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A Robust Group Multiple Attributes Decision-Making Method Based on Risk Preference of the Decision Makers

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Abstract: In this paper, we propose a robust multiple attributes decision-making (MADM) method based on prospect theory to reflect the decision behavior of a decision maker in face of risk. Instead of identifying the reference points, the decision makers only need to determine the feasible ranges for each attribute by their knowledge and experience in the beginning of the decision process. The psychological value distances are defined to measure the overall prospect values of each alternative reference to extreme feasible solutions using the value function and the additive weighting method. This study further extends the method to a group decision environment. The preferences of more than one decision maker are internally aggregated into the decision procedure. Performance of the proposed algorithms is comparatively analyzed and sensitivity analysis is conducted. The results show that it is an appropriate and robust MADM method.

Keywords: Decision support systems; MADM; prospect theory; value function

1. Introduction

MADM is an approach that consists of dealing with structuring and solving decision problems involving multiple attributes. A typical MADM requires comparing the aggregated performance ratings. These comparison processes, however, can be quite complex and produce results that may be unreliable. Over many years researchers have been developing many popular methods for structuring and solving multiple-attributes decision problems [1-4]. The evaluations of decision attributes for alternatives made by these methods are based on expected utility theory, with the assumption that the decision maker is rational, and perhaps risk averse, which implies that their utility functions are concave and show diminishing marginal utility. Thus, practically these methods start from the premise that the decision maker always looks for the solution corresponding to the maximum utility [5]. According to the principle of expected utility function, the utility of a risky prospect has linear outcome probability. However, it has been argued that utility theory is not able to capture or take into account the risk preferences of the decision makers [6-7]. Kahneman and Tversky [6] provided evidence that people have nonlinear preferences and tend to take risks to avoid losses. In this paper, we use value function to describe the risk-averse and risk-seeking behavior of decision makers, and as the basis for alternative ranking.

Considering the risk attitude of decision maker in MADM study gets more and more attention [8-12]. Prospect Theory predicts that the value assigned to an option is determined by comparison to other options. The reference points of each concerning attribute use in this comparison is therefore of critical importance. But, what are the origins of the decision makers’reference point?

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In a group decision process, how does the group attain consensus on the reference point? Most of the methods base on the assumption that the neutral points of reference can be identified first. However, few studies have explored how to obtain the reference points.

The present study proposes a new MADM method called EBVD (election based on value distances), which is based on the prospect theory. Decision makers make decisions based on the potential value of losses and gains rather than to final assets. Instead of identifying the reference points, the decision makers only need to determine the feasible ranges for each attribute by their knowledge and experience. Two extreme feasible solutions, the extreme positive feasible solution (EPFS) and the extreme negative feasible solution (ENFS), are identified first. Then, we calculate the overall value distances (prospect values) of EPFS to each alternative and the distances of ENFS to each alternative using the value function and the additive weighting method. The value function from prospect theory is proposed to describe and explain user behavior in the decision-making under risk. Finally, According to the value distances, a multiple attributes ranking index is developed to evaluate the preference ranking. Through our numerical examples, the present study can demonstrate the EBVD method appears to be an appropriate and robust MADM method.

In practical, many decision-making problems within organizations will be a collaborative effort. This study also extends EBVD to a group decision environment to fit real work. To simplify the decision-making activities, we develop an integrated group EBVD procedure using internal preference aggregation method for solving group decision problems. A complete and efficient group decision procedure is provided.

The remaining of this paper is organized as follows: The related works of our study is given in section 2. Section 3 describes the EBVD methods. Some numerical examples, applying the EBVD method to evaluate the MADM problems, are presented. Section 4 introduces the group EBVD method. Conclusions are presented in Section 5.

2. Related works

MADM is a procedure that consists in finding the best alternative among a set of feasible alternatives. An MADM problem with m alternatives, $A_1,\ldots, A_m$, and n decision attributes, $C_1, \ldots, C_n$, can be expressed in the following matrix format:

$$
\begin{array}{cc}
 & \begin{array}{cccc} w_1 & w_2 & \cdots & w_n \end{array} \\
 & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\
A = \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{bmatrix}
\end{array}
$$

where d_{ij} represents the rating of alternative A_i under attribute C_j, and w_j is the relative weight of attribute C_j.

