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An Adaptive Finite Element Framework for Fatigue Crack Propagation under Constant Amplitude Loading

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Abstract: An adaptive Finite Element framework for fatigue crack propagation analysis under constant amplitude loading is proposed. This framework combines the simplicity of standard industrial fatigue crack propagation analysis with the generality and accuracy of a full Finite Element analysis and can be implemented by combining standard existing computational tools. The equivalent domain integral method has been used to predict the fatigue crack direction as well as the corresponding stress-intensity factors is estimated at each small crack increment. The propagation is modeled by successive linear extensions, which are determined by the stress intensity factors under linear elastic assumption. The procedure is applied to the fatigue analysis of two internal parallel cracks specimen. The fatigue life cycle analysis is based upon Paris' equation. The proposed methodology is implemented in an interactive graphics computational scheme for 2D finite element analysis, which includes modeling, analysis, and visualization capabilities. The numerical results are validated with other relevant researcher's results.

Keywords: Finite element; fatigue crack propagation; adaptive mesh; constant amplitude loading; stress intensity factor.

1. Introduction

The analysis of fatigue crack propagation is very important to ensure the reliability of structures under cyclic loading conditions. The fatigue life of components is mainly predicted by traditional strength based theories. The fracture based numerical simulations have extensive application to quantify and predict the fatigue life of component under constant or variable amplitude loading condition. An accurate evaluation of fracture parameters such as stress intensity factors (SIFs) becomes quite essential for the simulation based life cycle design analysis. To simulate cracked structures, a number of methods such as boundary element method [1–3], meshfree methods [4–7], finite element method (FEM), and finite difference method (FDM) are available. Finite element modelling has been in the forefront of numerical methods used for the simulation of fatigue fracture problems. A number of approaches have been developed in FEM over the years, which makes it as a most suited method for analyzing the asymptotic stress fields at the crack tip. However, FEM requires that the crack surface should coincide with the edge of the finite elements, i.e. a conformal mesh is needed besides special elements to handle crack tip asymptotic stresses.

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Nevertheless, with regard to the finite element approach, the fatigue crack path and its associated stress-intensity factors K_I and K_{II} can be conveniently calculated using the appropriate crack tip elements, the mesh regeneration schemes and the crack increment criteria (Miranda et al. [8]). The finite element method has also been proven to be very well suited for the study of fracture mechanics, nevertheless modelling the propagation of a crack through a finite element mesh turns out to be difficult because of the modification of the mesh topology.

In this paper a finite element program is developed and used to analyze the fatigue crack growth of 2D structural components under constant amplitude loading. Based on the linear elastic fracture mechanics, the fatigue crack propagation is modeled by successive linear extensions, which are determined by the stress intensity factors at each crack increment. The program will directly predict the fatigue life cycles after the stress intensity factors history is completely recorded. Cracks can be introduced arbitrarily by the user at any position in the model. The system regenerates the meshes automatically taking into account the new created crack surfaces. The self-adaptive procedure takes into account the arbitrarily generated crack geometry and the finite element error estimation analysis. The computational scheme of the fatigue crack propagation program is adopted from Alshoaibi [9] as shown in Figure 1.

