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Reflection of Thermoelastic Waves at a Non-free Thermally Insulated Surface

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Abstract: Sinha and Sinha (J. Phys. Earth, 22, 237-244, 1974) studied a problem on the reflection of thermoelastic waves at a stress free thermally insulated solid half-space in context of the Lord and Shulman theory of generalized thermoelastcity. He showed the existence of three plane waves (two longitudinal waves and a shear wave) in a homogeneous, linear and isotropic thermoelastic medium. He also obtained the reflection coefficients of reflected waves theoretically and numerically for the incident plane waves. Due to the engineering applications, a problem on the reflection of thermoelastic waves at a non-free boundary surface is considered in this paper. The reflection coefficients of various reflected waves are obtained by considering the new boundary conditions at non-free surface. For a particular material representing the half-space, the reflection coefficients are also computed numerically and are shown graphically against the angle of incidence for different values of boundary parameters.

Keywords: Generalized thermoelasticity; Non-free surface; Reflection coefficients; Thermal relaxation.

1. Introduction

Lord and Shulman [1] and Green and Lindsay [2] extended the classical dynamical coupled thermoelasticity of Biot [3] to generalized thermoelastic theories. In these generalized thermoelastic theories, the field equations are hyperbolic to describe the heat in the form of a wave. Finite speed of heat propagation is predicted due to these generalized thermoelastic theories, whereas Biot's coupled thermoelasticity admits an infinite speed of heat propagation. Green and Naghdi [4] also gave a generalized theory of thermoelasticity without energy dissipation with isothermal displacement gradients among its independent constitutive variables. Chandrasekharaiah [5] developed a dual-phase-lag theory of thermoelasticity. Hetnarski and Ignaczak [6] revisited the representative generalized theories of thermoelasticity. Recently Ignaczak and Ostoja-Starzewski [7] presented some problems based on these theories in their book.

The phenomenon of wave propagation has many applications in the fields of mineral and oil exploration, geophysical exploration and seismology. Using Lord and Shulman theory, a problem on reflection of thermoelastic waves at a stress free thermally insulated solid half-space was studied by Sinha and Sinha [8]. The phenomena of the reflection and refraction of generalized thermoelastic waves at an interface were studied by Sinha and Elsibai [9, 10], Singh [11, 12] and Sharma et al. [13]. Taking into account various other parameters present in the earth, the reflection phenomena at free surface and interfaces of thermoelastic solid half-spaces were studied by many

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authors in [14-25].

In case of real engineering problems, the boundary surface may be considered as non-free with distributed elastic constraint or support, where each mass point is subjected to the normal and tangential translation constraint. In this paper, a problem on reflection of thermoelastic waves at a non-free surface is considered. The reflection coefficients of all reflected waves are derived for the new boundary conditions at a non-free surface. The numerical computations of the reflection coefficients are performed for a particular material. These reflection coefficients are depicted graphically against the angle of incidence to show the impact of non-free surface parameters.

2. Governing equations of linear thermoelasticity

We consider a system of rectangular Cartesian axes $Ox_i (i = 1, 2, 3)$. We consider a linear, isotropic and homogeneous thermally conducting elastic medium in undeformed state at uniform temperature T_0. Following Lord and Shulman [1], the governing equations of linear, isotropic and homogeneous generalized thermoelastic medium in absence of body forces and heat sources, are

(a) Constitutive equations

$$e_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}), \tag{1}$$

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma T)\delta_{ij}, \tag{2}$$

(b) Equations of motion

$$\mu \frac{\partial^2 u_i}{\partial x_j^{\,2}} + (\lambda + \mu)\frac{\partial^2 u_j}{\partial x_i \partial x_j} - \gamma \frac{\partial T}{\partial x_i} = \rho \frac{\partial^2 u_i}{\partial t^2}, \tag{3}$$

(c) Heat Equations

$$K \frac{\partial^2 T}{\partial x_i^{\,2}} = \rho c_E (\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2}) + \gamma T_0 (\frac{\partial e_{ii}}{\partial t} + \tau_0 \frac{\partial^2 e_{ii}}{\partial t^2}), \tag{4}$$

where $i, j = 1, 2, 3$; x_i are cartesian coordinates, t is time; $u_i(x_i, t)$ are cartesian components of the displacement vector; $T(x_i, t)$ is increment in reference temperature T_0, e_{ij} are components of the strain tensor; $e_{kk} = e_{11} + e_{22} + e_{33}$ is an invariant, σ_{ij} are components of the stress tensor; δ_{ij} is Kronecker delta; ρ is density of the medium; λ, μ are Lame's elastic constants; K is thermal conductivity; τ_0 is relaxation time; c_E is the specific heat at constant strain; $\gamma = (3\lambda + 2\mu)\alpha_0$ is thermal parameter and α_0 is coefficient of thermal expansion. In the following sections, the cartesian axes x_1, x_2 and x_3 are renamed as x, y and z axes, respectively.