One of the most popular MADM methodologies is the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method, first introduced in Hwang and Yoon [1]. It is based on the intuitive concept that the elected alternative must have the shortest Euclidian distance to a positive ideal solution (PIS) and be as far as possible from the negative-ideal solution (NIS). TOPSIS is a utility-based method that compares each alternative directly depending on data in the evaluation matrices and weights [13]. Zanakis *et al.* [14] made comparisons between eight MADM methods and found that TOPSIS has the fewest rank reversals. However, Wang and Luo [15] note the problem of rank reversal in TOPSIS when the alternatives are close. Opricovic and Tzeng [16] believe the problem is caused by the definition of closeness coefficient. García-Cascales and Lamata [17] pointed out that it is caused by two factors. The first is related to the norm utilized in the algorithm and the other to the definition of the PIS and the NIS. TOPSIS method has been extended to deal with different decision problems in the past few years. For example, Yue [18] established an extended TOPSIS model to handle the decision problem that the attribute values are not precisely known but value ranges can be obtained. Baky and Abo-Sinna [19] proposed a fuzzy TOPSIS algorithm to solve bi-level multi-objective decision-making (BL-MODM) problems. Kahraman *et al.*, [20] summarize the fuzzy MADM methods.

Kahnema and Tversky [6] discovered that human decision behavior under uncertainty is actually relative, in the sense that some individuals are risk-seeking, and some are risk-averse. And in most of the situation facing gains, risk is to be avoided. Prospect theory [6] is a descriptive model of individual decision making under the condition of risk. In 1992, Tversky and Kahneman [21] developed the cumulative prospect theory, which captures psychological aspects of decision-making under risk. The value function in prospect theory is defined on deviations from a reference point. It assumed a S-shape value function. The concave part above the reference point reflects the aversion of risk in face of gains, and the convex part below the reference point reflects the seeking of risk in the case of losses. The value function defined in this paper is in form of a power law according to the following Equation [21]:

$$v(x)=\begin{cases} (x-\varphi)^{\alpha} & \text{if } x>\varphi \\ -\lambda(\varphi-x)^{\alpha} & \text{otherwise} \end{cases}, \tag{6}$$

where x is the potential outcome and φ is the reference point. Parameter λ represents the attenuation factor of the losses, which can be tuned according to the problem at hand. α is the diminishing sensitivity parameters. The diminishing sensitivity parameter $\alpha < 1$ yields an S-shape value function; $\alpha > 1$ produces an inverse S-shape value function. There are three characteristics of the value function [21]: reference dependence, diminishing sensitivity, and loss aversion. Reference dependence refers to the fact that human cognitive ability is based on relative value changes, the so-called endowment effect or status quo bias, and with the consideration for costs, current status tends to be kept unchanged. The diminishing sensitivity is defined by the decreasing utility as the gain increases. Finally, for loss aversion, individuals tend to feel a greater sense of pain on losses than the degree of happiness they sense on gains.

Prospect theory has been widely used as behavioral model of decision-making under risk, mainly in economics and finance [22-24]. One of the first MADM methods based on prospect theory was TODIM (an acronym in Portuguese for iterative multi-attributes decision making) proposed by Gomes and Lima [8]. Basically, TODIM is described in the following steps [5, 8]:

Step 1. Normalize the decision matrix.

Like the TOPSIS, the first step of TODIM is to obtain the normalized decision matrix $R=[r_{ij}]_{m\times n}$.

Step 2. Calculate the measurement of dominance of each alternative A_i over each alternative A_j.

For such calculations the decision makers need to define a reference attribute, which usually is the attribute with the most weight. In addition, let w_{pc} be equal to w_c divided by w_p, where p is the reference attribute. Then the dominance of each alternative A_i over each alternative A_j can be calculated as

$$\delta(A_i, A_j) = \sum_{c=1}^{n} \varphi_c(A_i, A_j) \ \forall(i, j), \tag{7}$$

where

$$\varphi_c(A_i, A_j) = \begin{cases} \sqrt{\dfrac{w_{pc}(r_{ic} - r_{jc})}{\sum_{c=1}^{n} w_{pc}}} & if(r_{ic} - r_{jc}) > 0, \\ 0 & if(r_{ic} - r_{jc}) = 0, \\ -\dfrac{1}{\theta} \cdot \sqrt{\dfrac{(\sum_{c=1}^{n} w_{pc}) \cdot (r_{jc} - r_{ic})}{w_{pc}}} & if(r_{ic} - r_{jc}) < 0. \end{cases}$$

The term represents the contribution of the attribute c to the function when comparing the alternative Ai with alternative Aj. θ is the attenuation factor of the losses; and different choices of θ lead to different shapes of the prospect theoretical value function in the negative quadrant. TODIM makes use of a Prospect Theory type of value function that is algebraically quite similar to Cumulative Prospect Theory's value function. The trade-off weighting factors of TODIM are interpreted as probabilities.