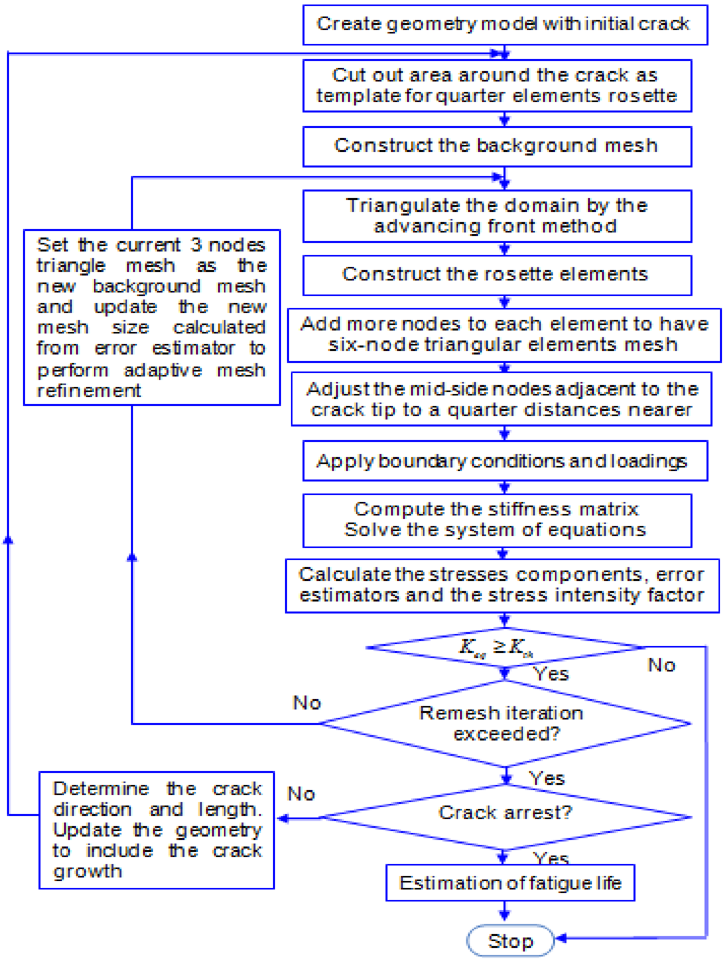

Figure 1. Computational scheme of the fatigue crack propagation program

2. Mesh generation and adaptive refinement

In this work, the unstructured triangle mesh is automatically generated by employing the advancing front method (Löhner [10]). The latest review of this method can be found in Zienkiewicz, and Taylor and Zhu [11].

The mesh must satisfy several conditions, depending on the problem.

- The mesh must conform to the boundary of the region, which may consist of more than one connected component.
- The mesh must be fine enough to produce an acceptable approximation to the original problem.
- The number of elements in the mesh should be small, because the complexity of solving the finite element problem depends on the mesh size.

An interactive graphics pre-processor is used to generate the initial mesh information and the boundary conditions of the finite element model. This initial model is solved by an incremental theory using von Mises yield criterion. After the solution has converged at the end of each load step, the solution errors are estimated. If the error at some point in the model exceeds a specified maximum error, the incremental analysis is interrupted and a new finite element model is constructed. The system decides automatically where to refine the mesh. If it is necessary, the system refines the mesh considering the initial boundary conditions. After the new mesh is generated, the solution variables (displacements, stresses, strains, etc.) are mapped from the old mesh to the new mesh. The analysis is then restarted from the current step and it is continued until the errors again become larger than the specified limit. In the final analysis or in each step, the user can visualize the responses using a graphics post-processor. The details description of the procedure can be referred to Alshoaibi [9] and Alshoaibi et al. [12].

The adaptive procedure provides a regular mesh refinement for the free-boundary curves including cracks. The adaptive process is based on *a posteriori* error estimation. An *h*-refinement type is utilized in this process. The strategy used to refine the mesh throughout analysis process is adopted from (Alshoaibi [9]).

Stress intensity factor: A major achievement in the theoretical foundation of linear elastic fracture mechanics was the introduction of the stress intensity factor as a parameter for the intensity of stresses near to the crack tip and associated to the energy release rate (Bazant and Planas [13]). Ingliss [14] studied the unexpected failure of naval ships, and Griffith [15] extended this work using thermodynamic criteria. Using this work, Irwing [16] developed the concept of the stress intensity factor. Stress intensity factors are a measure of the change in stress within the vicinity of the crack tip. Therefore, it is important to know the crack direction and when the crack stops propagating. The stress intensity factor is compared with the fracture toughness parameter K_{IC} to determine whether or not the crack will propagate in the case of static loading whereas the equivalent stress intensity factor is compared to the threshold stress intensity factor in the case of dynamic loading. In the present work the stress intensity factors is predicted by using the equivalent domain integral method as illustrated by Alshoaibi [17].

3. Fatigue crack propagation analysis

In order to simulate fatigue crack propagation under linear elastic condition, the crack path direction must be determined. There are several methods used to predict the direction of the crack trajectory such as the maximum circumferential stress theory, the maximum energy release rate theory and the minimum strain energy density theory. Bittencourt et al. [18] have shown that, if the crack orientation is allowed to change in automatic fracture simulation, the three criteria provide basically the same numerical results, since the maximum circumferential stress criterion is the simplest, presenting a closed form solution; it is briefly described by Alshoaibi [17] as follows:

The maximum circumferential stress theory Erdogan and Sih [19] asserts that, for isotropic materials under mixed-mode loading, the crack will growth in a direction normal to maximum tangential tensile stress. In this case, the tangential stress is given by

$$\sigma_\theta = \frac{1}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[K_I \cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] \tag{1}$$

