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MHD Flow past a Vertical Plate with Variable Temperature and Mass Diffusion in the Presence of Hall Current

U. S. Rajput and Neetu Kanaujia*

Department of Mathematics and Astronomy, University of Lucknow, UP, India

Abstract: In the present paper, MHD flow past an impulsively started oscillating vertical plate with variable temperature and constant mass diffusion in the presence of Hall current is studied. The fluid considered is an electrically conducting, absorbing-emitting radiation but a non-scattering medium. The Laplace transform technique has been used to find the solutions for the velocity profile and Skin friction. The velocity profile and Skin friction have been studied for different parameters like Schmidt number, Hall parameter, magnetic parameter, mass Grashof number, thermal Grashof number, Prandtl number, and time. The effect of parameters are shown graphically and the value of the Skin-friction for different parameters has been tabulated.

Keywords: MHD flow; Hall current; variable temperature; constant mass diffusion.

1. Introduction

The study of MHD flow and Hall effect plays an important role in engineering and biological science. It has many applications in science and technology like MHD generators, MHD pumps, biomechanics, irrigation engineering and aerospace technology. Furthermore, MHD flow problems frequently occur in petro-chemical industry, chemical vapor deposition on surfaces, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics. MHD flow models with Hall effect have been studied by a number of researchers, some of which are mentioned here. Datta and Jana [1] have studied oscillatory magnetohydrodynamic flow past a flat plate with Hall effects. Satya Narayana et al. [2] have studied the effects of Hall current and radiation absorption on MHD micro polar fluid in a rotating system. Watanabe and Pop [3] have studied Hall effects on magnetohydrodynamic boundary layer flow over a continuous moving flat plate. Satya Narayana et al. [4] have studied Hall current effects on free-convection MHD flow past a porous plate. Sudhakar, et al. [5] have studied Hall effect on unsteady MHD flow past along a porous flat plate with thermal diffusion, diffusion thermo and chemical reaction**.** Deka [6] has analyzed Hall effects on MHD flow past an accelerated plate. Some research articles related to MHD flow with oscillating plate are mentioned here. Muthucumaraswamy [7] have analyzed mass transfer effect on isothermal vertical oscillating plate in the presence of chemical reaction. Satya Narayana et al. [8] have studied chemical reaction and heat source effects on MHD oscillatory flow in an irregular channel. Devika et al. [9] have studied MHD oscillatory flow of a visco-elastic fluid in a porous channel with chemical reaction. Sharma and Deka [10] have analyzed thermal radiation and oscillating plate temperature effects on MHD unsteady flow past a semi-infinite porous vertical plate under suction and chemical reaction. Rajput and Kumar [11] have studied radiation effect on MHD flow through porous media past an impulsively started vertical plate with

*Corresponding author; e-mail: rajputneetulko@gmail.com

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variable heat and mass transfer. Attia [12] has analyzed unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties. Rajput and Kumar [13] have studied MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. Raptis et al. [14] have studied mass transfer effects subjected to variable suction or injection. We are considering MHD flow past an impulsively started oscillating vertical plate with variable temperature and constant mass diffusion in the presence of Hall current. The effect of Hall current on the velocity have been observed with the help of graphs, and the skin friction has been tabulated.

2. Mathematical Analysis

An unsteady viscous incompressible electrically conducting fluid past an impulsively started oscillating vertical plate is considered here. The plate is electrically non-conducting. A uniform magnetic field B is assumed to be applied on the flow. Initially, at time $t \leq 0$ the temperature of the fluid and the plate is T_∞; and the concentration of the fluid is C_∞. At time $t > 0$, the plate starts oscillating in its own plane with frequency ω, the temperature of the plate and the concentration of the fluid, respectively are raised to T_w and C_w. Using the relation $\nabla \cdot B = 0$ for the magnetic field $\bar{B} = (B_x, B_y, B_z)$, we obtain B_y (say B_0) = constant, i.e. $B = (0, B_0, 0)$, where B_0 is externally applied transverse magnetic field. The Geometry of the problem is given in Figure 1.

