prandtl number fluid is thick and consequently the temperature gradient decreases with the decrease in Prandtl number. The rate of heat transfer at the sheet increases due to increase in λ, which is due to the fact that thermal boundary thickness decreases due to increase in λ and is also shown in Figure 5. Figure 3 shows that with the increase in the value of ε, temperature profile increases hence considering the thermal conductivity constant would lead to lower approximation of the temperature profile. Figure 4 shows that the fluid temperature profile decreases due to decrease in the rate of heat generation. It is seen from Figure 5 that fluid temperature decreases due to increase in λ and Figure 6 depicted that with increase in Prandtl number, temperature profile decreases which shows that with increase in Prandtl number, thermal boundary layer thickness reduces. Figure 7 shows the effects of magnetic parameter on temperature profile which increases with increase in the magnetic parameter. The temperature of fluid near the sheet is higher because the magnetic field retards the velocity of the fluid.

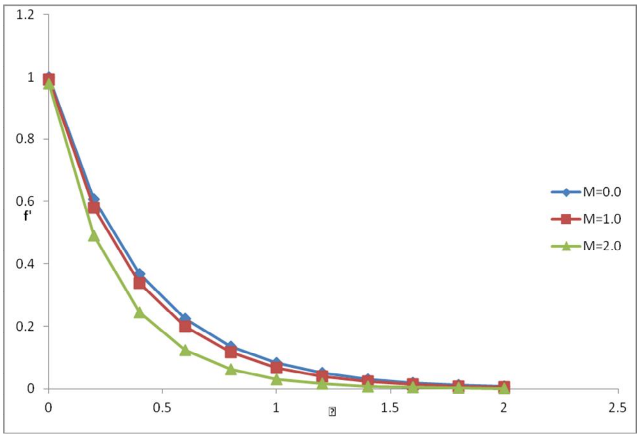

Figure 2. Velocity profiles for different values of magnetic parameter

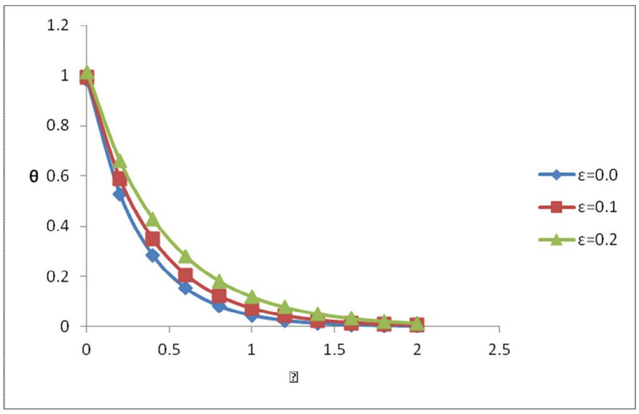

Figure 3. Temperature profiles for different values thermal conductivity parameter

