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Reliability Analysis of Embedded System with Different Modes of Failure Emphasizing Reboot Delay

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Abstract: Present paper discusses the reliability analysis of a complex system which consists of two repairable subsystems namely A and B connected in series. Subsystem A is a *k*-out of -*n*: G system and subsystem B is a circular consecutive 2-out of -3: F system. In this study an inspection shop has been taken into consideration along with a special type of delay viz. reboot delay. By employing supplementary variable technique, Laplace transformation and Gumbel-Hougaard family of copula various transition state probabilities, reliability, availability, M.T.T.F., cost analysis, sensitivity analysis and steady state behaviour of the system have been obtained. At the end some special cases of the system have been taken and important results have been derived as particular cases.

Keywords: Reliability; availability; mean time to failure; sensitivity; reboot delay; complex system; gumbel-hougaard family of copula.

1. Introduction

Many reliability studies assume stochastically independent components of a system, i.e. the condition of one component does not affect the lifetime distribution of other components. But in real world there are systems having dependent components in which a component failure may have some influence on the working components by causing an increased stress. The reliability characteristics of such systems depend not only on its components but also on the interactions among the components. In these systems a single failure event can propagate and cause failure of other components. For example, in a load sharing system if one component fails, the work load has to be shared by remaining working components, which can cause an increase in the failure rate of every working component. In general a system passes through three phases of failure viz. initial failure (due to designing problem), random failure (due to accident or any other reason) and wear out failure (due to over age of the system). Each system may have different failure properties. For instance, the failure rate of the system may be constant, increasing or decreasing and the types of the failures (partial failure, catastrophic failure, human failure etc.) may be different. Likewise it may be the case that in the system chances of initial failure is maximum. In this case once the system passes the initial phase successfully then it will work properly for a long period of time. Many useful results [3, 4, 5, 8, 9, 10, 11] have been published by researchers regarding various types of failure in a system. But it seems that a system having maximum initial chances of failure has not been studied widely. Researches [1, 2] carried out analysis of systems

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having k-out of-n: G system but they have not considered the concept of inspection shop where it is to be decided whether the system should go for repair or for replacement of the failed units. Furthermore, the system configuration also affects the failure properties of a system. The systems having different configurations possess different properties. A circular consecutive 2-out of-3: F is a type of system configuration having a sequence of 3 ordered components arranged along a circle such that the system will be in failure condition if at least 2 consecutive components in the system will fail. In these systems the order in which the components are arranged is also important. There are some studies [13] in which such systems with identical components have been considered. But if the components are not identical the system will have different properties. Thus, there is a scope for further work in this direction.

In real life it can be seen that the stoppage of a system due to any error takes some time to recover/restart, ofcourse with some recovery rate. The time between the system failure and restart is said to be reboot delay. The concept of reboot delay and its effect on the reliability and/or availability of a repairable system has been proposed by [12]. The above situation can be visualised in communication and computer science applications where there is a reboot delay. Recently, [6, 7] studied reliability measures of repairable systems in which switching failures, detection delay and reboot delay are considered. Reboot delay can be viewed as the unreliability of a recovery software. Further, the reliability of a system with inspection shop concept emphasizing reboot delay employs an increasingly important issue in power systems, manufacturing systems, and industrial systems.

Therefore, keeping all the above facts under consideration here researchers attempt to study the reliability and sensitivity measures of a complex system having different modes of failure incorporating reboot delay. This paper differs from previous works in the following sense: (i) it emphasizes the reliability problem with inspection shop concept emphasizing reboot delay (ii) it performs sensitivity analysis for the reliability with respect to different system parameters. Various reliability measures have also been computed for the considered reliability model for different cases. An attempt has also been made to provide performance analysis of the system by applying the reliability theoretic approaches. Several reliability characteristics of complex system evaluated in this study will be helpful to system designers as well as operations managers.

The present contribution is structured as follows: Section 2 describes the material and methods of the model; sub-sections 2.1, 2.2, 2.3, 2.4 and 2.5 cover the assumptions, notations, formulation of mathematical model and solution of the model respectively of the proposed problem. Section 3 covers results and discussion of the model; sub-sections 3.1, 3.2 and 3.3 cover asymptotic behaviour of the system, particular case in the absence of reboot delay in the system and numerical computations respectively. Again sub-section 3.3 sub divided into five sections 3.3.1, 3.3.2, 3.3.3, 3.3.4 and 3.3.5 which compute different reliability measures namely availability analysis, reliability analysis, M. T. T. F. analysis, cost analysis and sensitivity respectively. Section 4 presents the conclusions of the proposed analysis. Block diagram, transition diagram and state specification chart of the system are shown in Figures 1, 2 and Table 1 respectively.

2. Material and methods

In this study, the complex system considered consists of two repairable subsystems namely A and B. Subsystem A is a k-out of -n: G system, i.e if any k units out of n units work, subsystem A will be operational whereas subsystem B is a circular consecutive 2-out of -3: F system i.e, if consecutively two units out of three units fail, subsystem B will fail. The subsystems A and B are arranged in series configuration. The subsystem A can fail only from fully operational state. Once it passed through the initial phases it will be operational for sufficiently long period of time. During this period if any failure occurs, then it may lead to further failures in the system. Since subsystem A is a non repairable system so whenever there occurs any failure in subsystem A we go for replacement. It is also assumed that after failure in the subsystem B, the system goes to the inspection shop where it is to be decided whether the system should go for repair or for replacement of the failed units. We have assumed that after repair of the subsystem B it works at higher risk of failure. Further, the policy of the maintenance company providing the replacement or repair of the units is to replace the units if they further fail after repair. So any further failure in the subsystem B is being replaced by new subsystem without any inspection. The state S_6 in which the subsystems A and B both are in failed state is a critical degraded state. Any further failure in the system from a critical degraded state will be replaced by the new one. As mentioned earlier, many times failure takes some time to recover/restart. For example, when the power goes, the generator takes some time to start. Consequently power system takes some time to restart which is said to be reboot delay for this. This aspect is also incorporated in reliability modeling of the present system. The system is studied by using the supplementary variable technique, Laplace transform and Gumbel-Hougaard family of Copula. When two types of repair occur in present system, coupled repair rate evaluated by Gumbel-Hougaard family of copula. At last some numerical examples have been taken to highlight the reliability characteristics of the system. The following characteristics of the system have been analyzed:

a) Transition state probabilities of the system.
b) Asymptotic behaviour of the system.
c) Various measures such as reliability, availability, M.T.T.F. and cost effectiveness of the system.
d) Sensitivity analysis of the system with respect to different parameters.