The direction normal to the maximum tangential stress can be obtained by solving $d\sigma_\theta / d\theta = 0$ for θ. The nontrivial solution is given by:

$$K_I \sin\theta + K_{II}\left(3\cos\theta - 1\right) = 0 \tag{2}$$

which can be solved as:

$$\theta = \pm\cos^{-1}\left\{\frac{3K_{II}^2 + K_I\sqrt{K_I^2 + 8K_{II}^2}}{K_I^2 + 9K_{II}^2}\right\} \tag{3}$$

Since fatigue is a cyclic dissipation of energy, in the form of hysteretic loops, which are related to a collective damage process, the elapsed time for failure is expressed in terms of the number of fatigue cycles *(N)*. The control parameter that is used to evaluate this process is the rate of crack growth per cycle (*da/dN)*. Hence, *da/dN* depends on the applied stress intensity factor range and *N* is the well-known fatigue life term. For crack initiation, the threshold stress intensity factor and threshold stress range are associated as:

$$\Delta K_{th} = f\,\Delta\sigma_{th}\sqrt{\pi a} \tag{4}$$

Where: *f* is a function of geometry and loading and $\Delta\sigma_{th}$ is analogous to fatigue limit. This equation indicated that if $\Delta\sigma < \Delta\sigma_{th}$ crack growth does not occur. Practically, during the implementation we use the equivalent $\Delta K_{eq} \geq \Delta K_{th}$ as the condition for crack to propagate.

According to this criterion, the equivalent mode I stress intensity factor is obtained as:

$$K_{Ieq} = K_I\, cos^3(\theta/2) - 3K_{II}\, cos^2(\theta/2)\, sin(\theta/2) \tag{5}$$

To model the stable crack propagation, we use the generalized Paris' law:

$$\frac{da}{dN} = C\,(\Delta K_{Ieq})^m \tag{6}$$

where *C* and *m* are the material properties, *a* is the crack length, *N* is the number of loading cycles and ΔK_{Ieq} is obtained by equation (5) by substituting ΔK_I and ΔK_{II} to K_I and K_{II}. Then, the number of cycles N_{if} for crack propagation from the initial crack length a_i to the final crack length a_{if} can be integrated as:

$$N_{if} = \int_{a_i}^{a_f} \frac{1}{C\left(\Delta K_{Ieq}\right)^m}\, da \tag{7}$$

The developed program has safety features to automatically stop the calculation if, during any loading event, it detects that: (i) $K_{Ieq,max} = K_{Ic}$; (ii) the crack has reached its maximum specified size; (iii) one of the borders of the piece is reached by the crack front.

4. Numerical results and analysis

4.1. Two Internal Parallel Cracks

A rectangular Aluminum plate (90mm×180mm) with two internal, parallel and non-angled cracks (length = 10 mm for both) is submitted to a cyclic tension ($\sigma_{\max}$ = 160 N/mm, $\sigma_{\min}$ = 0) at both ends as shown in Figure 2a. The horizontal distance between the two tips close to each other is 15 mm and the vertical distance is 5mm. The material properties are modules of elasticity (E = 74 GPa), Poisson's ratio (ν = 0.3), fracture toughness (K_{Ic} = 60 $\mathrm{MPa}\sqrt{\mathrm{m}}$), threshold stress intensity factor (ΔK_{th} = 4 $\mathrm{MPa}\sqrt{\mathrm{m}}$), Paris constants (m = 3.32 and C = 2.087136×10^{-13}). The fatigue crack growth paths are presented in Figure 2b and the line contours representation of maximum principal stress distribution is also presented in Figure 2c.

In the beginning, both cracks show a pure mode I state with almost identical SIF values. Then, mode I factor at *A* increases higher than of that at *B* and mode II factor at *A* becomes negative so that the crack path curves towards the opposite crack. Finally, when the crack tips *A* get closer, the mode I factor at A tend to decrease, while mode I factor increases continuously at *B*. Finally, the equivalent mode I SIF at *B* exceeds the fracture toughness and unstable fracture occurs at the crack tips *B*.

An enlargement of the cracks tips counters representation of maximum principal stress distribution is shown in Figure 3. The evolution of the SIFs at the most interior crack tip (*A*) and at the crack tip near the edge (*B*) with the crack length is plotted in Figure 4.

