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Effects of Slip and Heat Transfer on the Peristaltic Pumping of a Williamson Fluid in an Inclined Channel

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Abstract: Effects of slip and heat transfer on the peristaltic flow of a Williamson fluid in an inclined channel is studied under long wavelength and low Reynolds number assumptions. The perturbation technique is used to solve the problem as the equations are non linear. The stream function, the temperature distribution and the pressure rise are calculated .The effect of various parameters on the pumping characteristics and on the temperature profiles is discussed with the help of graphs.

Keywords: Slip effects; heat transfer; peristaltic pumping; williamson fluid.

1. Introduction

Peristalsis is a well-known mechanism for pumping biological and industrial fluids. Even though it is observed in living systems for many centuries; the mathematical modeling of peristaltic transport has begun with important works by Fung and Yih [2] using laboratory frame of reference and Shapiro et al. [10] using wave frame of reference. Many of the contributors to the area of peristaltic pumping have either followed Shapiro or Fung.

Most of the studies on peristaltic flow deal with Newtonian fluids. The complex rheology of biological fluids has motivated investigations involving different non-Newtonian fluids. Peristaltic flow of non-Newtonian fluids in a tube was first studied by Raju and Devanathan [6]. Ravi Kumar et.al [9] studied the unsteady peristaltic pumping in a finite length tube with permeable wall. Y. V. K. Ravi Kumar et. al [8] studied the Peristaltic pumping of a magneto hydrodynamic casson fluid in an inclined channel. Ravi Kumar et.al [7] studied the Peristaltic pumping of a Jeffrey fluid under the effect of a magnetic field in an inclined channel. Mekheimer [4] studied the peristaltic transport of MHD flow in an inclined planar channel. Hayat et al. [3] extended the idea of Elshehawey et al.[1] for partial slip condition. Srinivas et al. [11] studied the Peristaltic transport in an asymmetric channel with heat transfer. Srinivas et al. [12] studied the non-linear peristaltic transport in an inclined asymmetric channel. Vajravelu et al. [13] analyzed peristaltic transport of a Casson fluid in contact with a Newtonian fluid in a circular tube with permeable wall. Nadeem and Akram [5] discussed peristaltic flow of a Williamson fluid in an asymmetric channel. It is observed that most of the physiological fluids (for example, blood) cannot be described by Newtonian model. Hence, several non-Newtonian models are being proposed by various researchers to investigate the flow behavior in physiological system of a living body. Among them Williamson model is expected to explain

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most of the features of a physiological fluid. Moreover, this model is nonlinear and Newtonian fluid model can be deduced as a special case from this model.

In this paper, the peristaltic pumping of Williamson fluid in an inclined channel under the influence of slip and heat transfer is investigated. The peristaltic waves are assumed to propagate on the walls of the channel with speed c. Using the wave frame of reference, boundary value problem is solved. The stream function, the temperature distribution and the pressure rise are calculated.

2. The williamson fluid model

For an incompressible fluid the balance of mass and momentum are given by

$$div V = 0 \tag{1}$$

$$\rho \frac{dV}{dt} = div S + \rho f \tag{2}$$

where ρ is the density, V is the velocity vector, S is the Cauchy stress tensor and f is the specific body force and d/dt represents the material derivative. The constitutive equation for Williamson fluid is given by

$$S = -PI + \tau \tag{3}$$

$$\tau = -\left[\mu_\infty + (\mu_0 + \mu_\infty)(1 - \Gamma\bar{\dot{y}})^{-1}\right]\bar{\dot{y}} \tag{4}$$

in which $-PI$ is the spherical part of the stress due to constraint of incompressibility, τ is the extra stress tensor, μ_∞ is the infinite shear rate viscosity, μ_0 is the zero shear rate viscosity, Γ is the time constant and $\bar{\dot{y}}$ is defined as

$$\bar{\dot{y}} = \sqrt{\frac{1}{2}\sum_i \sum_j \bar{\dot{y}}_{ij}\bar{\dot{y}}_{ji}} = \sqrt{\frac{1}{2}\Pi}\,. \tag{5}$$

Here Π is the second invariant strain tensor. We consider the constitutive equation (4), the case for which $\mu_\infty = 0\, and\, \Gamma\dot{y} < 1$. The component of extra stress tensor therefore, can be written as

$$\tau = -\mu_0\left[(1 - \Gamma\bar{\dot{y}})^{-1}\right]\bar{\dot{y}} = -\left[(1 + \Gamma\bar{\dot{y}})\right]\bar{\dot{y}} \tag{6}$$

3. Mathematical formulation

Let us consider the peristaltic flow of a Williamson fluid in an inclined symmetric channel as shown in Figure 1. The channel walls are lined with non erodible porous material. The thickness of the lining is very small when compared with the width of the channel. The lower permeable wall of the channel is maintained at temperature T_1 while the upper permeable wall has temperature T_o. The flow is generated by sinusoidal wave trains propagating with constant speed *c* along the channel. The geometry of the wall surfaces is defined as

$$Y = \pm H = \pm\left(\bar{d} + \bar{a}\cos\left[\frac{2\pi}{\lambda}\left(\bar{X} - c\bar{t}\right)\right]\right) \tag{7}$$

where $\bar{a}$ is the amplitudes of the wave, d is the mean width of the channel, λ is the wave length, c is the velocity of propagation, $\bar{t}$ is the time and $\bar{X}$ is the direction of wave propagation.