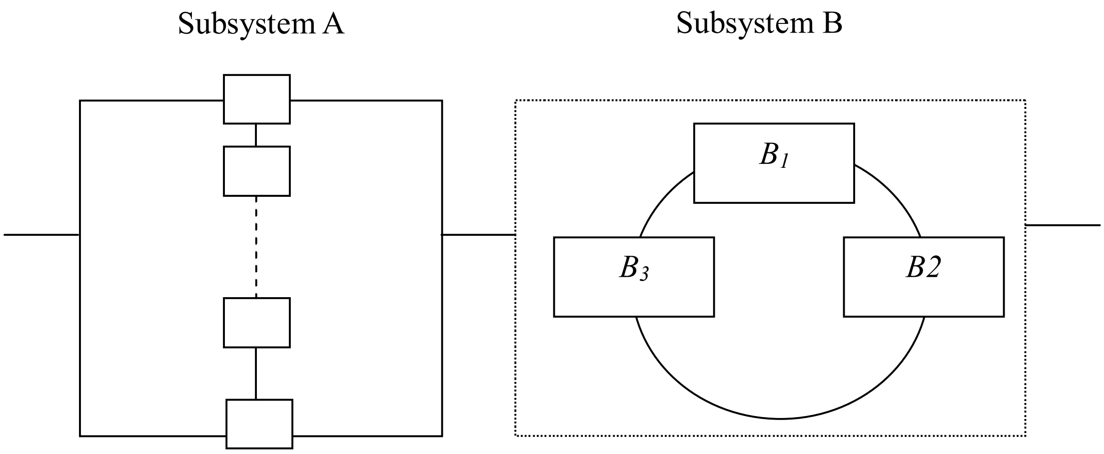

Figure 1. Diagram of investigated system

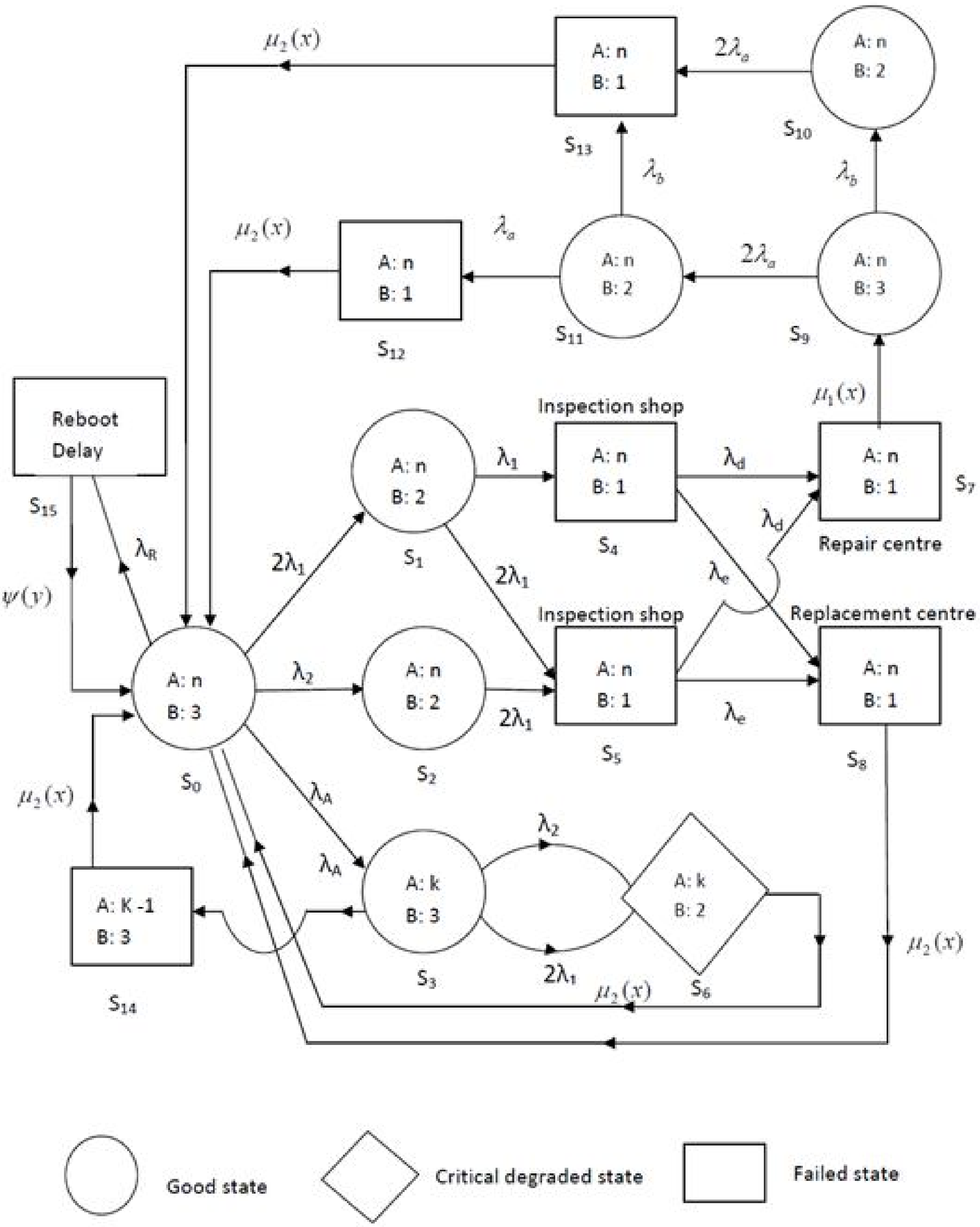

Figure 2. State transition diagram

Table 1. State specification

States	State of subsystem A	Number of good units of subsystem B	System state
S_0	G	3	G
S_1	G	2	G
S_2	G	2	G
S_3	G	3	G
S_4	G	1	F
S_5	G	1	F
S_6	G	2	D_R
S_7	G	1	F_r
S_8	G	1	F_R
S_9	G	3	G
S_{10}	G	2	G
S_{11}	G	2	G
S_{12}	G	1	F_R
S_{13}	G	1	F_R
S_{14}	F	3	F_R
S_{15}	F	0	F_R

G: Good state**,** F: Failed state; F_r: Failed under repair; F_R: Failed under replacement; D_R: Critical degraded under replacement.

2.1. Assumptions

a) Initially all components are functioning perfectly.
b) The system consists of two subsystems namely A and B.
c) System has three states good, critical degraded and failed.
d) Subsystem A and B are arranged in series configuration.
e) The state S_6 in which both subsystems A and B are in failed state, is a critical degraded state. From a critical degraded state we replace the failed units directly without inspection.
f) Subsystem A can fail only from fully operational state. Once it starts working properly then it will not fail for a long period of time. During this period if any failure occurs then it may lead to further failures.
g) The policy of the maintenance company providing the replacement or repair of the units is to replace the units without inspection if these units further fail after repair.
h) After repair of subsystem B it works at higher risk of failure, i.e. failure rate of subsystem B increases in this state.
i) Due to the reboot delay the system stops and restarts after some time with some recovery rate.

2.2. Notations

The following notations are associated with this model

λ_1	Failure rate of first and third unit of B.
λ_2	Failure rate of second unit of B.
λ_a	Increased failure rate of first and third unit of B.
λ_b	Increased failure rate of second unit of B.
λ_R	Reboot delay rate.
$\mu_1(x)$	General repair rate after detecting when individual repair is done.
$\mu_2(x)$	General repair rate after failure in critical degraded state when individual repair is done.
λ_A	Failure rate of subsystem A.
λ	Coupled repair rate i.e. repair rate when two types of repair occur. $C_\theta(2\lambda_1, \lambda_2) = \exp(-((-\log 2\lambda_1)^\theta + (-\log \lambda_2)^\theta)^{1/\theta}), \quad 1 \le \theta \le \infty$
$\psi(y)$	Recovery rate after reboot delay.
$P_i(t)$	Probability that the system is in S_i state at instant 't' for i = 1 to 15.
$P_i(x,t)$	The probability distribution function (system is in state S_i and is under repair, elapsed repair time is (x, t) where i = 4, 5, 6, 7, 8, 12, 13, 14.
$P_{15}(y,t)$	The probability distribution function (system is in state S_{15} and is under repair, elapsed repair time is (y, t)).
$\overline{P_i}(s)$	Laplace transformation of P_i *(t)*.
$E_p(t)$	Expected profit during the interval (0, 1).

