International Journal of Applied Science and Engineering
2015. 13, 3: 233-245

Determination of Probability Distribution Function for Modelling Global Solar Radiation: Case Study of Ibadan, Nigeria

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Abstract: In this paper, the most probable probability distribution for modelling the global solar radiation of Ibadan is determined. The data used for the analysis consists of daily average global solar radiation collected from International Institute of Tropical Agriculture (IITA) located in Ibadan. The data span over the periods of nine years (2000-2008). Various distribution functions are tested and most suitable one is determined using four different goodness of fit test. The parameters of the best fitted distribution are calculated and the variations in the characteristic of the global solar radiation are clearly shown. Some of the key results show that logistic distribution present the best probability distribution for modelling global solar radiation of Ibadan with Root Mean Square Error (RMSE) of 0.399 MJ/m^2/day, Mean Absolute Error (MAE) of 0.214 MJ/m^2/day, Mean Absolute Percentage Error (MAPE) of 3.26% and coefficient of determination (R^2) of 0.989. The location and the scale parameter for the distribution varies over a wide range from season to season and year to year: The location parameter varies from 8.25 MJ/m^2/day in August 2001 to 18.58 MJ/m^2/day in March 2001 and the scale parameter ranges from 0.61 in February 2003 to 4.19 in July 2007. The month of August present the lowest mean global solar radiation in most of the years. This paper is useful as first-hand information in the prediction of future global solar irradiation for Ibadan having known the past behavior and for fixing the missing data.

Keywords: Empirical models; probabilistic distribution function; global solar radiation; logistic distribution; Ibadan; Nigeria.

1. Introduction

The knowledge of available solar radiation data of a location is a fundamental requirement before embarking on any solar energy project such as photovoltaics farm, solar thermal systems, and passive solar design [1]. Therefore, the data should be dependable and readily available for design analysis, optimization and performance evaluation of solar technologies. Unfortunately, many of the developing countries do not have the facilities for continuous and accurate measurements of solar radiation because of the expensive measuring equipment and techniques required. It is therefore necessary to develop alternative methods to estimate the solar radiation of a potential location.

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Received 2 September 2014
Revised 9 June 2015
Accepted 9 October 2015

In the past, various models has been developed to model solar global radiation from other climatological data that are relatively easy to measured such as the maximum and minimum temperature, sunshine hour, relative humidity, atmospheric pressure etc. Some probability distribution functions have also been used to model global solar radiation: frequency distribution of global solar radiation of Taiwan was investigated by Chang [2], he concluded that lognormal distribution performs best in modelling the solar global radiation of Taiwan. In the same manner, normal distribution was found appropriate for Beer Sheva, Israel [3]. However, Arthur *et al* reported that different probability distribution may be used to model different month of the year judging from the case of Kumasi, Ghana [4]. Other probability distributions that have been reported for modelling global solar radiation in the literature include: Boltzmann distribution function [5, 6], Gamma distribution function [7], Beta distribution function [8] and Dirichlet distribution function [9]. In Nigeria, some authors have modelled global solar radiation using different empirical models for some parts of the country. For example, factorial design has been used in the modelling of global solar radiation in eight selected location in Nigeria [10], the global solar radiation in some Northern area of Nigeria has also been modelled using Angstrom model[11, 12]. Other empirical models that have been used for modeling global solar radiation in Nigeria include: Page model [13], Garcia Model and Hargreaves Model [14-17]. Although, the solar radiations for some part of Nigeria have been modelled using different empirical models, but, none has modelled the global solar radiation using probability density function to the best knowledge of the author. Knowing the probability distribution function and the parameters of the distribution of any time series data (such as solar radiation data, wind speed data etc.) of a location allows someone to be able to generate data that will have the same characteristic as the actual data of the location in the future. This is important as it can serves as the starting point for design analysis of any solar power project. In this study, the most probable probability distribution function that best model the solar radiation data of Ibadan is determined using statistical goodness of fit. The parameters of the distribution are determined and the variation in the solar radiation of Ibadan is presented.