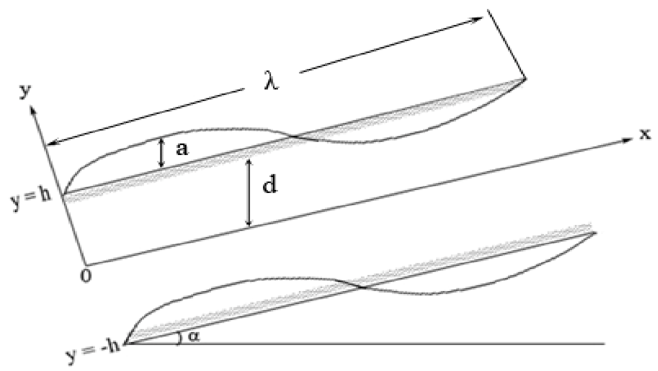

Figure 1. Physical model

3.1. Equation of motion

The equations governing the motion and energy of a Williamson fluid are given by

$$\frac{\partial \bar{U}}{\partial \bar{X}} + \frac{\partial \bar{V}}{\partial \bar{Y}} = 0 \tag{8}$$

$$\rho\left(\frac{\partial \bar{U}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{U}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{U}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{X}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{Y}} - \rho g \sin\alpha \tag{9}$$

$$\rho\left(\frac{\partial \bar{V}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{V}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{V}}{\partial \bar{Y}}\right) = -\frac{\partial \bar{P}}{\partial \bar{Y}} - \frac{\partial \bar{\tau}_{\bar{X}\bar{Y}}}{\partial \bar{X}} - \frac{\partial \bar{\tau}_{\bar{Y}\bar{Y}}}{\partial \bar{Y}} + \rho g \cos\alpha \tag{10}$$

$$C'\left[\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{U}\frac{\partial \bar{T}}{\partial \bar{X}} + \bar{V}\frac{\partial \bar{T}}{\partial \bar{Y}}\right] = \frac{K'}{\rho}\nabla^2 \bar{T} + \upsilon\Phi \tag{11}$$

Where $\tau_{\bar{X}\bar{X}}$, $\tau_{\bar{X}\bar{Y}}$, $\tau_{\bar{Y}\bar{Y}}$ are components of stress,

$$\nabla^2 = \frac{\partial^2}{\partial \bar{X}^2} + \frac{\partial^2}{\partial \bar{Y}^2} \qquad \Phi = \left[2\left(\frac{\partial \bar{U}}{\partial \bar{X}}\right)^2 + 2\left(\frac{\partial \bar{V}}{\partial \bar{Y}}\right)^2 + \left(\frac{\partial \bar{U}}{\partial \bar{Y}} + \frac{\partial \bar{V}}{\partial \bar{X}}\right)^2\right]$$

where α is the inclination of the channel with the horizontal, $\bar{U}, \bar{V}$ are the velocities in X and Y directions in fixed frame, ρ is the density, $\bar{P}$ is the pressure, υ is the kinematic viscosity, K' is the thermal conductivity, C' is the specific heat and $\bar{T}$ is the temperature.

We introduce a wave frame $(\bar{x}, \bar{y})$ moving with velocity c away from the fixed frame $(\bar{X}, \bar{Y})$ by the transformation.

$$\bar{x} = \bar{X} - c\bar{t},\ \bar{y} = \bar{Y},\ \bar{u} = \bar{U} - c,\ \bar{v} = \bar{V} \quad and \quad \bar{P}(x) = \bar{P}(X,t). \tag{12}$$

We define the non-dimensional quantities as follows

$$x=\frac{\bar{x}}{\lambda},y=\frac{\bar{y}}{\lambda},u=\frac{\bar{u}}{c},v=\frac{\bar{v}}{c},t=c\frac{\bar{t}}{\lambda},h=\frac{\bar{h}}{\bar{d}},\delta=\frac{\bar{d}}{\lambda}\,\tau_{xx}=\frac{\lambda}{\mu_0 c}\bar{\tau}_{\bar{xx}},\tau_{xy}=\frac{\bar{d}}{\mu_0 c}\bar{\tau}_{\bar{xy}},\tau_{yy}=\frac{\bar{d}}{\mu_0 c}\bar{\tau}_{\bar{yy}}$$

$$Re=\frac{\rho c\bar{d}}{\mu_0},P=\frac{\bar{d^2}}{c\lambda\mu_0}\bar{P},\gamma=Re=\frac{\bar{\gamma}\bar{d}}{c},\theta=\frac{T-T_0}{T_1-T_0}$$

$$Ec=\frac{c^2}{c'(T_1-T_0)},Pr=\frac{\rho c v'}{K'}\bar{P},\eta=\frac{-d^2\rho g}{c\mu_0} \tag{13}$$

