

Production-Distribution Planning for a Two-Echelon Decentralized Supply Chain Coordinated with Revenue Sharing Mechanisms

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Abstract: This study presents a model for multi-product, multi-period production-distribution planning with revenue sharing for a two-echelon decentralized supply chain. The model incorporates a revenue sharing mechanism that is used to coordinate the two echelons and considers the distributor's ordering behavior under uncertain demands which is formulated by a single-period news-vendor problem. Since the model is a mixed-integer, nonlinear programming problem, a heuristic is proposed. The heuristic determines not only the production quantities and product flows for the supply chain network but also the wholesale prices between the two echelons. The proposed heuristic is evaluated against the optimal solution values and the computation times obtained by solving the model as a mixed-integer, nonlinear programming problem. The experiments employ the solution gap ratio and time-saving ratio to measure the solution quality and the computation efficiency for the proposed heuristic. The average solution gap was 3.564%. The maximum average time-saving ratio was 870.651 at the large supply chain environment. The results show that the proposed heuristic is effective and efficient in terms of solution quality and computation time.

Keywords: Supply chain management; production-distribution planning; demand uncertainty; revenue sharing.

1. Introduction

A decentralized supply chain (SC) consists of autonomous members in various echelons. No unbiased decision maker leading the supply chain exists. Each member identifies its most effective strategy without considering the impacts on other members in different echelons. In practice, a supply chain often operates in a decentralized form [1, 2]. The supply chain involves multiple organizations with different concerns, and it is difficult for a single organization to dominate the whole supply chain [3]. According to Wang [4], decentralized supply chains can be classified into two kinds: intra-organization-coordination and inter-organization-cooperation. In a supply chain of intra-organization-cooperation, a central power may exist and control the whole supply chain to some extent. The central power is required to coordinate members in the supply chain toward a consensus goal. On the other hand, an inter-organization-cooperation supply chain contains no central powers but only self-interested members. All members are willing to cooperate to achieve supply chain efficiency based on carefully designed cooperative

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mechanisms such as contracts.

A revenue sharing contract is one of the cooperative mechanisms for members in a decentralized supply chain of the inter-organization-cooperation kind. The supplier reduces wholesale or transfer prices to retailers and the retailers share part of their revenues with the supplier. With a revenue sharing contract, the decentralized supply chain can achieve two main objectives: 1) increasing the total profits, drawing closer to those of the centralized supply chain, and 2) sharing risks among members [5]. The revenue sharing contract lets decision makers cooperate in the decentralized supply chain to work toward a consensual goal.

Revenue sharing (RS) mechanisms have been designed from many perspectives. Chauhan and Proth [6] studied an RS contract that is proportional to the risks undertaken by the involved parties. Gupta and Weerawat [7] studied three types of revenue sharing contracts for supplier-manufacturer coordination. In the first kind of contract, revenue sharing depends on the supply lead time. In the second kind of contract, the supplier guarantees a delivery lead time to the manufacturer and incurs an expedited shipping charge if the supplier can not meet the promised lead time. In the last kind of contract, the revenues shared with the supplier rely on the supplier's inventory level. Hua et al. [8] considered the impact of revenue sharing contracts on the quality level, retail price, and customer utility in the product design strategies. The contractual power of a member also influences the use of the RS mechanism. Lau et al. [9] studied how to design a purchase contract for a dominant retailer. Their results suggest that RS mechanisms are ineffective for dominant retailers. Yao et al. [10] investigated a RS mechanism with a manufacturer as a Stackelberg leader in a supply chain. As to who should be the leader, the study by Qin and Yang [11] suggests that the party that keeps more than half of the revenue should serve as the Stackelberg leader to make the supply chain more profitable.

The revenue share rate, the wholesale or transfer prices, and retailer prices are critical variables when implementing RS contracts in a supply chain. Linh and Hong [12] investigated setting the optimal revenue share rate and wholesale price for RS mechanisms in which a retailer has single-buying and two-buying opportunities respectively. Xuehao et al. [13] proposed revenue sharing contracts that can induce members' reliabilities to improve the supply chain performance. Their contracts contain a two-round profit allocation mechanism to distribute profits for members. The study by Giannoccaro and Pierpaolo [14] employs an agent-based system to determine a wholesale price for a distributor and an order quantity for a retailer. Nachiappan and Jawahar [15] developed a genetic algorithm to identify the optimal contract prices and the revenue sharing ratio between the vendor and the buyer. Giannoccaro and Pontrandolfo [16] built revenue sharing models for two- and three-stage supply chains. Their analytical solutions show that the transfer price for the distributor equals the revenue-keep-rate times the marginal cost in a two-stage supply chain.

The wholesale or transfer prices between manufacturers and distributors impact not only the implementation of RS contracts but also the product flows of production and distribution in the supply chain. Mula et al. [17] and Fahimnia et al. [18] provided a comprehensive review on mathematical models for production and distribution planning. Meixella and Gargeyab [19] and Kilibi et al. [20] reviewed optimization models for supply chain network design. Most studies of production and distribution planning treat the wholesale or transfer prices as model parameters. To consider prices between echelons as variables in the model, Gjerdrum et al. [21] proposed a model to determine the inter-firm transfer prices, production and inventory level, and flows of products between echelons to optimize the total supply chain profit while ensuring adequate rewards for each party. Miller and Matta [22] proposed a model that maximizes profits for a global supply chain. The model determines an optimal production and distribution plan and the

transfer prices between different echelons in a supply chain. Transfer prices are controlled by mark-up percentages when SC members transact items to others in the downstream echelon. Their model considers constraints of demand satisfaction for each echelon, capacity limitation, and the total markup price for each product family. Lakhali [23] investigated the problem of determining transfer prices and flows of goods for network-manufacturing companies. A profit sharing function is incorporated in their model. Vidal and Goetschalckx [24] and Perron et al. [25] studied the problem of determining transfer prices and flow of goods for global supply chain networks to maximize the supply chain total profit.

We summarize the contributions by literature from [21] to [25] because they are directly relative to the proposed production-distribution model for a global supply chain. In these works, the order quantities of retailers are price insensitive. However, members that collaborate in a supply chain are autonomous. They will pursue and maximize their own profits. So, when a retailer in the supply chain suffers demand uncertainty, the behavior of ordering for the retailer will depend on the transfer prices offered from the manufacturers [26, 10]. When determining the transfer prices in the global chain, the retailer's ordering behavior should be considered in the model.

Table 1. Summary of contributions from literatures on the production-distribution model for global supply chains

| Literatures | Contributions |
|-------------|---|
| [21] | Consider the transfer prices between echelons as variables in a global supply chain model. |
| [22][25] | Propose a global supply chain model to determine an optimal production and distribution plan (P-D plan) and the transfer prices between echelons. |
| [24] | Extend the model by considering the issue of the transportation allocation. |
| [23] | Extend the model by add a profit sharing function |

Since the prices between echelons affect the implementation of RS contracts and distribution of production flows of production in supply chains, determining the wholesale or transfer prices and planning production-distribution flows should be integrated when RS mechanisms are employed in the coordination of decentralized supply chains. Although numerous studies have provided analytical solutions for designing contracts and models for determining prices and product flows, few of them consider integrating production-distribution planning with RS mechanisms where demands of retailers are price-sensitive. Being self-interested, distributors order products depending on the wholesale prices. Distributor decrease order quantity to manufacturers with high wholesale prices, due to demand uncertainty. In such cases, different wholesale prices induce different order quantities from distributors and then produce different production and distribution costs for the manufacturers. That is, wholesale prices determine the manufacturer's production and distribution costs. Therefore, when employing a RS mechanism to coordinate manufacturers and retailers that are price sensitive, the wholesale prices should be considered with production-distribution planning.

This study proposes a model of production-distribution planning with revenue sharing (PDP/RS) to integrate a RS mechanism and a multi-period production-distribution plan given the order quantity of each distributor depends on the wholesale price. The ordering behavior of a

distributor is formulated by a news-vendor model. The PDP/RS model determines wholesale prices and the production-distribution plan maximizes total supply chain profit under a given revenue share rate. Table 2 compares the proposed model with the models of [23], [24], [25], and [22]. The main difference is that the PDP/RS model assumes that demand is uncertain and the distributor's ordering behavior depends on the demand uncertainty. Moreover, the PDP/RS model considers a multi-period planning horizon.

Table 2. Comparison of the PDP/RS model with other works

| Configurations of the SC environments | Models | | | |
|--|----------------|---------------|----------|---|
| | [23] | [24] and [25] | [22] | PDP/RS |
| Number of echelons | Four | Three | Two | Two |
| Number of products | 2 | Multiple | Multiple | Multiple |
| Number of periods | Single | Single | Single | Multiple |
| Demand pattern | Fixed | Fixed | Fixed | Distributed as a exponential distribution |
| Revenue/Profit Sharing | Profit sharing | - | - | Revenue sharing |
| Transportation modes | Single | Multiple | Single | Single |
| Downstream ordering behavior formulation | - | - | - | Single-period news vendor model |

Since the PDP/RS model is a mixed integer, nonlinear programming problem, a three-phase heuristic is proposed. In the first phase, the nonlinear total-profit function in the PDP/RS model is approximated by a linear one. Then, the heuristic identifies the order quantities for retailers and product flows in the supply chain to maximize the approximated total profit of the supply chain. In the second phase, the heuristic obtains the wholesale prices that result in the order of quantities and product flows from the first phase. In the last phase, the heuristic computes the total profit for the supply chain using results from the previous two phases. The heuristic is evaluated in terms of the solution quality and computation time. In the 60 trials executed in the experiment, the average solution gap was 3.564% with standard deviation of 3.451; the minimum and maximum solution gaps were 0.09% and 9.87% respectively. A solution gap is defined as one minus a ratio of a heuristic solution value to the optimal solution value for the same PDP/RS problem instance. As to the computation time, the proposed heuristic runs much faster in instances with large problem size. The maximum and minimum mean time-saving ratios were 870.651 and 0.392 respectively. A time-saving ratio is defined as the computation time at which the Lingo optimization software solved a PDR/RS problem instance divided by the amount of time that the proposed heuristic spent. Sources of variance that caused the deviations in the solution quality and computation time are identified.