3. Equations governing two-dimensional motions

We consider a half-space which occupies the region $z > 0$, where the origin is taken at plane surface and $z-$ axis is taken normal into the half-space. We also assume that the plane surface

$z = 0$ is non-free and thermally insulated. We consider motions in the (x, z) plane with displacement components u_1 and u_3, where u_1 and u_3 depend only on x, z and t. We choose the x-axis as the direction of propagation of waves. Using the following Helmholtz's representations

$$u_1 = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z},\ u_3 = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x}, \tag{5}$$

the equations (3) and (4) are specialized in the (x, z) plane as

$$(\lambda + 2\mu)(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2}) - \gamma T = \rho\frac{\partial^2\phi}{\partial t^2}, \tag{6}$$

$$K(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}) = (\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2})[\rho c_E T + \gamma T_0(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2})], \tag{7}$$

$$\mu(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2}) = \rho\frac{\partial^2\psi}{\partial t^2}, \tag{8}$$

where $\phi(x, z, t)$ and $\psi(x, z, t)$ are field potentials. We seek the plane wave solutions of equations (6) to (8) in the following form

$$\{\phi, T, \psi\} = \{A, B, C\}e^{ik(\sin\theta x + \cos\theta z - Vt)}, \tag{9}$$

where k is wave number, V is the complex wave speed, θ is the angle of propagation, and A, B and C are amplitude factors. With the use of (9) into equations (6) to (8), it is shown that there exists three plane waves in x-z plane namely longitudinal wave (*P* wave), thermal wave (*T* wave) and shear wave (*SV* wave) with speeds V_1, V_2 and V_3 given by

$$V_1 = \sqrt{\frac{G + \sqrt{G^2 - 4H}}{2}},\ V_2 = \sqrt{\frac{G - \sqrt{G^2 - 4H}}{2}},\ V_3 = \sqrt{\frac{\mu}{\rho}},$$

where

$$G = \bar{K} + c_1^{\,2} + \varepsilon c_1^{\,2},\ H = \bar{K}c_1^{\,2},\ c_1^{\,2} = \sqrt{\frac{\lambda + 2\mu}{\rho}},\ \bar{K} = \frac{K}{\rho c_E(\tau_0 + \frac{i}{\omega})},\ \varepsilon = \frac{\gamma^2 T_0}{\rho^2 c_E c_1^{\,2}},$$

and ω is frequency of the wave. If we write, $V_j^{-1} = v_j^{-1} - i\omega^{-1}q_j, (j = 1, 2)$, then clearly v_j and q_j are speeds of propagation and attenuations of the P and T waves.

4. Reflection at a non-free surface

An incident P or SV wave travels in half-space $z > 0$ making an angle θ_0 with normal to the half-space and impinges the non-free surface $z = 0$. The energy of incident wave is partitioned into three reflected waves, namely, P, T and SV waves as shown in Figure 1. The potentials representing the incident and reflected waves are expressed as

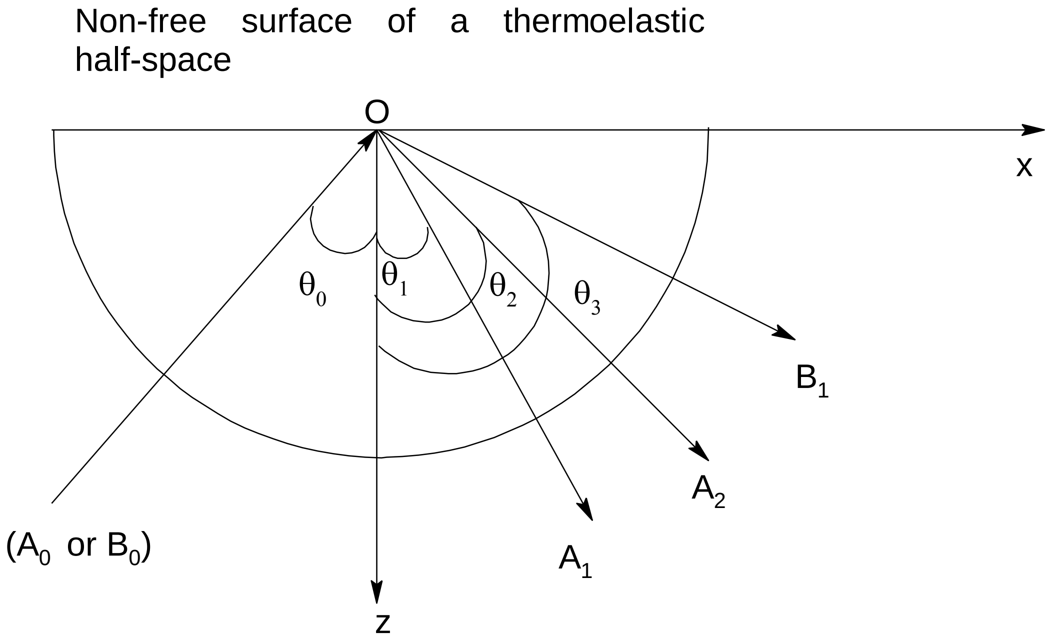

Figure 1. Geometry of the problem showing incident and reflected waves.

