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Using Multivariate Control Chart to Maintain the Quality of Drinking Water in Accordance with Standard

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Abstract: This research aims to find the multivariate control chart which is suitable for production process, determine the current capability process, and also find the dominant variable which is correspondent to the improvement of drinking water quality. This research use a secondary data collected from January 2016 until February 2017. There were four group of variable used in this research. The first group was consist of color, turbidity, maximum containing level of copper and manganese. The second group consist of level of total dissolved solid, hardness, calcium and magnesium. The third level was consist of ph. value and alkaline; and fourth group consist of nitrate and nitrite. The result of data processing showed us that Multivariate Exponentially Weighted Moving Average or MEWMA control chart can detect the out of control data in the each group of variable faster than T2 Hotelling control chart. Then, the capability process index for the first group of variable is 2.238, the capability process index for second group of variable is 4.208, the capability process index for third group of variable is 1.438, and capability process index for fourth group of variable is 1.346. All group of variable have capability process index more than 1. This condition indicated that process goes well in all group of variable although there were some data is out of control in each group variable.

Keywords: Drinking water; multivariate control chart; group of variable; capability process index.

1. Introduction

Water plays a vital role for maintaining the biological life in the earth and also for the several sector of the economy such as agriculture, livestock production, forestry, industrial power generation, fisheries and other creative activities [1]. About 97% percent of water is exists in oceans. This water is not suitable for drinking. Only 3% of water is a fresh water. Among 3%, 2.97% is contained by glaciers and ice caps and only remaining 0.3% is available as a surface and ground water for human use [2]. Maybe, in the next century, the water will become even more limiting due to increased population, urbanization and climate change [3]. Unfortunately, in developing countries (i.e. Indonesia) the resource of drinking water is frequently being polluted because of the high growth of population, expansion of industry, throwing away of waste water and chemical effluents into canals and other water sources. It make the drinking water dangerous for human use. According to the World Health Organization (WHO), in developing countries about 80% of water pollution is a caused by domestic waste. Moreover, the poor management of water systems can cause serious problems in the availability and quality of water [4]. So, to ensure the quality and also safety of drinking water, the accurate monitoring and controlling are required.

Tirta Moedal Regional Water Company (Perusahaan Daerah Air Minum Tirta Moedal or

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PDAM Tirta Moedal) is a regional company that has a function to monitor and control the quality of drinking water for Semarang area. The quality of drinking water should be met the requirement as stated in regulation of the Minister of Health of the Republic of Indonesia No. 492/MENKES/PER/IV/2010. The process of monitor and control the quality of drinking water is not easy since more than one variables parameters should be considered. As number of the parameters to be considered is more, the complexity of the process also becomes more. Currently, PDAM Tirta Moedal conduct the process of monitor and control the quality of drinking water in two ways. PDAM Tirta Moedal checks the pH value and hardness of drinking water every day and PDAM Tirta Moedal checks the content of physics, chemistry, and bacteriology in drinking water once a week. Totally, according to the current standard (Regulation of the Minister of Health No. 492 / Menkes / PER / IV / 2010 on Water Quality Requirements), there were about 25 variable should be checked. Although there were more than one variable should be accounted for the quality of drinking water, so far, PDAM Tirta Moedal only checks each variable independently and compares the value obtained with the maximum limit allowed by the current standard. PT Tirta has not paid any attention into the correlation of each variable. So, the result of checks can be wrong if there is any correlation between variable. According to Montgomery (2009), using an independent control chart for two variable which have correlation each other can create a distortion. If quality of two pair variable which is correlated each other are monitored separately as individuals, it might not be very effective in detecting process changes [5]. As example, the probability of the pair X1 and X2 individually exceed the limit of 3 σ is 0.0027 and the probability of the pair X1 and X2 simultaneously exceed the limit of 3 σ is 0.00000279 (0.0027*0.0027) which is significantly lower than 0.0027. In the other words, the probability of the pair X1 and X2 individually within the control limit is 0.9973 and the probability of the pair of X1 and X2 simultaneously within the control limit is 0.99460 (0.9973*0.9973). It can be seen that the probability of the pair X1 and X2 simultaneously within the control limit is lower than the probability of the pair X1 and X2 individually. The use of independent control charts creates a distortion in the simultaneous monitoring of the pair of X1 and X2 through the type I error and the probability of being within control limits not be adjusted to the actual value. So, to overcome this problem, many multivariate control charts [6-8] have been proposed to better utilize the abundant data for process monitoring.