This paper is organized as follows. Section two presents the production-distribution planning with revenue sharing model (PDP/RS). Then, a heuristic for solving the PDP/RS problem is proposed in section three. In section four, the simulation designs and results for evaluating the proposed heuristic are presented. Finally, a conclusion is drawn in the last section.

2. Model of production and distribution planning with revenue sharing

This study considers a decentralized supply chain consisting of manufacturer and distributor echelons. The two echelons interact with each other through the wholesale prices and the product orders. The manufacturers are required to decide the wholesale prices for distributors. Next, the

distributors identify the orders for manufacturers. Then, the manufacturers produce and distribute products according to the order. Also, based on a revenue sharing contract, distributors share part of their revenues with manufacturers.

To formulate the above interactions in decentralized supply chains, a model of production and distribution planning with revenue sharing (PDP/RS) is proposed. The PDP/RS model consists of two sub-models, as shown in Figure 1. The Production-Distribution Planning sub-model (or PDP sub-model) identifies the optimal production and distribution plan in terms of the orders given by the distributors. The Ordering Planning sub-model (or OP sub-model) determines the optimal order quantity, given the wholesale prices from manufacturers. The objective of the model is to maximize the total profits of the supply chain.

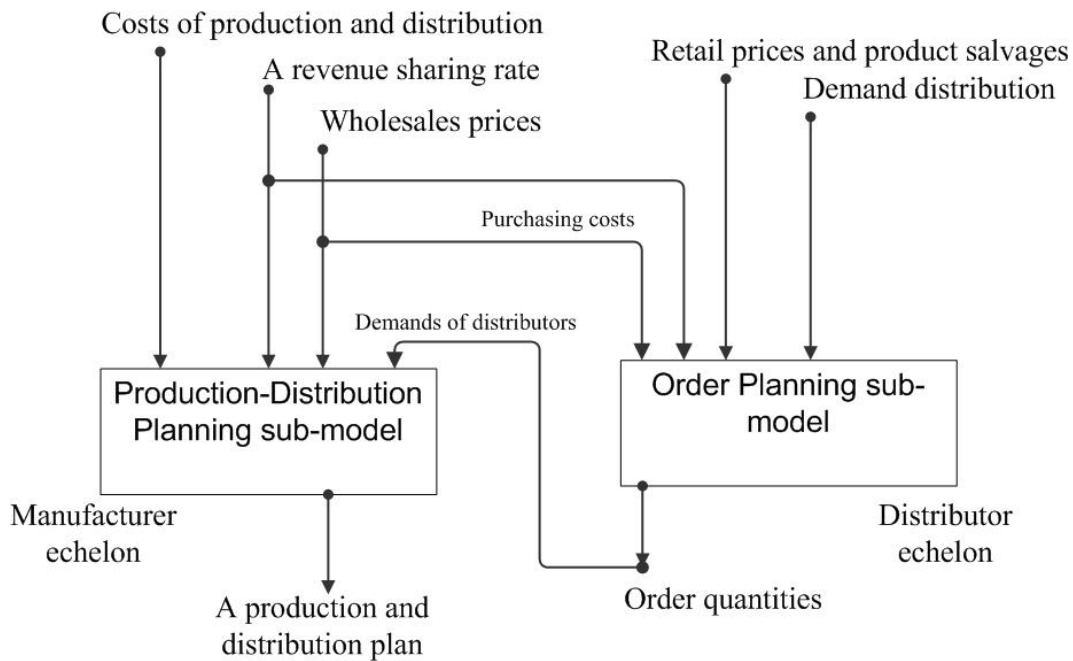


Figure 1. Architecture for the model of production and distribution planning with revenue sharing (PDP/RS)

Before presenting the two submodels, required notations are introduced first.

| <i>Indices/Sets</i> | |
|---------------------|---|
| i | Index for a product, $i = 1 \dots N_i$. |
| m | Index for a manufacturer, $m = 1 \dots N_m$. |
| s | Index for a distributor, $s = 1 \dots N_s$. |
| t | Index for a given period, $t = 1 \dots N_t$. |

| <i>Parameters</i> | |
|-------------------|---|
| α | Shortage penalty for a product. |
| β | Penalty for a unit of idle capacity. |
| γ | A very large number. |
| λ_{ist} | Mean of the random demand for a product of a distributor during a given period. |
| ϕ | Revenue sharing rate. |

| | |
|-------------------|--|
| BC_{is} | Purchasing cost of a product for a distributor. |
| \tilde{D}_{ist} | Demand of a product from a distributor during a given period. The demand is a random variable. |
| FC_{is} | Fixed charge of a product for a manufacturer. |
| HC_{im} | Holding cost of a product for a manufacturer. |
| MC_{mt} | Maximum capacity of a manufacturer during a given period. |
| PC_{im} | Production cost of a product for a manufacturer. |
| RP_{is} | Retail price for a product in a distributor. |
| SC_{ms} | Shipping cost of a product from a manufacturer to a distributor. |
| SV_{is} | Salvage value of a product for a distributor. |
| UC_{im} | Consumed capacity of a product in for manufacturer. |

| | |
|------------------|---|
| <i>Variables</i> | |
| I_{imt} | Inventory of a product for a manufacturer at the end of a given period. |
| Q_{ist} | Quantity of a product ordered by a distributor at the beginning of a given period. |
| QP_{imt} | Production quantity of a product for a manufacturer during a given period. |
| QS_{imst} | Shipping quantity of a product from a manufacturer to a distributor at the end of a given period. |
| W_{im} | Wholesale price of a product for a manufacturer. |
| X_{mt} | Promised capacity of a manufacturer to the supply chain during a given period. |
| Y_{imt} | Yes/No decision for producing a product by a manufacturer during a given period. |

2.1. Sub-model of order planning

The ordering sub-model describes the ordering behavior of a distributor. Given the uncertain demand and the wholesale prices from the manufacturers, a distributor determines the optimal ordering quantity. The study formulates the distributor's ordering problem as a news-vendor problem [27].

Assuming demands for the product of a distributor in a period fit an exponential distribution with mean $\frac{1}{\lambda_{ist}}$, the density function for the demands is:

$$f_{ist}(x) = \lambda_{ist} e^{(-\lambda_{ist}x)}, \forall i, s, t \tag{1}$$

When a distributor orders y products in a period, the expected total profit for the distributor is:

$$\begin{aligned}
 E_{ist}(y) &= -BC_{is}y + (1-\phi)RP_{is} \left(\int_0^y x f_{ist}(x) dx + \int_y^\infty y f_{ist}(x) dx \right) + SV_{is} \int_0^y (y-x) f_{ist}(x) dx \\
 &= -BC_{is}y + (1-\phi)RP_{is} \left(\frac{1}{\lambda_{ist}} \left(1 - e^{(-\lambda_{ist}y)} \right) \right) + SV_{is} \left(y + \frac{1}{\lambda_{ist}} \left(e^{(-\lambda_{ist}y)} - 1 \right) \right)
 \end{aligned} \tag{2}$$

Then, the optimal order that maximizes the total profits is

$$y^* = \frac{-1}{\lambda_{ist}} \ln \left(\frac{BC_{is} - SV_{is}}{(1-\phi)RP_{is} - SV_{is}} \right), \quad (3)$$

given that

$$(1-\phi)RP_{is} - SV_{is} \neq 0 \text{ and } (1-\phi)RP_{is} \geq BC_{is} \geq SV_{is} \geq 0, \forall i, s. \quad (4)$$

Purchasing costs depend on the wholesale prices of the products. Since more than one manufacturer can provide products to a distributor, we assume that the unit purchasing cost is the average of product wholesale prices:

$$BC_{is} = \frac{\sum_{m,t} W_{im} QS_{imst}}{\sum_{m,t} QS_{imst}}, \forall i, s. \quad (5)$$

2.2. Sub-model of production-distribution planning

The PDP sub-model identifies an optimal production and distribution plan to maximize the total profit of the manufacturer echelon given orders from the distributor echelon. The sub-model formulates a production and distribution planning problem with multiple products and multiple periods.