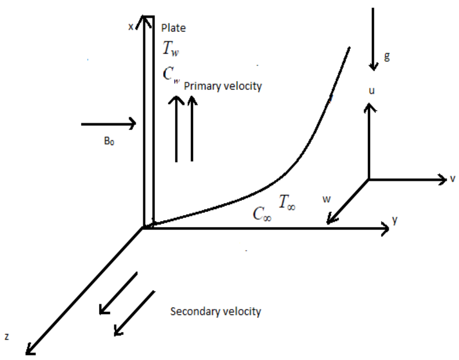

Figure 1. Geometry of the problem

Let V be the velocity vector, and u, v, w are respectively the velocity components along x, y and z-directions. The governing equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Since there is no variation of flow in the y- direction, therefore v = 0.

The generalized ohm's law including the effect of Hall current according to cowling (1957) is given as

$$\bar{J} + \frac{\omega_e \tau_e}{B_0}(\bar{J} \times \bar{B}) = \sigma(E + \bar{V} \times \bar{B})$$

The external electric field E = 0, since polarization of charges is negligible.

Let (j_x, j_y, j_z) be the components of current density J. Here j_x, j_y and j_z are the components of current density in the x, y, and z directions, respectively. Using above assumption, we get

$J_x = \frac{\sigma B_0^{\,2}}{1+m^2}(u+mw)$ and $J_z = \frac{\sigma B_0^{\,2}}{1+m^2}(mu-w)$.

The fluid model is as under –

$$\frac{\partial u}{\partial t} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T-T_\infty) + g\beta^*(C-C_\infty) - \frac{\sigma B_0^{\,2}}{\rho(1+m^2)}(u+mw), \quad (1)$$

$$\frac{\partial w}{\partial t} = \upsilon\frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_0^{\,2}}{\rho(1+m^2)}(w-mu), \quad (2)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2}, \quad (3)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}. \quad (4)$$

The following boundary conditions have been assumed:

$$\left.\begin{array}{l} t \le 0: u = 0, w = 0, C = C_\infty, T = T_\infty, \text{ for all values of } y \\ t > 0: u = u_0\cos\omega t, w = 0, C = C_w, T = T_\infty + \left(T_w - T_\infty\right)\frac{u_0^{\,2}t}{\upsilon} \text{ at } y = 0 \\ u \to 0, w \to 0, C \to C_\infty, T \to T_\infty \quad \text{as } y \to \infty. \end{array}\right\} \quad (5)$$

Here u is the velocity of the fluid in x- direction (primary velocity u), w - the velocity of the fluid in z- direction (secondary velocity w), m - Hall parameter, g - acceleration due to gravity, β - volumetric coefficient of thermal expansion, β^* - volumetric coefficient of concentration expansion, t- time, C_∞ - the concentration in the fluid far away from the plate, C - species concentration in the fluid , C_w - species concentration at the plate, D - mass diffusion, T_∞ - the temperature of the fluid near the plate, T_w - temperature of the plate, T - the temperature of the fluid, k - the thermal conductivity, υ - the kinematic viscosity, ρ - the fluid density, σ - electrical conductivity, μ - the magnetic permeability, and C_P - specific heat at constant pressure. Here $m = \omega_e\tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (4) in dimensionless from, we introduce the following non - dimensional quantities:

$$\left.\begin{array}{l} \bar{u} = \frac{u}{u_0}, \bar{w} = \frac{w}{u_0}, \bar{y} = \frac{yu_0}{\upsilon}, Sc = \frac{\upsilon}{D}, Pr = \frac{\mu C_P}{k}, M = \frac{\sigma B_0^2\upsilon}{\rho u_0^2}, \bar{t} = \frac{tu_0^{\,2}}{\upsilon}, \bar{\omega} = \frac{\omega\upsilon}{u_0^{\,2}}, \\ Gr = \frac{g\beta\upsilon(T_w - T_\infty)}{u_0^{\,3}}, Gm = \frac{g\beta\upsilon(C_w - C_\infty)}{u_0^{\,3}}, \bar{C} = \frac{C - C_\infty}{C_w - C_\infty}, \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \end{array}\right\} \quad (6)$$

where $\bar{u}$ is the dimensionless velocity of the fluid in x-direction, $\bar{w}$ is the dimensionless velocity of the fluid in z-direction, θ is the dimensionless temperature, $\bar{C}$ is the dimensionless concentration, Gr is the thermal Grashof number, Gm is the mass Grashof number, μ is the coefficient of viscosity, Pr is the Prandtl number, Sc is the Schmidt number, and M is the magnetic parameter.