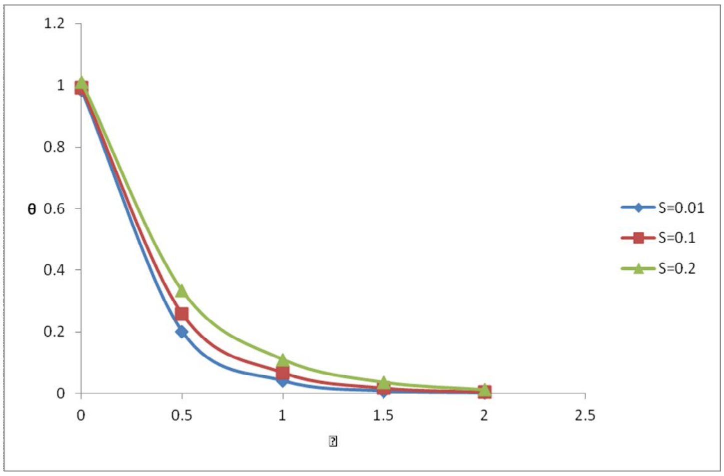

Figure 4. Temperature profiles for different values of heat source parameter, S

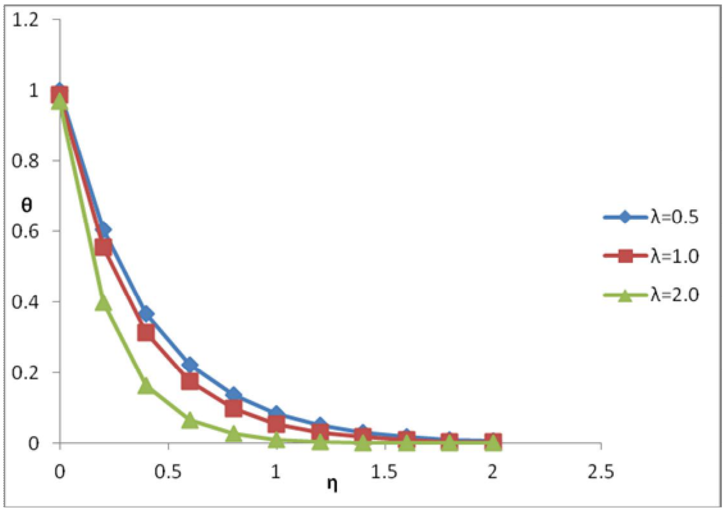

Figure 5. Temperature profiles for different values of ratio parameter, λ

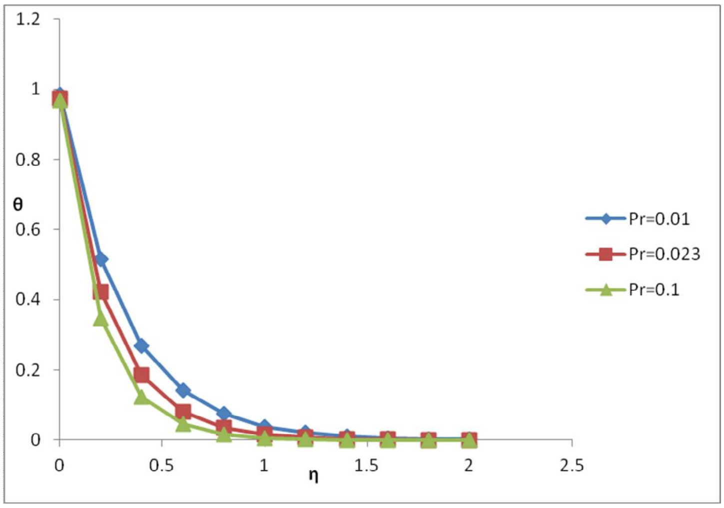

Figure 6. Temperature profiles for different values of prandth number, Pr

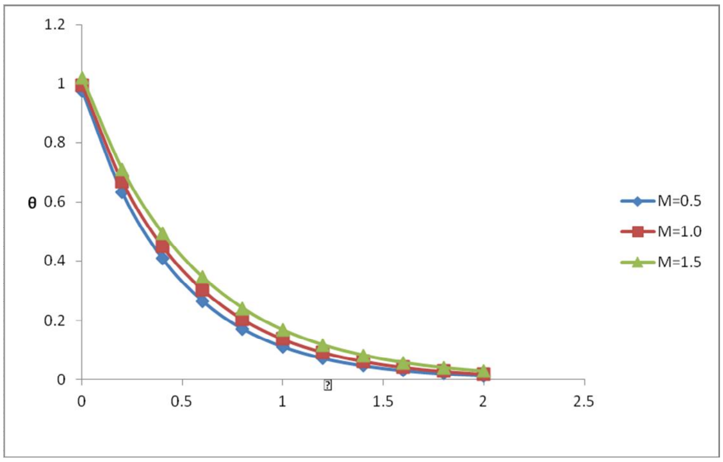

Figure 7. Temperature profiles for different values of magnetic parameter, M

6. Conclusion

The problem of hydro magnetic flow near a stagnation point on a stretching sheet with variable thermal conductivity and heat source/sink has been investigated. The effects of magnetic field, thermal conductivity, heat source, Prandtl number and ratio of free stream velocity to stretching sheet parameter on velocity and temperature profiles as well as on skin-friction and on rate of heat transfer has also been examined. The results indicated that these parameters have significant influences on the velocity, temperature, Skin-friction and on the rate of heat transfer. Generally, our results shows that:

a) The rate of heat transfer at the sheet increases due to increase in λ.
b) Temperature profile increases due to increase in thermal conductivity parameter.
c) Decrease in the rate of heat generation decreases temperature profile.
d) Temperature profile increases with increase in magnetic field parameter which shows that the temperature of fluid near the sheet is higher.

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