Using the above non-dimensional quantities in equations (3), (4) and (5), the resulting equations in terms of stream function $\psi\left(u=\partial\Psi/\partial y, v=-\delta\,\partial\Psi/\partial x\right)$ can be written as

$$\delta\,\mathrm{Re}\left[\left(\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\Psi}{\partial y}\right]=-\frac{\partial p}{\partial x}-\delta^2\frac{\partial\tau_{xx}}{\partial x}-\frac{\partial\tau_{xy}}{\partial y}+\eta\sin\alpha \tag{14}$$

$$-\delta^3\,\mathrm{Re}\left[\left(\frac{\partial\Psi}{\partial y}\frac{\partial}{\partial x}-\frac{\partial\Psi}{\partial x}\frac{\partial}{\partial y}\right)\frac{\partial\Psi}{\partial x}\right]=-\frac{\partial p}{\partial y}-\delta^2\frac{\partial\tau_{xy}}{\partial x}-\delta\frac{\partial\tau_{yy}}{\partial y}+\delta\eta\cos\alpha \tag{15}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}+Ec\frac{\partial^2\psi}{\partial y^2}=0 \tag{16}$$

where $\tau_{xx}=-2\left[1+\mathrm{We}\dot{\gamma}\right]\frac{\partial^2\Psi}{\partial x\partial y}$,

$$\tau_{xy}=-\left[1+\mathrm{We}\dot{\gamma}\right]\left(\frac{\partial^2\Psi}{\partial y^2}-\delta^2\frac{\partial^2\Psi}{\partial x^2}\right),$$

$$\tau_{yy}=2\delta\left[1+\mathrm{We}\dot{\gamma}\right]\frac{\partial^2\Psi}{\partial x\partial y}$$

$$\dot{\gamma}=\left[2\delta^2\left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2+\left(\frac{\partial^2\Psi}{\partial y^2}-\delta^2\frac{\partial^2\Psi}{\partial x^2}\right)^2+2\delta^2\left(\frac{\partial^2\Psi}{\partial x\partial y}\right)^2\right]^{\frac{1}{2}}$$

Here δ is the wave number, Re is the Reynolds number and We is the Weissenberg number. Under the assumptions of long wavelength δ << 1 and low Reynolds number, neglecting the terms of order δ and higher, equations (14) and (15) take the form

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left[\left(1+\mathrm{We}\frac{\partial^2\Psi}{\partial y^2}\right)\frac{\partial^2\Psi}{\partial y^2}\right] \tag{17}$$

$$\frac{\partial p}{\partial y}=0 \tag{18}$$

Elimination of pressure from equations (17) and (18), yield

$$\frac{\partial^2}{\partial y^2}\left[\left(1+\mathrm{We}\frac{\partial^2\Psi}{\partial y^2}\right)\frac{\partial^2\Psi}{\partial y^2}\right]=0 \tag{19}$$

The dimensionless mean flow $\bar{Q}$ is defined by

$$\bar{Q} = F + 1 \tag{20}$$

in which

$$F = \int_{-h(x)}^{h(x)} \frac{\partial \Psi}{\partial y} dy \tag{21}$$

where $h = 1 + \phi \cos 2\pi x$

The appropriate boundary conditions for the problem under consideration are

$$\Psi = \frac{F}{2} \quad \text{at} \quad y = h \tag{22}$$

$$\Psi = \frac{-F}{2} \quad \text{at} \quad y = -h \tag{23}$$

$$\frac{\partial \Psi}{\partial y} + L \frac{\partial^2 \Psi}{\partial y^2} = -1 \quad \text{at} \quad y = h \tag{24}$$

$$\frac{\partial \Psi}{\partial y} - L \frac{\partial^2 \Psi}{\partial y^2} = -1 \quad \text{at} \quad y = -h \tag{25}$$

$$\theta = 0 \quad \text{at y=h} \tag{26}$$

$$\theta = 1 \quad \text{at y=-h} \tag{27}$$

where L is permeability parameter including slip, Da is the Darcy number and F is the flux. Equations (24) and (25) are Saffman boundary conditions at the permeable walls of the channel.