Step 3. Calculate the overall value of alternative Ai through normalization of the corresponding dominance measurements by the following expression:

$$\varsigma_i = \frac{\sum_{j=1}^{m} \delta(A_i, A_j) - \min_{i \in \{1 \cdots m\}} \sum_{j=1}^{m} \delta(A_i, A_j)}{\max_{i \in \{1 \cdots m\}} \sum_{j=1}^{m} \delta(A_i, A_j) - \min_{i \in \{1 \cdots m\}} \sum_{j=1}^{m} \delta(A_i, A_j)}. \tag{8}$$

Step 4. Ordering the overall values ς_i provides the rank of each alternative. The preferred alternatives are those that have higher overall value.

According to prospect theory, decision makers decide which outcomes they consider equivalent set a reference point and then consider lesser outcomes as losses and greater ones as gains. TODIM, however, uses a different way, by using of pair comparisons between decision attributes and the reference points are not determined initially. When comparing alternative A_i with alternative A_j under a certain attribute, it represents a gain if the outcome of alternative A_i is larger than alternative A_j; and it represents a loss if the outcome of alternative A_i is smaller than alternative A_j. Although TODIM does not deal with risk directly, it deals with the risk attitude of decision maker [25]. Liu *et al.*, [9] and Fan *et al.*, [10] also develop MADM methods based on prospect theory. Different from TODIM, the gains and losses of alternatives are calculated by measuring perceived differences of attribute values from reference points. The overall prospect value of alternative is calculated by simple additive weighting method. Wei *et al.*, [11] and Wang *et al.*, [12] extend TODIM method using hesitant fuzzy linguistic numbers to describe the preferences of decision makers. They find the TODIM method is more practical than the TOPSIS method when solving practical decision-making problems. Lourenzutti and Krohling [26] bring the Hellinger distance to the MADM context to assist TOPSIS and TODIM to deal with the situation that the ratings of alternatives are not real numbers. Khamseh and Mahmoodi [27] use fuzzy TOPSIS to evaluate initial weight of each attribute, and then use TODIM to evaluate the final weight of each attribute against alternatives and the relationship between attributes.

3. The EBVD model

The EBVD is a multi-attribute decision making method incorporating prospect theory, which is used to reflect the decision behavior of a decision maker in face of risk. The use of the new model relies on a value function to replace the traditional expected utility function for multi-attributes. This function is built in parts, with their mathematical descriptions reproducing the gain/loss function of prospect theory. To prevent the valuations of the alternatives for each of the attributes can depend on the rest of the alternatives, the EPFS and ENFS are fixed, so any valuation with reference to them cannot change.

EBVD is based on the intuitive concept that not only EPFS should have the least relative prospect value for the elected alternative, but also the elected alternative should have the greatest relative prospect value for ENFS. The detailed steps are described as follows:

Step 1. Determine the EPFS and ENFS.

To determine the EPFS and ENFS, the feasible range of values of each attribute has to be agreed upon by decision maker. The range is based upon decision maker's experience with or knowledge of the particular attribute. We determine the EPFS (I^+) and ENFS (I^-) as follows:

$$I^+ = \{D_1^+, \dots, D_n^+\}, I^- = \{D_1^-, \dots, D_n^-\} \tag{9}$$

where D_j^+ and D_j^- are the "best feasible" and "worst feasible" values assigned to attribute j.

Skin friction is given in Table 1. The value of τ_x increases with the increase in thermal Grashof number, mass Grashof Number, Hall current parameter, radiation parameter and time; and it decreases with the angle of inclination of plate, the magnetic field, Prandtl number and Schmidt number. Similar effects are observed with τ_y except magnetic field and Hall parameter, in which case τ_y increases with magnetic field parameter, and decreases with Hall parameter. Nusselt number is given in Table 2. The value of Nu decreases with increase in Prandtl number, radiation parameter and time.