2.3. Formulation of mathematical model

By probability consideration and continuity arguments, we obtain the following set of integro-differential equations governing the behavior of the system.

$$\left[\frac{d}{dt} + 2\lambda_1 + \lambda_2 + \lambda_A + \lambda_R\right] P_0(t) = \int_0^\infty P_6(x,t)\mu_2(x)dx + \int_0^\infty P_8(x,t)\mu_2(x)dx + \int_0^\infty P_{12}(x,t)\mu_2(x)dx + \int_0^\infty P_{13}(x,t)\mu_2(x)dx + \int_0^\infty P_{14}(x,t)\mu_1(x)dx + \int_0^\infty P_{15}(y,t)\psi(y)dy \tag{1}$$

$$\left[\frac{d}{dt} + \lambda_1 + \lambda_2\right] P_1(t) = 2\lambda_1 P_0(t) \tag{2}$$

$$\left[\frac{d}{dt} + 2\lambda_1\right] P_2(t) = \lambda_2 P_0(t) \tag{3}$$

$$\left[\frac{d}{dt}+\lambda+\lambda_A\right]P_3(t)=\lambda_A P_0(t) \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_d+\lambda_e\right]P_4(x,t)=0 \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_d+\lambda_e\right]P_5(x,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_2(x)\right]P_6(x,t)=0 \tag{7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_1(x)\right]P_7(x,t)=0 \tag{8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_2(x)\right]P_8(x,t)=0 \tag{9}$$

$$\left[\frac{d}{dt}+2\lambda_a+\lambda_b\right]P_9(t)=\int_0^\infty P_7(x,t)\mu_1(x)dx \tag{10}$$

$$\left[\frac{d}{dt}+2\lambda_a\right]P_{10}(t)=\lambda_b P_9(t) \tag{11}$$

$$\left[\frac{d}{dt}+\lambda_a+\lambda_b\right]P_{11}(t)=2\lambda_a P_9(t) \tag{12}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_2(x)\right]P_{12}(x,t)=0 \tag{13}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_2(x)\right]P_{13}(x,t)=0 \tag{14}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_2(x)\right]P_{14}(x,t)=0 \tag{15}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\psi(y)\right]P_{15}(y,t)=0 \tag{16}$$

Boundary conditions

$$P_4(0,t)=\lambda_1 P_1(t) \tag{17}$$

$$P_5(0,t)=\lambda_2 P_1(t)+2\lambda_1 P_2(t) \tag{18}$$

$$P_6(0,t)=\lambda P_3(t) \tag{19}$$

$$P_7(0,t)=\lambda_D\left[P_4(t)+P_5(t)\right] \tag{20}$$

$$P_8(0,t)=\lambda_e\left[P_4(t)+P_5(t)\right] \tag{21}$$

$$P_{12}(0,t)=\lambda_a P_{11}(t) \tag{22}$$

$$P_{13}(0,t)=\lambda_b P_{11}(t)+2\lambda_a P_{10}(t) \tag{23}$$

$$P_{14}(0,t)=\lambda_A P_3(t) \tag{24}$$

$$P_{15}(0,t)=\lambda_R P_0(t) \tag{25}$$

where $\lambda=\exp(-((-\log 2\lambda_1)^{\theta}+(-\log\lambda_2)^{\theta})^{1/\theta}),\quad 1\le\theta\le\infty$

Initial condition

$P_0(0)=1$ and other probabilities are zero at t=0. (26)

2.4. Solution of the model

Taking Laplace transform of equations (1)-(25) and using initial condition, we get

$$\begin{aligned}\left[s+2\lambda_1+\lambda_2+\lambda_A+\lambda_R\right]\overline{P_0}(s)=1&+\int_0^\infty\overline{P_6}(x,s)\mu_2(x)dx+\int_0^\infty\overline{P_8}(x,s)\mu_2(x)dx\\&+\int_0^\infty\overline{P_{12}}(x,s)\mu_2(x)dx+\int_0^\infty\overline{P_{13}}(x,s)\mu_2(x)dx\\&+\int_0^\infty\overline{P_{14}}(x,s)\mu_2(x)dx+\int_0^\infty\overline{P_{15}}(y,s)\psi(y)dy\end{aligned} \tag{27}$$

$$\left[s+\lambda_1+\lambda_2\right]\overline{P_1}(s)=2\lambda_1\overline{P_0}(s) \tag{28}$$

$$\left[s+2\lambda_1\right]\overline{P_2}(s)=\lambda_2\overline{P_0}(s) \tag{29}$$

$$\left[s+\lambda+\lambda_A\right]\overline{P_3}(s)=\lambda_A\overline{P_0}(s) \tag{30}$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_d+\lambda_e\right]\overline{P_4}(x,s)=0 \tag{31}$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_d+\lambda_e\right]\overline{P_5}(x,s)=0 \tag{32}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_2(x)\right]\overline{P_6}(x,s)=0 \tag{33}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_1(x)\right]\overline{P_7}(x,s)=0 \tag{34}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_2(x)\right]\overline{P_6}(x,s)=0 \tag{35}$$

$$\left[s+2\lambda_a+\lambda_b\right]\overline{P_9}(s)=\int_0^\infty \mu_1(x)\overline{P_7}(x,s)dx \tag{36}$$

$$\left[s+2\lambda_a\right]\overline{P_{10}}(s)=\lambda_b\overline{P_9}(s) \tag{37}$$

$$\left[s+\lambda_a+\lambda_b\right]\overline{P_{11}}(s)=2\lambda_a\overline{P_9}(s) \tag{38}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_2(x)\right]\overline{P_{12}}(x,s)=0 \tag{39}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_2(x)\right]\overline{P_{13}}(x,s)=0 \tag{40}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_2(x)\right]\overline{P_{14}}(x,s)=0 \tag{41}$$

$$\left[s+\frac{\partial}{\partial y}+\psi(y)\right]\overline{P_{15}}(y,s)=0 \tag{42}$$

Boundary conditions

$$\overline{P_4}(0,s)=\lambda_1\overline{P_1}(s) \tag{43}$$

$$\overline{P_5}(0,s)=\lambda_2\overline{P_1}(s)+2\lambda_1\overline{P_2}(s) \tag{44}$$

$$\overline{P_6}(0,s)=\lambda\overline{P_3}(s) \tag{45}$$

$$\overline{P_7}(0,s)=\lambda_d\left[\overline{P_4}(s)+\overline{P_5}(s)\right] \tag{46}$$

$$\overline{P_8}(0,s) = \lambda_e\left[\overline{P_4}(s) + \overline{P_5}(s)\right] \tag{47}$$

$$\overline{P_{12}}(0,s) = \lambda_a \overline{P_{11}}(s) \tag{48}$$

$$\overline{P_{13}}(0,s) = \lambda_b \overline{P_{11}}(s) + 2\lambda_a \overline{P_{10}}(s) \tag{49}$$

$$\overline{P_{14}}(0,s) = \lambda_A \overline{P_3}(s) \tag{50}$$

$$\overline{P_{15}}(0,s) = \lambda_R \overline{P_0}(s) \tag{51}$$

Solving equations (27-42) and using equations (43-51), one can get the following transition state probabilities:

$$\overline{P_1}(s) = \frac{2\lambda_1}{(s+\lambda_1+\lambda_2)}\overline{P_0}(s) \tag{52}$$

$$\overline{P_2}(s) = \frac{\lambda_2}{(s+2\lambda_1)}\overline{P_0}(s) \tag{53}$$

$$\overline{P_3}(s) = \frac{\lambda_A}{(s+\lambda+\lambda_A)}\overline{P_0}(s) \tag{54}$$

$$\overline{P_4}(s) = \frac{2\lambda_1^2}{\left(s+\lambda_1+\lambda_2\right)}\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]\overline{P_0}(s) \tag{55}$$

$$\overline{P_5}(s) = 2\lambda_1\lambda_2\left[\frac{1}{(s+\lambda_1+\lambda_3)}+\frac{1}{\left(s+2\lambda_1\right)}\right]\left[\frac{1-\overline{S}_{\mu}(s)}{s}\right]\overline{P_0}(s) \tag{56}$$