2.2.1. Objective function

The objective function of the PDP sub-model is to maximize the total profit of the manufacturer echelon. The revenue for manufacturers includes sales for manufacturers and returns from distributors, as shown in Equations (6) and (7). Related costs include the production cost, inventory cost, transportation cost and idle capacity penalty, as shown in Equations (8) to (11).

$$\text{Sales for manufacturers} = \sum_{i,m,t} \left(W_{im} \sum_s QS_{imst} \right) \quad (6)$$

$$\text{Returns from distributors} = \sum_{i,s,t} \phi RP_{is} \left(\frac{1}{\lambda_{ist}} (1 - e^{-\lambda_{ist} O_{ist}}) \right) \quad (7)$$

$$\text{Production Cost} = \sum_{i,m,t} (FC_{im} Y_{imt} + PC_{im} QP_{imt}) \quad (8)$$

$$\text{Inventory Cost} = \sum_{i,m,t} HC_{im} I_{imt} \quad (9)$$

$$\text{Transportation Cost} = \sum_{i,m,s,t} SC_{ms} QS_{imst} \quad (10)$$

$$\text{Idle Capacity Penalty} = \beta \sum_{m,t} \left(X_{mt} - \sum_i UC_{im} QP_{imt} \right) \quad (11)$$

Therefore, the total profit for manufacturers becomes:

$$\text{Total profit for manufacturers} = (6) + (7) - ((8) + (9) + (10) + (11)) \quad (12)$$

2.2.2. Constraints

Maximization of the total profit for manufacturers suffers from the following constraints.

Capacity Related Constraints: Equation (13) disables the production quantity variable for a product when the sub-model decides not to produce the product in a period. Equation (14) requires the total consumed capacity to be less or equal to the promised capacity to the SC for each manufacturer in each period. Furthermore, Equation (15) constrains the promised capacity for each manufacturer.

$$QP_{imt} \leq \gamma Y_{imt}, \forall i, m, t. \quad (13)$$

$$\sum_i UC_{im} QP_{imt} \leq X_{mt}, \forall m, t. \quad (14)$$

$$X_{mt} \leq MC_{mt}, \forall m, t. \quad (15)$$

Inventory Balance: The inventory balance equation calculates the inventory balance at the end of each period for each product and each manufacturer, as shown in Equation (16).

$$I_{imt} = I_{im(t-1)} + QP_{imt} - \sum_s QS_{imst}, \forall i, m, t. \quad (16)$$

Demand fulfillment: The shipping quantity for a product to a supplier for each period should be less or equal to the order quantity of the supplier, as shown in Equation (17).

$$\sum_m QS_{imst} \leq O_{ist}, \forall i, s, t. \quad (17)$$

Available inventory for fulfilling demand: The following constant limits the available inventory for fulfilling distributors' orders. For each product in each period, the available quantity is the sum of the production quantity and the inventory balance at the start of the period.

$$\sum_s QS_{imst} \leq QP_{imt} + I_{imt}, \forall i, m, t. \quad (18)$$

Variable domain constraints: Variable domains are limited by the following two equations:

$$I_{imt}, Q_{ist}, QP_{imst}, W_{im}, X_{mt} \geq 0, \forall i, m, s, t. \quad (19)$$

$$Y_{imt} \in \{0, 1\}, \forall i, m, t. \quad (20)$$

2.3. The PDP/RS model

The PDP/RS model integrates the above two sub-models. The objective function for the PDP/RS model is to maximize the total supply chain profit. The total supply chain profit consists of the profits from the manufacturer echelon and distributor echelon. The total supply chain profit is defined as:

$$\begin{aligned} \text{Total SC profit} = & \sum_{i,s,t} \left\{ (1-\phi) \text{RP}_{is} \left(\frac{1}{\lambda_{ist}} (1 - e^{-\lambda_{ist} O_{ist}}) \right) \right\} + \sum_{i,s,t} \left\{ \text{SV}_{is} \left(O_{ist} + \frac{1}{\lambda_{ist}} (e^{-\lambda_{ist} O_{ist}} - 1) \right) \right\} \\ & + \sum_{i,s,t} \left\{ \phi \text{RP}_{is} \left(\frac{1}{\lambda_{ist}} \right) (1 - e^{-\lambda_{ist} O_{ist}}) \right\} - \{(8) + (9) + (10) + (11)\} \end{aligned} \quad (21)$$

The sum of the first and second terms in Equation (21) is the expected revenues in the distributor echelon. The third term is the returns shared with the manufacturer echelon. The last term represents the total cost for the supply chain. Note that the total purchasing cost in the distributor echelon and the total sales for manufacturers do not appear in the equation. Assuming that the unit purchasing cost equals the average product wholesale prices (Equation (5)), it is easy to verify that the total purchasing cost equals the total sales for manufacturers. That is,

$$\sum_{i,m,s,t} \text{BC}_{is} Q S_{imst} = \sum_{i,m,s,t} W_{im} Q S_{imst} . \quad (22)$$

Having the same amount but in different signs, the two terms can be cancelled out from Equation (21).

The constraints for the PDP/RS model include the constraints from the PDP and OP sub-models. The constraint for the OP sub-model includes Equation (4). The constraints for the PDP sub-model contain Equations (13) to (19). Therefore, the PDP/RS model for the decentralized supply chain can be formulated as shown in Figure 2.

Maximize Total supply chain profit: Equation (21)

Subject to:

- Constraint in the distributor echelon
 - Exist of an optimal order quantity: Equation (4).
 - Constraints in manufacturer echelon
 - Capacity related constraints in manufacturer echelon: Equations (13) to (15).
 - Inventory Balance: Equation (16).
 - Demand fulfillment: Equation(17).
 - Available inventory for fulfilling demand: Equation(18).
 - Variable domain: Equations (19) and (20).
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Figure 2. PDP/RS model

The PDP/RS model has the following features. Firstly, the variables of the wholesale prices are canceled out from the model. The model cannot identify a set of optimal wholesale prices for coordinating distributor and manufacturer echelons but can identify an optimal order quantity for each product and each distributor. Nevertheless, this study considers that the manufacturer and distributor echelons coordinate order quantities by wholesale prices in the decentralized supply chain. The wholesale prices need to be determined. Secondly, the PDP/RS model is a mixed integer, non-linear programming problem, because of the function which calculates the total supply chain profit. Therefore, solving the PDP/RS model directly involves high time complexity.

The following section presents a heuristic not only to solve PDP/RS problem as a mixed integer programming problem but also to obtain wholesale prices that ensure the resultant

production and distribution planning.

3. A heuristic approach for solving PDP/RS problem

A three-phase heuristic is proposed to solve the PDP/RS. The heuristic, in the first phase, identifies the order quantities for distributors and the product flows in the supply chain such that the approximated total profit of the supply chain is maximized. Then, the heuristic identifies the wholesale prices that contribute to the order quantities and the product flows in the first phase. Last, in the third phase, the heuristic computes the expected total profit for the supply chain. Details for the three phases are presented as follows.

3.1. Phase I: Identify the order quantities for distributors and the product flows in the supply chain

Phase I identifies the order quantities for distributors and the product flows in the supply chain to maximize the approximated total profit of the supply chain. Since the distributor's expected profit function is nonlinear, the heuristic employs a simple method to approximate the function in order to reduce the time complexity for solving the PDP/RS problem.

An order quantity O_{ist} can be divided into two parts:

$$O_{ist} = O_{ist}^+ + O_{ist}^-, \forall i, s, t \tag{23}$$

O_{ist}^+ denotes the part of the order that can be sold at a retail price; O_{ist}^- denotes the other part of the order that can be sold at a salvage value. Assume that customers demand $n\bar{D}_{ist}$ products where \bar{D}_{ist} denotes an average demand and $n \geq 0$ is a parameter called the Average Demand Multiplier (ADM). Then, it is reasonable to assume that O_{ist}^+ is bounded to

$$O_{ist}^+ \leq n\bar{D}_{ist}, \forall i, s, t. \tag{24}$$

With Equation (24), a function to approximate the expected profit for a distributor (Equation (2)) can be written as:

$$\tilde{E}(O_{ist}^+, O_{ist}^-) = -BC_{is}(O_{ist}^+ + O_{ist}^-) + (1-\phi)RP_{is}O_{ist}^+ + SV_{is}O_{ist}^-, \forall i, s, t. \tag{25}$$

Figure 3 illustrates the approximations to an expected profit given three different ADM values: 0.8, 1.0, and 1.2. Equation (25) appears as a convex function and its shape can be adjusted by the ADM value. Given a fixed product purchasing cost, the ADM values change the equation's skewness. Increasing the ADM values makes the shape move toward negative skewness.

An approximated total profit of the supply chain is obtained by replacing the function of the expected profits for distributors with the approximated one. Therefore, the approximated total profit of the supply chain can be written as Equation (26).

Approximated total SC profit =

$$\sum_{i,s,t} ((1-\phi)RP_{is}O_{ist}^+ + SV_{is}O_{ist}^-) + \sum \phi RP_{is}O_{ist}^+ - [(8) + (9) + (10) + (11)] \tag{26}$$

The model for determining the order quantities and the product flow in the supply chain to maximize the approximated total profit of the supply chain is presented in Figure 4. The model is called the Approximated PDP/RS model. With the formulation, the model determines optimal O_{ist}^+ and O_{ist}^- and the corresponding product flows to optimize the approximated total profit for the supply chain. The advantage is that it reduces the time complexity of the PDP/RS model.

While the PDP/RS model is a mixed integer, nonlinear programming problem, in contrast, the Approximated PDP/RS model is a mixed-integer programming problem.

