

Unsteady MHD Mixed Convection, Radiative Boundary Layer Flow of a Micro Polar Fluid Past a Semi-infinite Vertical Porous Plate with Suction

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Abstract: The present study investigates the thermal radiation effect on unsteady magneto hydro dynamic (MHD), mixed convection flow of a micro polar fluid past an infinite, steadily moving plate with variable suction. The dimensionless equations which govern the flow, such as conservation of mass, linear momentum, angular momentum, and energy, are solved by using a regular perturbation method. The expressions for velocity, micro rotation, and temperature are obtained. With the aid of these, the expressions for Skin-friction and Nusselt number are obtained and the behavior of various physical parameters on the above flow quantities is discussed with the help of the graphs. The existence of the magnetic field and as well as radiation decrease the velocity throughout the boundary layer. The effect of increase in the plate moving velocity and the radiation parameter, manifest in a linearly decreasing surface skin-friction on the porous plate, whereas the surface heat transfer from the porous plate tends to increase, with an increase in Prandtl number.

Keywords: Micro polar fluid; uniform magnetic field; mixed convection; suction; thermal radiation; vertical plate and porous medium

1. Introduction

The study of flow with heat transfer for an electrically conducting fluid past a porous plate has attracted the interest of many investigators in view of its applications in many engineering and industrial problems such as in oil exploration, geothermal energy extractions and boundary layer control in aerodynamics [1-7]. Raptis [8] studied, mathematically the case of time varying two dimensional natural convection flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Gorla [9] considered a problem of free convection from a vertical plate with non-uniform heat flux and embedded in a porous medium. Kafoussias et al. [10] studied the effects of free convection on the Stokes problem for an infinite vertical limiting surface with constant suction. Magneto hydrodynamic (MHD) flows have applications in meteorology, Solar physics, cosmic fluid dynamics, Astrophysics, Geophysics in the motion of earth core. In addition from technological point of view, MHD free

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convection flows have significant applications in the field of stellar and planetary magnetospheres, aeronautical plasma flows, chemical engineering and electronics. An excellent summary of applications is to be found in Huges and Young [11]. Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction was investigated by Kim [12]. In their study, Raju and Varma [13] considered MHD free convection oscillatory Couette flow through a porous medium. Helmy [14] studied, MHD unsteady free convection flow past a vertical porous plate embedded in a porous medium. Elbashbeshy [15] studied heat and mass transfer along a vertical plate in the presence of magnetic field. Heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in presence of temperature dependent heat source, were studied by Ravikumar et al. [16] Hall-current effects on unsteady MHD flow between stretching sheet and an oscillating porous upper parallel plate with constant suction, was studied by Raju et al. [17] Seshaiyah et al. [18] investigated, on Induced magnetic field effects on free convective flow of radiative, dissipative fluid past a porous plate with temperature gradient heat source.

The radiation effects are important under many non-isothermal situations. Raptis [19-20] studied numerically the case of steady two-dimensional flow of a micro polar fluid past a continuously moving plate with a constant velocity in the presence of thermal radiation. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment then radiation could be important. The need of radiation heat transfer in the system can perhaps lead to desired product with sought characteristic. Radiation effects on Newtonian and non-Newtonian fluids with and without magnetic field have been considered by many authors [21-26]. In most of the above studies, micro polar fluid was not considered. Kim [27] presented an unsteady MHD mixed convection with mass transfer flow for a micro polar fluid past a moving vertical plate via a porous medium. The effects of Joule heating on the MHD free convection flow of a micro polar fluid were studied by EL-Hakien et al [28] Sharma and Gupta [29] considered thermal convection in micro polar fluids in porous medium. Motivated by the above studies, in this paper we have made an attempt to analyze the thermal radiation effects on magneto hydro dynamic mixed convection flow of a micro polar fluid past an infinite, steadily moving porous plate with variable suction and constant viscosity. The micro polar fluid considered here is a gray, absorbing-emitting but non-scattering optically thick medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. It is also assumed that the porous plate moves with constant upward velocity.

2. Nomenclature

A	Real positive constant		
B_0	Magnetic induction	β	Spin gradient viscosity
g	Acceleration due to gravity	β^*	Coefficient of volumetric expansion
k	Thermal conductivity	δ	Exponential index
M	Magnetic parameter	μ	Fluid dynamic viscosity
Nu	Nusselt number	ν	Fluid kinematic viscosity
N	Dimensionless material parameter	ν_r	Fluid kinematic rotational viscosity
Pr	Prandtl number	θ	Non dimensional temperature
q_r	Radiation heat flux density	σ	Electrical conductivity
R	Radiation parameter	ρ	Fluid density
		ε	Scalar constant($\ll 1$)

T	Temperature	ω	Angular velocity vector
U_0	Scale of free stream velocity	κ	Vertex viscosity(micro-rotation)
V_0	Scale of suction velocity	Sub scripts	
u, v	Longitudinal and transverse components of velocity vector	P	Plate
x^*, y^*	Distances along and perpendicular to the plate	W	Wall condition
x, y	Non dimensional Cartesian coordinates	∞	Free stream condition

3. Mathematical analysis

A two-dimensional, unsteady, magneto hydrodynamic (MHD), mixed convection flow, of a viscous incompressible and electrically conducting micro polar fluid past a semi-infinite moving vertical plate is considered. The x^* -axis is taken along the plate in the upward direction and y^* -axis is taken normal to it. A magnetic field of uniform strength is applied transversely to the direction of the flow. The induced magnetic field can be neglected since the magnetic Reynolds number of the flow is assumed very small. The MHD term derived from an order-of-magnitude analysis of the full Navier-Stokes equations. The fluid properties are assumed to be constant in a limited temperature range except for influence of density variation with temperature which is considered only in the body force term. Since the plate is semi-infinite in length, the physical variables are functions of y^* and t^* only. Under the usual Boussinsq's approximation, the equations pertained to the conservation of mass, linear momentum, micro-rotation, and energy, are as follows:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + (v + v_r) \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta(T - T_\infty) - \frac{\sigma B_0^2}{\rho} u + 2v_r \frac{\partial w^*}{\partial y^*} \tag{2}$$

$$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} = \frac{\gamma}{\rho j^*} \frac{\partial^2 w^*}{\partial y^{*2}} \tag{3}$$