$$\overline{P_6}(s) = \frac{\lambda\lambda_A}{\left(s+\lambda+\lambda_A\right)}\left[\frac{1-\overline{S}_{\mu_2}(s)}{s}\right]\overline{P_0}(s) \tag{57}$$

$$\overline{P_7}(s) = \frac{2\lambda_1\lambda_d}{(s+\lambda_d+\lambda_e)}\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{\left(s+\lambda_1+\lambda_2\right)}+\frac{1}{\left(s+2\lambda_1\right)}\right\}\right]\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]\overline{P_0}(s) \tag{58}$$

$$\overline{P_8}(s) = \frac{2\lambda_1\lambda_e}{(s+\lambda_d+\lambda_e)}\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{\left(s+\lambda_1+\lambda_2\right)}+\frac{1}{\left(s+2\lambda_1\right)}\right\}\right]\left[\frac{1-\overline{S}_{\mu_2}(s)}{s}\right]\overline{P_0}(s) \tag{59}$$

$$\overline{P_9}(s) = \frac{2\lambda_1\lambda_d\overline{S}_{\mu_1}(s)}{(s+\lambda_d+\lambda_e)\left(s+2\lambda_a+\lambda_b\right)}\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right] \left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{\left(s+\lambda_1+\lambda_2\right)}+\frac{1}{\left(s+2\lambda_1\right)}\right\}\right]\overline{P_0}(s) \tag{60}$$

$$\overline{P_{10}}(s) = \frac{2\lambda_1\lambda_d\lambda_b\overline{S}_{\mu_1}(s)}{(s+\lambda_d+\lambda_e)\left(s+2\lambda_a+\lambda_b\right)\left(s+2\lambda_a\right)}\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right] \left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{\left(s+\lambda_1+\lambda_2\right)}+\frac{1}{\left(s+2\lambda_1\right)}\right\}\right]\overline{P_0}(s) \tag{61}$$

$$\overline{P_{11}}(s)=\frac{4\lambda_1\lambda_a\lambda_d\overline{S}_{\mu_1}(s)}{(s+\lambda_d+\lambda_e)(s+2\lambda_a+\lambda_b)(s+\lambda_a+\lambda_b)}\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]$$
$$\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{(s+\lambda_1+\lambda_2)}+\frac{1}{(s+2\lambda_1)}\right\}\right]\overline{P_0}(s) \tag{62}$$

$$\overline{P_{12}}(s)=\lambda_a\overline{P_{11}}(s)\left[\frac{1-\overline{S}_{\mu_2}(s)}{s}\right] \tag{63}$$

$$\overline{P_{13}}(s)=\left(\lambda_b\overline{P_{11}}(s)+2\lambda_a\overline{P_{10}}(s)\right)\left[\frac{1-\overline{S}_{\mu_2}(s)}{s}\right] \tag{64}$$

$$\overline{P_{14}}(s)=\frac{\lambda_A^2}{(s+\lambda+\lambda_A)}\left[\frac{1-\overline{S}_{\mu_2}(s)}{s}\right]\overline{P_0}(s) \tag{65}$$

$$\overline{P_{15}}(s)=\lambda_R\left[\frac{1-\overline{S}_{\psi}(s)}{s}\right]\overline{P_0}(s) \tag{66}$$

where $\overline{S}_{\mu_1}(s)=\dfrac{\mu_1(x)}{s+\mu_1(x)}$, $\overline{S}_{\mu_2}(s)=\dfrac{\mu_2(x)}{s+\mu_2(x)}$, $\overline{S}_{\psi}(s)=\dfrac{\psi(y)}{s+\psi(y)}$

Putting all these values in equations (27) and simplifying, we get transition state probability at S_0 as

$$\overline{P}_0(s)=\frac{1}{D(s)} \tag{67}$$

where

$$D(s)=[s+2\lambda_1+\lambda_2+\lambda_A+\lambda_R]-\left[\left[\left\{\frac{\lambda\lambda_A}{(s+\lambda+\lambda_A)}\overline{S}_{\mu_2}(s)\right\}\right.\right.$$
$$\left.+\left\{\frac{2\lambda_1\overline{S}_{\mu_2}(s)}{(s+\lambda_d+\lambda_e)}\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{(s+\lambda_1+\lambda_2)}+\frac{1}{(s+2\lambda_1)}\right\}\right]\right]\right\}$$
$$\left\{\lambda_e+\frac{2\lambda_a^2\lambda_d\overline{S}_{\mu_1}(s)}{(s+\lambda_a+\lambda_b)(s+2\lambda_a+\lambda_b)}\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]+\frac{2\lambda_a\lambda_b\lambda_d\overline{S}_{\mu_1}(s)}{(s+2\lambda_a+\lambda_b)}\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]\right.$$
$$\left.\left.\left(\frac{1}{(s+2\lambda_a)}+\frac{1}{(s+\lambda_a+\lambda_b)}\right)\right\}+\left\{\frac{\lambda_A^2}{(s+\lambda+\lambda_A)}\overline{S}_{\mu_2}(s)\right\}+\left\{\lambda_R\overline{S}_{\psi}(s)\right\}\right] \tag{68}$$

Transition state probability that the system is in the up state is given by

$$\overline{P}_{up}(s)=\overline{P}_0(s)+\overline{P}_1(s)+\overline{P}_2(s)+\overline{P}_3(s)+\overline{P}_6(s)+\overline{P}_9(s)+\overline{P}_{10}(s)+\overline{P}_{11}(s)$$

$$=\frac{1}{D(s)}\left[1+\frac{2\lambda_1}{(s+\lambda_1+\lambda_2)}+\frac{\lambda_2}{(s+2\lambda_1)}+\frac{\lambda_A}{(s+\lambda+\lambda_A)}+\frac{\lambda\lambda_A}{(s+\lambda+\lambda_A)}\left[\frac{1-\overline{S}_{\mu_2}(s)}{s}\right]\right.$$
$$+\frac{2\lambda_1\lambda_d\overline{S}_{\mu_1}(s)}{(s+\lambda_d+\lambda_e)(s+2\lambda_a+\lambda_b)}\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{(s+\lambda_1+\lambda_2)}+\frac{1}{(s+2\lambda_1)}\right\}\right] \tag{69}$$
$$\left.\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]\left(1+\frac{\lambda_b}{(s+2\lambda_a)}+\frac{2\lambda_a}{(s+\lambda_a+\lambda_b)}\right)\right]$$

Transition state probability that the system is in the down state is given by

$$\overline{P}_{down}(s)=\overline{P}_4(s)+\overline{P}_5(s)+\overline{P}_7(s)+\overline{P}_8(s)+\overline{P}_{12}(s)+\overline{P}_{13}(s)+\overline{P}_{14}(s)+\overline{P}_{15}(s)$$

$$=\frac{1}{D(s)}\left[\left\{\frac{2\lambda_1^2}{(s+\lambda_1+\lambda_2)}+2\lambda_1\lambda_2\left(\frac{1}{(s+\lambda_1+\lambda_3)}+\frac{1}{(s+2\lambda_1)}\right)\right\}\left\{\frac{1-\overline{S}_{\mu_1}(s)}{s}\right\}\right.$$
$$+\left\{\frac{2\lambda_1\lambda_d}{(s+\lambda_d+\lambda_e)}\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{(s+\lambda_1+\lambda_2)}+\frac{1}{(s+2\lambda_1)}\right\}\right]\right\} \tag{70}$$
$$\left\{\lambda_d\left(\frac{1-\overline{S}_{\mu_1}(s)}{s}\right)+\lambda_e\left(\frac{1-\overline{S}_{\mu_2}(s)}{s}\right)\right\}+\left\{(\lambda_a+\lambda_b)\overline{P_{11}}(s)+2\lambda_a\overline{P_{10}}(s)\right\}$$
$$\left.\left\{\frac{1-\overline{S}_{\mu_2}(s)}{s}\right\}+\left\{\frac{\lambda_A^2}{(s+\lambda+\lambda_A)}\left(\frac{1-\overline{S}_{\mu_2}(s)}{s}\right)\right\}+\left\{\lambda_R\left(\frac{1-\overline{S}_{\psi}(s)}{s}\right)\right\}\right]$$