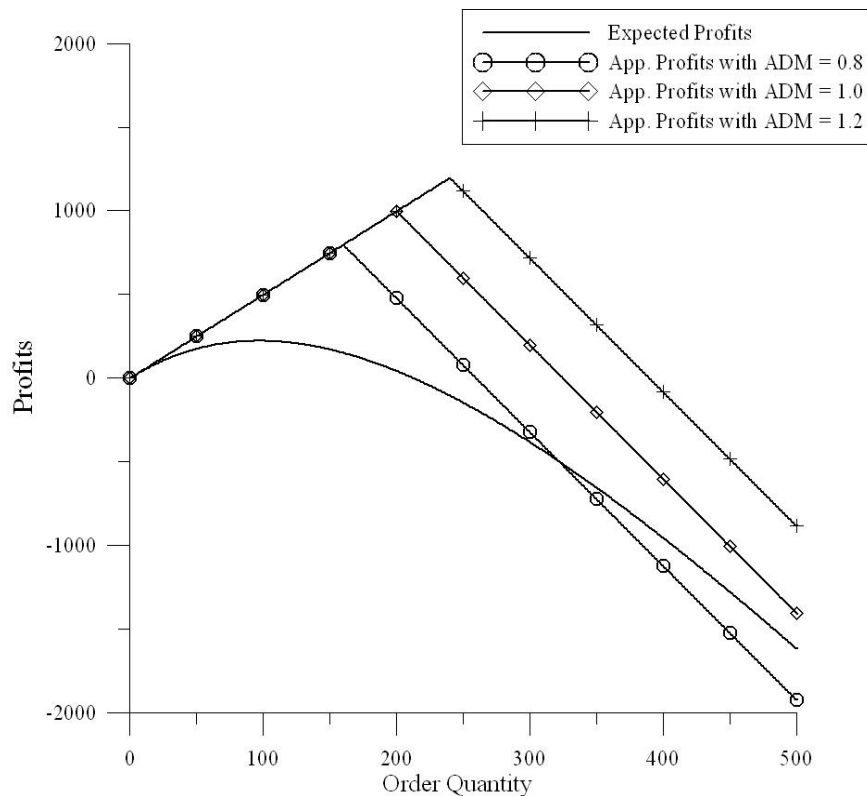


Figure 3. Approximating an expected profit with different ADM values. The product purchasing cost is 10, salvage value is 2, and retail price is 15. The revenue share rate is 0.001 and demand rate is 0.005

3.2. Phase II: Determine wholesale prices

Use the following steps to calculate wholesale prices of products for distributors given the product flows in Phase I.

Step II.1: Calculate the amount of products allocated to each distributor and use the amount as the order quantity. The quantity of a product received by a distributor is:

$$QR_{ist} = \sum_m QS_{imst}, \forall i, s, t. \quad (27)$$

Maximize Approximate total supply chain profit: Equation (26)

Subject to:

- Constraint in the distributor echelon
 - Exist of an optimal order quantity: Equation (4).
 - Upper bounds for expected sales variables: Equation (24).
- Constraints in manufacturer echelon
 - Capacity constraint in manufacturer echelon: Equations (13) to (15).
 - Inventory Balance: Equation (16).
 - Demand fulfillment against $O_{ist}^+ + O_{ist}^-$:

$$\sum_m QS_{imst} \geq O_{ist}^+ + O_{ist}^-, \forall i, s, t. \tag{38}$$

- Available inventory for fulfilling demand: Equation(18).
- Variable domain:

$$* \quad O_{ist}^+, O_{ist}^- \geq 0, \forall i, s, t. \tag{39}$$

* Equations (19) and (20).

Figure 4. Approximated PDP/RS model

Step II.2: Identify the purchase cost of a product for a distributor in a period. Given a distributor's receiving quantity at the end of a period, the purchasing cost must satisfy Equation (3) to order QR_{ist} products. So, the purchasing cost can be expressed as:

$$BC_{ist} = ((1-\phi)RP_{is} - SV_{is})e^{(-\lambda_{ist}QR_{ist})} + SV_{is}, \forall i, s, t. \tag{28}$$

Step II.3: Identify the purchase cost of a product for a distributor that can apply to all periods. The purchase cost that is independent of periods must satisfy the following equation:

$$\sum_t E_{ist}(BC_{is}) = \sum_t E_{ist}(BC_{ist}), \tag{29}$$

where E_{ist} is defined as Equation (2). Hence, the purchase cost is

$$BC_{is} = \begin{cases} \frac{\sum_t (BC_{ist} QR_{ist})}{\sum_t QR_{ist}} & \text{if } \sum_t QR_{ist} \geq 0, \\ \infty & \text{Otherwise.} \end{cases} \tag{30}$$

Step II.4: Identify the wholesale price of a product for a manufacturer that can apply to all periods. The wholesale price for a given period must satisfy Equation (5). Therefore, the wholesale price for a given period is:

$$W_{imt} = \begin{cases} \frac{\sum_s BC_{is} QS_{imst}}{\sum_s QS_{imst}} & \text{if } \sum_s QS_{imst} \geq 0, \\ \infty & \text{Otherwise.} \end{cases} \quad (31)$$

A wholesale price that can be used to replace the wholesale prices for periods for a manufacturer must satisfy

$$W_{im} \sum_{t,s} QS_{imst} = \sum_t \left(W_{imt} \sum_s QS_{imst} \right), \forall i, m. \quad (32)$$

As a result, the wholesale price of a product for periods for a manufacturer is

$$W_{im} = \begin{cases} \frac{\sum_t \left(W_{imt} \sum_s QS_{imst} \right)}{\sum_{t,s} QS_{imst}} & \text{if } \sum_{t,s} QS_{imst} \geq 0, \\ \infty & \text{Otherwise.} \end{cases} \quad (33)$$

3.3. Phase III: Calculate total expected profit for the supply chain

Step III.1: Calculate expected profits for distributors. With the purchasing cost defined by Equation (30), the total expected profit for all distributors is

$$\text{Total expected profits for distributors} = \sum_{i,s,t} E_{is} (QR_{ist}), \quad (34)$$

where E_{is} is defined by Equation (2).

Step III.2: Calculate expected profits for manufacturers. Substitute QR_{ist} to Equation (12) to obtain the total expected profit for manufacturers.

Step III.3: Calculate the total profit for the supply chain. Sum the expected profits from Step III.1 and III.2 to obtain a total supply chain profit.

4. Performance evaluation

This section presents the simulation environments for evaluating the proposed heuristic and the results of the evaluation in terms of the solution quality and computation time. The solution quality is measured by solution-gap ratio as defined by equation (37). The benchmark values are the optimal solution values obtained by solving the PDP/RS model as a mixed integer, nonlinear programming problem. The solution values generated from the proposed heuristic is evaluated against the benchmark ones. The solution value is the total supply chain profit represented by equation (21).

4.1. Simulation environments

The simulation considered supply chains of three different sizes: small, medium and large, as shown in Table 3. The small supply chain was a supply chain with short planning periods. The supply chain consisted of three manufacturers and two distributors. The supply chain sold two products and the planning horizon was two periods. The medium supply chain considered a supply chain with short planning periods. Five manufacturers and ten distributors formed the medium supply chain that sold five products and had a planning horizon of four periods. The large supply chain was operated as the medium one except it had longer planning horizon of ten periods.

In the simulation we first conducted an experiment of the robust parameter design using the small supply chain environment. The experiment identified a robust value for the ADM parameter in the proposed heuristic, in terms of the solution quality. Then, all three supply chains were used in simulation experiments to evaluate the heuristic against the solution quality and computation time. Equation (35) is used to measure the problem size of a supply chain:

$$SC\text{-Size} = \ln(V + C) + B \ln 2, \tag{35}$$

where V denotes the number of variables, C denotes the number of constraints, and B denotes the number of binary variables. The problem sizes of all supply chains are summarized in Table 3.

Table 3. Summary of the three supply chain environments for performance evaluation

| SC Environ- ments | Descriptions | Constraints | Variables | Binary Variables | SC-Sizes |
|----------------------|---|-------------|-----------|---------------------|----------|
| Small | 3 manufacturers, 2 distributors, 2 products, and 2 planning peri- ods | 44 | 68 | 12 | 13.036 |
| Medium | 5 manufacturers, 10 distributors, 5 products, 4 planning periods | 440 | 1445 | 100 | 76.856 |
| Large | 5 manufacturers, 10 distributors, 5 products, 10 planning periods | 1100 | 3575 | 250 | 181.737 |

SC-Size was calculated by Equation (35).

4.1.1. Factors for creating simulation scenarios

Two factors were controlled to create various testing scenarios for each supply chain environment, as shown in Table 4. The first factor was the ratio of the retail price to the salvage value (or P/S ratio). The retail price and salvage value go directly into the equation that determines the order quantity. Hence, various ratios between the two values are considered to generate different simulation scenarios. The levels for P/S ratio were 1.5 and 5. The P/S = 1.5 represented a situation in which product values depreciated slowly; the P/S = 5 represented a situation where product values depreciated fast.

The other factor was a supply-demand ratio (or S/D ratio) that controlled the ratio of a total supply to a total demand. The S/D ratio satisfies the following equation:

$$\sum_{m,t} MC_{mt} \frac{N_i \times N_m}{\sum_{i,m} UC_{im}} = S/D \times \sum_{i,s,t} \lambda_{ist} \tag{36}$$

If the S/D ratio is less than one, the total supply is less than the total demand. The levels for the S/D ratio were 0.5 and 2, which represents that total demands were greater and less than the total supply respectively.