Step 2. Construct the normalized decision matrix R.

To compare the alternatives on each attribute, an interval scale transformation is used to transform the various attributes scale into a comparable scale. The norm that the TOPSIS approach establishes may cause the problem of rank reversal. This is because that after normalization, the new scale depends not only on the initial value but also on the valuation obtained by the other alternatives [17]. It can change the scale when an alternative is added to or removed from the decision problem. In the circumstance, the normalized value r_{ij} of the decision matrix is obtained by the following equation:

$$\begin{aligned} r_{ij} &= \frac{d_{ij}-D_i^-}{D_i^+-D_i^-},\ j \in \text{benefit type attributes (the larger the better)}, \\ r_{ij} &= \frac{D_i^- - d_{ij}}{D_i^- - D_i^+},\ j \in \text{cost type attributes (the smaller the better)}. \end{aligned} \tag{10}$$

The normalized decision matrix $R=[r_{ij}]_{m\times n}$. Note that we presume the available data to be completed in the given decision matrix, including quantitative and qualitative information. The normalization of qualitative data or linguistic data could be first transformed to a linear scale, e.g., 1–10; and then the above method will be applicable. Taking into account that the EPFS $I^+ = \{D_1^+, \cdots, D_n^+\}$ and the ENFS $I^- = \{D_1^-, \cdots, D_n^-\}$, the vectors of the normalized values are $\{1,1,\ldots,1\}$ and $\{0,0,\ldots,0\}$, respectively.

Step 3. Calculate the overall prospect values.

According to the prospect theory, people make decisions based on the potential value of losses and gains rather than the final outcome. The value distances represent the dominance of one alternative under an attribute over another alternative. It can be a concave function and then it implies that decision maker tends to risk averse in a domain of gain and a convex function reflects the seeking of risk in the case of losses. Kahneman and Tversky [6] provided evidence that people have nonlinear preferences but not linear preferences. The vale distances present a nonlinear preference of the decision maker. Instead of using Euclidean distances, we apply value distances to represent the separation measures of alternative A_i from the EPFS and ENFS. To calculate the value distance from EPFS to alternative A_i, we use the following expression.

$$S_i^+ = \sum_{j=1}^{n} w_j \cdot (1 - r_{ij})^{\alpha}, \ i = 1, \ldots, m. \tag{11}$$

It's a weighted value function. Compared with A_i, I^+ can provide more prospect value to the decision maker. The value distance represents the dominance of I^+ over alternative A_i. The concave function implies that decision maker tend to risk averse in a domain of gain. It estimates the overall prospect value of the EPFS can provide to the decision maker when alternative A_i is selected.

A different function [Equation (12)] is used to measure the value distance from alternative A_i to ENFS.

$$S_i^- = \sum_{j=1}^{n} w_j \cdot (-\lambda \cdot (r_{ij})^{\alpha}), \ i = 1, \ldots, m. \tag{12}$$

The function is a convex function that implies that decision maker tends to risk seeking in a domain of loss. It is used to estimate the loss when the worst feasible solution I^- is selected to replace A_i. It is noted that the output value of S_i^- is negative.

Step 4. Calculate the ranking index.

The ranking index of each alternative is calculated as

$$\phi_i = \frac{|S_i^-|}{|S_i^+| + |S_i^-|}, \quad i = 1, \ldots, m. \tag{13}$$

where ϕ_i is a number between 0 and 1; the larger this values is, the less prospect value the EPFS can provide to the decision maker when the alternative is selected, but the selected alternative can provide higher prospect value to the ENFS, and so this alternative should be ranked higher.

The following examples are given to illustrate the proposed model and evaluate the performance of EBVD.

Example 1.

One problem with TOPSIS is that it may derive a false preference ranking when two alternatives are very close. In this experiment, a decision maker must choose an alternative from a set of five alternatives, *i.e.* $\{A_1, A_2, A_3, A_4, A_5\}$. Two benefit attributes with equal weights are used to evaluate the five alternatives. The original data is from [28] and the evaluation results are presented in Table 1.