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \alpha \left(\frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{k} \frac{\partial q_r}{\partial y^*} \right) \tag{4}$$

The magnetic and viscous dissipations are neglected in this study. The 3rd and 4th terms on the RHS of the momentum equation (2) denote the thermal buoyancy effect and transverse magnetic field effect respectively. Also the 2nd term on the RHS of the energy equation (3) represents the radiation effect. It is supposed that the permeable plate moves with a variable velocity in the direction of fluid flow. In addition, it is assumed that the temperature at the wall as well as the suction velocity are exponentially varying with time.

By using the Roseland approximation [30], the radiative heat flux in y direction is given by

$$q_r = -\frac{4}{3} \frac{\sigma_s}{k_e} \frac{\partial T^4}{\partial y^*} \tag{5}$$

where σ_s and k_e are the Stefan-Boltzmann constant and the mean absorption coefficient,

respectively. It should be noted that by using the Roseland approximation we limit our analysis is limited to optically thick fluids. If the temperature differences within the flow sufficiently small, then equation (5) can be linearized by expanding T^4 into Taylor series about T , and neglecting higher order terms to give:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

The heating due to viscous dissipation is neglected for small velocities in energy conservation equation (4) and Boussinesq approximation is used to describe buoyancy force in equation (2). It is assumed that the free stream velocity (U_∞^*) the suction velocity (v^*) and the plate temperature follow an exponentially increasing or decreasing small perturbation law.

Under these assumptions, the appropriate boundary conditions for the velocity, micro rotation, temperature and concentration fields are

$$\begin{aligned} \text{at } y^* = 0 ; \quad & u^* = u_p^*, \quad v^* = -V_0(1 + \varepsilon A e^{\delta^* t^*}), \quad T = T_\infty + \varepsilon(T_w - T_\infty)e^{\delta^* t^*}, \quad w^* = -n \frac{\partial u^*}{\partial y^*} \\ \text{as } y^* \rightarrow \infty ; \quad & u^* \rightarrow U_\infty^* = U_0(1 + \varepsilon A e^{\delta^* t^*}), \quad T \rightarrow T_\infty, \quad w^* \rightarrow 0 \end{aligned} \tag{7}$$

The last equation in (7) is the boundary condition for micro rotation variable that describes its relationship with the surface stress. In this equation, the parameter n is a number between 0 and 1 that relates the micro-rotation vector to the shear stress, the value $n = 0$ corresponds to the case where the particle density is sufficiently large so that micro elements close to the wall are unable to rotate. The value $n = 0.5$ is indicate of weak concentrations, and when $n = 1$ flows are believed to represent turbulent boundary layers, outside the boundary layer, equation (2) is reduced to

$$-\frac{1}{\rho} \frac{dp^*}{dx^*} = \frac{dU_\infty^*}{dt^*} + \frac{\sigma}{\rho} B_0^2 u_\infty^* \tag{8}$$

We now introduce the following dimensionless quantities.

$$\begin{aligned} u = \frac{u^*}{U_0}, \quad v = \frac{v^*}{V_0}, \quad U_\infty = \frac{U_\infty^*}{U_0}, \quad y = \frac{V_0 y^*}{\nu}, \quad U_p = \frac{u_p^*}{U_0}, \quad w = \frac{\nu w^*}{U_0 V_0}, \quad t = \frac{V_0^2 t^*}{\nu}, \quad \delta = \frac{\nu \delta^*}{V_0^2}, \quad j = \frac{V_0^2 j^*}{\nu}, \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \text{Pr} = \frac{\rho \nu C_p}{k} = \frac{\nu}{\alpha}, \quad \text{Gr} = \frac{\nu \beta g (T_w - T_\infty)}{U_0 V_0^2}, \quad R = \frac{k k_e}{4 \sigma_s T_\infty^3}, \quad M = \frac{\sigma B_0^2}{\rho V_0^2} \end{aligned} \tag{9}$$

Furthermore, the spin-gradient viscosity γ which defines the relationship between the coefficients of viscosity and micro-inertia is given by:

$$\gamma = \left(\mu + \frac{k}{2} \right) j^* = \mu j^* \left(1 + \frac{1}{2} \beta \right); \beta = \frac{k}{\mu} \tag{10}$$

where β denotes the dimensionless viscosity ratio.

In view of equations (8)-(10), the governing equations (2)-(4) reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A e^{\delta t}) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + (1 + \beta) \frac{\partial^2 u}{\partial y^2} + Gr\theta - M(u - U_\infty) + 2\beta \frac{\partial w}{\partial y} \quad (11)$$

$$\frac{\partial w}{\partial t} - (1 + \varepsilon A e^{\delta t}) \frac{\partial w}{\partial y} = \frac{1}{\eta} \frac{\partial^2 w}{\partial y^2} \quad (12)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{\delta t}) \frac{\partial \theta}{\partial y} = \frac{1}{\Gamma} \frac{\partial^2 \theta}{\partial y^2} \quad (13)$$

where $\eta = \frac{\mu J^*}{\gamma} = \frac{2}{2 + \beta}$; $\Gamma = \left(1 - \frac{4}{3R + 4}\right) Pr$

The relevant boundary conditions in dimensionless form are:

$$\begin{aligned} \text{at } y=0 \quad u &= U_p, \quad \theta = 1 + \varepsilon^{\delta t}, \quad w = -n \frac{\partial u}{\partial y} \\ \text{as } y \rightarrow \infty \quad u &\rightarrow U_\infty = (1 + \varepsilon A e^{\delta t}), \quad \theta \rightarrow 0, \quad w \rightarrow 0 \end{aligned} \quad (14)$$