The fatigue life diagram is presented in Figure 5. This prediction of the fatigue crack growth path by present study is in good agreement with the experimental results in a similar structure reported by Tu and Cai [20] as shown in Figure 6 and also with the numerical results obtained by Duflot and Dang [6] and Yan and Dang [2]. The fatigue life of the structure is evaluated as 6840 cycles, which is in agreement with the results obtained by Duflot and Dang [6] using a meshless method.

Special representation of the final step of fatigue crack propagation of this geometry with enlargement including the node number and the element number is shown in Figure 7.

5. Conclusion

The developed program using an adaptive finite element mesh generation strategy is used to simulate the fatigue crack propagation of two internal parallel cracks geometry under constant amplitude cyclic loading and to predict the fatigue life cycles based on the generalized Paris' equation. The results of the developed program have been successfully validated through direct comparisons with the relevant experimental data and similar calculations performed using numerical simulation performed by other researchers. The developed program demonstrated good predictions of fatigue life and crack propagation paths for 2D structural components under linear elastic fracture condition.

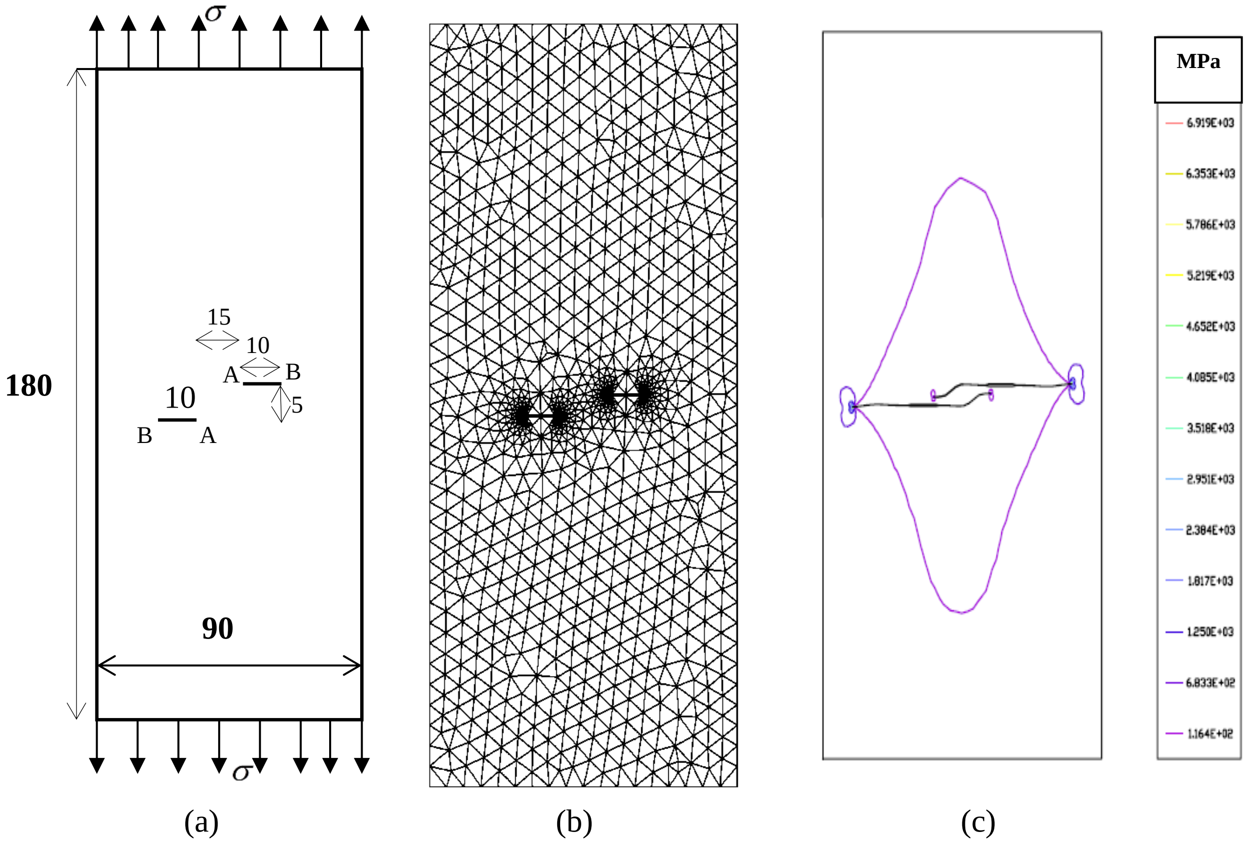

Figure 2. (a) Two internal parallel cracks plate (b) Crack path and final adaptive mesh (c) countors representation of maximum principal stress