Table 1. The decision matrix and results obtained by TOPSIS and EBVD of example 1

Alternatives	Criteria		TOPSIS				EBVD			
	C1	C2	S_i^+	S_i^-	ϕ_i	Rank	S_i^+	S_i^-	ϕ_i	Rank
A1	85	50	0.08	0.14	0.6	3	0.36	-1.586	0.8	3
A2	40	64	0.15	0.04	0.2	5	0.52	-1.260	0.7	5
A3	50	70	0.11	0.07	0.3	4	0.44	-1.433	0.7	4
A4	80	70	0.02	0.14	0.8	1	0.29	-1.746	0.8	2
A5	75	76	0.03	0.14	0.8	2	0.29	-1.757	0.8	1
Weights	0.5	0.5								

The TOPSIS and EBVD provide different ranking results. Using the TOPSIS method, A_4 should be the best alternative since it has the highest closeness coefficient value with respect to the other alternatives. However, if we remove alternative A_1, A_2 and A_3 from the alternative set and conduct TOPSIS method in this experiment again. The final closeness coefficient values of A_4 and A_5 are 0.44 and 0.56, respectively. The results show A_5 is a better solution than A_4, which violate the invariance principle of utility theory. To prevent the problem, absolute positive ideal solution and absolute negative ideal solution are introduced to the new method EBVD. It makes any valuation with reference to them cannot change. The concave and convex functions deal with the risk attitude of decision maker. It can be observed when we only consider A_4 and A_5 in the choice set and use the EBVD method (assume EPFS={100,100} and ENFS={0,0}), then A_5 still has higher closeness coefficient value than A_4 (A_4:0.856 and A_5:0.858).

An alternative model of decision making, in this case the TODIM method, is applied to this experiment too. The first thing to be noted is that the ranking results are consistent with TOPSIS (see Table 2) in case 1: $A_4 > A_5 > A_1 > A_3 > A_2$. However, if we only consider A_4 and A_5 in the choice set (case 2) then the ranking order is reversed, too.

Table 2. The final values and rank obtained by TODIM

Alternatives	Criteria		Case 1		Case2	
	C1	C2	ς_i	Rank	ς_i	Rank
A1	85	50	0.485	3	-	-
A2	40	64	0.000	5	-	-
A3	50	70	0.409	4	-	-
A4	80	70	1.000	1	0.000	2
A5	75	76	0.990	2	1.000	1

Example 2.

We use the numerical example provided by García-Cascales and Lamata [17] to additional verify our model. This example considers the evaluation of three candidates to occupy a certain position. The candidates complete two questionnaires that will both be evaluated. The questionnaires have the same weight. Table 3 presents the basic data of the decision problem. The total ratings for all three candidates are equal based on weighted sum method.

Table 3. Decision matrix of example 2 [17]

	C1	C2
A1	1	5
A2	4	2
A3	3	3
Weights	0.5	0.5

TOPSIS and EBVD are applied to solve the MADM problem. The evaluation results are shown in Table 4.

Table 4. Results obtained by TOPSIS and EBVD of example 2

Alternatives	TOPSIS				EBVD			
	S_i^+	S_i^-	ϕ_i	Rank	S_i^+	S_i^-	ϕ_i	Rank
A1	0.294	0.243	0.453	3	0.211	-	0.601	3
A2	0.243	0.294	0.547	1	0.173	-	0.648	2
A3	0.190	0.212	0.528	2	0.202	-	0.682	1

Now, suppose that one new alternative is added for evaluation with a valuation of (5, 1) for (C1, C2), respectively. Table 5 represents the new evaluation results.

Table 5. Results obtained by TOPSIS and EBVD of example 2 with the incorporation of a new alternative

Alternatives	Criteria		TOPSIS				EBVD			
	C1	C2	S_i^+	S_i^-	ϕ_i	Rank	S_i^+	S_i^-	ϕ_i	Rank
A1	1	5	0.2	0.32	0.5	1	0.17	-0.462	0.6	3
A2	4	2	0.2	0.22	0.4	3	0.21	-0.446	0.6	2
A3	3	3	0.2	0.21	0.5	2	0.20	-0.468	0.6	1
A4	5	1	0.3	0.28	0.4	4	0.20	-0.399	0.6	4

It can be observed that candidate A_1, who was previously the worst, has now become the best when we apply TOPSIS in this case. Thus the introduction of a new alternative makes the order totally reversed. The other 24 simulated new alternatives with valuations from (1,2) to (5,5) have been conducted to verify the EBVD model. It is noteworthy that the EBVD has no problem evaluating the performance of all the alternatives, and there is no rank reversal problem in all the cases. However, there are 5 cases (20%) with rank reversal problem when TOPSIS is applied in this experiment.