2. Area under Study and the Data Used

The location of the study area is Ibadan and it is situated in the South West of Nigeria on latitude 7.4^0N and longitude 3.5^0S. It is a tropical region with population of about 3.8million people. The inability of the electricity utility company to meet up with the required electricity demand and the availability of good solar radiation throughout the years make the city of Ibadan a good candidate for various solar power projects. The solar radiation data used for the analysis was measured over a period of nine years (2000-2008) and was collected from the International Institute of Tropical Agriculture (IITA) located in Ibadan. The data for these periods were characterized by very few missing data; hence, it is believed that the pyrometer used for capturing the solar irradiation was good and dependable during these periods. The long period of observed data will allow the understanding of how solar radiation varies from month to month, season to season and year to year.

3. Materials and Methods

It is impractical to report all statistical distribution functions as they are numerous to be accommodated in the paper. However, several statistical distribution functions were tested and the best seven among them were reported. They are: gamma, generalized extreme value, Weibull, extreme value, logistic, normal and lognormal distributions. The other statistical distribution functions that are not listed are far from fitting the data.

3.1 Gamma Distribution

The probability density function of a Gamma distribution is given by equation (1)

$$f_{GM} = \frac{x^{a-1}}{b^{a}\Gamma(a)} exp\left[-\left(\frac{x}{b}\right)\right] \quad a,b>0 \tag{1}$$

where x is the measured solar radiation data, a is the scale parameter while b is the shape parameter of the distribution. The cumulative distribution function is defined as

$$F_{GM} = \frac{1}{b^{a}\Gamma(a)}\int_{0}^{x} t^{a-1} exp\left[-\frac{t}{b}\right]dt \tag{2}$$

where $\Gamma(.)$ is the gamma function.

3.2 Extreme Value Distribution

Extreme value distribution is also known as Gumbel distribution. It has the probability density function $f_{ex}(x)$ and the cumulative distribution function $F_{ex}(x)$ given by equation (3) and (4) respectively.

$$f_{ex}(x,\lambda,\beta) = \frac{1}{\beta} e^{-\left(\frac{x-\lambda}{\beta}\right)} e^{-e^{-\left(\frac{x-\lambda}{\beta}\right)}} \tag{3}$$

$$F_{ex}(x,\lambda,\beta) = e^{-e^{-\left(\frac{x-\lambda}{\beta}\right)}} \tag{4}$$

where λ is the location parameter and β is the scale parameter. The parameters can be determined using the method of moment as given by (5) and (6)

$$\lambda = E(x) - 0.5772\beta \tag{5}$$

$$\beta = \frac{\sqrt{6.var(x)}}{\pi} \tag{6}$$

The mean, $E(x)$, and the variance, $var(x)$ of the global solar radiation, x are calculated as (7) and (8) respectively

$$E(x) = \frac{1}{n}\left[\sum_{i=1}^{n} x\right] \tag{7}$$

$$var(x) = \frac{1}{n}\left[\sum_{i=1}^{n}\left(x - E(x)\right)^{2}\right] \tag{8}$$

where n is the number of observations.

3.3 Generalized Extreme Value Distribution

The generalized extreme value is a family of continuous probability distributions developed within value theory to combine the Gumbel, Frechet and Weibull families. The probability density function for the generalized extreme value distribution with location parameter α, scale parameter ξ, and shape parameter $\eta \neq 0$ is given by (9) [18, 19]

$$f_{ge}(x,\eta \neq 0) = \left(\frac{1}{\xi}\right) exp\left(-\left(1+\eta\frac{(x-\alpha)}{\xi}\right)^{-\frac{1}{\eta}}\right)\left(1+\eta\frac{(x-\alpha)}{\xi}\right)^{-1-\frac{1}{\eta}} \tag{9}$$

For

$$1+\eta\frac{(x-\alpha)}{\xi} > 0$$

$\eta > 0$ is the type II case and it correspond to distribution whose tail decreases as a polynomial. $\eta < 0$ is the type III case and it correspond to distribution whose tail are finite.