Equations (1), (2), (3) and (4) are transformed into the following dimensionless forms:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gr\theta + Gm\overline{C} - \frac{M(\bar{u} + m\overline{w})}{(1+m^2)}, \tag{7}$$

$$\frac{\partial \overline{w}}{\partial \bar{t}} = \frac{\partial^2 \overline{w}}{\partial \bar{y}^2} - \frac{M(\overline{w} - m\bar{u})}{(1+m^2)}, \tag{8}$$

$$\frac{\partial \overline{C}}{\partial \bar{t}} = \frac{1}{Sc}\frac{\partial^2 \overline{C}}{\partial \bar{y}^2}, \tag{9}$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial \bar{y}^2}. \tag{10}$$

The corresponding boundary conditions become-

$$\left.\begin{array}{l} \bar{t} \le 0, \bar{u} = 0, \vec{w} = 0, \overline{C} = 0, \theta = 0, \text{ for all values of } \bar{y} \\ \bar{t} > 0, \bar{u} = \cos \overline{\omega} \bar{t}, \overline{w} = 0, \theta = \bar{t}, \overline{C} = 1 \text{ at } \bar{y} = 0 \\ \bar{u} \to 0, \overline{C} \to 0, \theta \to 0, \overline{w} \to 0 \text{ as } \bar{y} \to \infty. \end{array}\right\}. \tag{11}$$

Dropping the bars and combining equations (7) and (8), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + Gr\theta + GmC - \left(\frac{M}{1+m^2}(1-mi)\right)q, \tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}, \tag{13}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}, \tag{14}$$

where $q = u + iw$.
The boundary conditions now become:

$$\left.\begin{array}{l} t \le 0, q = 0, C = 0, \theta = 0, , \text{for all values of } y \\ t > 0, q = \cos \omega t, \theta = t, C = 1 \text{ at } y = 0 \\ q \to 0\, C \to 0, \theta \to 0, \text{as } y \to \infty. \end{array}\right\} \tag{15}$$

Equations (12), (13) and (14), subject to boundary conditions (15), are solved by Laplace transform technique. The solution obtained is as follows,

$$\begin{aligned} q = {} & \frac{1}{4}e^{-it\omega} + A_0 + \frac{1}{4a^2}\, y\, Gr\{A_{13}(1 - Pr - at)\} + \sqrt{a}\, e^{-\sqrt{a}y}(A_1 - e^{2\sqrt{a}y}) + B_{13}\{A_{14}(1-Pr)\} \\ & + \frac{1}{2a}Gm(-e^{-\sqrt{a}y}A_1 + e^{\frac{at}{-1+Sc}\sqrt{\frac{aSc}{-1+Sc}}}(1 + B_{11} + e^{2\sqrt{\frac{aSc}{-1+Sc}}}B_{12})) - \\ & \frac{1}{2a^2\sqrt{\pi}}Gr\sqrt{Pr}\, y(-B_{14}\{-1 + \sqrt{Pr} + at\} + a(2e^{\frac{-\mathrm{Pr}\, y^2}{4t}}\sqrt{t} - \sqrt{\pi}\sqrt{Pr}\, y\{1 - Erf[\frac{\sqrt{Pr}\, y}{2\sqrt{t}}]\}) \\ & + \frac{1}{y}e^{\frac{at}{-1+Pr}\sqrt{\frac{a}{-1+Pr}}\sqrt{Pr}\, y}\sqrt{\pi}B_{17}\frac{1}{\sqrt{Pr}}Pr - 1) \\ & - \frac{1}{2a}Gm[-2Erfc[\frac{\sqrt{Sc}y}{2\sqrt{t}}] + e^{\frac{at}{-1+Sc}\sqrt{\frac{a}{-1+Sc}}\sqrt{Sc}y}(1 + B_{18} + e^{2\sqrt{\frac{aSc}{-1+Sc}}y}B_{19})] \end{aligned} \tag{16}$$

$$\theta = \left[\left(t + \frac{Pry^2}{2}\right)Erf\left(\frac{\sqrt{Pr}\, y}{2\sqrt{t}}\right) - e^{-\frac{Pry^2}{4t}}\frac{\sqrt{Prt}\, y}{\sqrt{\pi}}\right] \tag{17}$$

$$C = Erfc\left[\frac{\sqrt{Sc}\,y}{2\sqrt{t}}\right]. \tag{18}$$

More details of the notations are described in the appendix.