It is worth mentioning that

$$\overline{P}_{\text{up}}(s)+\overline{P}_{\text{down}}(s)=\frac{1}{s} \tag{71}$$

3. Results and discussion

3.1. Asymptotic behaviour of the system

Using Abel's lemma, $\lim_{s\to 0}\{s\overline{F}(s)\}=\lim_{t\to\infty}F(t)$ in equation (69) through (70), one can obtain the following time independent probabilities

$$\overline{p}_{up}(s)=\frac{1}{D(0)}\left[1+\frac{2\lambda_1}{\lambda_1+\lambda_2}+\frac{\lambda_2}{(2\lambda_1)}+\frac{\lambda_A}{(\lambda+\lambda_A)}+\frac{\lambda\lambda_A}{(\lambda+\lambda_A)}+\frac{2\lambda_1\lambda_d\overline{S}_{\mu 1}(0)}{(\lambda_d+\lambda_e)(2\lambda_a+\lambda_b)}\right]$$
$$\left[\frac{\lambda_1}{\lambda_1+\lambda_2}+\lambda_2\left\{\frac{1}{\lambda_1+\lambda_2}+\frac{1}{(2\lambda_1)}\right\}\right]\left(1+\frac{\lambda_b}{2\lambda_a}+\frac{2\lambda_a}{(\lambda_a+\lambda_b)}\right) \tag{72}$$

Transition state probability that the system is in the down state is given by

$$\overline{P}_{down}(s)=\frac{1}{D(0)}\left[\left\{\frac{2\lambda_1^2}{(\lambda_1+\lambda_2)}+2\lambda_1\lambda_2\left(\frac{1}{(\lambda_1+\lambda_3)}+\frac{1}{(2\lambda_1)}\right)\right\}\right.$$
$$\left\{\frac{2\lambda_1^2}{(\lambda_1+\lambda_2)}+2\lambda_1\lambda_2\left(\frac{1}{(\lambda_1+\lambda_3)}+\frac{1}{(2\lambda_1)}\right)\right\} \tag{73}$$
$$\left.\{\lambda_d+\lambda_e\}+\{(\lambda_a+\lambda_b)\overline{P_{11}}(0)+2\lambda_a\overline{P_{10}}(0)\}+\left\{\frac{\lambda_A^2}{(\lambda+\lambda_A)}\right\}+\lambda_R\right]$$

where $\overline{S}_{\mu_1}(0)=\frac{1}{1+s}$, $\overline{S}_{\mu_2}(0)=\frac{1}{1+s}$, $\overline{S}_{\psi}(0)=\frac{1}{1+s}$ (74)

$$D(0)=\lim_{s\to 0}D(s) \tag{75}$$

3.2. Particular case

When reboot delay does not occur in the system then the transition state probabilities can be obtained by putting $\lambda_R = 0$ in (52) to (66) which are given below

$$\overline{P}_{up}(s)=\frac{1}{D_1(s)}\left[1+\frac{2\lambda_1}{(s+\lambda_1+\lambda_2)}+\frac{\lambda_2}{s+2\lambda_1}+\frac{\lambda_A}{(s+\lambda+\lambda_A)}+\frac{\lambda\lambda_A}{s+\lambda+\lambda_A}\left[\frac{1-\overline{S}_{\mu_2}(s)}{s}\right]\right.$$
$$+\frac{2\lambda_1\lambda_d\overline{S}_{\mu_1}(s)}{(s+\lambda_d+\lambda_e)(s+2\lambda_a+\lambda_b)}\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{(s+\lambda_1+\lambda_2)}+\frac{1}{(s+2\lambda_1)}\right\}\right] \tag{76}$$
$$\left.\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]\left(1+\frac{\lambda_b}{(s+2\lambda_a)}+\frac{2\lambda_a}{(s+\lambda_a+\lambda_b)}\right)\right]$$

$$\overline{P}_{down}(s)=\frac{1}{D_1(s)}\left[\left\{\frac{2\lambda_1^2}{(s+\lambda_1+\lambda_2)}+2\lambda_1\lambda_2\left(\frac{1}{(s+\lambda_1+\lambda_3)}+\frac{1}{(s+2\lambda_1)}\right)\right\}\left\{\frac{1-\overline{S}_{\mu_1}(s)}{s}\right\}\right.$$
$$\left.+\left\{\frac{2\lambda_1\lambda_d}{(s+\lambda_d+\lambda_e)}\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{(s+\lambda_1+\lambda_2)}+\frac{1}{(s+2\lambda_1)}\right\}\right]\right\}\right.$$
$$\left\{\lambda_d\left(\frac{1-\overline{S}_{\mu_1}(s)}{s}\right)+\lambda_e\left(\frac{1-\overline{S}_{\mu_2}(s)}{s}\right)\right\}+\{(\lambda_a+\lambda_b)\overline{P_{11}}(s)+2\lambda_a\overline{P_{10}}(s)\} \tag{77}$$
$$\left.\left\{\frac{1-\overline{S}_{\mu_2}(s)}{s}\right\}+\left\{\frac{\lambda_A^2}{(s+\lambda+\lambda_A)}\left(\frac{1-\overline{S}_{\mu_2}(s)}{s}\right)\right\}+\left\{\lambda_R\left(\frac{1-\overline{S}_{\psi}(s)}{s}\right)\right\}\right]$$

where $\overline{P}_0(s)=\frac{1}{D_1(s)}$ (78)

$$D_1(s)=[s+2\lambda_1+\lambda_2+\lambda_A]-\left[\left\{\frac{\lambda\lambda_A}{(s+\lambda+\lambda_A)}\overline{S}_{\mu_2}(s)\right\}\right.$$
$$+\left\{\frac{2\lambda_1\overline{S}_{\mu_2}(s)}{(s+\lambda_d+\lambda_e)}\left[\frac{\lambda_1}{(s+\lambda_1+\lambda_2)}+\lambda_2\left\{\frac{1}{(s+\lambda_1+\lambda_2)}+\frac{1}{(s+2\lambda_1)}\right\}\right]\right\}$$
$$\left\{\lambda_e+\frac{2\lambda_a^2\lambda_d\overline{S}_{\mu_1}(s)}{(s+\lambda_a+\lambda_b)(s+2\lambda_a+\lambda_b)}\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]+\frac{2\lambda_a\lambda_b\lambda_d\overline{S}_{\mu_1}(s)}{(s+2\lambda_a+\lambda_b)}\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]\right.$$
$$\left.\left.\left(\frac{1}{(s+2\lambda_a)}+\frac{1}{s+\lambda_a+\lambda_b}\right)\right\}+\left\{\frac{\lambda_A^2}{(s+\lambda+\lambda_A)}\overline{S}_{\mu_2}(s)\right\}\right] \tag{79}$$

3.3. Numerical computations

3.3.1. Availability analysis

Assuming repair rate to be exponential then one can obtain

where $\overline{S}_{\mu_1}(s)=\frac{\mu_1(x)}{s+\mu_1(x)}$, $\overline{S}_{\mu_2}(s)=\frac{\mu_2(x)}{s+\mu_2(x)}$, $\overline{S}_{\psi}(s)=\frac{\psi(y)}{s+\psi(y)}$ (80)