The results of the literature review indicated that there were any correlation between variables used to measure the quality of drinking water. Therefore, the use of individual control charts for each variable was not appropriate. Tirta Moedal PDAM requires a multivariate control chart that can assist the company to monitor and control the condition of each variables related with the quality of drinking water simultaneously. In this case, this research will use T^2 Hotelling control chart which is introduced by Harold Hotelling (1947) and Multivariate Exponentially Weighted Moving Average (MEWMA) which is introduced by Lowry et al. [5] to monitor and control the condition of each variables related with the quality of drinking water simultaneously. Then, this research will use the average value of the distribution "Run Length" (Average Run Length or ARL) to compare the performance of T^2 Hotelling control chart and MENWA control chart. The multivariate control chart with the best performance will be selected. Then, after the best performance multivariate control chart is selected, the next step is analyze the uncontrollable causes of the process using the MYT Decomposition Method. Research conducted by Mason et al (1997) has developed an approach about the decomposition method that can determine the contribution of the variables that cause the process out of control [9]. After analyze the uncontrollable causes of the process, this research will calculate the multivariate process capability and proposed some recommendation to reduce the process out of control.

2. Literature review

2.1. T^2 Hotelling control chart

Hotelling (1947) is the first author who introduce the multivariate quality control was introduced [10]. In his paper "Multivariate Quality Control", Hotelling was the first researcher who consider the problem of analyzing a correlated set of variables in his analysis. Hotelling developed a control procedure based on a concept referred to as statistical distance, a generalization of the T-statistic. The statistic was later named Hoteling's T^2 in his honor. The application of Hoteling's T2 control chart can be divided into two different phases of operation, i.e. retrospective or Phase I and prospective or Phase II operations. Under retrospective or Phase I operation, Hoteling's T^2 chart is used to identify outliers in the historical data set, mean shifts in the new subgroups of data, and other distributional deviations from in-control distributions. While Phase II operations detect shifts in process when new observations are drawn [11]. Let X be a random matrix of size m and normally distributed p-variate with the normal multivariate density function of X denoted by $X_i \sim N_p(\mu, \Sigma)$ with i= 1, 2, 3, ... m. So, the matrix data can be seen as follow. The notation of m indicates the number of samples and the notation of p indicates the number of quality characteristics

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} \cdots & x_{1p} \\ x_{21} & x_{22} \cdots & x_{2p} \\ \vdots & \vdots \ \ddots & \vdots \\ x_{m1} & x_{m2} \cdots & x_{mp} \end{bmatrix} \tag{1}$$

1) The first step: retrospective

a. For the I^{th} sample vector contains observations on each of the p variables of quality Xi1, Xi2, …, Xip, hence, the sample mean vector is defined as,

$$\bar{X} = \frac{1}{m}\sum_{i=1}^{m} x_i \quad i = 1,2,\dots,m \tag{2}$$

and the sample covariance matrix is

$$S = \frac{1}{m-1}\sum_{i=1}^{m}(X_i - \bar{X})(X_i - \bar{X})' \tag{3}$$

b. Calculate the statistic value of T2 Hotelling with the equation (4).

$$T^2 = (x - \bar{x})' S^{-1} (x - \bar{x}) \tag{4}$$

c. The retrospective limits of the T^2 control chart using Equation (4).is based on a beta distribution and the UCL of this T^2 statistic is given as below (Mason & Young 2001)

$$UCL = \frac{(m-1)^2}{m} \beta_{\alpha, \frac{p}{2}, (m-p-1)/2} \tag{5}$$

where $\beta_{\alpha,\frac{p}{2},(m-p-1)/2}$ is the upper a percentage point of a beta distribution with parameters $\frac{p}{2}$ and$(m-p-1)/2$. If an observation vector is found to have T2 value greater than the UCL, it is regarded as an outlier and is discarded from the data set. Other observation vectors whose T^2 values are less than or equal to the upper control limit (UCL) remain in the data set.

d. Then, calculate again the sample mean vector and the sample covariance matrix using the remain data in the set (after excluded the data belongs to outlier).

2) The second step: prospective

During the prospective phase, the use of multivariate chart is to test whether the process remains in-control when future observations are sampled. With the sample mean vector $\bar{X}$ and sample covariance matrix S, computed in the retrospective phase, the T^2 statistic of the prospective T2 control chart is defined as [11].

2.2. Multivariate exponentially weighted moving average

The MEWMA is a logical extension of the univariate EWMA and is defined as follows [12].