4. Solution of the problem

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we perform an asymptotic analysis by representing the linear velocity, micro rotation, and temperature as

$$u = u_0(y) + \varepsilon e^{\delta t} u_1(y) + O(\varepsilon^2), \quad w = w_0(y) + \varepsilon e^{\delta t} w_1(y) + O(\varepsilon^2) \quad \& \quad \theta = \theta_0(y) + \varepsilon e^{\delta t} \theta_1(y) + O(\varepsilon^2) \quad (15)$$

Substituting equation (15) into equations (11)-(13), neglecting the terms of $O(\varepsilon^2)$, we obtain the following pairs of equations for (u_0, w_0, θ_0) and (u_1, w_1, θ_1) .

$$(1 + \beta) u_0'' + u_0' - M u_0 = -Gr\theta_0 - 2\beta w_0' - M \quad (16)$$

$$(1 + \beta) u_1'' + u_1' - (M + \delta) u_1 = -\delta - Gr\theta_1 - 2\beta w_1' - A u_0' - M \quad (17)$$

$$w_0'' + \eta w_0' = 0 \quad (18)$$

$$w_1'' + \eta w_1' - \delta \eta w_1 = -A \eta w_0' \quad (19)$$

$$\theta_0'' + \Gamma \theta_0' = 0 \quad (20)$$

$$\theta_1'' + \Gamma \theta_1' - \delta \Gamma \theta_1 = -A \Gamma \theta_0' \quad (21)$$

Here the primes denote differentiation with respect to y . The corresponding boundary conditions can be written as follows.

$$\begin{aligned} \text{at } y=0 \quad u_0 &= U_p, u_1 = 0, \quad \theta_0 = 1, \theta_1 = 1, \quad w_0 = -n u_0', w_1 = -n u_1' \\ \text{as } y \rightarrow \infty, u_0 &= 1, u_1 = 1, w_0 \rightarrow 0, \quad w_1 \rightarrow 0, \quad \theta_0 \rightarrow 0, \quad \theta_1 \rightarrow 0, \end{aligned} \quad (22)$$

The solution of Equations (14)-(21) satisfying boundary conditions (22) and is given by

$$u_0(y) = 1 + R_9 e^{-h_3 y} + R_6 e^{-\Gamma y} + R_7 e^{-Scy} + R_8 e^{-\eta y} \tag{23}$$

$$u_0(y) = 1 + R_{17} e^{-h_2 y} + R_{10} e^{-h_4 y} + R_{11} e^{-h_3 y} + R_{12} e^{-h_4 y} + R_{13} e^{-h_5 y} + R_{14} e^{-\Gamma y} + R_{15} e^{-Scy} + R_{16} e^{-\eta y} \tag{24}$$

$$w_0(y) = C_{11} e^{-\eta y} \tag{25}$$

$$w_1(y) = C_{12} e^{-h_4 y} - C_{11} \left(\frac{A\eta}{\delta} \right) e^{-\eta y} \tag{26}$$

$$\theta_0(y) = e^{-\Gamma y} \tag{27}$$

$$\theta_1(y) = \left(1 + \frac{A\Gamma}{\delta} \right) e^{-h_4 y} - \frac{A\Gamma}{\delta} e^{-\Gamma y} \tag{28}$$

In view of the above solution, the velocity, micro rotation, temperature and concentration distributions in the boundary layer become

$$u(y,t) = 1 + R_9 e^{-h_3 y} + R_6 e^{-\Gamma y} + R_7 e^{-Scy} + R_8 e^{-\eta y} + \varepsilon e^{\delta t} \left[1 + R_{17} e^{-h_2 y} + R_{10} e^{-h_4 y} + R_{11} e^{-h_3 y} + R_{12} e^{-h_4 y} + R_{13} e^{-h_5 y} + R_{14} e^{-\Gamma y} + R_{15} e^{-Scy} + R_{16} e^{-\eta y} \right] \tag{29}$$

$$w(y,t) = C_{11} e^{-\eta y} + \varepsilon e^{\delta t} \left[C_{12} e^{-h_4 y} - C_{11} \left(\frac{A\eta}{\delta} \right) e^{-\eta y} \right] \tag{30}$$

$$\theta(y,t) = e^{-\Gamma y} + \varepsilon e^{\delta t} \left[\left(1 + \frac{A\Gamma}{\delta} \right) e^{-h_4 y} - \frac{A\Gamma}{\delta} e^{-\Gamma y} \right] \tag{31}$$

Knowing the velocity field in the boundary layer, we can calculate the Skin friction coefficient at the plate, which in the non-dimensional form is given by

$$C_f = \frac{\tau_w}{\rho U_0 V_0}, \text{ where } \tau_w = \left[\mu \frac{\partial u^*}{\partial y^*} \right]_{y^*=0} = \left[\frac{\partial u}{\partial y} \right]_{y=0} = \left[\frac{\partial u_0}{\partial y} + \varepsilon e^{\delta t} \frac{\partial u_1}{\partial y} \right]_{y=0} = - \left[R_9 h_2 + R_6 \Gamma + R_7 Sc + R_8 \eta \right] - \varepsilon e^{\delta t} \left[R_{17} h_2 + R_{10} h_1 + R_{11} h_3 + R_{12} h_4 + R_{13} h_5 + R_{14} \Gamma + R_{15} Sc + R_{16} \eta \right] \tag{32}$$