Let the failure rate $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\lambda_a=0.4$, $\lambda_b=0.5$, $\lambda_d = 0.6$, $\lambda_e = 0.7$, $\lambda_R = 0.1$, $\lambda_A=0.8$, and taking repair rate $\mu_1 = 1$, $\mu_2 = 1$, $\theta = 1$, $x = 1$ and $y=1$. Substituting these values in equation (69) and computing inverse Laplace Transform, we get

$$\begin{aligned}P_{up}(t)=\;&0.01239745844\, e^{(-1.693201163\, t)}\\&+0.1462227159\, e^{(-1.525861861\, t)}\cos(0.6415743038\, t)\\&+0.4583892963\, e^{(-1.525861861\, t)}\sin(0.6415743038\, t)\\&+0.1478972584\, e^{(-1.185619179\, t)}\cos(0.5920698014\, t)\\&+0.02013120970\, e^{(-1.185619179\, t)}\sin(0.5920698014\, t)\\&-0.0001908063947\, e^{(-0.8335117469\, t)}\\&+0.02726873924\, e^{(-0.4572796620\, t)}\cos(0.2540008768\, t)\\&+0.02936475130\, e^{(-0.4572796620\, t)}\sin(0.2540008768\, t)\\&-0.004140676722\, e^{(-0.2357656843\, t)}+0.6705453111\end{aligned} \tag{81}$$

3.3.2. Reliability analysis

a) Let the failure rate of system be $\lambda_1=0.1$, $\lambda_2=0.2$, $\lambda_a=0.4$, $\lambda_b=0.5$, $\lambda_d=0.6$, $\lambda_e=0.7$, $\lambda_R=0.1$, $\lambda_A=0.8$ and repair rate be $\mu_1=1$, $\mu_2=2$, $\theta=1$, $x=1$ and $y=1$. Also assuming repair rate to be exponential and putting these values in equation (69) and using equation (80), one can obtain Table 3.
b) If the system cannot be recovered, i.e. $\lambda_R=0$ and failure rates be $\lambda_1=0.1$, $\lambda_2=0.2$, $\lambda_a=0.4$, $\lambda_b=0.5$, $\lambda_d=0.6$, $\lambda_e=0.7$, $\lambda_A=0.8$ and repair rate be $\mu_1=1$, $\mu_2=2$, $\theta=1$, $x=1$ and $y=1$. Putting these values in equation (69) and using equation (80), one can obtain Table 3. In both the cases (a) and (b), Figure 4 shows how reliability varies with respect to time.

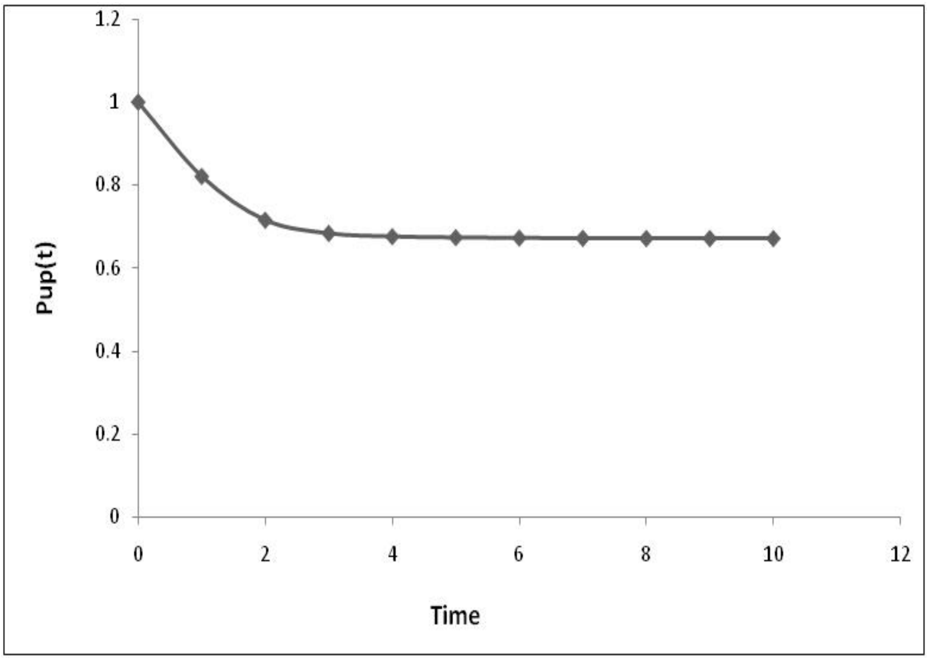

Figure 3. Time vs. availability

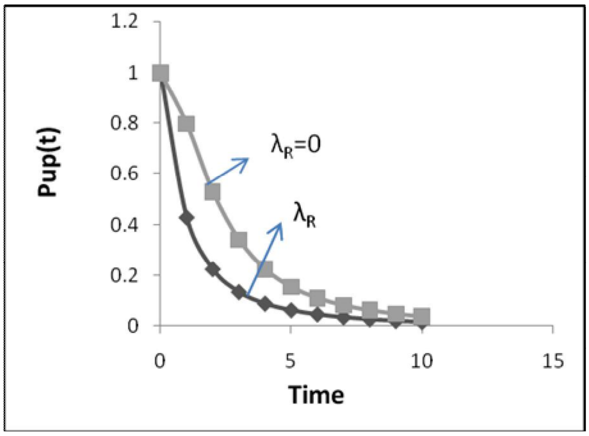

Figure 4. Time vs. reliability

Table 2. Time vs. availability

Time	$P_{up}(t)$
0	1.000000000
1	0.820414477
2	0.714528962
3	0.682572282
4	0.674770014
5	0.672478281
6	0.671345355
7	0.670656079
8	0.670278986
9	0.670120571
10	0.670093811

Table 3. Time vs. reliability

Time	$P_{up}(t)$	$P_{up}(t)(\lambda_R=0)$
0	1.000000000	1.000000000
1	0.429654774	0.798665249
2	0.226612179	0.530794097
3	0.136291145	0.342686642
4	0.090494265	0.226617719
5	0.064492291	0.156266310
6	0.048118733	0.112369612
7	0.036911566	0.083607849
8	0.028779985	0.063765638
9	0.022658435	0.049458495
10	0.017947116	0.038793036

3.3.3. M. T. T. F. analysis

Let the repair follows exponential distribution, i.e. (80) holds then M. T .T. F. of the system is given by

$$M.T.T.F. = \lim_{s \to 0} \overline{P}up(s)$$

$$= \frac{1}{D(0)}\left[1 + \frac{2\lambda_1}{(s+\lambda_1+\lambda_2)} + \frac{\lambda_2}{(s+2\lambda_1)} + \frac{\lambda_A}{(s+\lambda+\lambda_A)} + \frac{\lambda\lambda_A}{(s+\lambda+\lambda_A)}\left[\frac{1-\overline{S}\mu_2(s)}{s}\right]\right.$$
$$+ \frac{2\lambda_1\lambda_d\overline{S}_{\mu_1}(s)}{(s+\lambda_d+\lambda_e)(s+2\lambda_a+\lambda_b)}\left[\frac{\lambda_1}{s+\lambda_1+\lambda_2} + \lambda_2\left\{\frac{1}{(s+\lambda_1+\lambda_2)} + \frac{1}{(s+2\lambda_1)}\right\}\right] \tag{82}$$
$$\left.\left[\frac{1-\overline{S}_{\mu_1}(s)}{s}\right]\left(1 + \frac{\lambda_b}{s+2\lambda_a} + \frac{2\lambda_a}{(s+\lambda_a+\lambda_b)}\right)\right]$$

a) Setting λ_2=0.02, λ_A=0.02, λ_R=0.02, x=1, y=1,θ=1 and varying the value of λ_1 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, in equation (82), we can obtain Table 4.
b) Taking λ_1=0.02, λ_A=0.02, λ_R=0.02, x=1, y=1, θ=1 and varying the value of λ_2 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, in equation (82), computational values obtained are given in Table 4.
c) Keeping the values as λ_1=0.02, λ_2=0.02, λ_A=0.02, x=1, y=1, θ=1 and varying the value of λ_R as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10, in equation (82), one can get Table 4. Figure 5 shows variation of M. T. T. F. with respect to λ_1, λ_2 and λ_R.