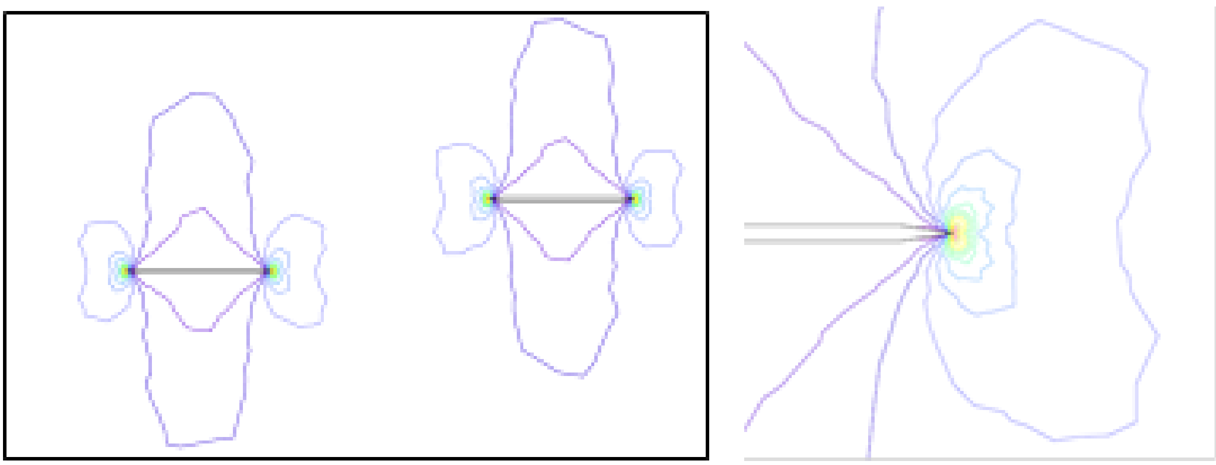

Figure 3. An enlargement of the cracks tips counters representation of maximum principal stress

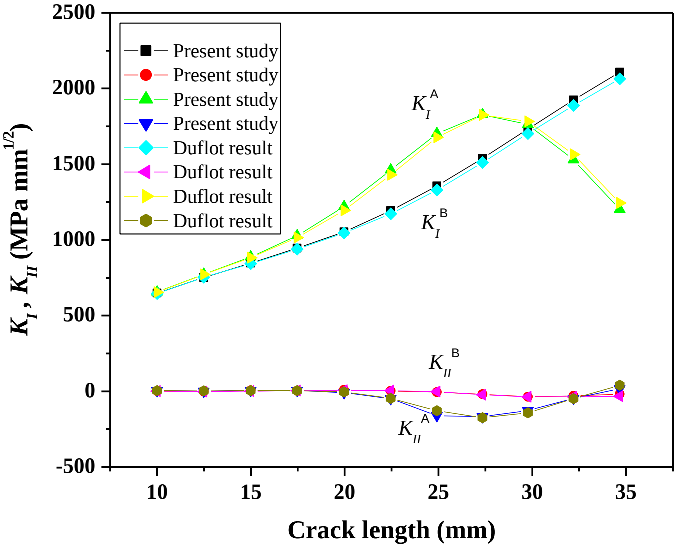

Figure 4. Comparisons of stress intensity factors for two internal parallel cracks