Knowing the micro rotation field in the boundary layer, we can calculate the couple stress coefficient at the plate, which in the non-dimensional form is given by

$$C_m = \frac{\mu_w}{\mu j U_0}, \text{ where } \mu_w = \left[\gamma \frac{\partial w^*}{\partial y^*} \right]_{y^*=0} = \left(1 + \frac{1}{2} \beta \right) \left[\frac{\partial w_0}{\partial y} + \varepsilon e^{\delta t} \frac{\partial w_1}{\partial y} \right]_{y=0} = \left(1 + \frac{1}{2} \beta \right) \left[-C_{11} \eta + \varepsilon e^{\delta t} \left\{ -C_{12} \eta + C_{11} \left(\frac{A\eta^2}{\delta} \right) \right\} \right] \tag{33}$$

Knowing the temperature field in the boundary layer we can calculate the heat transfer coefficient at the plate which in non-dimensional form in terms of the Nusselt number is given by

$$Nu_x = -\theta^1(0) = \left[\frac{\partial \theta_0}{\partial y} + \varepsilon e^{\delta t} \frac{\partial \theta_1}{\partial y} \right]_{y=0} = \Gamma + \varepsilon e^{\delta t} \left\{ h_4 \left(1 + \frac{A\Gamma}{\delta} \right) - \frac{A\Gamma^2}{\delta} \right\} \quad (34)$$

5. Results and discussions

In order to know the physical behavior of various parameters of the flow quantities derived in equations (29)-(33), the numerical study has been carried out and the effects of various parameters are presented through graphs in Figures 1-9. For various values of the magnetic field parameter M the velocity profiles are plotted in Figure 1. It is obvious that the existence of the magnetic field decreases the velocity. This is due to application of the transverse magnetic field, which acts as a drag force known as Lorentz force. Thus the existence of the magnetic field reduces the momentum boundary layer. It is observed in Figure 2, as expected that an increase in the radiation parameter R results a decrease in velocity within the boundary layer. This is because the large R values correspond to an increase dominance of conduction over radiation than by decreasing buoyancy force (thus vertical velocity). The effect of viscosity ratio β on the stream velocity is presented in Figure 3. From the numerical results we deduce that the velocity lower for a Newtonian fluid ($\beta = 0$) for the same flow conditions and the fluid properties, as compared with a micro polar fluid. When the viscosity ratio less than 1. When the viscosity ratio β takes values greater than 1 i.e., the gyro viscosity is larger than the translational viscosity. However, the velocity distribution shows a decelerating nature near the porous plate. Figure 4 shows the angular velocity curves for different values of magnetic parameter M . From this it is observed that the angular velocity increase with the increasing M values. In Figure 5 the angular velocity decreases as viscosity ratio β increases from 0 to 1 and suddenly it increases as for viscosity ratio increase from 1 to 2 and then decreases further for increasing values of β . Typical variation in the temperature profiles along the span wise coordinate are shown in Figure 6 for different values of the Prandtl number Pr .

As expected the numerical results show that an increase in the Prandtl number results in a decrease of the boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr . Hence in the case of smaller Prandtl numbers the thermal boundary layer is thicker and the rate of heat transfer is reduced. For different values of radiation parameter R the temperature profiles are plotted in Figure 7. It is obvious that an increase in the radiation parameter R results in the decreasing temperature within the boundary layer, as well as the decreased thickness of the temperature boundary layers. In Figure 8 the variation of surface Skin-friction (Sf) with plate moving velocity for different values of radiation parameter R has been shown. Numerical results show that for given flow conditions and fluid properties, which are listed in the figure, the effect of increasing the plate moving velocity and the radiation parameter manifest in a linearly decreasing surface skin-friction on the porous plate. Figure 9 shows that the variation of surface heat transfer (Nusselt number Nu) with suction velocity parameter A for different values of Prandtl number Pr . Numerical results show that for given flow and material parameters which are listed in the figure. The surface heat transfer from the porous plate tends to increase, with the increase on Prandtl number.

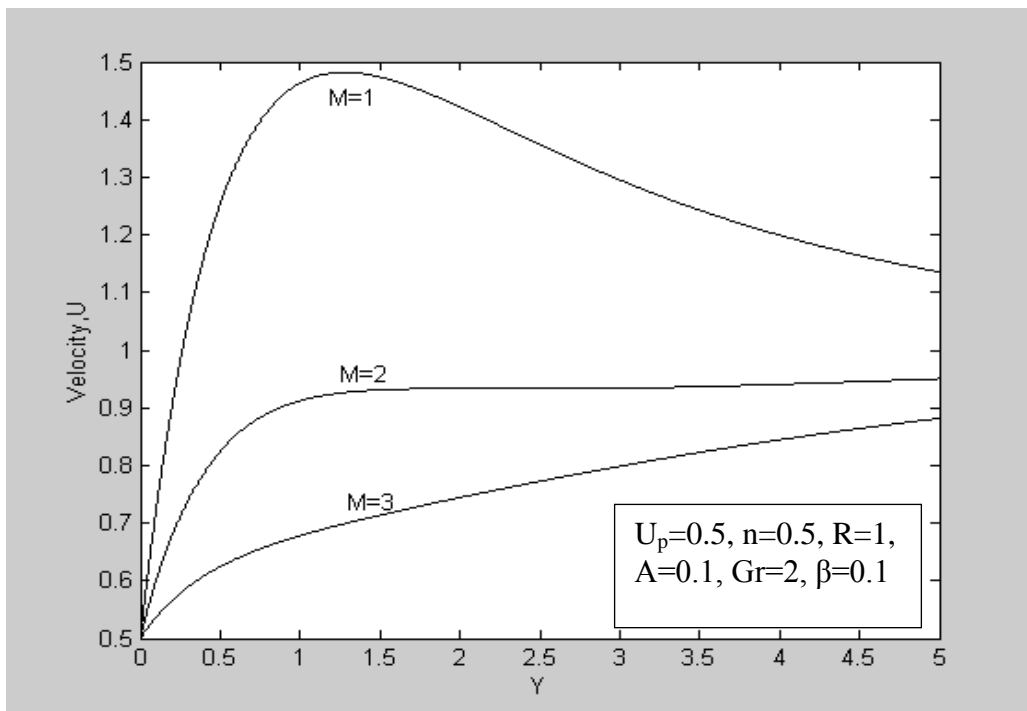


Figure 1. Velocity profiles against span wise coordinate y for different values of Magnetic parameter M