$$Z_i = RX_i + (I - R)Z_{i-1} \tag{6}$$

with $Z_0 = \mu = 0$ and $R = \text{diag}(r_1, r_2, r_3, \ldots, r_p)$, for $0 < r_j \leq 1,\ j = 1,2,3,..,p$; in this case, *r* is the weighted value for the MEWMA control chart, *p* is corresponding to number of the observed quality characteristics, and the sample size is one (m = 1). For observations with the characteristics of quality p and number of observation *i*, the equation 6 can written in the form of a matrix as follows

$$\begin{bmatrix} Z_{1i} \\ Z_{2i} \\ \vdots \\ Z_{pi} \end{bmatrix} = \begin{bmatrix} r_1 & 0\ldots & 0 \\ 0 & r_2\ldots & 0 \\ \vdots & \vdots\ \ddots & \vdots \\ 0 & 0\ \cdots & r_p \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{2i} \\ \vdots \\ x_{pi} \end{bmatrix} + \left(\begin{bmatrix} 1 & 0\ldots & 0 \\ 0 & 1\ldots & 0 \\ \vdots & \vdots\ \ddots & \vdots \\ 0 & 0\ \ldots & 1 \end{bmatrix} - \begin{bmatrix} r_1 & 0\ldots & 0 \\ 0 & r_2\ldots & 0 \\ \vdots & \vdots\ \ddots & \vdots \\ 0 & 0\ \cdots & r_p \end{bmatrix} \right) \begin{bmatrix} Z_{1(i-1)} \\ Z_{2(i-1)} \\ \vdots \\ Z_{p(i-1)} \end{bmatrix} \tag{7}$$

For example, the quality characteristic used p = 2, with X_1 is the first quality characteristic, X_2 is second the quality characteristic, and the sample size is one (m = 1), the vector Z_i for the first observation can be seen as follow.

$$\begin{bmatrix} Z_{11} \\ Z_{21} \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} + \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \right) \begin{bmatrix} Z_{10} \\ Z_{20} \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} + \left(\begin{bmatrix} 1 - r_1 \\ 1 - r_2 \end{bmatrix} \right) \begin{bmatrix} Z_{10} \\ Z_{20} \end{bmatrix} \tag{8}$$

For the second observation

$$\begin{bmatrix} Z_{12} \\ Z_{22} \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} + \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \right) \begin{bmatrix} Z_{11} \\ Z_{21} \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} x_{12} \\ x_{22} \end{bmatrix} + \left(\begin{bmatrix} 1 - r_1 \\ 1 - r_2 \end{bmatrix} \right) \begin{bmatrix} Z_{11} \\ Z_{21} \end{bmatrix} \tag{9}$$

For the i^{th} observation

$$\begin{bmatrix} Z_{1i} \\ Z_{2i} \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} + \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \right) \begin{bmatrix} Z_{1(i-1)} \\ Z_{2(i-1)} \end{bmatrix} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{2i} \end{bmatrix} + \left(\begin{bmatrix} 1 - r_1 \\ 1 - r_2 \end{bmatrix} \right) \begin{bmatrix} Z_{1(i-1)} \\ Z_{2(i-1)} \end{bmatrix} \tag{10}$$

In previous studies, the weighted value for r always same ($r_1 = r_2 = r_3 = \cdots = r_p = r$), so the vector of MEWMA control chart can be written as follow.

$$Z_i = rX_i + (I - r)Z_{i-1} \quad i - 1,2, \ldots .m \tag{11}$$

For observations with the p^{th} characteristic of quality and i^{th} observation, the equation 11 can be written in form of the matrix as follow.

$$\begin{bmatrix} Z_{1i} \\ Z_{2i} \\ \vdots \\ Z_{pi} \end{bmatrix} = \begin{bmatrix} r & 0\ldots & 0 \\ 0 & r\ldots & 0 \\ \vdots & \vdots\ \ddots & \vdots \\ 0 & 0\ \ldots & r \end{bmatrix} \begin{bmatrix} x_{1i} \\ x_{2i} \\ \vdots \\ x_{pi} \end{bmatrix} + \left(\begin{bmatrix} 1 & 0\ldots & 0 \\ 0 & 1\ldots & 0 \\ \vdots & \vdots\ \ddots & \vdots \\ 0 & 0\ \ldots & 1 \end{bmatrix} - \begin{bmatrix} r & 0\ldots & 0 \\ 0 & r\ldots & 0 \\ \vdots & \vdots\ \ddots & \vdots \\ 0 & 0\ \ldots & r \end{bmatrix} \right) \begin{bmatrix} Z_{1(i-1)} \\ Z_{2(i-1)} \\ \vdots \\ Z_{p(i-1)} \end{bmatrix} \tag{12}$$

If the sample size more than 1 ($m > 1$), the vector $\boldsymbol{Z_i}$ from MEWMA can be defined as follow

$$Z_i = R\bar{X}_i + (I - R)Z_{i-1} \quad (13)$$

with $\bar{X}_i$ is the vector mean of each sample. So, the equation (13) can be written in form of matrix as follow.