When, $\eta = 0$, then it is a the type I case and its density function can be written as (10). It correspond to distribution whose tail decreases exponentially

$$f_{ge}(x,\eta = 0) = \left(\frac{1}{\xi}\right) exp\left(-exp\left(\frac{(x-\alpha)}{\xi}\right) - \frac{(x-\alpha)}{\xi}\right) \tag{10}$$

Type I, II and III are sometimes refer to Gumbel, Frechet and Weibull types respectively. The cumulative distribution function for $\eta \neq 0$ and $\eta = 0$ can be expressed as (11) and (12) respectively

$$F_{ge}(x,\eta \neq 0) = exp\left(-\left(1+\eta\frac{(x-\alpha)}{\xi}\right)^{-\frac{1}{\eta}}\right) \tag{11}$$

$$F_{ge}(x,\eta = 0) = exp\left(-exp\left(\frac{(x-\alpha)}{\xi}\right)\right) \tag{12}$$

3.4 Weibull Distribution

The Weibull probability density function (pdf) and its cumulative distribution function can be expressed as (13) and (14) respectively

$$f_W(T_I) = \frac{k}{c}\left(\frac{x}{c}\right)^{k-1} exp\left[-\left(\frac{x}{c}\right)^k\right] \tag{13}$$

$$F_W\left(x\right)=1-exp\left[-\left(\frac{x}{c}\right)^k\right] \tag{14}$$

where $f_W(x)$ is the Weibull probability distribution of solar radiation, x. Weibull shape and scale parameters are denoted by k and c respectively. Weibull k and c can be determined using maximum likelihood method [20, 21] as given by (15) and (16) respectively.

$$k=\left[\frac{\sum_{i=1}^{n}T_I^k ln\left(x\right)}{\sum_{i=1}^{n}x^k}-\frac{\sum_{i=1}^{n}ln\left(x\right)}{n}\right]^{-1} \tag{15}$$

$$c=\left[\frac{1}{n}\sum_{i=1}^{n}x^k\right]^{1/k} \tag{16}$$

3.5 Lognormal Distribution

Lognormal distribution probability density function and the cumulative distribution function are given by equations (17) and (18) respectively.

$$f_{LN(x)}=\frac{1}{x\omega\sqrt{2\pi}}exp\frac{-(lnx-\mu)^2}{2\omega^2} \tag{17}$$

$$F_{LN}\left(x\right)=\frac{1}{2}erfc\left[-\frac{lnx-\mu}{\omega\sqrt{2}}\right] \tag{18}$$

where $x\geq 0$ is global solar radiation, $\omega>0$ is the lognormal shape parameter, $\mu>0$ is the lognormal scale parameter, and erfc (.) is the complimentary error function. The ω and μ can be estimated using (19) and (20) respectively.

$$\omega=\sqrt{ln\left(1+\frac{var(x)}{E[x^2]}\right)} \tag{19}$$

$$\mu=ln\,E(x)-\frac{1}{2}ln\left(1+\frac{var(x)}{E(x^2)}\right) \tag{20}$$

3.6 Logistic Distribution

Logistic distribution has the shape of normal distribution but with heavier tail i.e. higher kurtosis. [22]. The PDF and the CDF of this distribution can be expressed as (21) and (22) respectively.

$$f_{LGS(v)} = \frac{exp\left(-\frac{x-\psi}{s}\right)}{s\left(1+exp\left(-\frac{(x-\psi)}{s}\right)\right)^2} \tag{21}$$

$$F_{LGS(v)} = \frac{1}{1+exp\left(-\frac{x-\psi}{s}\right)} \tag{22}$$

where ψ is the location parameter and s is the scale parameter of the distribution. Both parameters can be evaluated using equation (23) and (23) respectively.

$$\psi = E(x) \tag{23}$$

$$s = \sqrt[3]{\frac{var(x)}{\pi^2}} \tag{24}$$

3.7 Normal Distribution

The normal probability density function and the cumulative distribution function can be expressed as (25) and (26) respectively.

$$f_N(T_I) = \frac{1}{\sqrt{2\pi\sigma}} exp\left[\frac{-(T_I-\mu)^2}{2\sigma^2}\right] \tag{25}$$

$$F_N(T_I) = \frac{1}{2}\left[1+erf\left(\frac{T_I-\mu}{\sqrt{2\sigma}}\right)\right] \tag{26}$$

The parameters μ and σ are as given in (7) and (8), respectively.