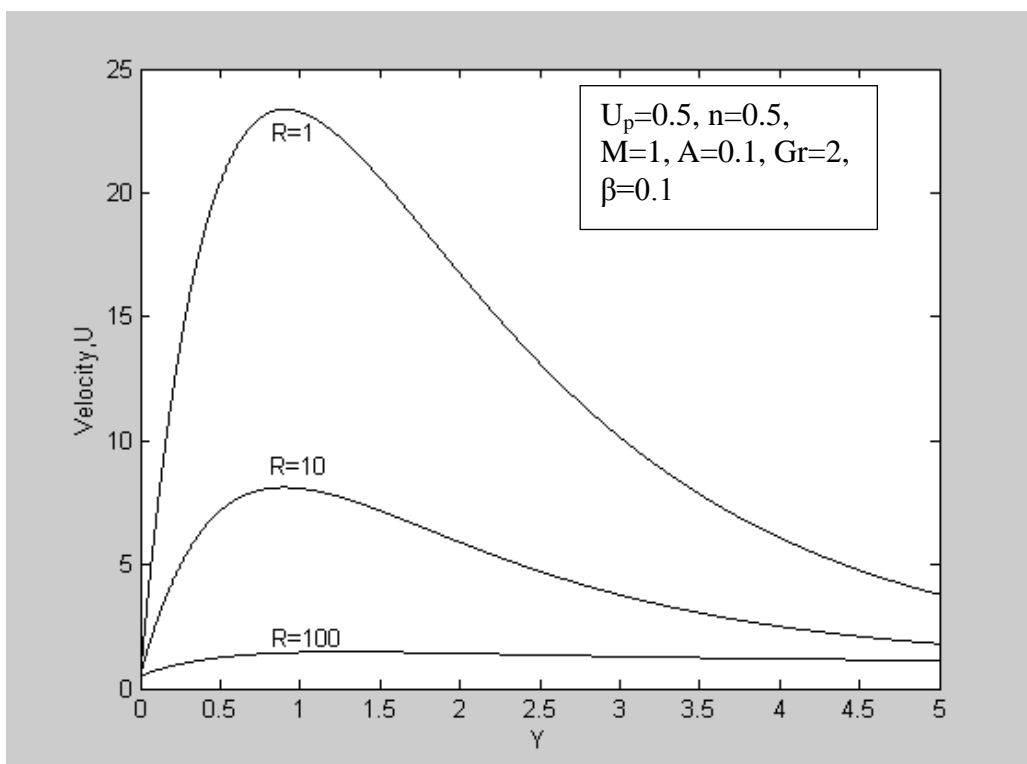


Figure 2. Velocity profiles against span wise coordinate y for different values of Radiation parameter R

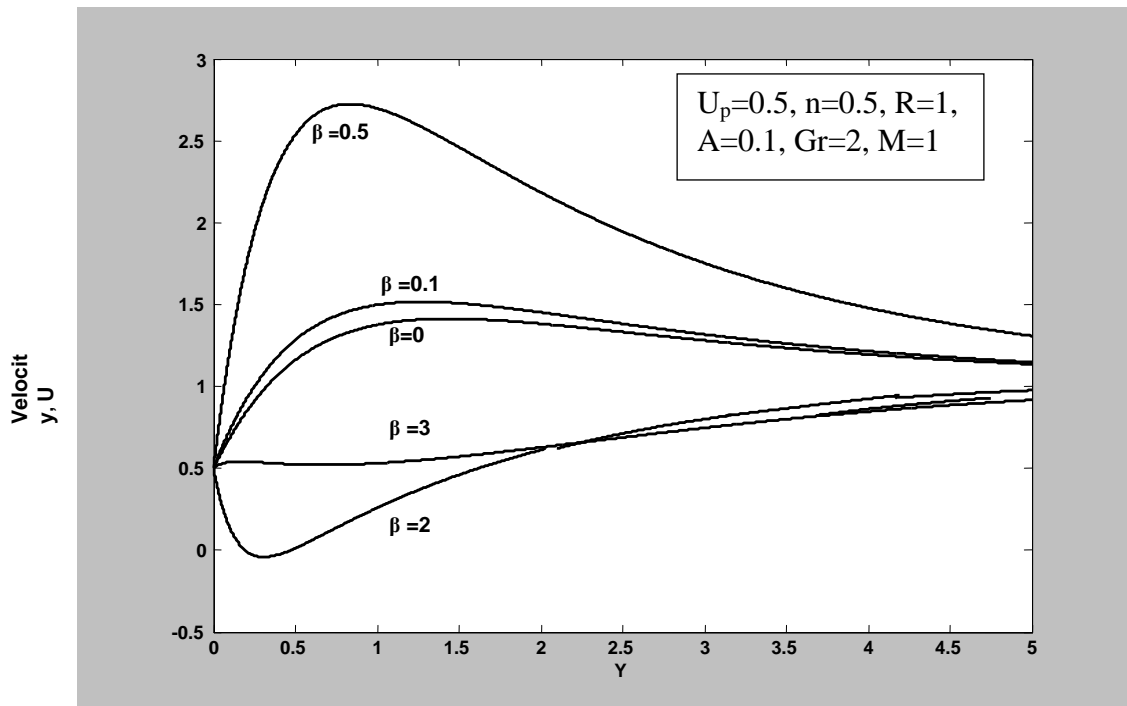


Figure 3. Velocity profiles against span wise coordinate y for different values of Viscosity ratio β