$$\begin{bmatrix} Z_{1i} \\ Z_{2i} \\ \vdots \\ Z_{pi} \end{bmatrix} = \begin{bmatrix} r_1 & 0 \dots & 0 \\ 0 & r_2 \dots & 0 \\ \vdots & \vdots \ddots & \vdots \\ 0 & 0 \cdots & r_p \end{bmatrix} \begin{bmatrix} \bar{x}_{1i} \\ \bar{x}_{2i} \\ \vdots \\ \bar{x}_{pi} \end{bmatrix} + \left(\begin{bmatrix} 1 & 0 \dots & 0 \\ 0 & 1 \dots & 0 \\ \vdots & \vdots \ddots & \vdots \\ 0 & 0 \dots & 1 \end{bmatrix} - \begin{bmatrix} r_1 & 0 \dots & 0 \\ 0 & r_2 \dots & 0 \\ \vdots & \vdots \ddots & \vdots \\ 0 & 0 \cdots & r_p \end{bmatrix} \right) \begin{bmatrix} Z_{1(i-1)} \\ Z_{2(i-1)} \\ \vdots \\ Z_{p(i-1)} \end{bmatrix} \quad (14)$$

If it is assumed that the multivariate observations x_i with $i = 1,2,3, \dots, n$ are identically independent and normally distributed $N_p(\mu, \Sigma)$ with μ is the vector mean of controlled process and Σ is covariance matrix, the MEWMA control chart is formed by plotting the value of statistic test Q_i^2 on the chart. The value of Q_i^2 can be obtained trough this equation

$$Q_i^2 = Z' \Sigma_{Z_i}^{-1} Z_i \quad (15)$$

where the covariance matrix is

$$\Sigma_i = \left(\frac{r}{2-r}\right) [1 - (1 - r)^{2i}] \Sigma \quad (16)$$

It can be said out of control if $Q_i^2 = Z' \Sigma_{Z_i}^{-1} Z_i > H$. H is the limit based on the value of Average Run Length (ARL) (in this case H> 0, H is upper limit and 0 is lower limit of control chart of MEWMA because the value of Q_i^2 always positive).

3. Method of research

3.1. Variable selection

There were 25 variables used by PDAM Tirta Moedal to control and monitor the quality of drinking water. According to literature review and also interview with the representative of PDAM Tirta Moedal, not all variable have correlation each other's. Among 25 variables, only 12 variables have the correlation each other. Moreover, according to the result of research conducted by [13-15], the twelve variables can be grouped into four group. The first group was consist of color, turbidity, maximum containing level of copper and manganese. The second group consist of level of total dissolved solid, .hardness, calcium and magnesium. The third level was consist of ph. value and alkaline; and fourth group consist of nitrate and nitrite. Moreover, to prove the correlation between variables in each group, this research also conduct the Pearson correlation and the Bartlett's test. Bartlett's statistic is designed to test for equality of variances across groups against the alternative that variances are unequal for at least two groups, assuming the populations are normally distributed (Mu, 2006). In addition to the twelve variables, or thirteen other variables can be controlled with the univariate control chart because there is no correlation between those variables.

3.2. Data of the research

Historical data for this research was taken from the final stage of the water production process whereby the water is in the pipe after the reservoir. There were 56 weekly data from each of 12 variables (from January 2016 until February 2017) used for creating the multivariate control chart. The number of data as the sample of this research is enough since it more than 20 [12].

4. Result of data processing and discussion

The result of data processing will consist of the result of statistic descriptive for each group of variable, the result of Pearson Correlation and Bartlett's test for each group of variable, the result of ARL simulation from T2 Hotelling and MEWMA control chart, the result of MYT decomposition and capability process of multivariate for each group of variable. First, the result of statistic descriptive for each group of variable can be seen in the Table 1.Second, the result of the Pearson Correlation and Bartlett's test for each group of variable can be seen in Table 2. The result of Pearson Correlation and Bartlett's test for each group of variable indicated that there were strong correlation within a group of variable. Then, third, the result of ARL simulation from T^2 Hotelling and MEWMA control chart can be seen in Table 3.

Table 1. The result of statistic descriptive for each group of variable

Group	Variable	Unit of Measurement	Mean	Standard Deviation	Minimum Value	Maximum Value
Group 1	Color	NTU	5.536	4.576	0.00	27.00
	Turbidity	TCU	1.030	0.337	0.53	2.46
	Copper	mg/l	0.027	0.019	0.00	0.09
	Manganese	mg/l	0.039	0.025	0.00	0.12
Group 2	Total Dissolved Solid	mg/l	148.661	30.171	99.00	251.00
	Hardness	mg/l	111.249	23.683	80.04	180.56
	Calcium	mg/l	73.738	18.968	45.76	136.40
	Magnesium	mg/l	37.156	13.219	13.48	88.49
Group 3	pH value	pH	7.203	0.133	6.74	7.52
	Alkaline	mg/l	85.768	18.708	50.00	127.00
Group 4	Nitrate	mg/l	0.868	0.809	0.04	3.34
	Nitrite	mg/l	0.140	0.138	0.013	0.69

Table 2. The result of Pearson correlation and Bartlett's test for each group of variable