4. Performance Evaluation of the Distribution Functions

Four different statistical goodness of fits were used to evaluate the distribution function that best fit the global solar radiation of Ibadan. They are: Mean Absolute Error (MAE) in ($MJm^{-2}day^{-1}$), Root Mean Square Error (RMSE) in ($MJm^{-2}day^{-1}$), Mean Absolute Percentage Error (MAPE) in (%) and Coefficient of Determination (R^2). The criteria can be evaluated as given by equations (27)-(30) respectively. The lower the value of RMSE, MAE, MAPE, the better the goodness of fit. Similarly, R^2 is simply the square of correlation coefficient. It is used to determine to what extent a prediction can be made from a model. The relationship between the variables is determined as $0 \le R^2 \le 1$ with 1 being the perfect fit. The closer the value of R^2 to 1, the better the fit to the actual variables.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_a(i)-x_p(i)\right)^2} \tag{27}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left(/ \, x_a(i) - x_p(i) / \right) \tag{28}$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\left(x_a(i) - x_p(i)\right)}{x_a(i)}\right| * 100\% \tag{29}$$

$$R^2 = \left(\frac{\sum_{i=1}^{n}\left(\left[x_a(i) - E\left(x_a(i)\right)\right].\left[x_p(i) - E\left(x_p(i)\right)\right]\right)}{\sqrt{\left(\left[x_a(i) - E\left(x_a(i)\right)\right]^2.\left[x_p(i) - E\left(x_p(i)\right)\right]^2\right)}} \right)^2 \tag{30}$$

where $x_a(i)$ is the i^{th} measured global solar radition, $x_p(i)$ is the i^{th} predicted global solar radiation, n is the number of observed global solar radiation.

5. Result and Discussion

Figure 1 depicts the comparison in the monthly mean global solar radiation for each year from 2000-2008. The figures revealed that the month of August present the least global solar radiation in most of the years under study followed by the month of July and September. The average values of global solar radiation for these months as furnished in Table 1 are 10.7, 12.0 and 12.9 MJ/m^2/day, respectively with the standard deviation of 3.5, 3.3 and 3.3 MJ/m^2/day. This is expected as these months fall within the wet season of the year when it is always cloudy and dull as a result of rainfall. The highest value of global solar radiation occurs in the month of March with mean value of 16.8 MJ/m^2/day and standard deviation of 2.67 MJ/m^2/day follows by the month of April with the average value of 16.6 MJ/m^2/day and standard deviation of 3.7 MJ/m^2/day.