Skin friction

$$\left(\frac{dq}{dy}\right)_{y=0} = \tau_x + i\tau_z.$$

3. Discussion and Results

The numerical values of velocity and skin friction are computed for different parameters like Hall parameter (m), mass Grashof number (Gm), Schmidt number (Sc), time (t), thermal Grashof number (Gr), magnetic field parameter (M), Prandtl number (Pr), and phase angle (ωt) .The values of the main parameters considered are

m = 1, 5; Gm = 10, 20, 30; Sc= 2.01, 5, 10; t = 0.15, 0.2, 0.25;
$\omega t = 30^\circ, 45^\circ, 60^\circ$; Gr =10, 20, 30; M = 2, 3, 4; Pr = 0.71, 7.

It has been observed from Figures 2, 3, 4, and 5 that primary velocity (u) increases when m, Gm, Gr, and t are increased. It means, Hall current has increasing effect on the flow of the fluid along the plate. However, Figures 6, 7, 8, and 9 show that u decreases when Pr, M, Sc and ωt are increased. Almost similar pattern is observed for secondary velocity. Figures 10, 11, 12, 13, and 14 show that the secondary velocity (w) increases when Gm, M, Pr, Gr and t are increased. However, Figures 15, 16 and 17 show that w decreases when Sc, m and ωt are increased. This implies that the Hall parameter slows down the transverse velocity. Table 1 shows that Skin fraction τ_x decreases with increase in Sc, Pr and M; and it increases with Gr, Gm, m, t and ωt . On the other hand, τ_z increases with increase in Gr, Gm, t, and M; and it decreases with Pr, m, Sc and ωt . The results obtained are in agreement with the actual flow.

The results of the model can suitably be applied in the industries and organizations dealing with eclectically conducting fluid in the presence of magnetic field.