Group			Color	Turbidity	Copper	Manganese
Group 1	Color	*Pearson Correlation*	1	0.580^{**}	0.079^{**}	0.138^{**}
		Sig. (2-tailed)		0.000	0.000	0.002
		N	56	56	56	56
	Turbidity	*Pearson Correlation*	0.580^{**}	1	0.280^{**}	0.272^{**}
		Sig. (2-tailed)	0.000		0.000	0.001
		N	56	56	56	56
	Copper	*Pearson Correlation*	0.079^{**}	0.280^{**}	1	0.290^{**}
		Sig. (2-tailed)	0.000	0.000		0.001
		N	56	56	56	56
	Manganese	*Pearson Correlation*	0.138^{**}	0.272^{**}	0.290^{**}	1
		Sig. (2-tailed)	0.002	0.001	0.001	
		N	56	56	56	56
	KMO and Bartlett's Test					
	Kaiser-Meyer-Olkin Measure of Sampling Adequacy.					0.459
	Bartlett's Test of Sphericity			*Approx. Chi-Square*		89.662
				df		6
				Sig.		0.000

Table 3. The result of Pearson correlation and Bartlett's test for each group of variable (Cont')

Group	Variable	Statistic	TDS	Hardness	Calcium	Magnesium
Group 2	Total Dissolved Solid (TDS)	*Pearson Correlation*	1	0.310**	0.703**	0.013**
		Sig. (2-tailed)		0.000	0.000	0.923
		N	56	56	56	56
	Hardness	Pearson Correlation	0.310**	1	0.579**	0.560**
		Sig. *(2-tailed)*	0.000		0.000	0.000
		N	56	56	56	56
	Calcium	*Pearson Correlation*	0.703**	0.579**	1	0.044
		Sig. (2-tailed)	0.000	0.000		0.747
		N	56	56	56	56
	Magnesium	*Pearson Correlation*	0.013	0.560**	0.044	1
		Sig. (2-tailed)	0.923	0.000	0.747	
		N	56	56	56	56
	KMO and Bartlett's Test					
	Kaiser-Meyer-Olkin Measure of Sampling Adequacy.					0.376
	Bartlett's Test of Sphericity	*Approx. Chi-Square*				325.352
		df				6
		Sig.				0.000

Group	Variable	Statistic	Alkaline	pH
Group 3	Alkaline	Pearson Correlation	1	0.265*
		Sig. (2-tailed)		0.048
		N	56	56
	pH	Pearson Correlation	0.265*	1
		Sig. (2-tailed)	0.048	
	KMO and Bartlett's Test			
	Kaiser-Meyer-Olkin Measure of Sampling Adequacy.			0.500
	Bartlett's Test of Sphericity	*Approx. Chi-Square*		3.903
		df		1
		Sig.		0.048

Group	Variable	Statistic	NIitrate	Nitrite
Group 4	Nitrate	*Pearson Correlation*	1	0.712**
		Sig. (2-tailed)		0.000
		N	56	56
	Nitrite	*Pearson Correlation*	0.712**	1
		Sig. (2-tailed)	0.000	
		N	56	56
	KMO and Bartlett's Test			
	Kaiser-Meyer-Olkin Measure of Sampling Adequacy.			0.500
	Bartlett's Test of Sphericity	*Approx. Chi-Square*		37.898
		df		1
		Sig.		0.000

Table 4. The result of ARL simulation from T^2 hotelling and MEWMA control chart for each group of variable