3.3.4. Cost analysis

Let the failure rates be λ_1=0.1, λ_2=0.2, λ_a=0.4, λ_b=0.5, λ_d=0.6, λ_e=0.7, λ_A=0.8, λ_R=0.1 and repair rates be μ_1=1, μ_2=1, θ=1 and x=1,y=1. Also letting the repair follows exponential distribution.

Substituting all these values in equation (69) and using equation (80) then taking inverse Laplace transform, one can obtain equation (81). Let the service facility be always available, then expected profit during the interval (0, t] is given by

$$E_P(t) = c_1 \int_0^t P_{up}(t)dt - c_2 t \tag{83}$$

where c_1 and c_2 are revenue rate per unit time and service cost per unit time, respectively.

Using (81) and (83) for the above mentioned parameters, we get

$$\begin{aligned} E_P(t) = c_1\big(&-0.0073219052\,24\ e^{(-1.693201163\,t)} \\ &-0.1950926773\ e^{(-1.525861861\,t)}\cos(0.6415743038\ t) \\ &-0.2360782826\ e^{(-1.525861861\,t)}\sin(0.6415743038\ t) \\ &-0.1066306211\ e^{(-1.185619179\,t)}\cos(0.5920698014\ t) \\ &+0.03626928591\ e^{(-1.185619179\,t)}\sin(0.5920698014\ t) \\ &+0.0002289186630\,e(-0.8335117469\ t) \\ &-0.07283104232\ e^{(-0.4572796620\,t)}\cos(0.2540008768\ t) \\ &-0.02376139504\ e^{(-0.4572796620\,t)}\sin(0.2540008768\ t) \\ &+0.01756267768\ e^{(-0.2357656843\,t)} \\ &+0.6705453111\ t + 0.3640846495\big) - t\,c_2 \end{aligned} \tag{84}$$

Taking C_1=1 and C_2=0.1, 0.2, 0.3, 0.4, 0.5 and using equation (81) one can obtain the variation of $E_P(t)$ with respect to time. The computational values obtained are given in Table 5 and shown in Figure 6.

Table 4. Failure rates vs. M. T. T. F.

λ_1	M. T. T. F.	λ_2	M. T. T. F.	λ_R	M. T. T. F.
0.1	45.83333334	0.1	39.81481482	0.1	38.88889
0.2	35.00000000	0.2	35.00000000	0.2	35.00000
0.3	29.44444445	0.3	32.27272728	0.3	31.81818
0.4	25.59523810	0.4	30.55555555	0.4	29.16667
0.5	22.67857143	0.5	29.39560440	0.5	26.92308
0.6	20.37037038	0.6	28.57142858	0.6	25.00000
0.7	18.49206350	0.7	27.96296298	0.7	23.33333
0.8	16.93181818	0.8	27.5000000	0.8	21.87500
0.9	15.61447812	0.9	27.13903742	0.9	20.58824
1.0	14.48717949	1.0	26.85185185	1.0	19.44444

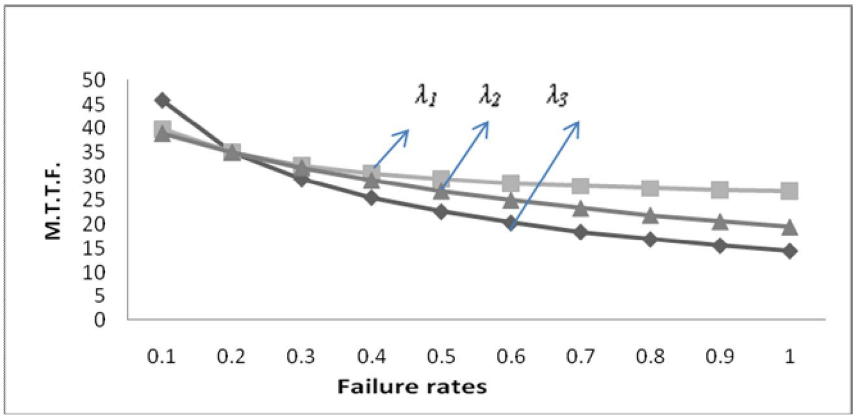

Figure 5. Failure rates vs. M. T. T. F.

Table 5. Time vs. expected profit

Time	Ep(t)				
	C2 = 0.1	C2 = 0.2	C2 = 0.3	C2 = 0.4	C2 = 0.5
0	0	0	0	0	0
1	0.813299	0.713299	0.613299	0.513299	0.413299
2	1.471865	1.271865	1.071865	0.871865	0.671865
3	2.066879	1.766879	1.466879	1.166879	0.866879
4	2.644667	2.244667	1.844667	1.444667	1.044667
5	3.218116	2.718116	2.218116	1.718116	1.218116
6	3.789979	3.189979	2.589979	1.989979	1.389979
7	4.360950	3.660950	2.960950	2.260950	1.560950
8	4.931396	4.131396	3.331396	2.531396	1.731396
9	5.501581	4.601581	3.701581	2.801581	1.901581
10	6.071680	5.071680	4.071680	3.071680	2.071680

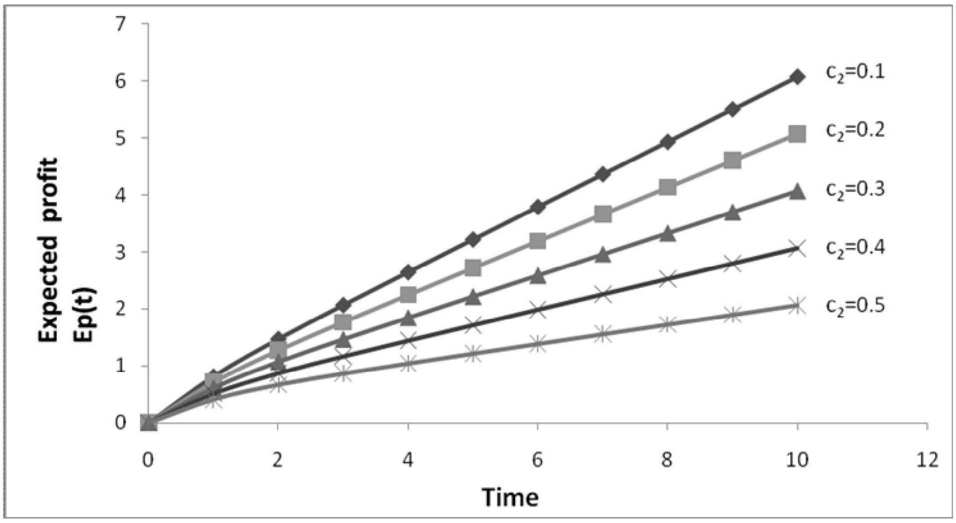

Figure 6. Time vs. expected profit

3.3.5. Sensitivity

Tables 6 and 7 are corresponding to the sensitivity analysis of the system reliability with respect to change in λ_1 and λ_2 respectively. The same is shown in Figures 7 and 8 respectively. One can easily conclude by these figures that sensitivity of the system reliability decreases with the increase in the value of λ_1 and λ_2. Also one can visualize that the system reliability is more sensitive with respect to λ_2 than λ_1.