Table 4. Control and noise factors in the experiment of the robust parameter design

| Factors | Descriptions | Levels |
|---------------|---|----------------------------|
| Control: | | |
| ADM parameter | A parameter to adjust the upper bound on the part of the order that can be sold at a retail price | 0.25, 0.5, 1, 1.25, 1.5, 2 |
| Noises: | | |
| P/S ratio | Ratio of the retail price to the salvage value | 1.5 and 5 |
| S/D ratio | Ratio of the total supply to total demand | 0.5 and 2 |

4.2. Robust parameter design

The proposed heuristic requires an appropriate ADM parameter value in order to obtain good solution quality. Recall that the ADM parameter determines the upper bound of the expected sales O_{ist}^+ in the expected profit function (Equation (24)). Setting the ADM parameter value too high might result in a large order quantity for a distributor. The large order quantity might decrease the expected profit for the distributor because the expected profit function is convex. Hence, the parameter directly impacts the solution quality of the proposed heuristic. The experiment of the robust parameter design discerns a robust value for the ADM parameter for different supply chain scenarios.

In the robust parameter design experiment, the P/S and S/D ratios were treated as noise factors. The ADM parameter was considered as a control factor. The levels for the ADM parameter were 0.25, 0.5, 1, 1.25, 1.5, and 2. There were 24 ($6 \times 2 \times 2$) treatments and each was replicated five times. The small supply chain environment in Table 3 was used for the experiment. The optimal solutions from Lingo 10 for solving PDP/RS problem instances were used as benchmark solutions to evaluate the solution quality for the proposed heuristic. The solution quality was measured by the gap ratio defined by Equation (37). Table 5 reports resultant solution values, benchmark values, and gap ratios.

$$\text{gap ratio} = 1 - \frac{\text{value of heuristic solution}}{\text{benchmark value}} \quad (37)$$

The ADM parameter interacted with S/D and P/S ratios, as shown in Figure 5. When the product prices depreciated slowly ($P/S = 1.5$), gap ratios were small and stable. The average gap ratio for all ADM values was 0.786% with standard deviation of 0.435 for $S/D = 0.5$; and was 0.807% with standard deviation of 0.861 for $S/D = 2.0$. On the contrary, when the product prices depreciated fast ($P/S = 5$), the average and variation of the gap ratios increased. The gap ratio for all ADM values increased to 10.302% on average with standard deviation of 6.421 when $S/D = 0.5$. The average changed to 9.035% with standard deviation of 7.264 when $S/D = 2.0$.

The above results show that the proposed heuristic generates better and more stable solution quality when product prices depreciate slowly. Secondly, in the case in which the product prices depreciate fast, the value of the ADM parameter determines the solution quality. Inadequate setting of the ADM value causes inferior solution quality. Setting ADM values depends on S/D values. When the supply chains had supply greater than demand ($S/D = 2$), the smallest gap ratio

occurred at $ADM = 1.5$. When the supply chain had supply less than demand, the smallest gap ratio occurred at $ADM = 1.0$.

Table 5. Average solution values, benchmark solution values, and gap ratios for the robust parameter design experiment. Each treatment replicated five times

| ADM | P/S | S/D | Mean | | |
|------|-----|-----|-------------|-------------|---------------|
| | | | Solutions | Benchmarks | Gap Ratios(%) |
| 0.25 | 1.5 | 0.5 | 1154237.467 | 1163573.768 | 0.805 |
| | | 2.0 | 838011.378 | 851331.780 | 1.637 |
| | 5.0 | 0.5 | 561153.956 | 650382.214 | 13.838 |
| | | 2.0 | 287309.519 | 358946.574 | 20.384 |
| 0.50 | 1.5 | 0.5 | 1160406.887 | 1163573.768 | 0.278 |
| | | 2.0 | 843224.925 | 851331.780 | 1.054 |
| | 5.0 | 0.5 | 572513.328 | 650382.214 | 11.979 |
| | | 2.0 | 308144.081 | 358946.574 | 14.425 |
| 1.00 | 1.5 | 0.5 | 1154203.155 | 1163573.768 | 0.803 |
| | | 2.0 | 847707.857 | 851331.780 | 0.489 |
| | 5.0 | 0.5 | 621274.846 | 650382.214 | 4.435 |
| | | 2.0 | 334900.550 | 358946.574 | 6.744 |
| 1.25 | 1.5 | 0.5 | 1150505.026 | 1163573.768 | 1.090 |
| | | 2.0 | 847701.710 | 851331.780 | 0.439 |
| | 5.0 | 0.5 | 606259.204 | 650382.214 | 6.655 |
| | | 2.0 | 342531.768 | 358946.574 | 4.604 |
| 1.50 | 1.5 | 0.5 | 1152379.620 | 1163573.768 | 0.941 |
| | | 2.0 | 846970.070 | 851331.780 | 0.470 |
| | 5.0 | 0.5 | 588142.850 | 650382.214 | 9.384 |
| | | 2.0 | 345558.836 | 358946.574 | 3.680 |
| 2.00 | 1.5 | 0.5 | 1155065.232 | 1163573.768 | 0.792 |
| | | 2.0 | 843414.152 | 851331.780 | 0.756 |
| | 5.0 | 0.5 | 550203.660 | 650382.214 | 15.523 |
| | | 2.0 | 342241.620 | 358946.574 | 4.375 |

To choose a robust value for smaller-the-better problem, Wu and Hamada [28] suggested a two-step procedure. The first step is to select the levels of the location factors to minimize the location. The second step is to select the levels of the dispersion factors that are not location factors to minimize dispersion. Table 6 shows the mean and the standard deviation for each level of the ADM parameter. Based on their approach, the robust value for the ADM parameter was set to 1. Setting ADM parameter to 1 resulted in the smallest average and dispersion in gap ratios, which were 3.118 and 3.321 respectively.

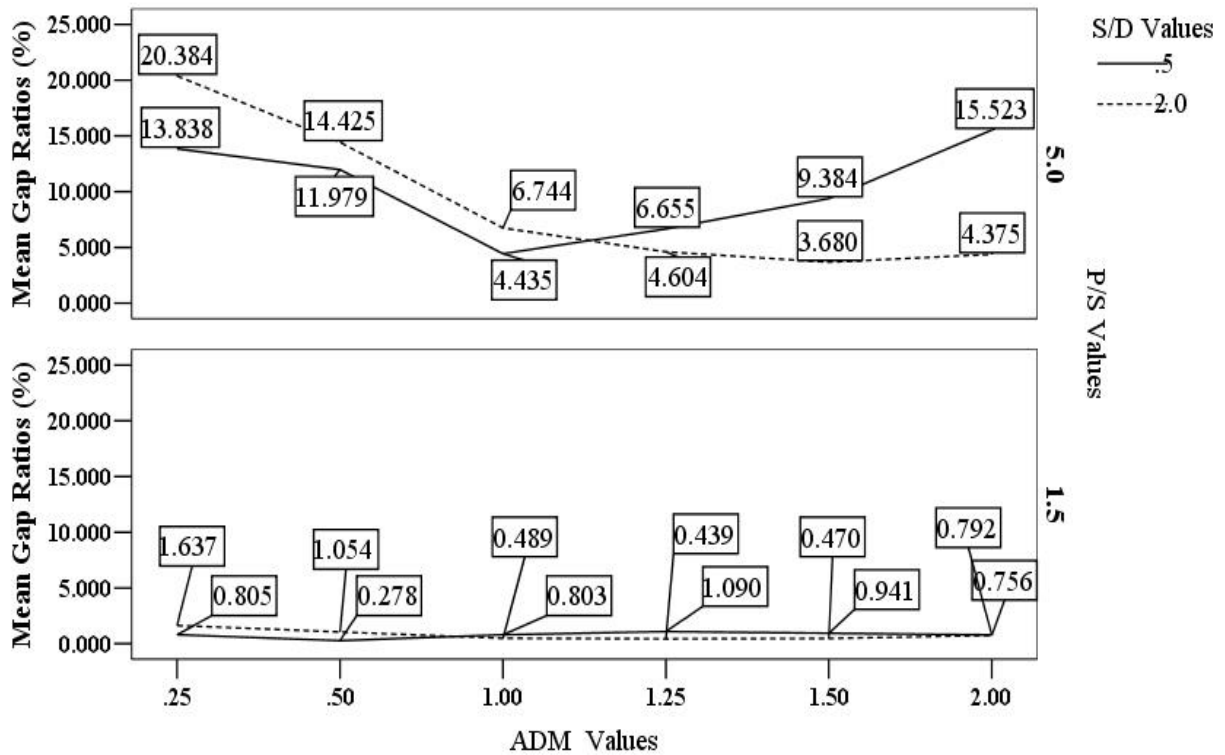


Figure 5. Interactions among ADM parameter, P/S ratio, and S/D ratio

Table 6. Means and standard deviations to identify a robust value for the ADM parameter

| ADM values | Gap Ratios (%) | |
|------------|----------------|-----------|
| | Means | Std. Dev. |
| 0.25 | 9.166 | 9.108 |
| 0.50 | 6.934 | 7.206 |
| 1.00 | 3.118 | 3.321 |
| 1.25 | 3.197 | 3.565 |
| 1.50 | 3.619 | 4.934 |
| 2.00 | 5.361 | 7.507 |

4.3. Effects analysis on solution quality

This subsection presents factors that affect the solution quality given the setting of the ADM parameter at the robust value. Three factors were considered: SC-Size, P/S, and S/D. All three supply chains in Table 3 were used for evaluation. The evaluation was done with a desktop with CPU Xeon 2.40G, 2G memory. Lingo 10 was employed to implement the proposed heuristic and to solve instances of the PDP/RS problem. The computation time was limited to 3600 seconds. The ADM parameter was set to one in all supply chain environments. In each supply chain environment, there were four simulation scenarios and each of them had five instances. There were a total of 60 trials ($3 \times 2 \times 2 \times 5$) in the experiment.

