Inelastic Buckling of Rectangular Panel with a Simply Supported Edge and Three Clamped Edges under Uniaxial Loads

D. O. Onwuka, U. G. Eziefula*, and O. M. Ibearugbulem

Department of Civil Engineering, Federal University of Technology, Owerri, Imo State, Nigeria

Abstract: This paper presents solutions to the inelastic buckling problem of a thin flat rectangular isotropic panel under uniform uniaxial in-plane compression. The case of boundary conditions studied is a panel clamped along three edges with one simply supported longitudinal edge (CCCS). Stowell's and Bleich's plasticity approaches are used in deriving the governing equations. A theoretical formulation based on Taylor's series is used in estimating the shape function which satisfied the boundary conditions and resulted to a peculiar shape function for the CCCS panel. Values of the panel buckling coefficient are calculated for aspect ratios from 0.1 to 2.0 at intervals of 0.1. The results are compared with the solutions from previous studies and the percentage differences are found to be consistent. Therefore, the proposed method can be used for the inelastic buckling analysis of thin flat rectangular isotropic panels with mixed boundary conditions subjected to uniform uniaxial in-plane loads.

Keywords: Boundary conditions; deflection function; plastic buckling; rectangular panel; Taylor's series; variational principles.

Nomenclature

a	length of panel
b	width of panel
h	thickness of panel
k	panel buckling coefficient
m	number of half-waves of the buckling mode along the x-direction
n	number of half-waves of the buckling mode along the y-direction
p	aspect ratio
w	transverse deflection
w'^{ξ}, w'^{η}	the first derivative of the deflection in the ξ and η coordinates respectively
w''^{η}	the second derivative of the deflection in the η coordinate
x,y	Cartesian coordinates in the horizontal and vertical direction respectively
A	amplitude of the shape function
C	clamped edge
CCCS	rectangular panel with three clamped edges and one longitudinal simply supported edge
D	panel flexural rigidity in the elastic range

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$\overline{D}$	panel flexural rigidity in the plastic range
E	Young's modulus
E_{sec}	secant modulus
E_{tan}	tangent modulus
H	buckling curve expression
J, K	unknown constants in the power series
N_x	uniaxial in-plane compressive load on x-plane
$N_{x,CR}$	critical buckling load
S	simply supported edge
ν	Poisson ratio
ξ, η	non-dimensional parameters of the x and y coordinates respectively
σ_x	in-plane compressive stress in the x-direction
τ	moduli ratio defined as $\tau = E_{tan}/E$

1. Introduction

Thin rectangular panels or plates are widely used in modern engineering structures to transmit in-plane and lateral loads. Examples include aircraft wings, bridge decks, ship hulls and grillages, platform of offshore structures, tank bottoms, walling units and building roofs. When a thin rectangular panel is subjected to in-plane compressive loads and the loads are gradually increased, the panel becomes prone to buckling even though lateral loads may not be present. Buckling of panels may be classified as elastic buckling or inelastic/plastic buckling depending on the stress-strain relationship. Inelastic buckling may likely occur if the panel material possesses a low proportional limit when compared with the yield stress e.g. aluminum alloys [1]. Inelastic buckling loads are always less than their elastic counterparts and in many cases, buckling occurs in the inelastic range. It is therefore important to analyze the inelastic buckling behavior of panels in order to accurately predict the critical loads in the inelastic range.

Various plasticity theories have been proposed in the literature to consider the inelastic effect but the more commonly used theories are the deformation theory and the flow or incremental theory [2]. Comparison of the two theories shows that the deformation theory has a weak theoretical formulation and contains fundamental mathematical inconsistencies which are not present in the flow theory. However, there is a well-known paradox of inelastic plate buckling in which values obtained by the deformation theory of plasticity are generally in better agreement with experimental results. Some researchers [3, 4] provided solutions of the inelastic buckling paradox by proposing modifications to the flow theory. Although several theoretical and experimental studies have been conducted for the inelastic buckling of panels in the past decades, a universally accepted solution to the plastic buckling paradox has not been presented. Hence, some researchers have continued to use the deformation theory in the inelastic buckling analysis of thin rectangular panels in spite of its weak mathematical background. Other plasticity theories have been proposed in the literature apart from the deformation and the flow theories e.g. [5].