Example 3.

The parameter λ represents the attenuation factor of the losses, which can be tuned according to the problem at hand. Kahneman and Tversky [6] suggest that the value of λ should be between 2.0 and 2.5. It is of interest to test the effects of parameter λ on the final ranking order. On the basis of the same decision information as for experiment 1, we further investigate the effects of λ on the proposed model. This experiment varies λ, from 2.0 to 2.5. α is set to be 0.88. The results (see Table 6) demonstrate that EBVD is a robust methodology. The relative coefficient will increase as λ increases. However, λ in the suggested range, 2.0 to 2.5, does not affect the final ranking order at all.

Table 6. Results obtained by EBVD with attenuation factor of the losses varied

	λ=2.0		λ=2.1		λ=2.25		λ=2.3		λ=2.4		λ=2.5	
	ϕ_i	Ra	ϕ_i	Ra	ϕ_i	Ra	ϕ_i	Ra	ϕ_i	Ran	ϕ_i	Ra
A1	0.79	3	0.80	3	0.81	3	0.81	3	0.82	3	0.82	3
A2	0.68	5	0.69	5	0.70	5	0.71	5	0.72	5	0.72	5
A3	0.74	4	0.75	4	0.76	4	0.76	4	0.77	4	0.78	4
A4	0.84	2	0.84	2	0.85	2	0.85	2	0.86	2	0.86	2
A5	0.84	1	0.85	1	0.85	1	0.86	1	0.86	1	0.87	1

According to the above experiments, EBVD provides outstanding performance. It prevents the problem of rank reversals and provides effective analysis when the alternatives are very close.

4. The Group EBVD

The EBVD can be further extended to include the multiple preferences of more than one decision maker. The preferences of more than one decision maker are internally aggregated into the EBVD process. The detailed procedure is illustrated in the following.

Step 1. Construct decision matrix A^k, k=1,…,K, for each decision maker.

The structure of the matrix can be expressed as follows:

$$A^k = \begin{array}{c} \\ \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} w_1^k & \cdots & w_n^k \\ C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} d_{11}^k & \cdots & d_{1n}^k \\ \vdots & \ddots & \vdots \\ d_{m1}^k & \cdots & d_{mn}^k \end{bmatrix} \end{array}$$

where d_{ij}^k indicates the performance rating of alternative A_i with respect to attribute C_j by decision maker k, k=1,…,K. w_j^k is the relative weight of attribute C_j assigned by decision maker k. It should be noted that there are K decision matrices in the decision problem.

Step 2. Determine the EPFS and ENFS for each decision maker.

For decision maker k, his or her EPFS (I^{k+}) and ENFS (I^{k-}) are

$$I^{k+} = \{D_1^{k+}, \dots, D_n^{k+}\}, I^{k-} = \{D_1^{k-}, \dots, D_n^{k-}\}.$$

where D_j^{k+} and D_j^{k-} are the “best feasible” and “worst feasible” values assigned to attribute j by decision maker k. However, in this stage, the decision makers may agree on the same setting of EPFS and ENFS based on a negotiation process.

Step 3. Construct the normalized decision matrix R^k, k=1…,K, for each decision maker.

We consider the normalized value r_{ij}^k of decision matrix R^k is obtained by the following interval scale transformation

$$r_{ij}^k = \frac{d_{ij}^k - D_j^{k-}}{D_j^{k+} - D_j^{k-}},\ j \in \text{ benefit attributes,} \tag{14}$$

$$r_{ij}^k = \frac{D_j^{k-} - d_{ij}^k}{D_j^{k-} - D_j^{k+}},\ j \in \text{ cost attributes,} \tag{15}$$

where i = 1,…, m; j=1,…, n; and k=1,…,K.

Step 4. Calculate the overall prospect values individually.

For decision maker k, his/her separation measures from EPFS and ENFS are computed by a S-Shape value function. The individual separation measures of each alternative from the EPFS and ENFS are

$$S_i^{k+} = \sum_{j=1}^{n} w_j^k \,(1 - r_{ij}^k)^{\alpha} \quad \text{, for alternative } i,\ i = 1,\ldots,m \tag{16}$$

and

$$S_i^{k-} = \sum_{j=1}^{n} w_j^k \left(-\lambda \cdot \left(r_{ij}^k\right)^{\alpha}\right) \quad \text{, for alternative } i,\ i = 1,\ldots,m. \tag{17}$$

Step 5. Aggregate individual separation measures.