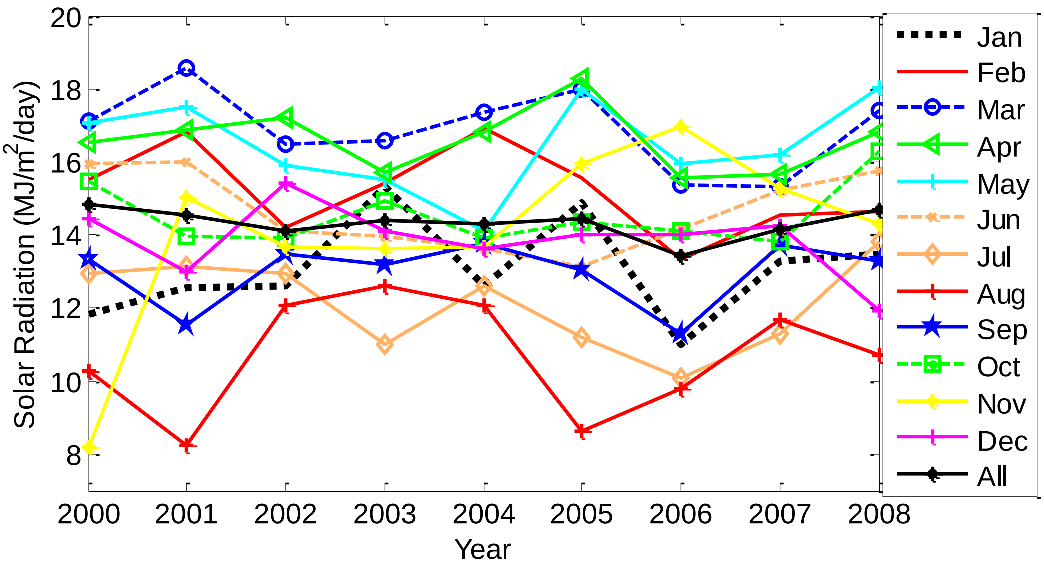

Figure 1. Comparison of the monthly average global solar radiation for each of the year (2000-2008)

Table 1. Average Value of Global Solar Radiation Over Nine Years Period

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean	13.4	15.1	16.8	16.5	15.9	14.5	12	10.7	12.9	14.3	14.9	13.7
std	2.4	2.2	2.67	3.7	3.5	3.3	3.3	3.5	3.3	3.9	2.2	2.8

In order to determine the most probable statistical distribution that can accurately model the global solar radiation of Ibadan, various statistical distributions were tested. However, only seven of the tested distributions that best approximate the global solar radiation data are presented in this paper due to lack of space and the results are presented in Table 2. It can be observed from the table that logistic distribution present the best statistical goodness of fit in modelling the global solar radiation data of Ibadan with RMSE of 0.399 MJ/m^2/day, MAE of 0.214 MJ/m^2/day, MAPE of 3.26% and R^2 of 0.989. Logistic distribution is competitively followed by normal distribution with RMSE of 0.475 MJ/m^2/day, MAE of 0.346 MJ/m^2/day, MAPE of 5.88% and R^2 of 0.981. The statistical distribution functions are fitted to the actual global solar radiation data as shown in Figure 2. It can be observed from the figure that logistic distribution closely matches with the measured data followed by the normal distribution. This confirms the accuracy of the statistical test.

The location (ψ) and the scale (s) parameters of logistic distribution (most suitable distribution) are determined for various months of the year from 2000- 2008, the result is furnished in Table 3. From the table, it can be observed that both the location and the scale parameter vary over a wide range of values indicating the wide variations in the global solar radiation of Ibadan. The location parameter varies from 8.25 MJ/m^2/day in August 2001 to 18.58 MJ/m^2/day in March 2001 and the scale parameter ranges from 0.61 in February 2003 to 4.19 in July 2007. The monthly variation in the probability distribution of the global solar radiation over the period of nine years is depicted in Figure 3. It can be inferred from the figure that many years of observed data are required to make a reasonable conclusion on the implementation of any solar power project as a result of wide variation in solar characteristic.

Table 2. Performance Evaluation of Statistical Distribution for Modelling Global Solar Radiation in Ibadan

Statistical Distributions	Performance Evaluation using statistical goodness of fit				
	RMSE (MJ/m^2/day)	MAE (MJ/m^2/day)	MAPE (%)	R^2	Order of good fit
Logistic	0.399	0.214	3.26	0.989	1st
Weibull	0.522	0.382	6.10	0.979	3rd
Lognormal	0.681	0.510	9.35	0.949	7th
Gamma	0.583	0.429	7.36	0.970	5th
Extreme Value	0.619	0.452	7.99	0.966	6th
GEV	0.548	0.422	6.89	0.974	4th
Normal	0.475	0.346	5.88	0.981	2nd