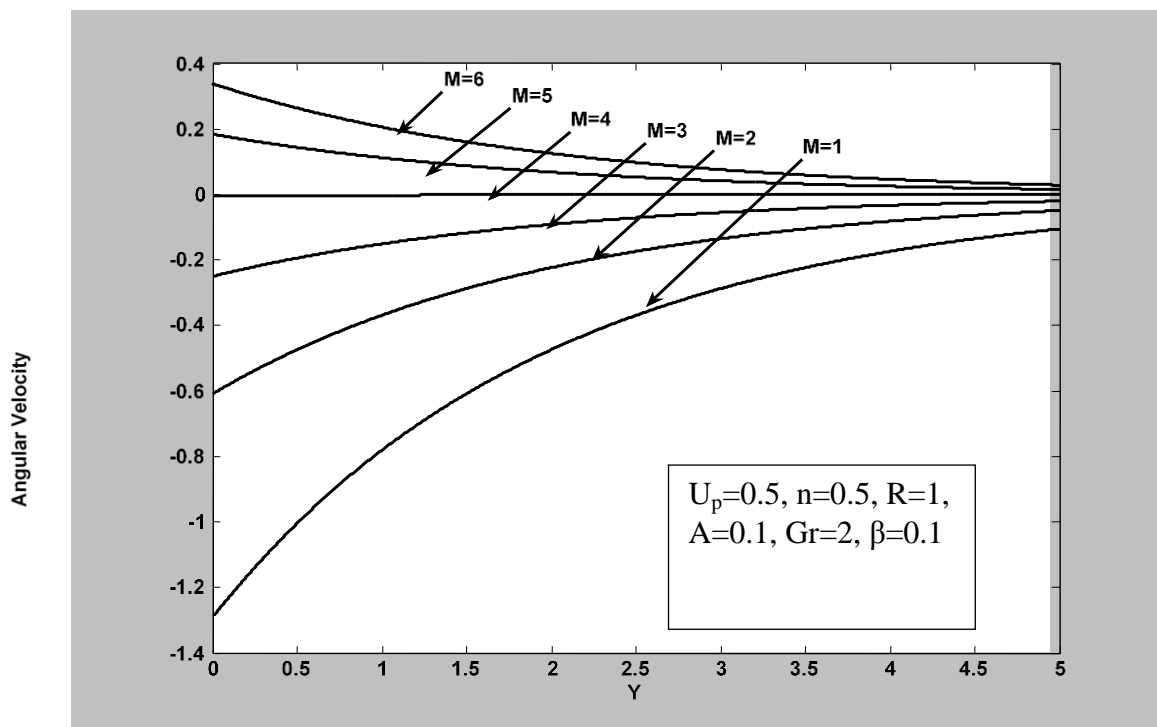


Figure 4. Angular velocity profiles against span wise coordinate y for different values of magnetic parameter M

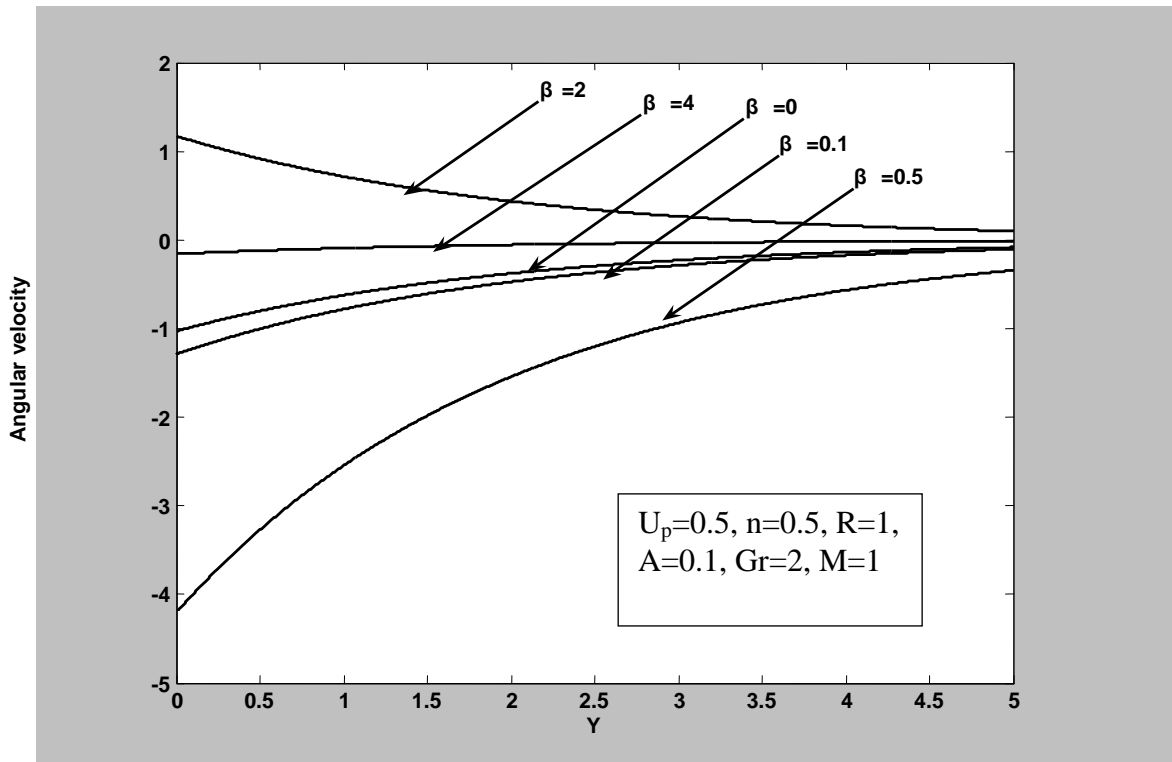


Figure 5. Angular Velocity profiles against span wise coordinate y for different values of Viscosity ratio β