Group	ARL	MEWMA					T2 Hotelling
	Shift	r=0.1 H=272.27	r=0.2 H=153.2	r=0.4 H=86.98	r=0.6 H=62.19	r=0.8 H=48.52	H=15.77
Group 1	0.00	300.60	299.59	299.84	299.10	300.41	298.47
	0.25	3.00	2.00	1.54	1.00	1.00	2.00
	0.50	2.00	1.00	1.00	1.00	1.00	2.00
	1.00	1.00	1.00	1.00	1.00	1.00	2.00
	1.50	1.00	1.00	1.00	1.00	1.00	2.00
	2.00	1.00	1.00	1.00	1.00	1.00	2.00
	2.50	1.00	1.00	1.00	1.00	1.00	2.00
	3.00	1.00	1.00	1.00	1.00	1.00	2.00
Group 2	ARL	MEWMA					T2 Hotelling
	Shift	r=0.1 H =737.1	r=0.2 H=398.4	r=0.4 H=200.6	r=0.6 H=133.5	r=0.8 H=97.4	H=15.87
	0.00	299.95	300.27	299.77	300.46	299.40	301.06
	0.25	299.10	288.80	286.78	289.77	286.81	284.86
	0.50	267.41	265.34	256.43	253.10	252.87	253.93
	1.00	218.23	175.59	165.34	155.23	152.23	157.51
	1.50	99.45	90.23	85.44	81.12	80.12	91.55
	2.00	50.12	46.45	40.12	37.46	32.14	47.41
	2.50	25.41	21.12	19.21	18.98	20.13	27.08
	3.00	9.12	8.65	7.69	7.47	7.87	15.29
Group 3	ARL	MEWMA					T2 Hotelling
	shift	r=0.1 H =58924.11	r=0.2 H =28234.6	r=0.4 H=12772.3	r=0.6 H=7565.9	r=0.8 H=4942.59	H=11.41
	0.00	300.81	299.32	301.94	299.80	300.17	298.01
	0.25	35.57	18.29	9.52	6.68	5.56	10.18
	0.50	27.66	13.78	6.68	4.17	2.34	2.86
	1.00	21.00	10.28	5.00	3.00	2.00	2.00
	1.50	17.48	8.87	4.00	2.89	1.44	2.00
	2.00	15.01	7.63	3.98	2.00	1.00	2.00
	2.50	13.55	7.00	3.00	2.00	1.00	2.00
	3.00	12.00	6.00	3.00	2.00	1.00	2.00
Group 4	ARL	MEWMA					T2 Hotelling
	shift	r=0.1 H =53.28	r=0.2 H=36.10	r=0.4 H=25.53	r=0.6 H=21.1	r=0.8 H=18.48	H=11.41
	0.00	300.58	298.65	300.96	301.95	299.98	300.61
	0.25	8.40	5.58	4.18	4.08	4.51	6.45
	0.50	4.20	2.70	1.89	1.40	1.18	2.09
	1.00	2.09	1.59	1.00	1.00	1.00	2.00
	1.50	1.99	1.00	1.00	1.00	1.00	2.00
	2.00	1.00	1.00	1.00	1.00	1.00	1.99
	2.50	1.00	1.00	1.00	1.00	1.00	2.00
	3.00	1.00	1.00	1.00	1.00	1.00	2.00

Average Run Length (ARL) is the average number of samples to be drawn before a sample point indicates an uncontrolled condition [12]. ARL is usually used as an important indicator for evaluating control chart performance. The result of ARL simulation in Table 4 indicated that MEWMA control chart with r=0.1 and H=272.27 is the best control chart for group 1. Then, the MEWMA control chart with r=0.8 dan H=97.40 is the best control chart for group2, the MEWMA control chart with r=0.8 dan H=4942.59 is the best control chart for group 3, and the MEWMA control chart with r=0.2 dan H=36.11is the best control chart for group 4. Moreover, all the result indicated MEWMA control chart is more sensitive in detecting small mean shift than T2 Hotelling control chart. So, based on this condition, the MEWMA control chart will be used as a tool for monitoring the quality variable of drinking water in each group. The result of MEWMA control chart for each group of variable can be seen in the Figure 1 until Figure 4.