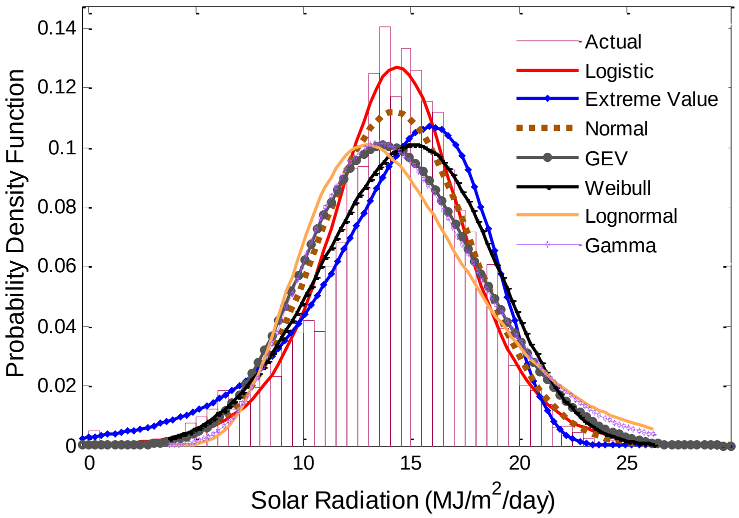

Figure 2. Fitting of Probability distributions to the global solar radiation data of Ibadan

Table 3. Monthly Logistic location parameter ψ (MJ/m2/day), and scale parameters s (MJ/m2/day) for all the years

Month	para	2000	2001	2002	2003	2004	2005	2006	2007	2008	Whole
Jan	ψ	11.82	12.56	12.58	15.33	12.60	14.87	11.02	13.26	13.47	13.36
	s	1.17	0.68	0.80	1.42	0.97	0.70	1.74	0.72	1.07	1.33
Feb	ψ	15.51	16.80	14.19	15.40	16.91	15.56	13.33	14.56	14.66	15.18
	s	0.83	1.08	0.66	0.61	1.09	1.31	1.12	1.11	0.87	1.18
Mar	ψ	17.12	18.58	16.48	16.58	17.38	17.99	15.39	15.34	17.39	16.95
	s	1.15	1.13	1.48	0.84	0.73	1.38	1.64	1.82	1.34	1.41
Apr	ψ	16.53	16.89	17.19	15.70	16.82	18.29	15.56	15.65	16.84	16.74
	s	1.76	1.72	1.89	0.92	1.80	1.46	1.40	3.19	1.89	2.00
May	ψ	17.07	17.52	15.91	15.52	14.13	17.99	15.94	16.17	18.03	16.05
	s	1.98	1.45	1.41	0.73	0.74	1.38	1.81	1.73	2.21	1.81
Jun	ψ	15.96	16.00	14.09	13.96	13.62	13.15	14.15	15.21	15.75	14.53
	s	1.76	1.66	0.97	1.31	0.86	2.00	1.94	2.23	2.16	1.78
July	ψ	12.96	13.12	12.94	10.99	12.60	11.18	10.08	11.29	13.81	12.08
	s	2.36	2.19	0.90	1.82	0.97	1.69	1.81	1.71	2.35	1.91
Aug	ψ	10.27	8.25	12.08	12.62	12.06	8.64	9.77	11.67	10.71	10.76
	s	2.27	1.29	0.98	1.07	0.83	1.82	2.95	1.62	2.14	1.93
Sept	ψ	13.34	11.53	13.50	13.17	13.76	13.04	11.28	13.72	13.28	12.99
	s	1.69	2.04	0.86	1.59	1.21	1.57	2.35	2.65	1.93	1.86
Oct	ψ	15.47	13.97	13.91	14.92	13.93	14.36	14.09	13.83	16.30	14.59
	s	1.91	1.50	0.96	1.56	1.53	2.27	2.19	4.19	1.77	2.05
Nov	ψ	8.20	15.05	13.68	13.60	13.66	15.97	16.99	15.29	14.31	14.85
	s	0.69	0.91	0.58	0.53	0.47	1.54	1.13	1.23	1.55	1.26
Dec	ψ	14.45	12.97	15.40	14.10	13.61	13.99	14.00	14.23	11.94	13.86
	s	1.05	1.25	1.65	0.74	0.84	0.98	0.98	1.62	1.75	1.46
Year	ψ	14.85	14.52	14.09	14.40	14.30	14.47	13.43	14.15	14.71	14.34
	s	2.00	2.30	1.42	1.41	1.42	2.16	2.41	2.23	2.17	1.97