The distributions of the gap ratios for all trials are shown in Figure 6. The average solution gap was 3.564% with standard deviation of 3.451. The minimum solution gap was 0.09% and the

maximum solution gap was 9.87%. Table 7 shows the average solution gap for each combination of the factor levels. The best and worst average solution gaps across all combinations were 0.196% and 7.929%, respectively. The smallest average solution gap occurred at supply chain 2 with S/D = 2 and P/S = 1.5. The largest average solution gap occurred at supply chain 3 with S/D value 0.5 and P/S value 5.

Table 8 presents the analysis of the variance for the average solution gap ratios on Table 7. Effects of P/S, S/D and their interactions significantly determined the gap ratios. The three effects contributed to up to 81% of the total variance. The effect of the SC-Size was not significant.

The individual effects of P/S and S/D values are shown in Figures 7(a) and 7(b), respectively. Rising of P/S vales from 1.5 to 5 increased the mean gap ratios from 0.549% to 6.579%, which lost 6.030% in the solution quality. Rising of S/D values from 0.5 to 2 decreased the mean gap ratios from 3.568% to 3.561%, which gained a small increment 0.007% in solution quality. In addition, as shown in Figure 7(c), P/S and S/D factors interacted to influence the mean gap ratios. When P/S = 5, lifting S/D value caused the mean gap ratios to increase. The mean gap ratios rose from 6.340% to 6.819% when S/D value changed from low to high levels. However, when P/S = 1.5, raising S/D value from low to high levels caused the mean gap ratios to decrease from 0.795% to 0.302%.

It appears that the proposed heuristic generates good solution quality and is independent of the problem sizes. Another finding is that the solution gap ratios seem to increase slightly with larger P/S values, as shown in Figure 7(c). The main cause would be the value of the ADM parameter in Equation (24). When P/S value is large and other things are equal, allocating more to O_{ist}^+ and less to O_{ist}^- can increase the profits of distributors. Since O_{ist}^+ is limited by the ADM parameter value, increasing the ADM parameter value can raise the profits of distributors. On the contrary, when P/S value is small, allocating more to O_{ist}^+ or O_{ist}^- makes no significant difference because the salvage value is close to the retail price. Based on the above findings, the ADM parameter values should be adjusted dynamically according to different P/S and S/D values to gain better solution quality.

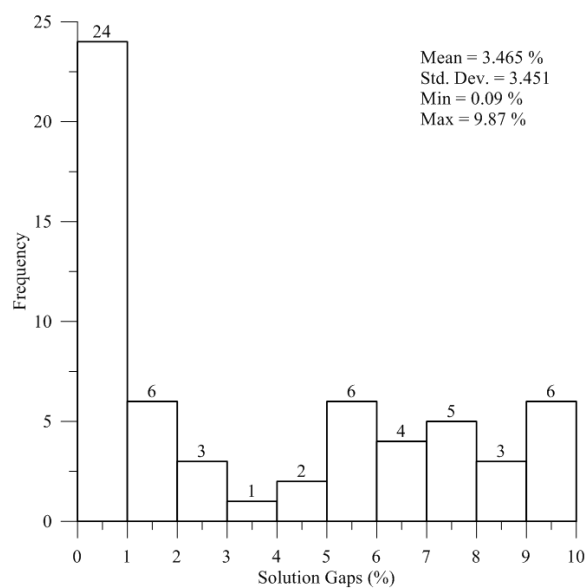
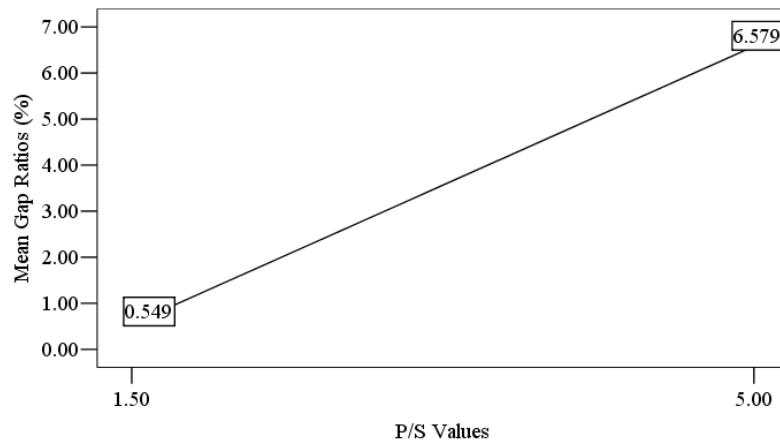
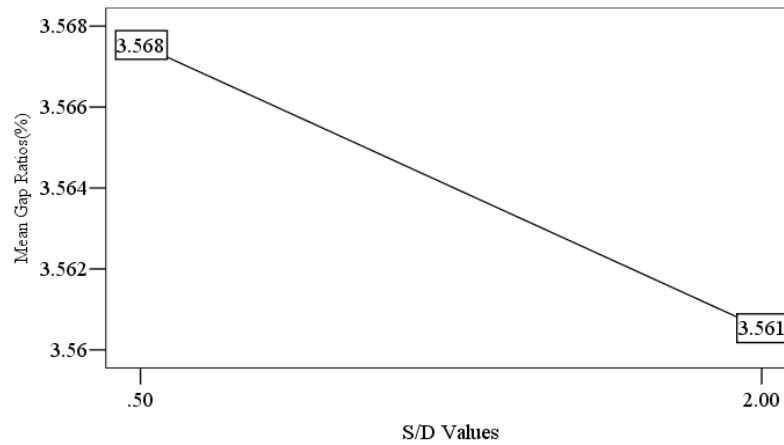


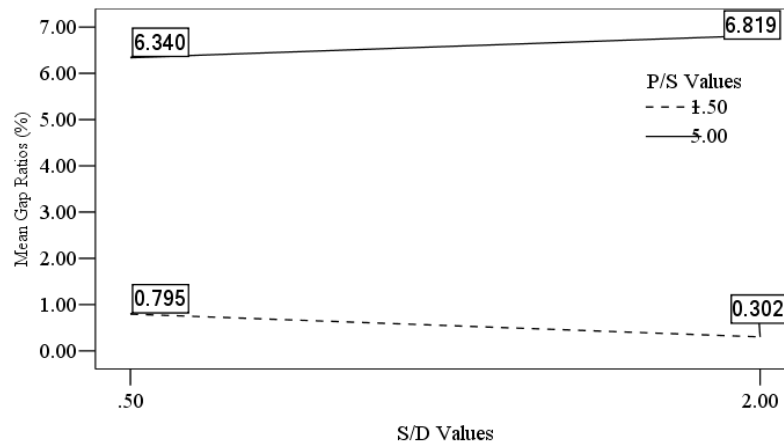
Figure 6. Distribution of all solution gap ratios



(a) Effects of the P/S ratio



(b) Effects of the S/D ratio



(c) Effects of the P/S ratio

Figure 7. Effects on gap ratios for P/S and S/D ratios and their interactions

Table 7. Average solution gap ratios and time-saving ratios under various combinations of the factor levels

| SC Environments | SC-Sizes | S/D | P/D | Rep. | Gap Ratio(%) | | Time-Saving Ratio | |
|-----------------|----------|-----|-----|------|--------------|-----------|-------------------|-----------|
| | | | | | Means | Std. Dev. | Means | Std. Dev. |
| Small | 13.036 | 0.5 | 1.5 | 5 | 0.803 | 0.533 | 0.392 | 0.117 |
| | | | 5 | 5 | 4.435 | 3.126 | 1.206 | 1.011 |
| | | 2 | 1.5 | 5 | 0.489 | 0.482 | 0.448 | 0.225 |
| | | | 5 | 5 | 6.744 | 2.857 | 2.094 | 1.628 |
| Medium | 76.856 | 0.5 | 1.5 | 5 | 0.818 | 0.317 | 138.568 | 177.367 |
| | | | 5 | 5 | 6.655 | 1.720 | 283.603 | 224.502 |
| | | 2 | 1.5 | 5 | 0.196 | 0.068 | 220.100 | 410.065 |
| | | | 5 | 5 | 7.695 | 1.825 | 719.434 | 379.840 |
| Large | 181.737 | 0.5 | 1.5 | 5 | 0.765 | 0.292 | 791.637 | 712.851 |
| | | | 5 | 5 | 7.929 | 1.656 | 161.745 | 224.912 |
| | | 2 | 1.5 | 5 | 0.222 | 0.093 | 870.651 | 628.843 |
| | | | 5 | 5 | 6.017 | 1.048 | 322.314 | 350.174 |

Table 8. Analysis of variance on average gap ratios.