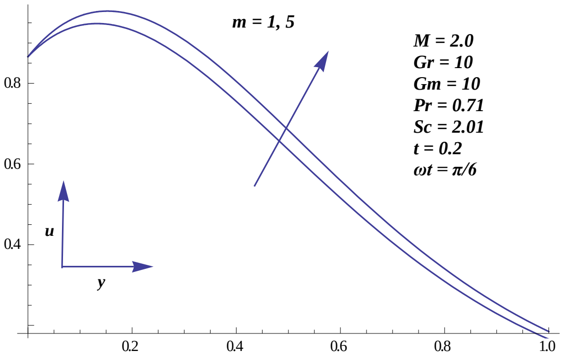

Figure 2. Velocity u for different values of *m*

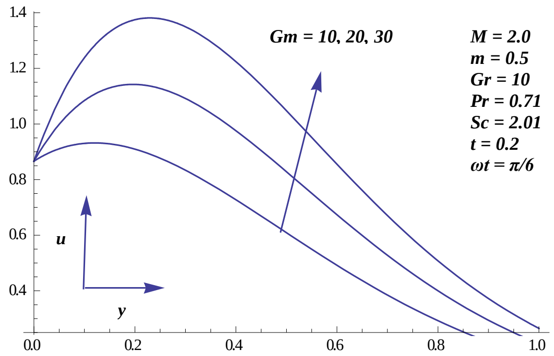

Figure 3. Velocity u for different values of *Gm*

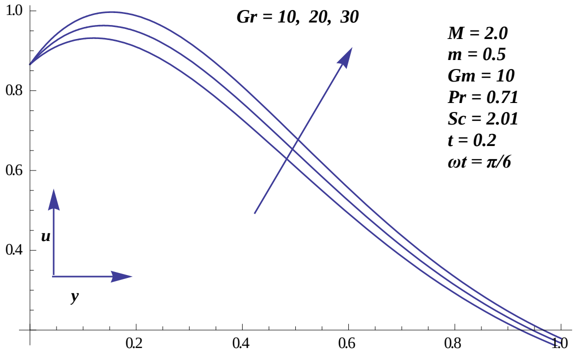

Figure 4. Velocity u for different values of *Gr*

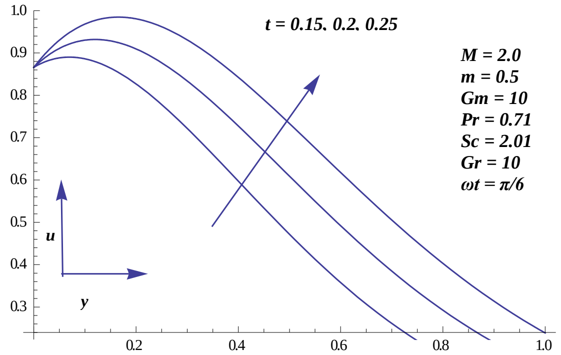

Figure 5. Velocity u for different values of *t*

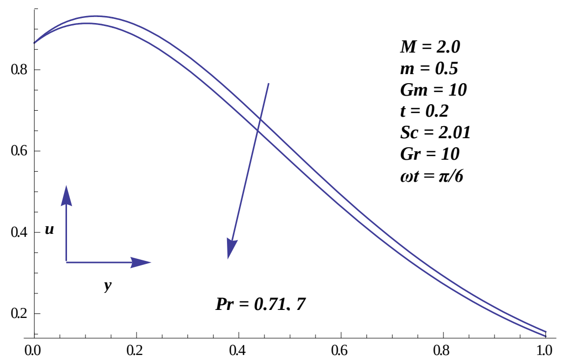

Figure 6. Velocity u for different values of *Pr*

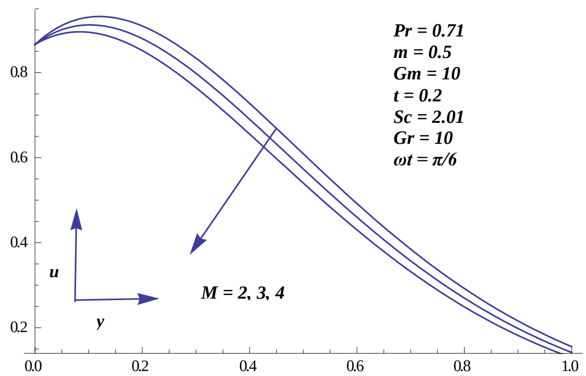

Figure 7. Velocity u for different values of *M*

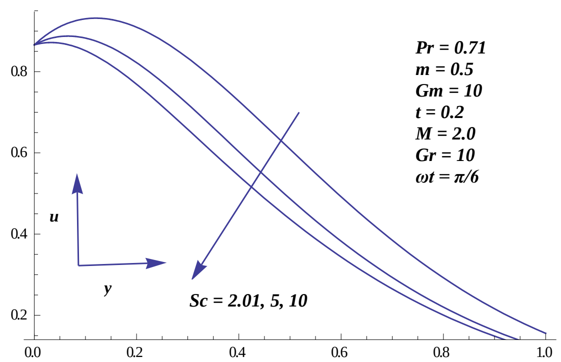

Figure 8. Velocity u for different values of *Sc*

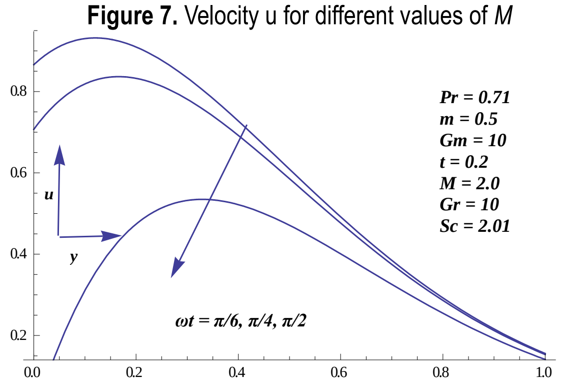

Figure 9. Velocity u for different values of ωt

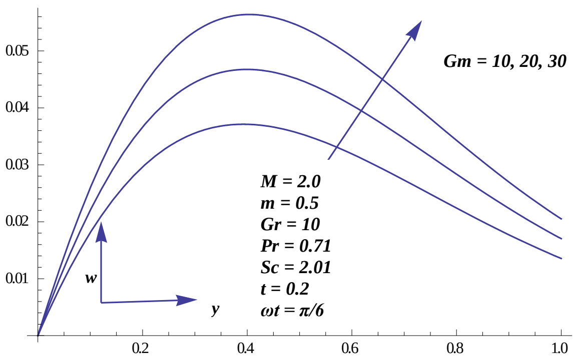

Figure 10. Velocity w for different values of *Gm*

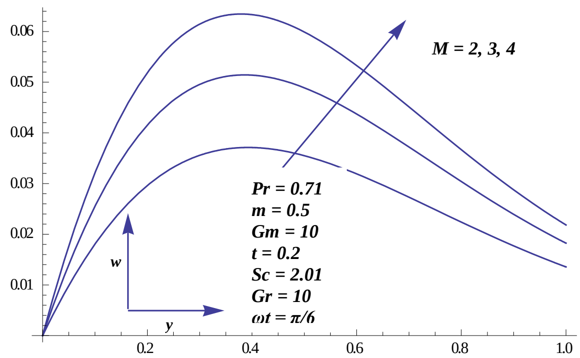

Figure 11. Velocity w for different values of *M*

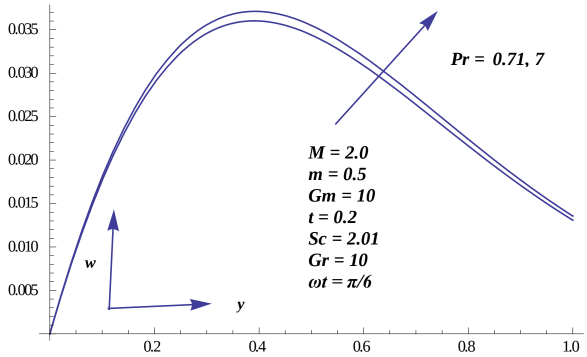