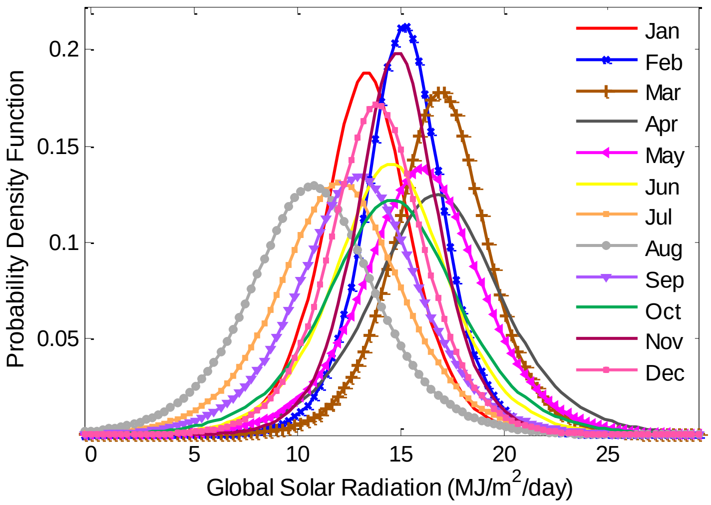

Figure 3. Plot showing monthly variation in global solar radiation of Ibadan

There are two main seasons in the country: the wet season which starts from the month of May to October with the peak in August and the dry season which span over the period of November to March with the peak in March. The yearly seasonal logistic parameters are computed and the results are displayed in Table 4. The table shows that ψ vary from 12.83 MJ/m^2/day in 2006 to 14.6 MJ/m^2/day in year 2000 while s vary from 1.61 in year 2003 to 2.67 in year 2001 in the dry season. In the case of dry season, ψ vary from 14.19 MJ/m^2/day in 2006 to 15.55 MJ/m^2/day in 2005 while s vary from 1.09 in 2003 to 2.04 in 2006. The dry season has higher value of location parameters compared to the wet season. This is because the observed values of solar radiation are higher during the dry season compared to the wet season. The variation in the seasonal distribution and yearly distribution are shown in Figures 4 and 5, respectively.

6. Conclusion

The most probable probability distribution function for modeling the global solar radiation of Ibadan, South West Nigeria has been determined and the parameters of the distribution calculated. From the study, it is concluded that logistic distribution is the most appropriate probability distribution function for modelling the global solar radiation of Ibadan with Root Mean Square Error (RMSE) of 0.399 MJ/m^2/day, Mean Absolute Error (MAE) of 0.214 MJ/m^2/day, Mean Absolute Percentage Error (MAPE) of 3.26% and coefficient of determination (R^2) of 0.989.

Table 4. Seasonal Logistic location parameter (ψ) (MJ/m^2/day), and scale parameters (s) (MJ/m^2/day) for each of the year (2000-2008)

		2000	2001	2002	2003	2004	2005	2006	2007	2008	whole
Wet	ψ	14.60	14.06	13.97	13.92	13.65	13.43	12.83	13.87	14.31	13.93
	s	2.36	2.67	1.48	1.61	1.38	2.51	2.61	2.74	2.47	2.25
Dry	ψ	15.13	15.00	14.24	14.93	15.22	15.55	14.19	14.47	15.02	14.81
	s	1.45	1.73	1.33	1.09	1.30	1.42	2.04	1.41	1.69	1.52