4. Perturbation solution

The equation (17) is non-linear and hence its exact solution is not possible. We use the perturbation technique to find the solution. For perturbation solution, we expand Ψ, $F$and p as

$$\Psi = \Psi_0 + \text{We}\,\Psi_1 + O\left(\text{We}^2\right) \tag{28}$$

$$F = F_0 + \text{We}\,F_1 + O\left(\text{We}^2\right) \tag{29}$$

$$p = p_0 + \text{We}\,p_1 + O\left(\text{We}^2\right) \tag{30}$$

Substituting the above expressions in equations (17)-(18) and equations (22)-(25), we get the following system of equations

4.1. System of order We^0

$$\frac{\partial^4 \Psi_0}{\partial y^4} = 0 \tag{31}$$

$$\frac{\partial p_0}{\partial x} = \frac{\partial^3 \Psi_0}{\partial y^3} + \eta \sin\alpha \quad (32)$$

$$\Psi_0 = \frac{F_0}{2} \quad \text{at } y = h \quad (33)$$

$$\Psi_0 = \frac{-F_0}{2} \quad \text{at } y = -h \quad (34)$$

$$\frac{\partial \Psi_0}{\partial y} + L\frac{\partial^2 \Psi_0}{\partial y^2} = -1 \quad \text{at } y = h \quad (35)$$

$$\frac{\partial \Psi_0}{\partial y} - L\frac{\partial^2 \Psi_0}{\partial y^2} = -1 \quad \text{at } y = -h \quad (36)$$

4.2. System of order We^1

$$\frac{\partial^4 \Psi_1}{\partial y^4} = -\frac{\partial^2}{\partial y^2}\left(\frac{\partial^2 \Psi_0}{\partial y^2}\right)^2 \quad (37)$$

$$\frac{\partial p_1}{\partial x} = \frac{\partial^3 \Psi_1}{\partial y^3} + \frac{\partial}{\partial y}\left(\frac{\partial^2 \Psi_0}{\partial y^2}\right)^2 \quad (38)$$

$$\Psi_1 = \frac{F_1}{2} \quad \text{at } y = h \quad (39)$$

$$\Psi_1 = -\frac{F_1}{2} \quad \text{at } y = -h \quad (40)$$

$$\frac{\partial \Psi_1}{\partial y} + L\frac{\partial^2 \Psi_1}{\partial y^2} = 0 \quad \text{at } y = h \quad (41)$$

$$\frac{\partial \Psi_1}{\partial y} - L\frac{\partial^2 \Psi_1}{\partial y^2} = 0 \quad \text{at } y = -h \quad (42)$$

4.3. Solution for system of order We^0

On solving equations (31) and (32) together with the boundary conditions (33)-(36), we get the solution to the zeroth-order problem as

$$\psi_0 = -\frac{\left(6h+3F_0\right)}{2h^3+6Lh^2}\frac{y^3}{6} + \frac{\left(3h+6L\right)F_0+2h^2}{4h^2+12hL} \quad (43)$$

The axial pressure gradient of the zeroth order is

$$\frac{\partial p_0}{\partial x} = -\frac{(6h+3F_0)}{(2h^3+6h^2L)} + \eta Sin\alpha \quad (44)$$

4.4. Solution for system of order We^1

Substituting the zeroth-order solution (41) into (37), the solution of the first-order problem satisfying the equations (39)-(42) is

$$\psi_1 = -c_1^2 \frac{y^4}{12} + d_1 \frac{y^3}{6} + d_2 \frac{y^2}{2} + d_3 y + d_4 \tag{45}$$

$$c_1 = -\frac{(6h+3F_0)}{2h^3+6Lh^2} \quad d_1 = -\frac{3F_1}{2h^3+6Lh^2} \quad d_2 = \frac{c_1^2 h^2 (3L+h)}{3(L+h)} \quad d_3 = \frac{3F_1(2L+h)}{4h^2+12hL} \quad d_4 = -\frac{c_1^2 h^4 (5L+h)}{12(h+L)}$$

The axial pressure gradient for the first order is

$$\frac{\partial p_1}{\partial x} = -\frac{3F_1}{(2h^3+6h^2L)} \tag{46}$$

Summarizing the perturbation results for small parameter *We*, the expression for stream function and pressure gradient can be written as

$$\psi = -\frac{(6h+3F)y^3}{(2h^3+6h^2L)6} + \frac{(3h+6L)F+2h^2}{4h^2+12Lh} y -$$

$$-we\frac{9(2h+F)^2}{(2h^3+6h^2L)^2}[\frac{y^4}{12} - \frac{h^2(3L+h)y^2}{3(L+h)2} + \frac{h^4(5L+h)}{12(L+h)} \tag{47}$$

$$\frac{\partial p}{\partial x} = -\frac{(6h+3F)}{(2h^3+6h^2L)} + \eta Sin\alpha \tag{48}$$

The dimensionless pressure rise and frictional force per one wavelength in the wave frame are defined, respectively as