The integrated process will obtain multiple sources of knowledge and experience from different decision makers. There are several different methods to aggregate the separation measures of EPFS and ENFS for the group, for example, using geometric mean, arithmetic mean, or other modification. They are ease of use in a MADM process. We take the geometric mean of all individual measures. The group separation measures from EPFS and ENFS are

$$\overline{\overline{S_i^+}} = \left(\prod_{k=1}^{K} S_i^{k+}\right)^{\frac{1}{K}} \quad \text{, for alternative } i,\ i= 1,\ldots,m \tag{18}$$

And

$$\overline{\overline{S_i^-}} = \left(\prod_{k=1}^{K} |S_i^{k-}|\right)^{\frac{1}{K}} \quad \text{, for alternative } i,\ i= 1,\ldots,m. \tag{19}$$

It should be mentioned again this is not necessary using geometric mean in EBVD.

Step 6. Calculate the aggregated ranking index for the group.

The ranking index of each alternative for the group is calculated as

$$\overline{\overline{\phi_i}} = \frac{\overline{\overline{S_i^-}}}{\overline{\overline{S_i^+}} + \overline{\overline{S_i^-}}} \quad \text{, for alternative } i,\ i= 1,\ldots,m, \tag{20}$$

where $0 \le \overline{\overline{\phi_i}} \le 1$. It is clear that the larger the index, the better the performance of the alternative. Now the alternatives can be preference ranked according to the descending order of $\overline{\overline{\phi_i}}$.

Next, we illustrate the approach by means of a case study. The decision problem is provided by Shih *et al.* [29]. A chemical company is going to choose an on-line manager from 17 qualified candidates. Four managers are responsible to evaluate the 17 alternatives. Due to space limitations, just one of the initial decision matrices is illustrated in Table 7. The weights of attributes, elicited by the decision maker, are also shown in the same Table.

In this example, criteria C1 to C5 are objective attributes. There is no difference in these attributes among the group. The other two attributes, C6 and C7, are subjected attributes. And we can find all the 7 attributes are benefit type attributes.

Following the proposed procedure, assume that all the decision makers agree on the EPFS and ENFS for all attributes are 100 and 30, respectively. The normalized decision matrices can be constructed and the separation measures from EPFS and ENFS can be calculated. In this example, parameters α and λ in our value function are set to be 0.88 and 2.0, respectively. The separation measures for each individual are calculated as illustrated in Table 8. In the next step, two different aggregation methods are taken to group individual measures to verify the effect of aggregation method. One of the methods is geometric mean and the other is arithmetic mean. Table 9 illustrates the final results. We can figure out the rankings of the 17 alternatives are consistent when two different aggregation methods are applied. Both of the results show that candidate A_{16} is ranked first, and candidate A_{11} is ranked last. The best-selected candidate is consistent with the original paper. It should be mentioned that the original paper has demonstrated various aggregation methods may affect the overall ranking result. Compare with the group TOPSIS method proposed by original paper, the group EBVD is more efficient and robust.

5. Conclusions

EBVD is a straightforward method. Instead of identifying the reference points, the decision makers only need to determine the feasible ranges for each attribute by their knowledge and experience in the beginning of the decision process. An extension of EBVD to a group decision environment is also investigated in this study. The group preferences are aggregated within the procedure. The difference between tradition MADM methods and EBVD is that EBVD uses the value function from the Cumulated Prospect Theory to transform the original rating to a relative prospect value. It uses the separation measures to measure the preference of alternatives and capture the risk preferences of the decision makers. The weighted value distance is determined to calculate the separation measures. The idea of EBVD is the best alternative should have the shortest value distance from the EPFS and the farthest value distance from the ENFS. The separation measures are created by the following concept. Once an alternative has been selected. What is the additional prospect value that EPFS can provide to the decision maker? In addition, if the decision maker reselects ENFS as a new solution, how much the prospect value will be reduced. They are the definition of our separation measures.