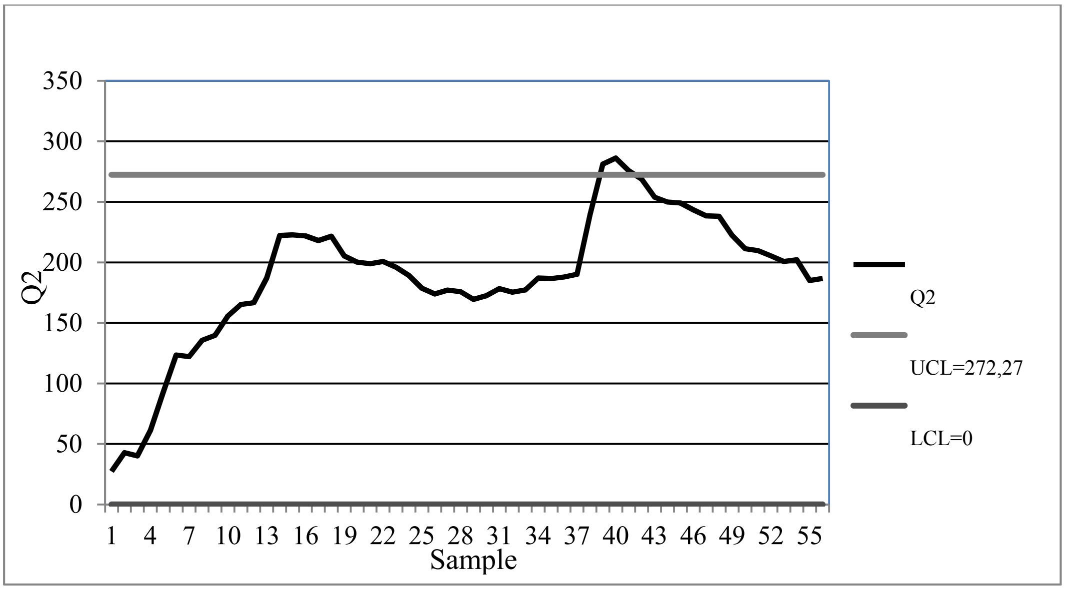

Figure 1. MEWMA Control Chart of Group of Variable 1

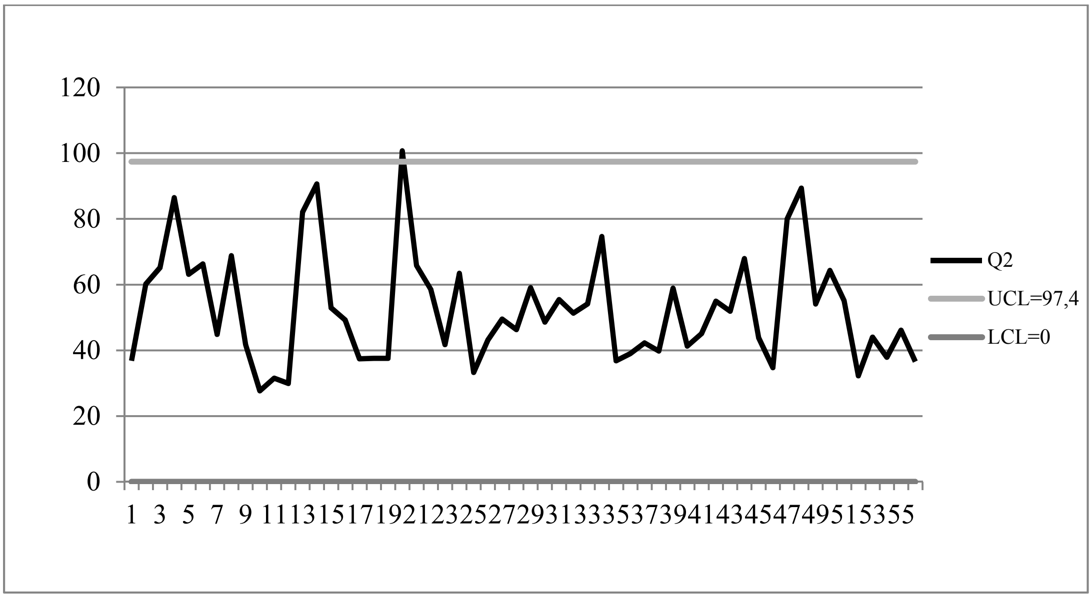

Figure 2. MEWMA Control Chart of Group of Variable 2

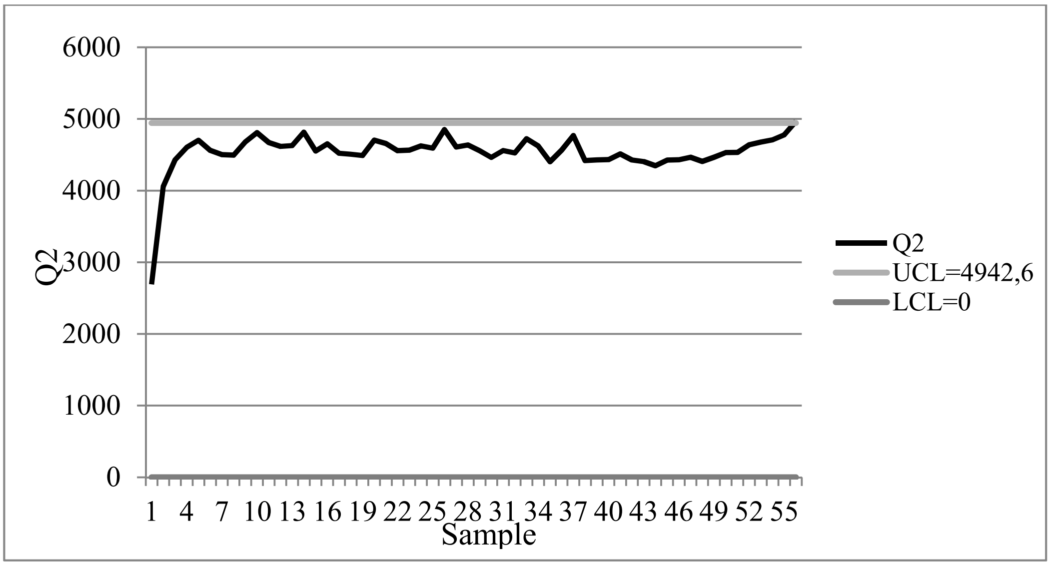

Figure 3. MEWMA Control Chart of Group of Variable 3

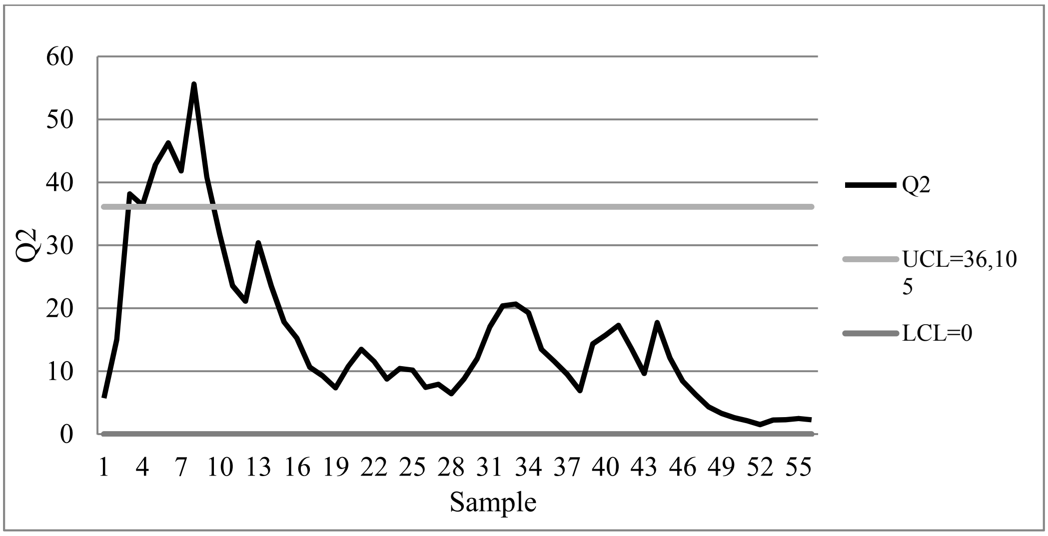

Figure 4. MEWMA Control Chart of Group of Variable 4

The MEWMA control chart for group of variable 1 indicated that the control chart have upper limit 272.27 and lower limit 0. Based on their upper and lower limit, there are 3 data out of control in group of variable 1. The MEWMA control chart for group of variable 2 indicated that the control chart have upper limit 97.40 and lower limit 0. Based on their upper and lower limit, there are 1 data out of control in group of variable 2. The MEWMA control chart for group of variable 3 indicated that the control chart have upper limit 4,942.6 and lower limit 0. Based on their upper and lower limit, there are 1 data out of control in group of variable 3. Last, the MEWMA control chart for group of variable 4 indicated that the control chart have upper limit 36.105 and lower limit 0. Based on their upper and lower limit, there are 7 data out of control in group of variable 4. After we know the data out of control from each MEWMA control chart, the next step is trying to find the cause of out of control with the MYT decomposition method. The result of MYT 1 indicated that color and turbidity are the main cause of out of control for group of variable 1, total dissolved solid, hardness, and calcium are the main cause of out of control for group of variable 2, pH value is the main cause of out of control for group variable 3, and nitrate is the cause of out of control for group variable 4.

The last step is calculating the capability process of multivariate. The result of calculation indicated that group of variable 1 has capability process of multivariate 22.77, group of variable 2 has capability process of multivariate 49.59, group of variable 3 has capability process of multivariate 1.448, and group of variable 2 has capability process of multivariate 1.346. All of the value of capability process of multivariate have value more than 1. This condition indicated that all of the process is running well and can produce the quality of drinking water accordance with the specification of the company and also the standard, although there is still one or more data out of control in each group of variable if we use the upper and lower limit from the MEWMA control chart.