| Dependent Variable: Ln(gap-ratio) | | | | | | |
|-----------------------------------|----------------|----|-------------|---------|-------|--|
| Source | Sum of Squares | df | Mean Square | F | Sig. | |
| P/S | 110.501 | 1 | 110.501 | 407.887 | 0.000 | |
| S/D | 3.646 | 1 | 3.646 | 13.457 | 0.001 | |
| SC-Size | 0.103 | 2 | 0.052 | 0.191 | 0.827 | |
| P/S * S/D | 5.571 | 1 | 5.571 | 20.562 | 0.000 | |
| P/S * SC-Size | 0.993 | 2 | 0.496 | 1.832 | 0.171 | |
| S/D * SC-Size | 1.338 | 2 | 0.669 | 2.47006 | 0.095 | |
| P/S * S/D * SC-Size | 0.256 | 2 | 0.128 | 0.472 | 0.627 | |
| Error | 13.004 | 48 | 0.270911 | | | |
| Total | 135.411 | 59 | | | | |

R Squared = .904 (Adjusted R Squared = .882)

4.4. Effects analysis on computation time

The time-saving ratios are presented in Table 7. The time required for Lingo 10 to solve the PDP/RS problem as a mixed integer, nonlinear problem was employed as a benchmark. A time-saving ratio is defined as dividing the benchmark time by the heuristic's computation time for the same problem instance. A higher time-saving ratio represents greater time-saving. The maximum average time-saving ratio was 870.651 which occurred at the large supply chains, S/D = 2, and P/S = 1.5. The minimum average time-saving ratio was 0.392, which occurred at the small supply chains, S/D = 0.5, and P/S = 1.5. Table 9 shows sources of the variances impacting the average time-saving ratios. Effects of SC-Size and the interaction between SC-Size and P/S values significantly affected the average time-saving ratios.

Table 9. Analysis of variances for time-saving ratios

| Dependent Variable: Ln(time-saving-ratio) | | | | | |
|---|----------------|----|-------------|--------|-------|
| Source | Sum of Squares | df | Mean Square | F | Sig. |
| SC-Size | 382.284 | 2 | 191.142 | 80.033 | 0.000 |
| P/S | 2.892 | 1 | 2.892 | 1.211 | 0.277 |
| S/D | 7.593 | 1 | 7.593 | 3.179 | 0.081 |
| SC-Size * P/S | 30.334 | 2 | 15.167 | 6.351 | 0.004 |
| SC-Size * S/D | 5.329 | 2 | 2.665 | 1.116 | 0.336 |
| P/S * S/D | 5.131 | 1 | 5.131 | 2.148 | 0.149 |
| SC-Size * P/S * S/D | 1.103 | 2 | 0.552 | 0.231 | 0.795 |
| Error | 114.638 | 48 | 2.388 | | |
| Total | 549.304 | 59 | | | |

R Squared = .791 (Adjusted R Squared = .743)

The SC-Size effect contributed 69.594% of the total variance. Figure 8 shows the average time-saving ratios at each SC-Size value. The time-saving ratio increased from 1.035 to 536.587 as the SC-Size changed from small to large. The interaction between SC-Size and P/S contributed 5.522% of the total variance. Figure 9 shows the average time-saving ratios by SC-Size and P/S factor. When P/S = 1.5, the time-saving ratios were improved in an upward trend. The ratios increased from 0.420 to 831.144 with increasing SC-Size. However, this was not the case when P/S = 5.0. The time-saving ratios first rose from 0.420 to 510.519 and then declined to 242.029 when the SC-Size increased from small to large. Thus, the P/S factor alters the time-saving ratios under different SC-Size values. The main reason would be that the proposed heuristic spends more computation time on large problem sizes at high P/S values than on small problem sizes at low P/S values. Figure 10 shows the benchmarking computation time and the time for the proposed heuristic. Both of them increased as the SC-Size increased. However, as shown in Figure 10(b), there was a sharp increase in average computation time for the proposed heuristic when SC-Size changed from medium to large given P/S = 5. Given SC-Size value was large, the average computation time was 235.155 seconds when P/S = 5, which was about 75 times the case as when P/S = 1.5. Therefore, the time-saving ratios decline with large SC-Size at high P/S values. Although time-saving ratio declined in this case, the proposed heuristic was still much faster than the benchmark for large problem sizes. According to above findings, it appears that using the proposed heuristic saves a great deal of computation time, with good solution quality of about 92% to 99% of the optimal solution on average. The proposed heuristic is more efficient than using Lingo to solve the PDP/RS problem.

The proposed heuristic has been evaluated in terms of the solution quality and computation time. The results of robust parameter analysis suggest that the ADM parameter should be set to one for the heuristic to generate good solution quality. The average gap ratios ranged from 0.196% to 7.929%, depending on the P/S and S/D values. The heuristic's solution quality is altered mainly by the P/S value. High P/S value makes the solution quality decline. Dynamically adjusting the ADM parameter according to different P/S and S/D values would be able to overcome this issue. The proposed heuristic is much faster than benchmark in cases of large problem instances in general. The average time-saving ratios ranged from 0.392 to 870.651, depending on the SC-Size and P/S values. The proposed heuristic can solve the PDP/RS problem effectively and efficiently.