Solutions to inelastic buckling problems of thin rectangular panels subjected to in-plane edge loadings have been provided using various methods [2, 3, 6-10]. The use of Fourier series or trigonometric series seems to be predominant in studies on plastic buckling of panels. Most solutions to inelastic buckling problems of thin rectangular panels were presented for panels of common boundary conditions such as rectangular panels with four simply supported edges. In many practical cases, however, other combinations of boundary conditions do exist. Due to the complicated mathematical structure of the other combination of boundary conditions, obtaining

closed form solutions are generally difficult, if not impossible, and using trigonometric series to formulate the shape function for certain boundary conditions may be rigorous [11, 12]. Wang and Huang [9] used the differential quadrature (DQ) method for the elastoplastic buckling analysis of thin rectangular panels subjected to biaxial distributed in-plane loadings and found new solutions for CCCC, SSCC and SCCC square panels. Ibearugbulem [13] used the Ritz method and Taylor's series shape function formulation to provide new solutions for the elastic buckling analysis of thin rectangular panels subjected to uniaxial in-plane loading under various support conditions. Due to the limitations of the trigonometric series, the Taylor's series may be used to formulate the shape function of thin rectangular panels. From available literature, there is dearth of literature in the use of Taylor's series in formulating the shape function in plate buckling analysis, and the Taylor's series has not been used to analyze the inelastic buckling of thin rectangular panels with three clamped edges and one simply supported edge in the longitudinal direction (CCCS).

In this paper, we present a solution to the inelastic buckling of a thin flat rectangular isotropic CCCS panel subjected to uniform in-plane uniaxial compression. The shape function which satisfied the boundary conditions was formulated using Taylor's series truncated at the fifth term. In order to take care of the inelastic effect, we modified both Stowell's plasticity theory and Bleich's plasticity theory by applying a work principle and variational principles.

2. Analytical method

2.1. Formulation of the stability problem

Consider a homogenous, rectangular, flat, isotropic panel and assume that the thickness of the panel in the z-axis is small compared to the other characteristic dimensions in the x-axis and y-axis respectively as postulated in the classical thin panel theory. The thin rectangular panel is subjected to uniform in-plane compressive loads along the x-axis. The panel is simply supported along one edge in the x-axis and the remaining edges are clamped. This means that in the panel, edges 1, 2, and 3 are clamped while edge 4 is simply supported. The edge numbers are shown in Figure 1 and the problem definition is illustrated in Figure 2.

To facilitate the solution of the problem, the Cartesian coordinates are expressed in non-dimensional axes as:
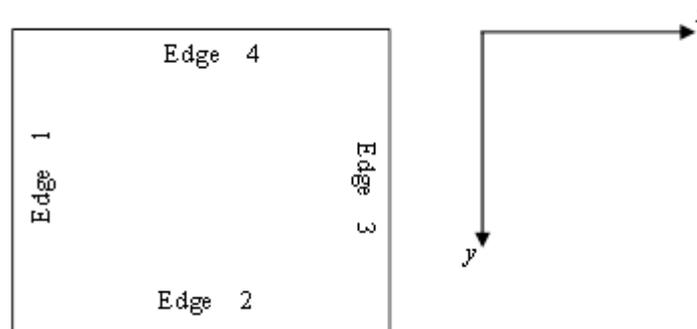
$$\xi = x/a; \quad \eta = y/b \tag{1}$$

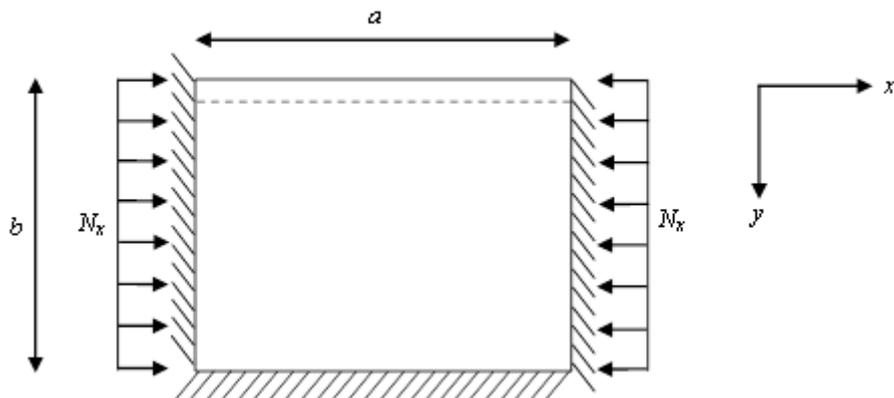**Figure 1.** Sketch of a rectangular panel showing the edge numbers