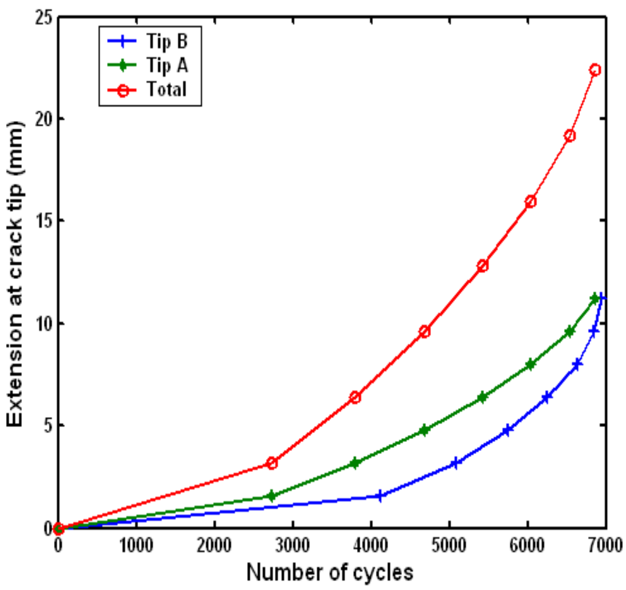

Figure 5. Fatigue life diagram for two internal parallel cracks

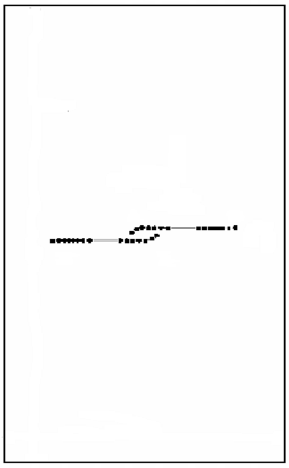

Figure 6. Experimental results of Tu and Cai [20]

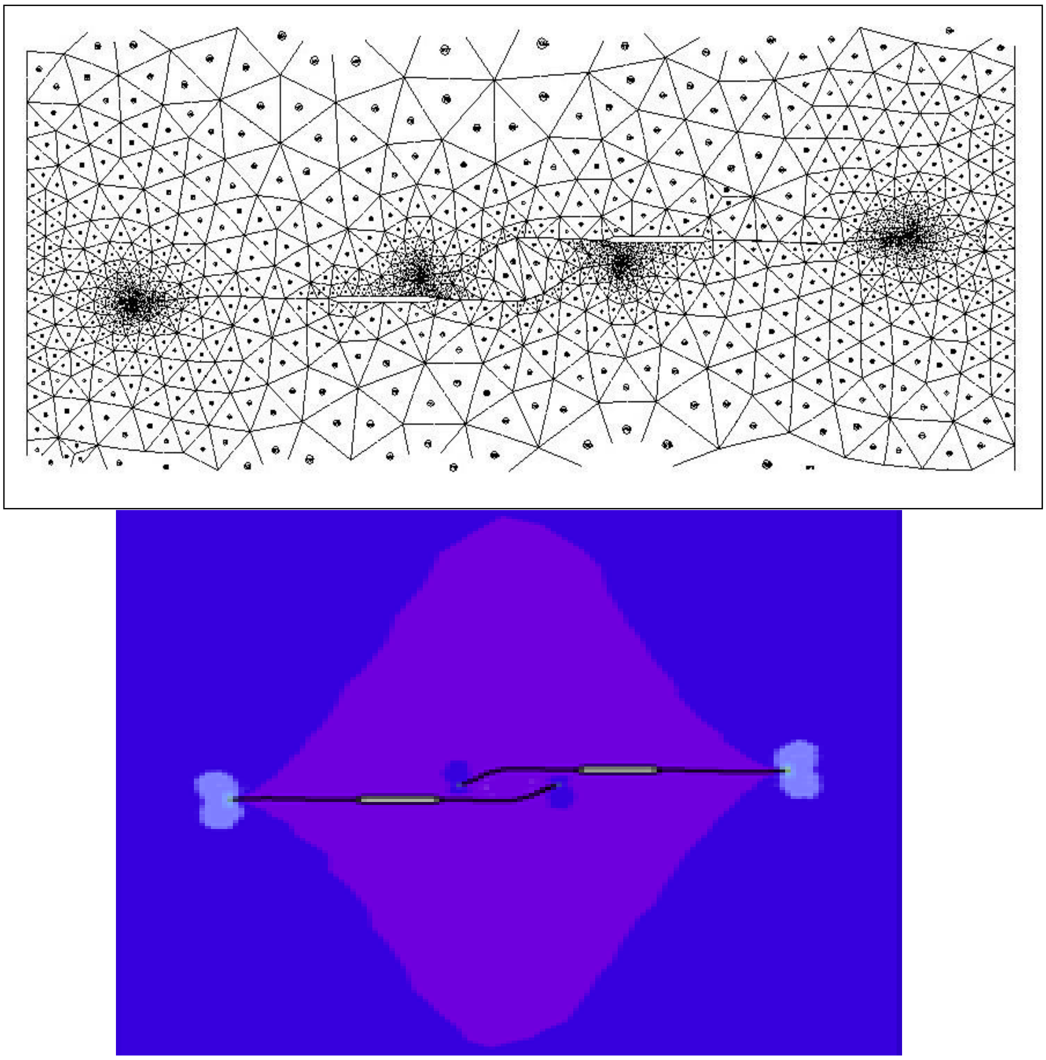

Figure 7. enlargement of the final step of fatigue crack growth

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