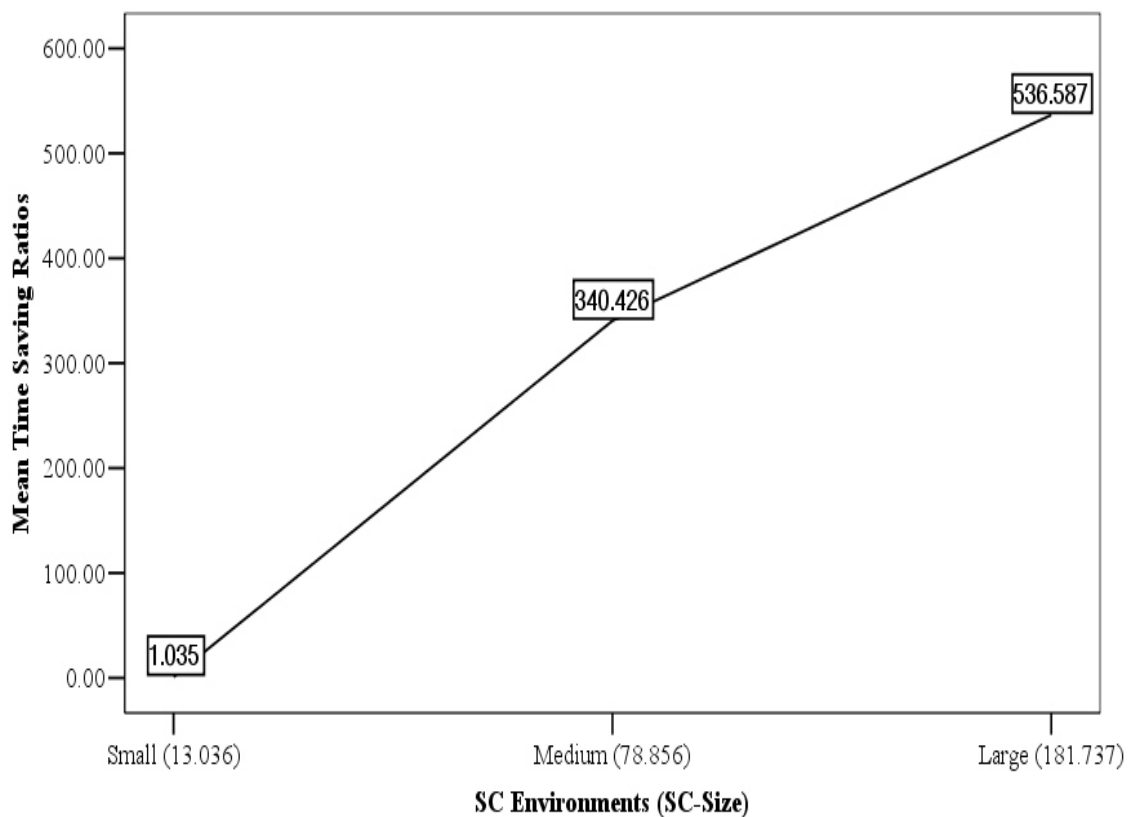


Figure 8. Average time-saving ratios by SC-Size values

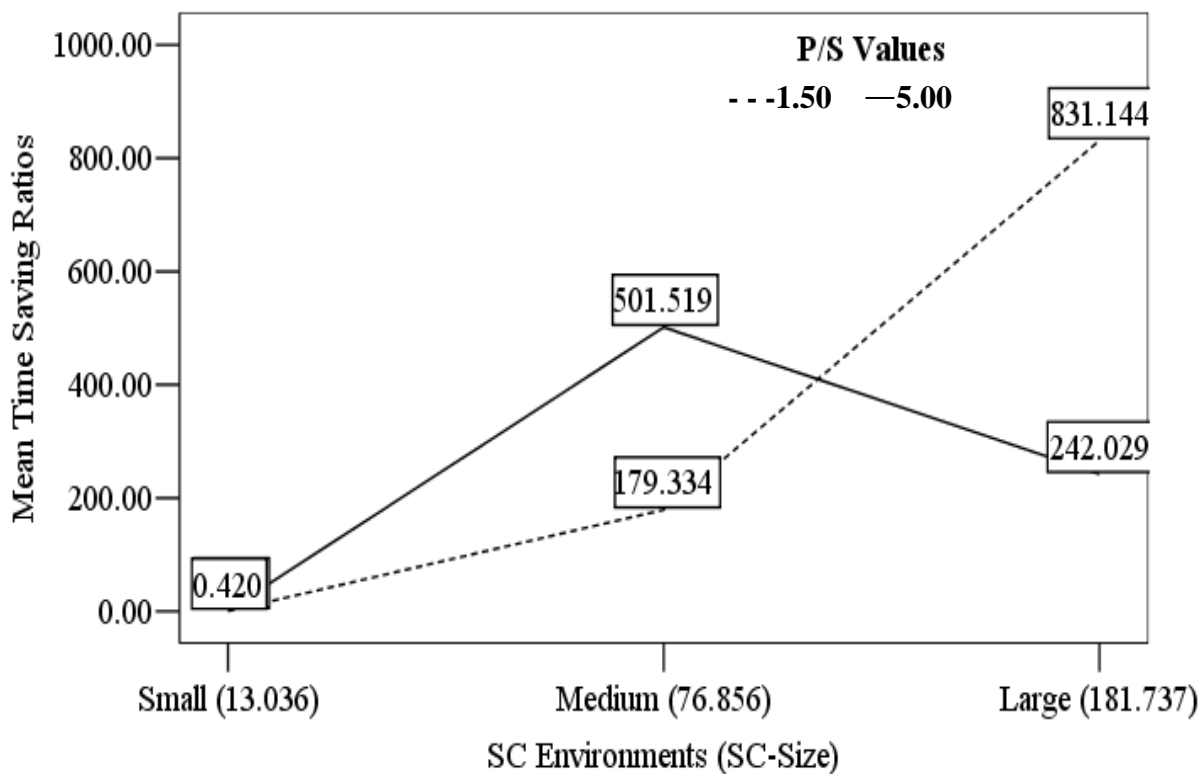
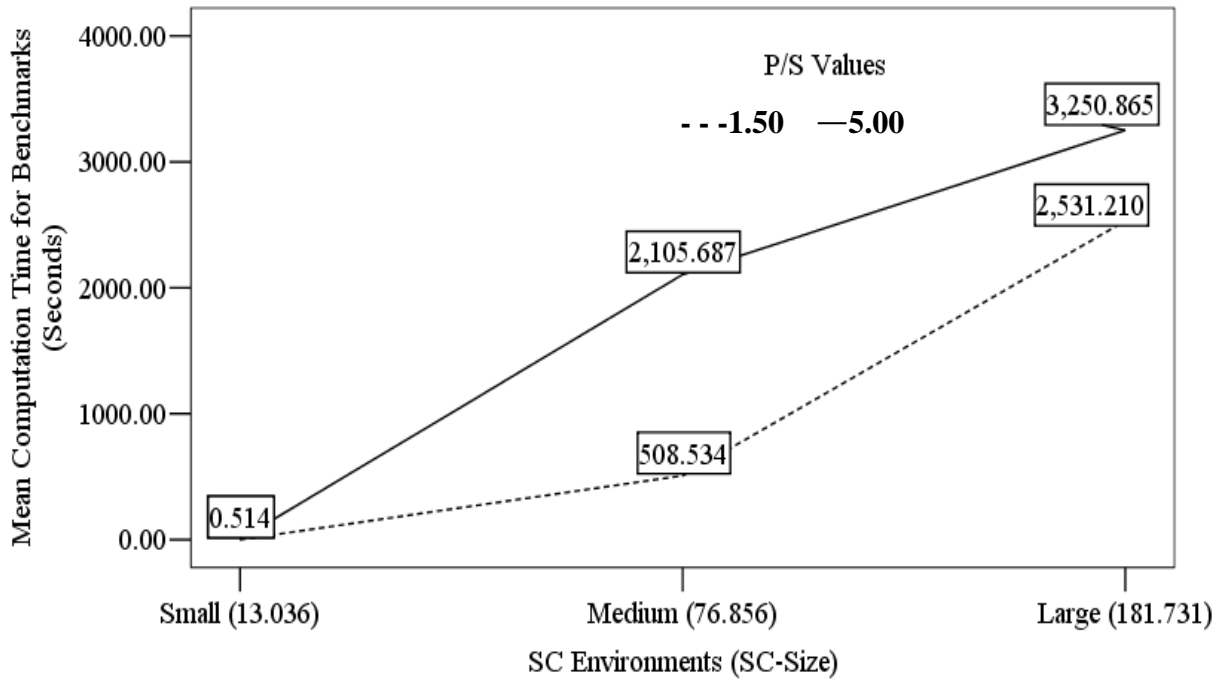
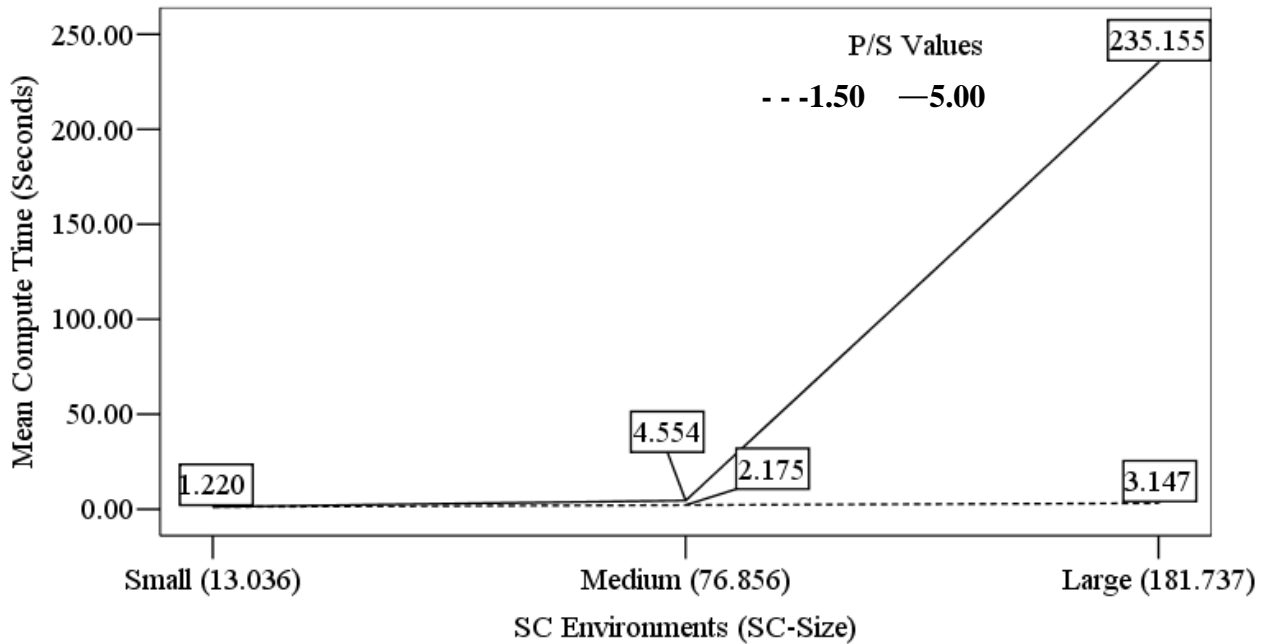


Figure 9. Average time-saving ratios by SC-Size and P/S values



(a) Computation time for benchmark solution values



(b) Computation time of the heuristic solution values

Figure 10. Computation time for the benchmark and heuristic solution values in various problem sizes

5. Conclusion

This study contributes to the literature of production-distribution planning in three ways. Firstly, the PDP/RS model incorporates the uncertain demand and the ordering behavior of the distributor to multi-period production-distribution planning. Secondly, the PDP/RS model integrates a revenue sharing mechanism to a production-distribution planning problem. Thirdly, an efficient and effective heuristic is proposed to solve the PDP/RS problem.

The proposed PDP/RS model for a decentralized supply chain consists of two sub-models. The OP sub-model formulates the ordering behavior of a distributor under demand uncertainty, which is a news-vendor problem. The PDP sub-model formulates a production and distribution planning problem with multiple products and periods. The PDP sub-model considers the returns from distributors in the objective function given a revenue sharing rate.

This study has proposed a heuristic to solve the PDP/RS problem. The heuristic contains three phases. In the first phase, the heuristic employs an approximated total supply chain function to identify optimal order quantities for distributors and product flows in the supply chain. A parameter called ADM parameter can be used to adjust the approximation. Then, the heuristic determines wholesale prices that lead to order quantities from the first phase. In the last phase, the heuristic identifies the expected total profits for manufacturers, distributors, and the supply chain. According to the results of the experiment, the heuristic can provide good solution quality with time efficiency. The maximum and minimum mean solution gap ratios were 0.196% and 7.929% respectively. The maximum and minimum time saving ratios were 870.651 and 0.392 respectively. Sources which cause variances in solution quality and computation time have been analyzed. The solution quality of the heuristic was determined by the ADM parameter, P/S ratio, S/D ratio, and the interaction between P/S and S/D ratios. However, the solution quality was independent of problem size. As to the computation time, the proposed heuristic depended on the problem size and P/S ratio. In general, the time saving ratios showed an upward trend when problem sizes increased.

The proposed model can be extended to other demand uncertainty patterns. It is also important to develop an efficient heuristic that is independent of the different uncertainty patterns. Moreover, integrating production-distribution planning model with other cooperative mechanisms for decentralized supply chains should be addressed in the future.

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