Figure 2. CCCS rectangular panel under uniaxial in-plane compression

Eight boundary conditions – four along the *x*-axis and four along the *y*-axis – are required to obtain a unique solution for the CCCS panel. For all the edges of the panel, the deflection at the corners must be zero. The moments at the corners of the simply supported edge are equal to zero since simply supported edges are free to rotate. The slope varnish along the corners of the clamped edges because clamped edges do not rotate. Thus, the first derivatives of *w* with respect to ξ and η are zero for the corners of the clamped edges, while the second derivative of *w* with respect to η is zero for the corners of the simply supported edge. The boundary conditions of the CCCS panel in non-dimensional parameters are therefore written as:

$$w(\xi = 0) = w'^{\xi}(\xi = 0) = 0 \tag{2}$$
$$w(\xi = 1) = w'^{\xi}(\xi = 1) = 0 \tag{3}$$
$$w(\eta = 0) = w''^{\eta}(\eta = 0) = 0 \tag{4}$$
$$w(\eta = 1) = w'^{\eta}(\eta = 1) = 0 \tag{5}$$

2.2. Theoretical development

2.2.1. Stowell's approach

The theoretical development starts with the derivation of the governing equation using the deformation theory of plasticity based on Stowell's approach. Stowell [14] expressed the governing differential equation of equilibrium for the inelastic buckling of a thin, flat, rectangular panel under uniform uniaxial in-plane compression along the *x*-axis as:

$$\left(\frac{1}{4} + \frac{3}{4}\frac{E_{tan}}{E_{sec}}\right)\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{h\sigma_x}{\bar{D}}\frac{\partial^2 w}{\partial x^2} = 0 \tag{6}$$

In deriving Equation (6), Stowell adopted ½ as the numerical value of the Poisson ratio in the inelastic range. The panel material is assumed to be incompressible and isotropic. In Equation (6):

$$\sigma_x = N_x/h \tag{7}$$
$$\bar{D} = E_{sec}h^3/9 \tag{8}$$

Expressing Equation (6) in terms of the non-dimensional parameters given in Equation (1) and applying Equation (7), we get:

$$\left(\frac{1}{4} + \frac{3}{4}\frac{E_{tan}}{E_{sec}}\right)\frac{\partial^4 w}{\partial \xi^4} + 2\frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \frac{\partial^4 w}{\partial \eta^4} - \frac{N_x}{\bar{D}}\frac{\partial^2 w}{\partial \xi^2} = 0 \tag{9}$$

Equation (9) can be simplified to:

$$\frac{1}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_{tan}}{E_{sec}}\right)\frac{\partial^4 w}{\partial\xi^4}+2\frac{\partial^4 w}{\partial\xi^2\partial\eta^2}+p^2\frac{\partial^4 w}{\partial\eta^4}-\frac{N_x b^2}{\overline{D}}\frac{\partial^2 w}{\partial\xi^2}=0 \tag{10}$$

$$p = a/b \tag{11}$$

Equation (10) can be transformed using a work principle. Bending problems of thin isotropic rectangular panels have been solved using a work principle and direct integration [15]. Instead of using traditional equilibrium and variational methods where the shape functions are first assumed, Ibearugbulem [15] carried out direct integration of the governing differential equation of the panels to obtain suitable shape functions. This approach requires the derivation of deflection equations by the simple principle of equilibrium of works performed by the load and the panel reaction (resistance). They multiplied the equation of equilibrium of force by the deflection and integrated the resulting equation in a closed domain. Applying the approach used in [15], Equation (10) gives:

$$N_x = \frac{\overline{D}\int_0^1\int_0^1\left[\frac{1}{p^2}\left(\frac{1}{4}+\frac{3E_{tan}}{4E_{sec}}\right)\frac{H\partial^4 H}{\partial\xi^4}+2\frac{H\partial^4 H}{\partial\xi^2\partial\eta^2}+p^2\frac{H\partial^4 H}{\partial\eta^4}\right]\partial\xi\partial\eta}{b^2\int_0^1\int_0^1\frac{H\partial^2 H}{\partial\xi^2}\partial\xi\partial\eta} \tag{12}$$

where

$$w = AH \tag{13}$$

Equation (12) is the inelastic buckling equation of a thin, flat, rectangular, isotropic panel using Stowell's plasticity approach and a work principle. Note that from Equation (13), the product of the terms on the right-hand side of the equation will be zero if the deflection is zero. This condition can only be satisfied if either $A = 0$ or $H = 0$. If $A = 0$, then we will arrive at a trivial solution where the panel maintains a straight configuration and remains flat under the action of any in-plane load. Therefore, A cannot be zero if the deflection is zero.