Figure 12. Velocity w for different values of *Pr*

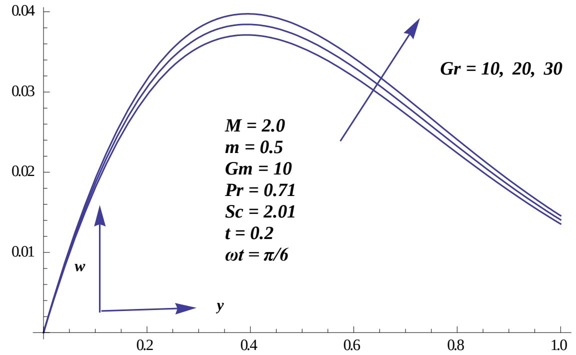

Figure 13. Velocity w for different values of *Gr*

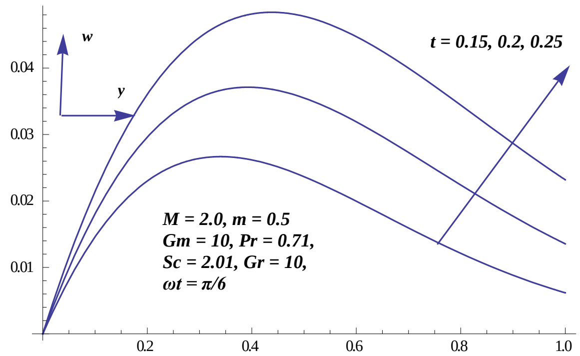

Figure 14. Velocity w for different values of *t*

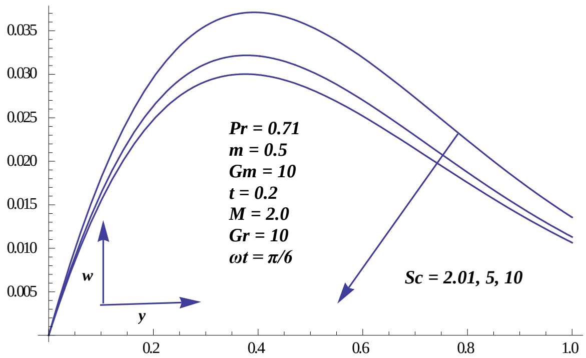

Figure 15. Velocity w for different values of *Sc*

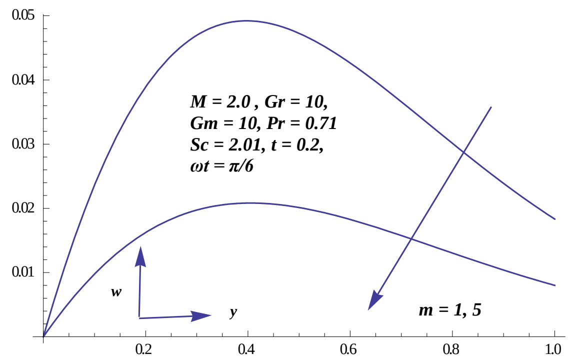

Figure 16. Velocity w for different values of *m*

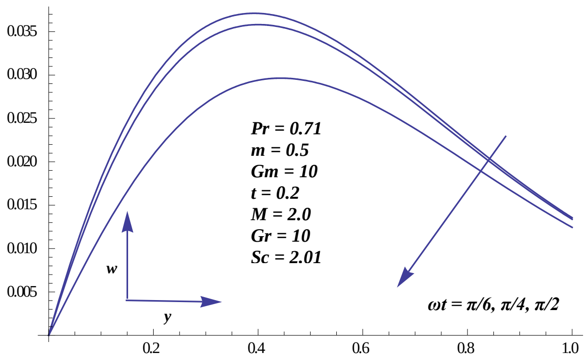

Figure 17. Velocity w for different values of ωt

4. Conclusion

The effect of Hall current is observed on both, the primary and the secondary velocities. It has been observed that the primary velocity increases with Hall parameter. On the other hand, secondary velocity decreases when Hall parameter is increased. Similar effect is observer for drag at boundary. That is τ_x increases with Hall parameter; and in contrary, τ_z decreases when Hall parameter is increased.