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x} dx$$

$$\Delta p = \int_0^1 \left(-\frac{(6h+3F)}{2h^3+6Lh^2} + \eta \sin\alpha \right) dx \tag{49}$$

$$F = \int_0^1 h(-\frac{\partial p}{\partial x})dx \tag{50}$$

Now on solving equation (16) with the help of equations (47), (26) and (27), we get the temperature as

$$\theta = -Pr.Ec[(\frac{6h+3F}{2h^3+6h^2L})^2 \frac{y^4}{12} + 54We\frac{(2h+F)^3}{(2h^3+6h^2L)^3}[\frac{y^5}{20} -$$

$$\frac{h^3(3L+h)}{3(L+h)}\frac{y^3}{6}]] + k_1 y + k_2 \tag{51}$$

$$k_1 = -\frac{1}{2h} - 3\Pr.Ec.We\left[\frac{h^4(2h+F)^3(21L+h)}{10(2h^3+6Lh^2)^3(L+h)}\right]$$

$$k_2 = \frac{1}{2} + 3\Pr.Ec\left[\frac{(2h+F)^2}{16(3L+h)^2}\right]$$

5. Results and discussion

The variation in pressure rise Δp with the mean flow $\bar{Q}$ is calculated from equation (49) and is shown in Figure 2 for different values of the slip parameter L for fixed η=1, $\alpha = \frac{\pi}{6}$ and ϕ=0.6. It is noticed that the pumping curves intersect at a point in the first quadrant and to the left of this point, $\bar{Q}$ decreases and to the right of this point, $\bar{Q}$ increases with an increase in L. We observe that in free pumping $(\Delta p=0)$ and co-pumping $(\Delta p<0)$ $\bar{Q}$ increases with increasing L.

From equation (49) we have calculated the pressure difference as a function of $\bar{Q}$ for different values of the angle of inclination of the channel α for fixed L=0.2, η=1, ϕ=0.6 and is shown in Figure 3. We observe that for a given ΔP, the flux $\bar{Q}$ increases with increasing α. We observe that for a given $\bar{Q}$, pressure rise increases with increasing α.

The variation of pressure rise with $\bar{Q}$ is calculated from equation (49) for different values of the gravity parameter η, for fixed L=0.2, $\alpha = \frac{\pi}{6}$, ϕ=0.6 and is shown in Figure 4. It is clear that the pressure rise increases with the increase in $\bar{Q}$. We find that for fixed $\bar{Q}$, pressure rise increases with increasing η. Also for a given Δp, the increase in η increases the mean flow.

The variation of pressure rise with $\bar{Q}$ is calculated from equation (49) for different values of the amplitude ratio ϕ, for fixed L=0.2, $\alpha = \frac{\pi}{6}$, η=1 and is shown in Figure 5. It is found that, the flux $\bar{Q}$ increases with an increase in ϕ in both pumping and free pumping regions. But in the co-pumping region, the pumping curves intersect at a point. After this point $\bar{Q}$ decreases with an increase in amplitude ratio ϕ.

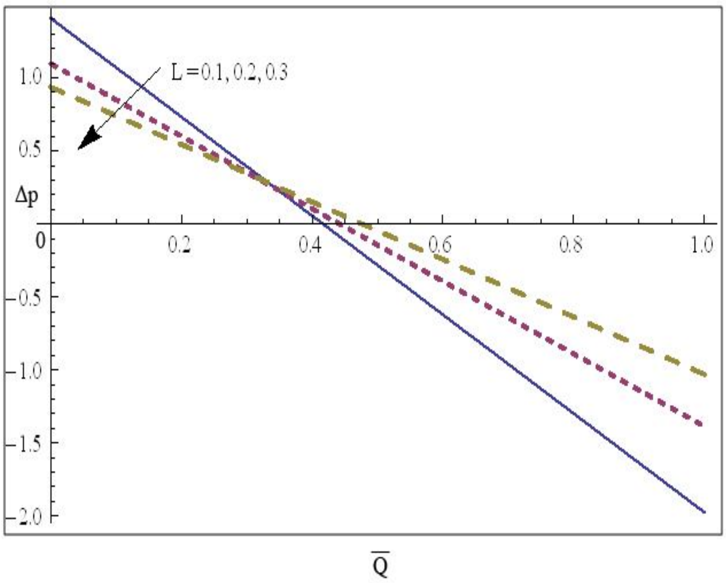

Figure 2. The variation of Δp with $\bar{Q}$ for different values of L with η=1, $\alpha = \frac{\pi}{6}$, ϕ=0.6