Since there is still out of control data in MEWMA control chart, this research propose some recommendations to PDAM Tirta Moedal to make the process more stable. The proposed recommendation are clean the basin periodically (at least once a week), check the volume of silica sand during washing or cleaning of dirt, check the blower pump periodically, repair the sludge/density or disposal periodically, clean the reservoir periodically (at least once a week), replace the pipes periodically (at least once a month), prepare the appropriate PCA application, and prepare the appropriate doses of alkaline and lime to neutralize the PH.

5. Conclusion

This research aims to find the multivariate control chart which is suitable for production process, determine the current capability process, find the dominant variable which is correspondent to the improvement of drinking water quality, and propose some recommendation to improve the quality of drinking water. The twelve variables related with the quality of drinking water can be grouped into four group. In each of group of variable, MEWMA control chart is more sensitive in detecting small mean shift compare to T^2 Hotelling control chart. So, based on this condition, the MEWMA control chart is a recommended tools for monitoring the quality variable of drinking water in each group of variable. Although the process is going well (based on the value of capability process of multivariate of each group of variable which is more than 1), the MEWMA control chart indicated that there were some data out of control compared with the value of upper limit. This condition told us that although the process in each of group of variable is going well, the process is still need improvement. So, in this research we propose some recommendation to improve the quality of drinking water such as clean the basin periodically (at least once a week), check the volume of silica sand during washing or cleaning of dirt, check the blower pump periodically, repair the sludge/density or disposal periodically, clean the reservoir periodically (at least once a week), replace the pipes periodically (at least once a month), appropriate PCA application, and prepare the appropriate doses of alkaline and lime to neutralize the pH.

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