Assume that the shape function of thin rectangular panels expressed in the form of Taylor's series is both continuous and differentiable. Truncating the infinite series at the fifth term i.e. $m = n = 4$, the Taylor's series formulated shape function can be written as:

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4}J_m K_n \xi^m\eta^n \tag{14}$$

Expanding Equation (14) in non-dimensional parameters gives:

$$w = A(J_0 + J_1\xi + J_2\xi^2 + J_3\xi^3 + J_4\xi^4)(K_0 + K_1\eta + K_2\eta^2 + K_3\eta^3 + K_4\eta^4) \tag{15}$$

The boundary conditions expressed in Equations (2)–(5) are now applied in Equation (15). This gives:

$$w = J_4 K_4[(\xi^2 - 2\xi^3 + \xi^4)(0.5\eta - 1.5\eta^3 + \eta^4)] \tag{16}$$

Equation (16) is the peculiar shape function and the expression for the transverse deflection of the CCCS panel. Comparing Equations (13), (15) and (16), it can be shown that:

$$H = (\xi^2 - 2\xi^3 + \xi^4)(0.5\eta - 1.5\eta^3 + \eta^4) \tag{17}$$

In order to determine the critical buckling load of the panel, numerical values of the integrals in Equation (12) are first calculated. Applying variational principles for the CCCS boundary conditions yields:

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial\xi^4}\partial\xi\partial\eta = 0.006031746 \tag{18}$$

$$\int_0^1 \int_0^1 2H \frac{\partial^4 H}{\partial \xi^2 \partial \eta^2} \partial \xi \partial \eta = 0.003265306 \tag{19}$$

$$\int_0^1 \int_0^1 H \frac{\partial^4 H}{\partial \eta^4} \partial \xi \partial \eta = 0.002857143 \tag{20}$$

$$\int_0^1 \int_0^1 H \frac{\partial^2 H}{\partial \xi^2} \partial \xi \partial \eta = 0.0001436130 \tag{21}$$

Substituting the numerical values of the integrals in Equations (18)–(21) into Equation (12) gives:

$$N_x = \frac{\bar{D}}{b^2} \left[\frac{\frac{0.006031746}{p^2} \left(\frac{1}{4} + \frac{3E_{tan}}{4E_{sec}} \right) + 0.003265306 + 0.002857143p^2}{0.0001436130} \right] \tag{22}$$

Inelastic buckling equation can be expressed as:

$$N_x = \frac{\bar{D}\pi^2}{b^2} k \tag{23}$$

Expressing Equation (22) in form of Equation (23) gives Equation (24).

$$k = \frac{4.25549}{p^2} \left(\frac{1}{4} + \frac{3}{4} \frac{E_{tan}}{E_{sec}} \right) + 2.30372 + 2.01576p^2 \tag{24}$$

2.2.2. Bleich's approach

Bleich [5] proposed a simplified theory for plate plastic buckling and expressed the differential equation governing the behaviour of the plate as:

$$\tau \frac{\partial^4 w}{\partial x^4} + 2\sqrt{\tau} \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{N_x}{D} \frac{\partial^2 w}{\partial x^2} = 0 \tag{25}$$

where

$$\tau = E_{tan}/E \tag{26}$$

It should be noted that in Equation (25), the flexural rigidity in the elastic range is used in the analysis, and the elastic Young's modulus is used instead of the secant modulus in the moduli ratio. Expressing Equation (25) in non-dimensional coordinates gives:

$$\frac{\partial^4 w}{\partial \xi^4} + 2\sqrt{\tau} \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \frac{\partial^4 w}{\partial \eta^4} - \frac{N_x}{D} \frac{\partial^2 w}{\partial \xi^2} = 0 \tag{27}$$

Applying the technique used in deriving Equation (12) in Equation (27) yields:

$$N_x = \frac{\frac{D}{b^2} \int_0^1 \int_0^1 \left[\frac{\tau}{p^2} \frac{H \partial^4 H}{\partial \xi^4} + 2\sqrt{\tau} \frac{H \partial^4 H}{\partial \xi^2 \partial \eta^2} + p^2 \frac{H \partial^4 H}{\partial \eta^4} \right] \partial \xi \partial \eta}{\int_0^1 \int_0^1 \frac{H \partial^2 H}{\partial \xi^2} \partial \xi \partial \eta} \tag{28}$$