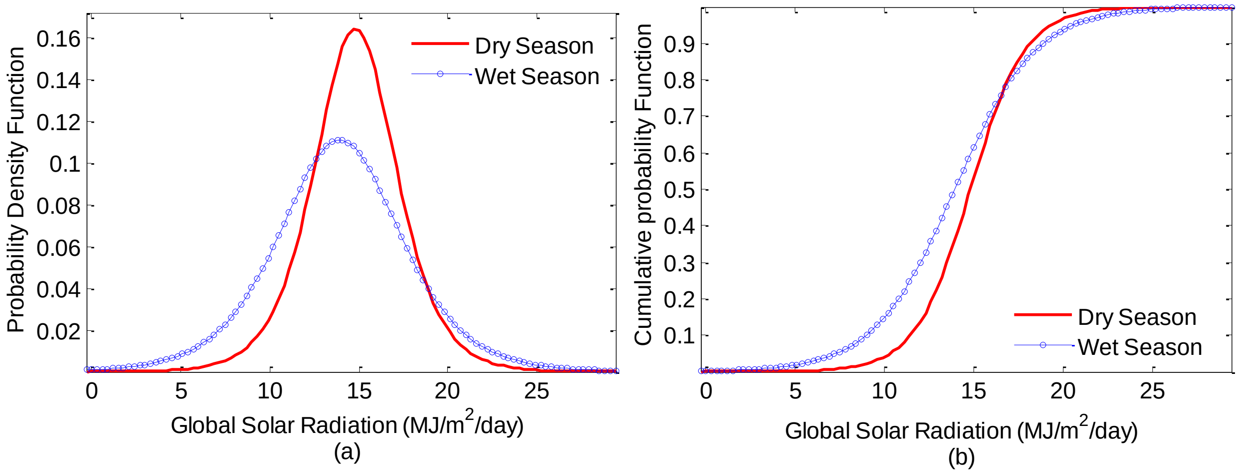

Figure 4. Seasonal variation in the logistic (a) probability density function (b) cumulative probability function of global solar radiation of Ibadan

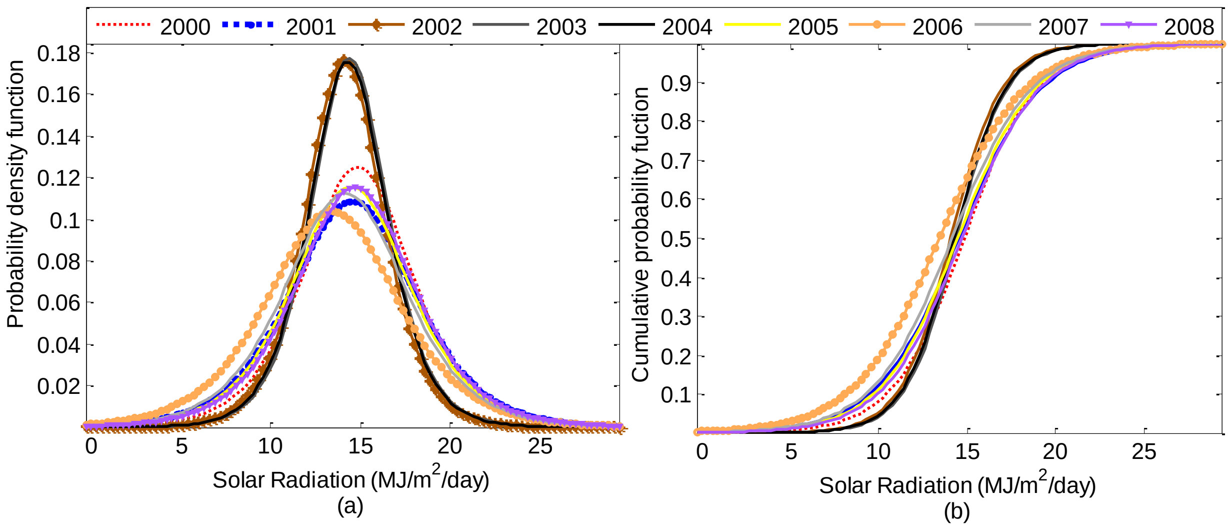

Figure 5. Yearly Variation in the logistic (a) probability density function (b) cumulative probability function of daily average global solar radiation for the city of Ibadan

Global solar radiation has the highest value in the month of March with the average value of 16.8 MJ/m^2/day and standard deviation of 2.67 MJ/m^2/day. The lowest value occurs in the month of August with the average value of 10.7 MJ/m^2/day and standard deviation of 10.7 MJ/m^2/day. The location parameter of the distribution (logistic distribution) over the period of 9 years is calculated to be 14.34 MJ/m^2/day while the scale parameter is determined to be 1.97. The results of this study is useful as a first-hand information to the engineers, investors, individuals and government policy makers who are interested in solar power project in Ibadan, Nigeria. Future research emanating from this study will predict the future global solar radiation of Ibadan using Logistic distribution

Acknowledgments

The author wants to thank International Institute of Tropical Agriculture (IITA) Ibadan for making the data used in the study available.

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