Table 6. Sensitivity of the system reliability w. r. t. λ_1

Time	Value of $\partial R(t)/\partial\lambda_1$		
0	0	0	0
10	-1.149940144	-0.519914911	-0.131185454
20	-0.405927773	-9.23E-02	-2.34E-02
30	-0.10489699	-2.31E-02	-8.24E-03
40	-2.68E-02	-7.66E-03	-3.03E-03
50	-7.49E-03	-2.76E-03	-1.11E-03
60	-2.33E-03	-1.01E-03	-4.10E-04
70	-7.85E-04	-3.72E-04	-1.51E-04
80	-2.77E-04	-1.37E-04	-5.54E-05
90	-1.00E-04	-5.04E-05	-2.04E-05
100	-3.65E-05	-1.85E-05	-7.50E-06

Table 7. Sensitivity of the system reliability w. r. t. λ_2

Time	Value of $\partial R(t)/\partial\lambda_2$		
0	0	0	0
10	-1.74914	-1.57E+00	-1.25E+00
20	-1.57939	-1.29E+00	-8.53E-01
30	-8.52E-01	-6.30E-01	-3.44E-01
40	-3.88E-01	-2.60E-01	-1.16E-01
50	-1.63E-01	-9.86E-02	-3.62E-02
60	-6.52E-02	-3.58E-02	-1.08E-02
70	-2.53E-02	-1.26E-02	-3.10E-03
80	-9.64E-03	-4.33E-03	-8.75E-04
90	-3.61E-03	-1.47E-03	-2.43E-04
100	-1.34E-03	-4.92E-04	-6.65E-05

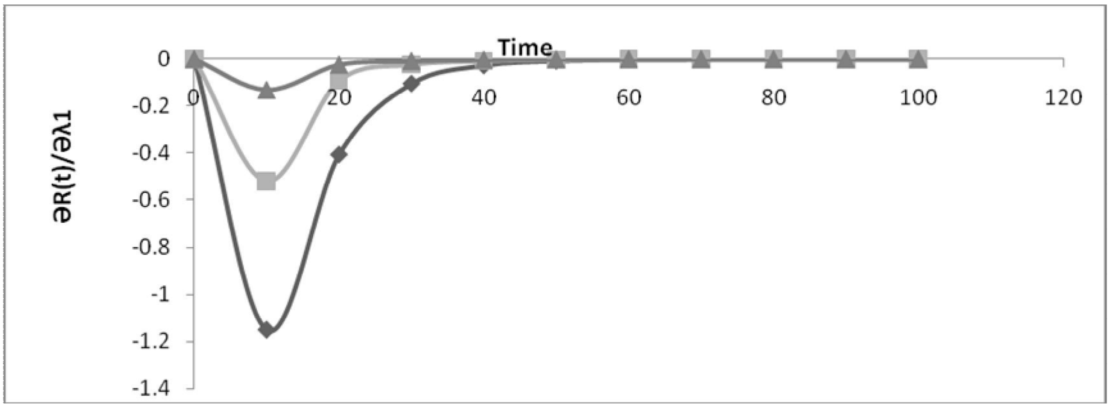

Figure 7. Sensitivity of the system reliability w. r. t. λ_1

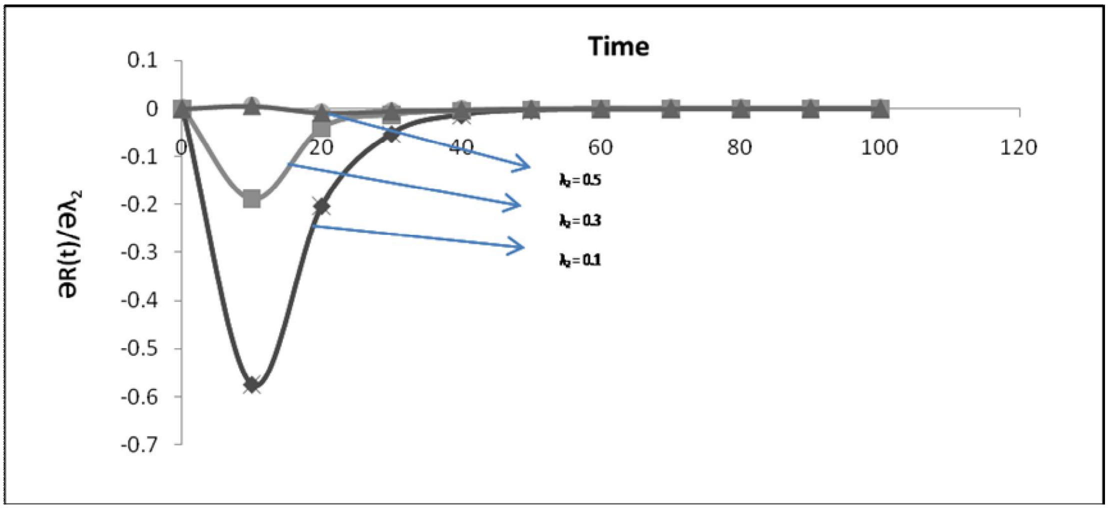

Figure 8. Sensitivity of the system reliability w. r. t. λ_2

4. Conclusions

In the present study various reliability measures have been computed for the reliability model with reboot delay for different cases. In this model different reliability measures such as transition state probabilities, availability, reliability, M.T.T.F., expected profit and sensitivity with respect to different parameters have been obtained. From Figure 3 we can see the variation of availability with respect to time when failure rates are fixed at different values. When failure rates are assumed as λ_1=0.1, λ_2=0.2, λ_a=0.4, λ_b=0.5, λ_d=0.6, λ_e=0.7, λ_A=0.8 and λ_R=0.1, one can observe that initially availability of the system decreases rapidly with respect to time but as the time increases it tends to stabilize at 0.670093811.

The Figure 4 provides the variation of reliability with respect to time. It shows the change in reliability with respect to time in both conditions: (1) when Reboot Delay present in the system and (2) when Reboot Delay does not occur in the system. By observing the figure one can visualize that it decreases sharply during initial stage and uniformly in the later. One can also easily conclude that the reliability of the system is better in the absence of Reboot Delay especially during initial stages though later on they are tending towards the same values as time passes away.

Figure 5 and corresponding Table 4 shows the variation in M. T. T. F. of the system w.r.t. failure rates λ_1, λ_2 and λ_R keeping other parameters constant. As the value of failure rates λ_1, λ_2

and λ_R increases, M. T. T. F. of the system decreases. From Figure 5 one can easily interpret that initially the value of M. T. T. F. decreases sharply then it declines gradually with respect to λ_1, λ_2 and λ_R. Critical examination of the Figure 5 reveals an interesting fact that at one point where λ_1, λ_2 and λ_R become 0.2, the value of M. T. T. F. corresponding to λ_1, λ_2 and λ_R is the exactly same i.e. 35 Critical examination of this figure reveals that M. T. T. F. of the system is in the order: M. T. T. F. w. r. t. λ_1 > M. T. T. F. w. r. t. λ_2 > M. T. T. F. w. r. t. λ_R.

From the Table 5 one can observe the variation in effective profit with respect to time. The corresponding Figure 6 has been drawn by keeping the revenue per unit time C_1 set at 1.0, service cost C_2 varied and failure rates are kept at constant value. By the observation of Figure 6 one can see that expected profit decreases as service cost increases with respect to time whereas with respect to time expected profit increases continuously.

The sensitivities of various values of λ_1 and λ_2 on the system reliability are shown in Figures 7 and 8 respectively. We observe that influence of λ_1 and λ_2 on system reliability increases as they decrease. Several reliability measures of complex system obtained in this study will be helpful to system designers as well as operations managers.

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