The buckling load and the panel buckling coefficient are expressed as given in Equations (29) and (30) respectively.

$$N_x = \frac{D}{b^2} \left[\frac{\frac{0.006031746\tau}{p^2} + 0.003265306\sqrt{\tau} + 0.002857143p^2}{0.0001436130} \right] \tag{29}$$

$$k = \frac{4.25549\tau}{p^2} + 2.30372\sqrt{\tau} + 2.01576p^2 \tag{30}$$

3. Results and discussion

The critical inelastic buckling equation obtained from the present study using the approaches proposed by [14] and [5] are expressed in Equations (31) and (32) respectively.

$$N_{x,CR} = \frac{\bar{D}\pi^2}{b^2}\left[\frac{4.25549}{p^2}\left(\frac{1}{4} + \frac{3}{4}\frac{E_{tan}}{E_{sec}}\right) + 2.30372 + 2.01576p^2\right] \tag{31}$$

$$N_{x,CR} = \frac{D\pi^2}{b^2}\left[\frac{4.25549}{p^2}\left(\frac{E_{tan}}{E}\right) + 2.30372\left(\sqrt{\frac{E_{tan}}{E}}\right) + 2.01576p^2\right] \tag{32}$$

It may be noted from Equations (31) and (32) that the numerical values of the integrals of the governing equation are the same because both equations are approximate solutions obtained using Taylor's series shape function (for the CCCS plate). The differences in both equations exist as a result of differences in flexural rigidity and moduli ratio. Stowell [14] used plastic flexural rigidity, $\bar{D}$, and E_{tan}/E_{sec} as the moduli ratio. Bleich [5], on the other hand, used the flexural rigidity of the plate in the elastic range, D, and adopted E_{tan}/E as the moduli ratio. The $\bar{D}$ is a function of E_{sec} as seen in Equation (8), while D is a function of E. Thus, for a panel with a known aspect ratio, the buckling coefficient and the critical plastic buckling load can be calculated if the actual values of E_{tan}, E_{sec}, E and ν are known. A comprehensive knowledge of the stress-strain curve of the panel material in the inelastic region is required to calculate the values of E_{tan} and E_{sec} from the stress-strain curve. The factors E_{tan}/E_{sec} and E_{tan}/E are numerically equal to unity in elastic buckling but their values are always less than unity in inelastic buckling. For a typical aluminum alloy, E is greater than E_{tan} and E_{sec}, and in the plastic range, E_{sec} is greater than E_{tan}. The elastic flexural rigidity is mathematically expressed as:

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{33}$$

The critical load equation for the elastic buckling of uniaxially compressed thin rectangular isotropic panels of various boundary conditions was derived in [13] and the critical load for the CCCS panel is expressed as:

$$N_{x,CR} = \left(\frac{4.25}{p^2} + 2.015p^2 + 2.303\right)\frac{\pi^2 D}{b^2} \tag{34}$$

Table 1 shows values of k via Stowell's approach for aspect ratios ranging from 0.1 to 2.0 at increments of 0.1 for numerical values of E_{tan}/E_{sec} equal to 0.5, 0.6, 0.7, 0.8 and 0.9. From Table 1, it may be observed that the buckling coefficient increases with moduli ratio. It is also observed that the percentage difference between the two solutions improves as the aspect ratio increases from 0.1 to 2.0. The percentage differences between the present study and [13] for $E_{tan}/E_{sec} = 0.9$ range from -0.656 percent to -7.340 percent. Reference solutions were not available in open literature for validating the elastic buckling results for the CCCS boundary conditions [13]. Hence, the percentage differences obtained in this study are therefore compared with the percentage differences for the panel with four simply supported edges (SSSS plate) whose solutions have been validated. Table 2 shows the percentage differences between solutions for inelastic plate buckling ($E_{tan}/E_{sec} = 0.9$) and solutions from [13] for values of k using selected values of aspect ratio. The boundary conditions presented in Table 2 are CCCS and SSSS.

For the percentage differences shown in Table 2, the values at $p = 0.1$ represent the maximum percentage differences while the values at $p = 2.0$ are the minimum percentage differences. It can be observed that the maximum and minimum percentage differences for the CCCS and SSSS boundary conditions have similar numerical values. As reported in [13], the elastic buckling

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solutions for the SSSS panel boundary conditions compared favorably with those of previous investigations. According to [13], the mean percentage difference between [13] and [6] is 0.069% for $0.1 \leq p \leq 1.0$ at intervals of 0.1 for the SSSS boundary conditions. For the inelastic buckling of the SSSS panel, the mean percentage difference between [16] and [6] is 0.091%. These differences are very close and are acceptable in statistics.