Table 7. Decision matrix defined by decision maker 1 [29]

Alt.	Criteria						
	Language Test (C1)	Professional Test (C2)	Safety Rule Test (C3)	Professional Skills (C4)	Computer Skills (C5)	Panel Interview (C6)	1-on-1 Interviews (C7)
A1	80	70	87	77	76	80	75
A2	85	65	76	80	75	65	75
A3	78	90	72	80	85	90	85
A4	75	84	69	85	65	65	70
A5	84	67	60	75	85	75	80
A6	85	78	82	81	79	80	80
A7	77	83	74	70	71	65	70
A8	78	82	72	80	78	70	60
A9	85	90	80	88	90	80	85
A10	89	75	79	67	77	70	75
A11	65	55	68	62	70	50	60
A12	70	64	65	65	60	60	65
A13	95	80	70	75	70	75	75
A14	70	80	79	80	85	80	70
A15	60	78	87	70	66	70	65
A16	92	85	88	90	85	90	95
A17	86	87	80	70	72	80	85
Weights	0.066	0.196	0.066	0.130	0.130	0.216	0.196

Table 8. Separation measures

Alternatives	DM#1		DM#2		DM#3		DM#4	
	S_i^+	S_i^-	S_i^+	S_i^-	S_i^+	S_i^-	S_i^+	S_i^-
A1	0.380	-1.312	0.338	-1.359	0.417	-1.220	0.314	-1.428
A2	0.441	-1.174	0.462	-1.096	0.407	-1.209	0.434	-1.182
A3	0.315	-1.405	0.358	-1.292	0.334	-1.343	0.352	-1.339
A4	0.413	-1.189	0.449	-1.068	0.416	-1.144	0.447	-1.122
A5	0.375	-1.334	0.365	-1.329	0.369	-1.325	0.412	-1.265
A6	0.330	-1.375	0.325	-1.343	0.329	-1.345	0.373	-1.296
A7	0.440	-1.248	0.425	-1.266	0.448	-1.224	0.425	-1.277
A8	0.425	-1.204	0.412	-1.192	0.406	-1.212	0.305	-1.425
A9	0.276	-1.416	0.259	-1.387	0.272	-1.378	0.183	-1.571
A10	0.404	-1.338	0.389	-1.363	0.412	-1.319	0.458	-1.237
A11	0.657	-0.872	0.641	-0.914	0.611	-0.976	0.611	-0.970
A12	0.556	-1.061	0.506	-1.158	0.586	-0.998	0.656	-0.847
A13	0.366	-1.351	0.394	-1.274	0.415	-1.238	0.409	-1.270
A14	0.361	-1.326	0.375	-1.262	0.413	-1.192	0.376	-1.297
A15	0.453	-1.224	0.436	-1.245	0.466	-1.189	0.520	-1.093
A16	0.189	-1.549	0.182	-1.498	0.223	-1.443	0.199	-1.531
A17	0.339	-1.436	0.404	-1.304	0.364	-1.380	0.406	-1.312

Table 9. The aggregated relative closeness and rank by group EBVD

Alternatives	Arithmetical mean		Geometric mean	
	Relative closeness	Rank	Relative closeness	Rank
A1	0.786	5	0.787	5
A2	0.728	13	0.728	13
A3	0.798	3	0.798	3
A4	0.724	14	0.724	14
A5	0.775	7	0.776	7
A6	0.798	4	0.798	4
A7	0.743	12	0.743	12
A8	0.765	9	0.766	9
A9	0.853	2	0.855	2
A10	0.760	11	0.760	11
A11	0.597	17	0.597	17
A12	0.638	16	0.638	16
A13	0.764	10	0.764	10
A14	0.769	8	0.769	8
A15	0.717	15	0.717	15
A16	0.884	1	0.884	1
A17	0.782	6	0.782	6

By using the EPFS and ENFS, the proposed method prevents the rank reversal problem. This is because in the decision matrix normalization process, the new scale depends only on the EPFS and ENFS but not on the valuation obtained by the other alternatives. However, it should be notified that EBVD does not deal with risk directly since decision attribute values are assumed deterministic. It only deals with the risk attitude of decision maker when he/she evaluates the outcomes of decision attributes. Our examples have demonstrated the proposed approach is an appropriate and effective MADM method compared. The current EBVD method considers the attribute values as crisp numbers. We believe it can be extended to take the form of the uncertain linguistic variable for the attribute value by fuzzy number in the future study.

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