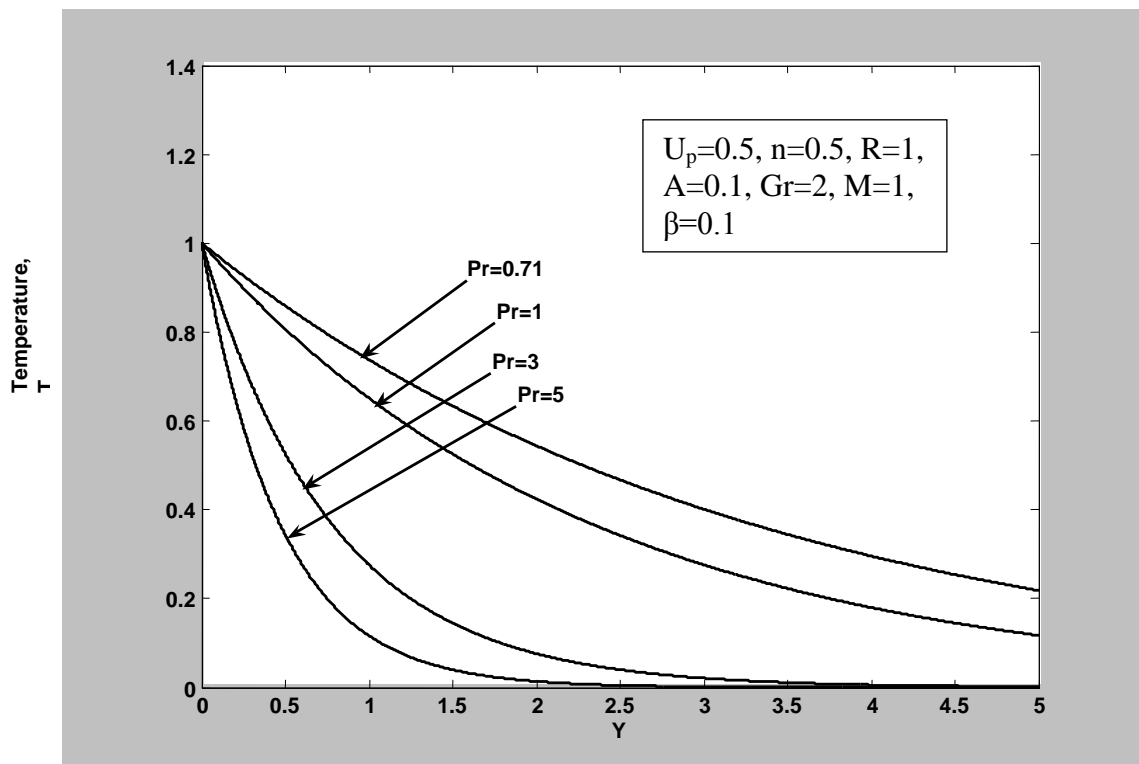


Figure 6. Temperature profiles against span wise coordinate y for different values of Prandtl number

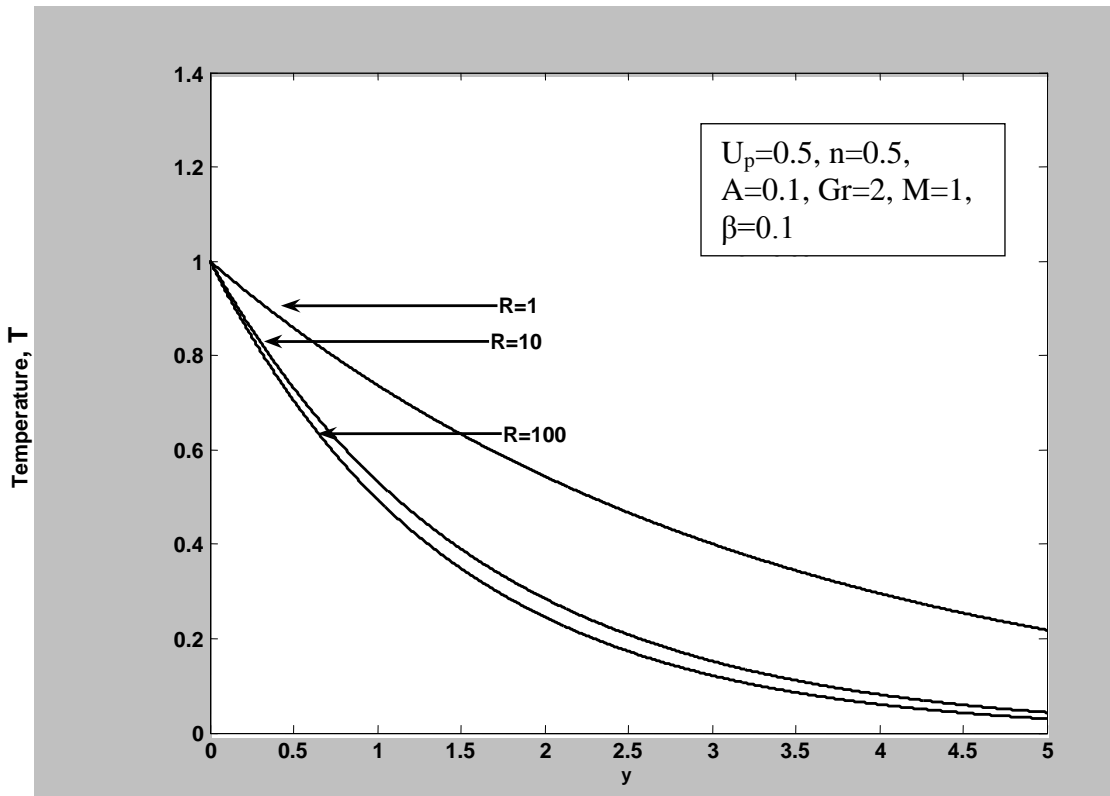


Figure 7. Temperature profiles against span wise coordinate y for different values of radiation parameter R