Table 1: Skin friction for different parameter

m	Gr	Gm	M	Sc	Pr	ωt (in degree)	t	τ_x	τ_z
0.5	10	10	2	2.01	0.71	30	0.2	1.1778	0.2150
0.5	10	10	2	2.01	7.00	30	0.2	1.0063	0.2107
0.5	20	10	2	2.01	0.71	30	0.2	1.5307	0.2208
0.5	30	10	2	2.01	0.71	30	0.2	1.8836	0.2265
1.0	10	10	2	2.01	0.71	30	0.2	1.3387	0.2795
5	10	10	2	2.01	0.71	30	0.2	1.6130	0.1146
0.5	10	20	2	2.01	0.71	30	0.2	3.1768	0.2560
0.5	10	30	2	2.01	0.71	30	0.2	5.1759	0.2971
0.5	10	10	3	2.01	0.71	30	0.2	0.9621	0.3062
0.5	10	10	4	2.01	0.71	30	0.2	0.7554	0.3884
0.5	10	10	2	5.00	0.71	30	0.2	0.6887	0.1972
0.5	10	10	2	10.0	0.71	30	0.2	0.3611	0.1881
0.5	10	10	2	2.01	0.71	30	0.15	0.7255	0.1798
0.5	10	10	2	2.01	0.71	30	0.25	1.5744	0.2489
0.5	10	10	2	2.01	0.71	45	0.2	1.7314	0.1983
0.5	10	10	2	2.01	0.71	90	0.2	4.0161	0.1207

References

[1] Datta, N. and Jana, R. N., 1976. Oscillatory magneto hydrodynamic flow past a flat plate with Hall effects. *Journal of the Physical Society of Japan*, 40, 5: 1469-1474.

[2] Satya Narayana, P.V., Venkateswarlu, B. and Venkataramana, S. 2013.Effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system. *Ain Shams Engineering Journal*, 4, 4:843-854.

[3] Watanabe, T. and Pop, I. 1995. Hall effects on magnetohydrodynamic boundary layer flow over a continuous moving flat plate.*ActaMechanica*,108, 1: 35-47.

[4] Satya Narayana, P.V., Rami Reddy, G. and Venkataramana, S. 2011. Hall current effects on free convection MHD flow past a porous plate. *International Journal of Automotive and Mechanical Engineering*, 3, 350-363.

[5] Sudhakar, K., Srinivasa Raju, R. and Rangamma, M. 2013. Hall effect on unsteady MHD flow past along a porous flat plate with thermal diffusion, diffusion thermo and chemical reaction. *International Journal of Physical and Mathematical Sciences*.4, 1:370-395.

[6] Deka, R. K., 2008. Hall effects on MHD flow past an accelerated plate. *Theoretical and Applied Mechanics*, 35, 4: 333-346.

[7] Muthucumaraswamy, R. and Janakiraman, B. 2008, Mass transfer effects on isothermal vertical oscillating plate in the presence of chemical reaction. *International Journal of Applied Mathematics and Mechanics*, 4, 1: 66–74.

[8] Satya Narayana, P. V., Venkateswarlu, B., and Devika, B. 2015. Chemical reaction and heat source effects on MHD oscillatory flow in an irregular channel. *Ain Shams Engineering Journal*, in press. Retrieved from http://dx.doi.org/10.1016/j.asej.2015.07.012.

[9] Devika, B., Satya Narayana,P.V., and Venkataramana, S. 2013. MHD Oscillatory flow of a visco elastic fluid in a porous channel with chemical reaction. *International Journal of Engineering Science Invention*, 2, 2: 26-35.

[10] Sharma, S. and Deka, R. K.2012. Thermal radiation and oscillating plate temperature effects on MHD unsteady flow past a semi-infinite porous vertical plate under suction and chemical reaction. *International Journal of Physics and Mathematical Sciences*, 2, 2: 33-52.