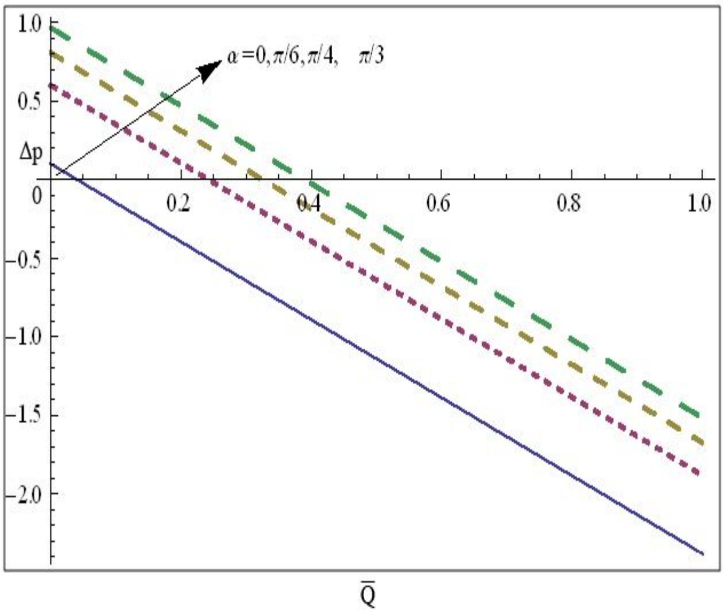

Figure 3. The variation of Δp with $\overline{\mathrm{Q}}$ for different values of α with L=0.2, η=1, ϕ=0.6

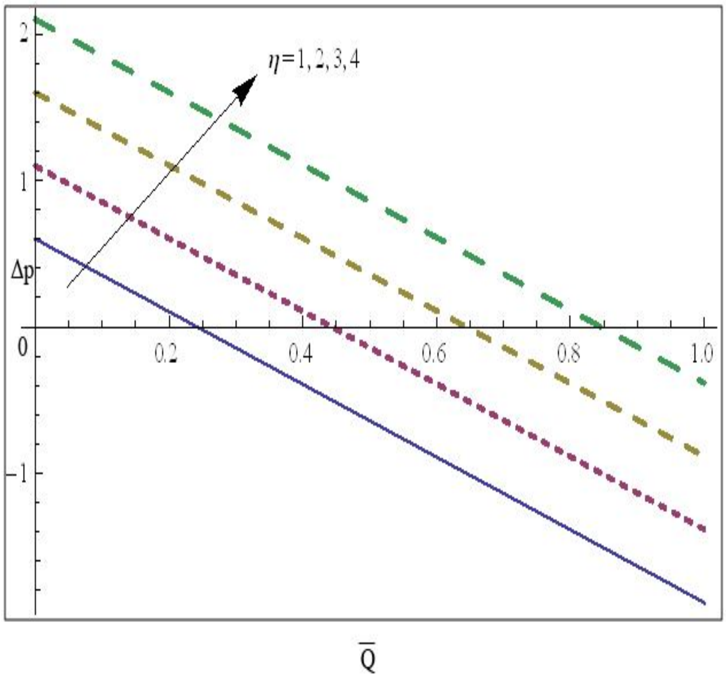

Figure 4. The variation of Δp with $\overline{\mathrm{Q}}$ for different values of η with L=0.2, $\alpha = \frac{\pi}{6}$, ϕ=0.6

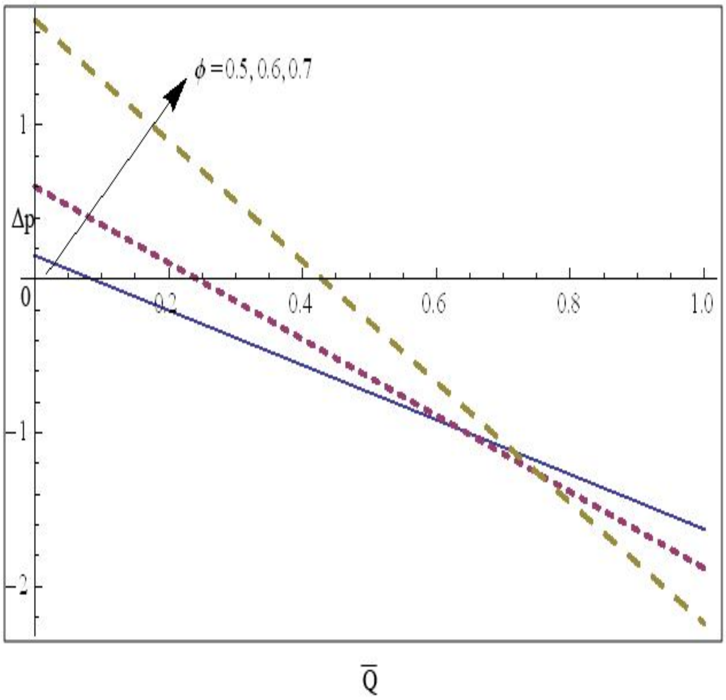

Figure 5. The variation of Δp with $\overline{Q}$ for different values of ϕ with L=0.2, $\alpha = \frac{\pi}{6}$, η=1

The variations of temperature field θ with y are calculated from equation (51) for different values of the slip parameter L is shown in Figure 6. It is observed that the temperature distribution θ decreases with an increase in L.