Table 1. Values of *k* for CCCS plate under uniform uniaxial in-plane loads.

p	Present Study using Stowell's approach					Ibearugbulem [13]	α[a]
	$E_{tan}/E_{sec}= 0.5$	$E_{tan}/E_{sec}= 0.6$	$E_{tan}/E_{sec}= 0.7$	$E_{tan}/E_{sec}= 0.8$	$E_{tan}/E_{sec}= 0.9$		
0.1	268.292	300.208	332.124	364.041	395.957	427.323	−7.340
0.2	68.876	76.855	84.834	92.814	100.793	108.634	−7.218
0.3	32.037	35.583	39.130	42.676	46.222	49.707	−7.011
0.4	19.249	21.244	23.239	25.234	27.228	29.188	−6.715
0.5	13.446	14.723	16.000	17.276	18.553	19.808	−6.336
0.6	10.417	11.304	12.191	13.077	13.964	14.832	−5.865
0.7	8.719	9.371	10.022	10.673	11.325	11.964	−5.341
0.8	7.750	8.248	8.747	9.246	9.744	10.233	−4.779
0.9	7.220	7.614	8.008	8.402	8.796	9.182	−4.204
1.0	6.979	7.298	7.617	7.937	8.256	8.568	−3.641
1.1	6.941	7.205	7.468	7.732	7.996	8.254	−3.126
1.2	7.053	7.275	7.497	7.718	7.940	8.156	−2.648
1.3	7.284	7.473	7.662	7.851	8.040	8.223	−2.225
1.4	7.612	7.774	7.937	8.100	8.263	8.421	−1.876
1.5	8.021	8.163	8.305	8.447	8.589	8.726	−1.570
1.6	8.503	8.628	8.752	8.877	9.002	9.122	−1.316
1.7	9.050	9.160	9.270	9.381	9.491	9.597	−1.105
1.8	9.656	9.754	9.853	9.951	10.050	10.143	−0.917
1.9	10.317	10.406	10.494	10.583	10.671	10.754	−0.772
2.0	11.032	11.111	11.191	11.271	11.351	11.426	−0.656

[a]α means percentage difference between *k* from Present Study ($E_{tan}/E_{sec} = 0.9$) and Ibearugbulem [13].

Table 2. Percentage differences between selected elastic and inelastic values of *k*.

p	CCCS panel			SSSS panel		
	Inelastic *k*	Elastic *k*	Difference (%)	Inelastic *k*[a]	Elastic *k*[b]	Difference (%)
0.1	395.957	427.323	−7.340	94.6305	102.110	−7.325
0.2	100.793	108.634	−7.218	25.1954	27.065	−6.908
0.5	18.553	19.808	−6.336	5.9554	6.254	−4.775
1.0	8.256	8.568	−3.641	3.9277	4.002	−1.857
2.0	11.351	11.426	−0.656	*	*	*

* Means not available;[a] means solutions from Onwuka, et al. [16];[b] means solutions from Ibearugbulem [13].

4. Summary and conclusions

A new method is presented and used to provide approximate solutions for the inelastic buckling analysis of a thin flat rectangular isotropic panel subjected to uniform uniaxial in-plane compression with one longitudinal simply supported edge and three clamped edges. The governing plasticity equations were derived using both Stowell's and Bleich's approaches. The Taylor's series truncated at the fifth term was used to formulate the shape function. The idea of the proposed method is to provide a simple alternative means of estimating the deflections and plastic buckling coefficients of thin rectangular isotropic panels. If the panel buckling coefficients for the aspect ratios are known, it would be possible to predict the inelastic buckling loads and stresses. It is expected that the new results will provide other researchers with data which can be used for comparing their results. The proposed method could be extended to inelastic buckling of thin flat rectangular isotropic panels of other boundary conditions under uniform in-plane loading.

From the results of the study, the following conclusions are drawn:

1. The principle of equilibrium of works can be used to derive the inelastic buckling load equation of a thin flat rectangular isotropic plate subjected to uniaxial in-plane loads.
2. Taylor's series function is adequate for formulating the shape function of a thin flat rectangular isotropic plate with three clamped edges and one simply supported edge in the longitudinal direction.
3. The values of the plastic buckling coefficient obtained by the present method are slightly less than those of elastic buckling coefficient.

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