[11] Rajput, U. S. and Kumar, S. 2013. Radiation effect on MHD flow through porous media past an impulsively started vertical plate with variable heat and mass transfer. *International Journal of Mathematical Archive*, 4, 10: 106-114.

[12] Attia, H., 2002. Unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties. *Applied Mathematical Modelling*, 26, 9: 863–875.

[13] Rajput, U. S. and Kumar, S. 2011. MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion. *Applied Mathematical Sciences*, 5, 3: 149-157.

[14] Kafousias, N. G. and Raptis, A. 1981. Mass transfer and free convection effects on the flow past an accelerated vertical plate with variable suction or injection. *Revue Roumaine des Sciences Techniques, Serie de Mecanique Appliquee*, 26: 11-22.

Appendix

$$A_0 = A + B + C_0 + D_0 - E_0 - F_0 - G_0 - H_0,$$

$$A_1 = A_{11} + A_{12},\ A = e^{y\sqrt{a-i\omega}},\quad B = e^{y\sqrt{a-i\omega}},\quad C_0 = e^{-y\sqrt{a+i\omega}+2it\omega},\quad D_0 = e^{y\sqrt{a+i\omega}+2it\omega},$$

$$E_0 = AErf\left[\frac{y-2t\sqrt{a-i\omega}}{2\sqrt{t}}\right],\quad F_0 = BErf\left[\frac{y+2t\sqrt{a-i\omega}}{2\sqrt{t}}\right],\quad G_0 = C_0 Erf\left[\frac{y-2t\sqrt{a+i\omega}}{2\sqrt{t}}\right],\quad H_0 = D_0 Erf\left[\frac{y+2t\sqrt{a+i\omega}}{2\sqrt{t}}\right],$$

$$A_{11} = 1 + Erf\left[\frac{2\sqrt{a}t - y}{2\sqrt{t}}\right],\quad A_{12} = e^{2\sqrt{a}y} Erfc\left[\frac{2\sqrt{a}t + y}{2\sqrt{t}}\right],\quad A_{14} = \frac{2e^{-\sqrt{a}y}\left(-1 - e^{2\sqrt{a}y} - A_{11} + A_{12}\right)}{y},$$

$$B_1 = \frac{Efr\left[2\sqrt{\frac{aPr}{-1+Pr}}t - y\right]}{2\sqrt{t}},\quad B_2 = \frac{Efr\left[2\sqrt{\frac{aPr}{-1+Pr}}t + y\right]}{2\sqrt{t}},$$

$$B_{11} = \frac{Efr\left[2\sqrt{\frac{aSc}{-1+Sc}}t - y\right]}{2\sqrt{t}},\quad B_{12} = \frac{Efr\left[2\sqrt{\frac{aSc}{-1+Sc}}t + y\right]}{2\sqrt{t}},\quad B_{13} = \left(-1 - e^{\frac{2\sqrt{aPr}}{-1+Pr}y} - B_1 + e^{\frac{2\sqrt{aPr}}{-1+Pr}y} B_2\right),\quad B_{14} = \frac{2\sqrt{\pi}\left(-1 + Erf\left[\frac{\sqrt{Pr}\,y}{2\sqrt{t}}\right]\right)}{\sqrt{Pr}\,y},$$

$$B_{15} = \frac{Efr\left[2\sqrt{\frac{a}{-1+\mathrm{Pr}}}t - \sqrt{Pr}\,y\right]}{2\sqrt{t}},\quad B_{16} = \frac{Efr\left[2\sqrt{\frac{a}{-1+\mathrm{Pr}}}t + \sqrt{Pr}\,y\right]}{2\sqrt{t}},\quad B_{17} = \left(1 + e^{2\sqrt{\frac{a}{-1+\mathrm{Pr}}}\sqrt{Pr}\,y} + B_{15} - e^{2\sqrt{\frac{a}{-1+\mathrm{Pr}}}\sqrt{Pr}} B_{16}\right),$$

$$B_{18} = Erf\,\frac{\left[2\sqrt{\frac{a}{1+Sc}}t - \sqrt{Sc}\,y\right]}{2\sqrt{t}},\quad B_{19} = Erfc\,\frac{\left[2\sqrt{\frac{a}{1+Sc}}t + \sqrt{Sc}\,y\right]}{2\sqrt{t}},\ a = \frac{M}{1+m^2}(1 - im),$$