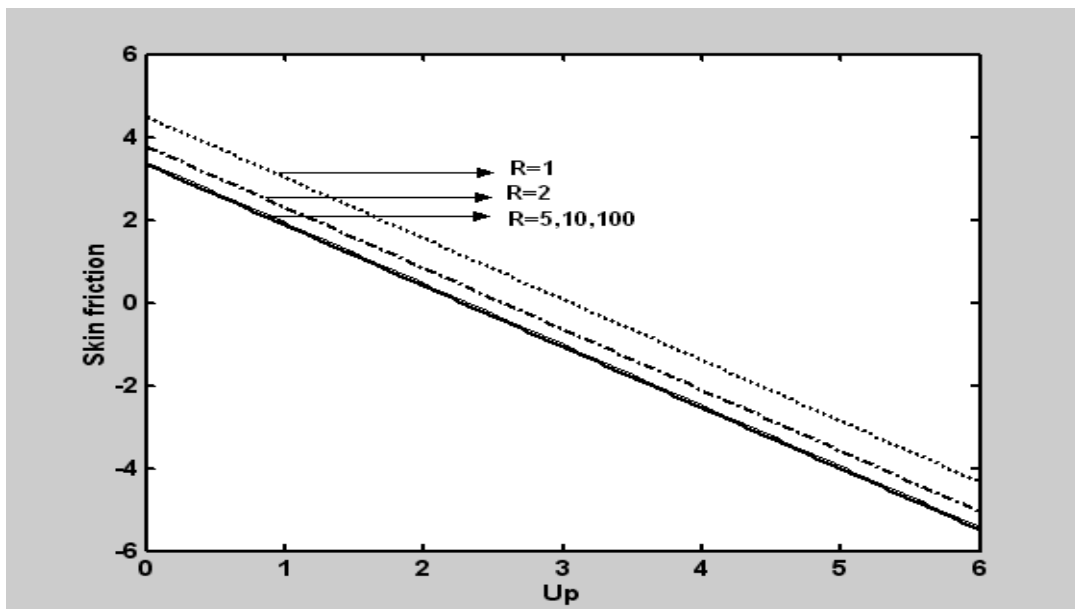


Figure 8. Skin-friction with plate moving velocity component U_p for various values of radiation parameter R

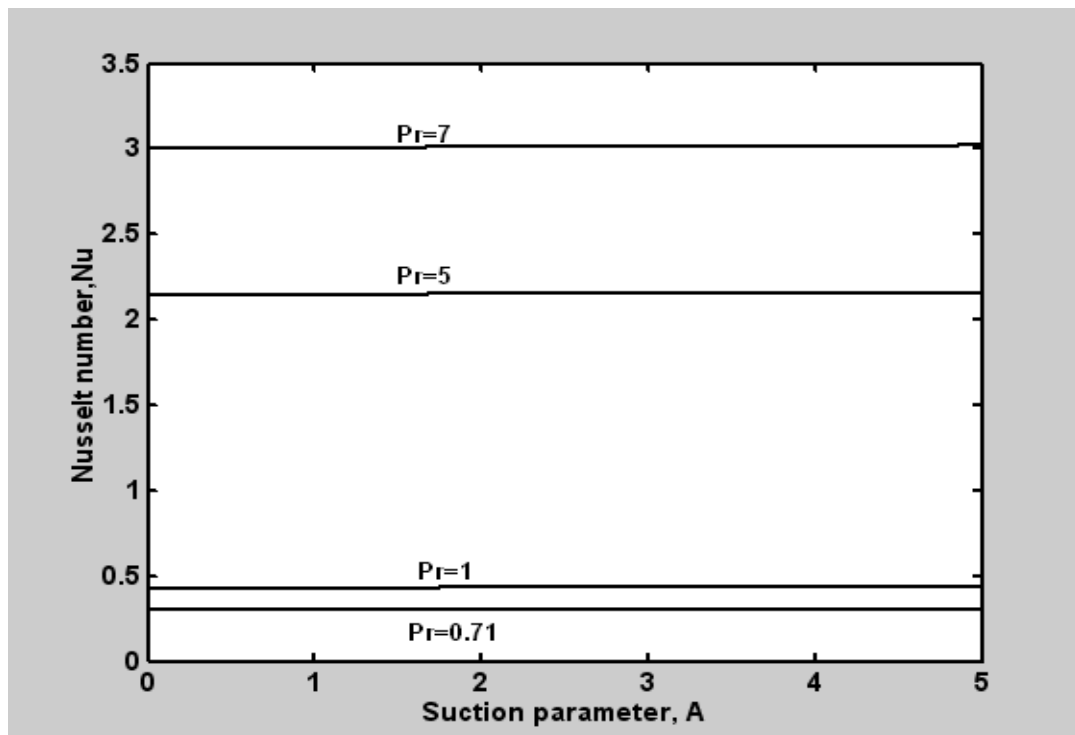


Figure 9. Surface heat transfer with suction parameter A effect for various values of prandtl number Pr

6. Conclusion

The this paper we have addressed the analytical solutions for a mixed convection flow problem and studied numerically the effect of the thermal radiation on magneto hydro dynamic mixed convection flow of a micro polar fluid past an infinite, steadily moving porous plate with variable suction. The dimensionless equations of continuity, linear momentum, angular momentum, and energy, which govern the flow, are solved by using a regular perturbation method. The behavior of various physical parameters on the velocity, micro rotation, temperature, Skin-friction, and Nusselt number has been discussed. In this analysis the following concluding remarks are made;

1. The existence of the magnetic field and as well as radiation parameter decrease the velocity
2. Angular velocity increases as the magnetic parameter increases.
3. An increase in the radiation parameter results a decrease in temperature within the boundary layer.
4. The effect of increasing the plate moving velocity and the radiation parameter manifest in a linearly decreasing surface Skin friction on the porous plate, the surface heat transfer from the porous plate tends to increase, with the increase on Prandtl number.

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