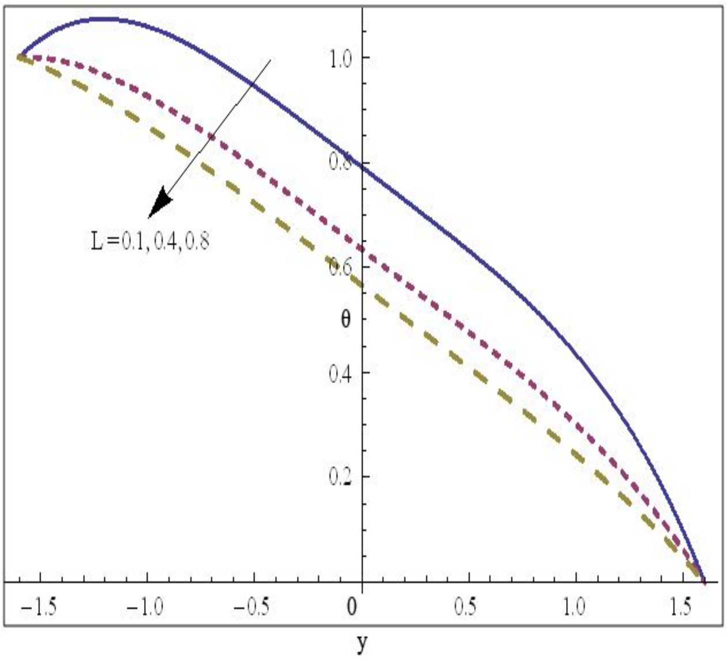

Figure 6. The variation of θ with y for different values of L

The variations of θ with y are calculated from equation (51) for different values of Prandtl number Pr is shown in Figure 7. It is found that the θ increases with increasing Pr.

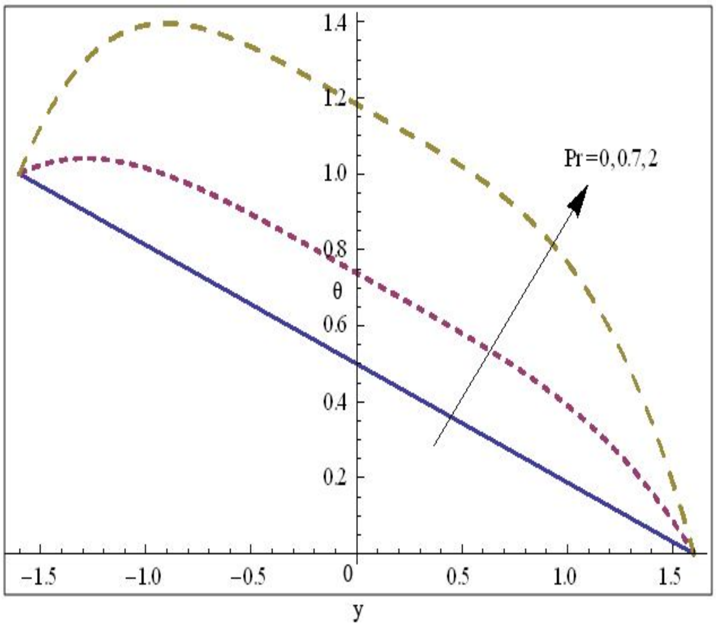

Figure 7. The variation of θ with y for different values of Pr

The variations of θ with y are calculated from equation (51) for different values of Eckert number Ec is shown in Figure 8.It is observed θ increases with increasing Ec.
Variations of θ with y are calculated from equation (51) for different values of the volume flow rate q and is presented in Figure 9. It is observed that the temperature θ increases with an increase q.

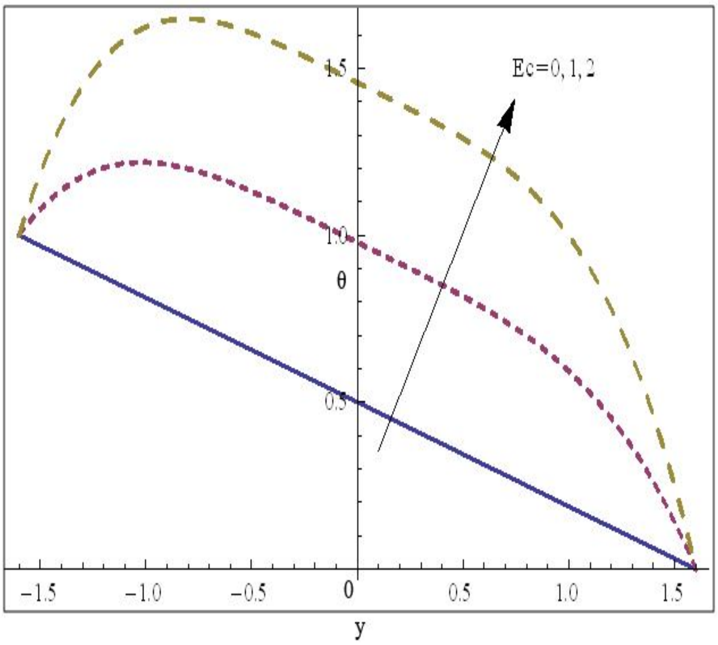

Figure 8. The variation of θ with y for different values of Ec

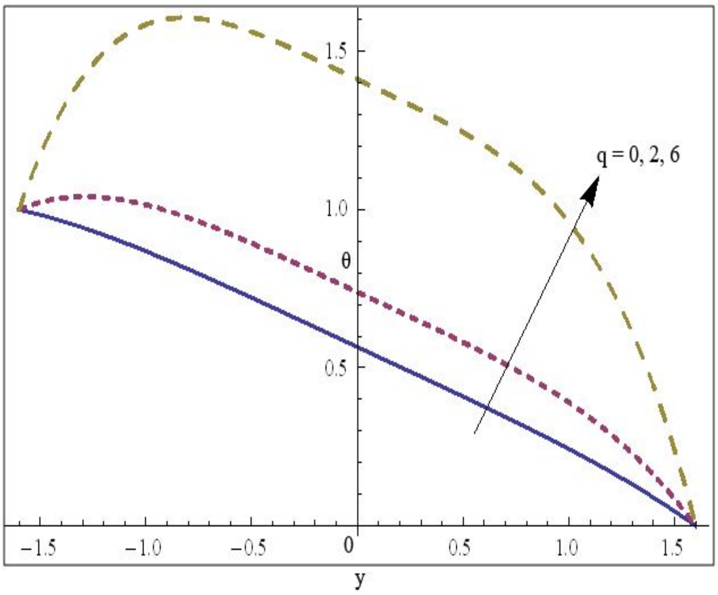

Figure 9. The variation of θ with y for different values of q

6. Conclusions

Effects of slip and heat transfer on the peristaltic pumping of a Williamson fluid in an inclined channel is studied. The effect of temperature, Prandtl number on the pumping characteristic in discusses.

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References

[1] Elshehawey, E. F., Eladabe, N. T., Elghazy, E. M., and Ebaid, A. 2006. Peristaltic transport in an asymmetric channel through a porous medium. *Applied Mathematics and Computation*, 182, 140-50.

[2] Fung and Yih, S.W. 1967. *"Dynamics of non-homogeneous fluids"*. Macmillan Co.

[3] Hayat, T., Hussain, Q., and Ali, N. 2008. Influence of partial slip on the peristaltic flow in a porous medium. *Physica A*, 38, 3399-409.

[4] Mekheimer, K. H. S. 2003. Non linear peristaltic transport of magneto-hydrodynamic flow in an inclined planar channel. *Arabian Journal for Science and Engineering*, 28: 183-201.

[5] Nadeem,S. and Akram, S. 2010. Peristaltic flow of a Williamson fluid in an asymmetric channel. *Communications in Nonlinear Science and Numerical Simulation*, 15: 1705-1716.

[6] Raju, K. K. and Devanatham, R. 1974. Peristaltic motion of a non-Newtonian fluid. *Rheologica Acta*, 13: 944-948.

[7] Ravi Kumar, Y. V. K., Krishna Kumari, P. S. V. H. N., Ramana Murthy, M. V., S., Sreenadh. 2011. Peristaltic pumping of a Jeffrey fluid under the effect of a magnetic field in an inclined channel. *Applied Mathematical Sciences*, 5, 9: 447-458.

[8] Ravi Kumar, Y. V. K., Krishna Kumari, P. S. V. H. N., Ramana Murthy, M. V., Chenna Krishna Reddy M. 2011. Peristaltic pumping of a magnetohydrodynamic Casson fluid in an inclined channel. *Advances in Applied Science Research*, 2(2): 428-436.
[9] RaviKumar, Y. V. K., Krishna Kumari, S. V. H. N., Raman Murthy, M. V., Sreenadh, S. 2010. Unsteady peristaltic pumping in a finite length tube with permeable wall, Trans. *ASME, Journal Fluids Engineering*, 32, 1012011-1012014.
[10] Shapir, A. H., Jaffrin, M. Y., and Weinberg, S. L. 1969. Peristaltic pumping with long wavelengths at low Reynolds Number. *Journal of Fluid Mechanics*, 37: 799-825.
[11] Srinivas, S. and Kothandapani, M. 2008. Peristaltic transport in an asymmetric channel with heat transfer. *International Communications in Heat and Mass Transfer*, 35: 514-22.
[12] Srinivas, S. and Pushparaj, V. 2008. Non-linear peristaltic transport in an inclined asymmetric channel. *Communications in Nonlinear Science*, 13: 1782-1795.
[13] Vajravelu, K., Sreenadh, S., Hemadri Reddy, R., and Murugeshan, K. 2009. Peristaltic Transport of a Casson fluid in contact with a Newtonian Fluid in a Circular Tube with permeable wall, *International Journal of Fluid Mechanics Research*, 36: 244-254.