$$\varphi = A_0 e^{\iota k_0(\sin\theta_0 x - \cos\theta_0 z - v_0 t)} + A_1 e^{\iota k_1(\sin\theta_1 x + \cos\theta_1 z - v_1 t)} + A_2 e^{\iota k_2(\sin\theta_2 x + \cos\theta_2 z - v_2 t)}, \tag{10}$$

$$T = \zeta_1 A_0 e^{\iota k_0(\sin\theta_0 x - \cos\theta_0 z - v_0 t)} + \zeta_1 A_1 e^{\iota k_1(\sin\theta_1 x + \cos\theta_1 z - v_1 t)} + \zeta_2 A_2 e^{\iota k_2(\sin\theta_2 x + \cos\theta_2 z - v_2 t)}, \tag{11}$$

$$\psi = B_0 e^{\iota k_0(\sin\theta_0 x - \cos\theta_0 z - v_0 t)} + B_1 e^{\iota k_3(\sin\theta_3 x + \cos\theta_3 z - V_3 t)}, \tag{12}$$

where $\iota = \sqrt{-1}$, A_0, A_1, A_2, B_0 and B_1 are amplitudes of incident P, reflected P, reflected T, incident SV and reflected SV waves, respectively. $\theta_0, \theta_1, \theta_2$ and θ_3 are angles of incident (P or SV), reflected P, reflected T and reflected SV waves with z-axis, respectively. k_0, k_1, k_2 and k_3 are wavenumbers of incident (P or SV), reflected P, reflected T and reflected SV waves, respectively. v_0, v_1, v_2 and V_3 are phase speeds of incident (P or SV), reflected P, reflected T and reflected SV waves, respectively. The thermo-mechanical coupling coefficients $\zeta_i = \dfrac{k_i^2(v_i^2 - c_1^2)}{\gamma}, (i = 1, 2)$. Here, for the case of incident P wave, $B_0 = 0$, $k_0 = k_1$, $v_0 = v_1$, $\theta_0 = \theta_1$ and for the case of incident SV wave, $A_0 = 0$, $k_0 = k_3$, $v_0 = V_3$, $\theta_0 = \theta_3$.

The normal force stress component t_{zz} and tangential force component t_{zx} are zero for the free surface. These components may have finite value and are proportional to displacement components for the non-free surface, namely,

$$t_{zz} = -iS_1 u_3, \; t_{zx} = -iS_2 u_1, \tag{13}$$

where

$$t_{zz} = \lambda(\frac{\partial^2\phi}{\partial x^2} - \frac{\partial^2\psi}{\partial x\partial z}) + (\lambda + 2\mu)(\frac{\partial^2\phi}{\partial z^2} + \frac{\partial^2\psi}{\partial x\partial z}) - \gamma(1 + v_0\frac{\partial}{\partial t})T, \quad (14)$$

$$t_{zx} = \mu(2\frac{\partial^2\phi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial z^2}), \quad (15)$$

and S_1 and S_2 are the proportional coefficients of normal and tangential stiffness, respectively. The free surface and fixed surface are two extreme cases of non-free surface. The free surface is recovered when S_1 and S_2 tend to zero, whereas, the fixed surface is recovered when S_1 and S_2 tend to infinity. A negative imaginary number $-i$ is multiplied on right hand side of above equations to remove the phase shift between the stress field and displacement field. For thermally insulated surface, we also need vanishing of normal component of heat flux across surface at $z = 0$, i.e.,

$$\frac{\partial T}{\partial z} = 0, \quad (16)$$

At any boundary point and at any time, we also assume that the circular frequency of each reflected wave is equal to that of an incident wave, i.e.,

$$k_0 v_0 = k_1 v_1 = k_2 v_2 = k_3 V_3, \quad (17)$$

and the apparent wave number of every wave is equal, i.e.,

$$k_0 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3. \quad (18)$$

Keeping in view of equations (14), (15), (17) and (18), the potentials given by equations (10) to (12) satisfy the boundary conditions (13) and (16) and we obtain the following non homogeneous system of three equations in reflection coefficients (amplitude ratios) Z_1, Z_2 and Z_3

$$\sum_{j=1}^{3} a_{ij} Z_j = b_i, (i = 1, 2, 3), \quad (19)$$

where

$$Z_1 = \frac{A_1}{A_0 \; or \; B_0}, Z_2 = \frac{A_2}{A_0 \; or \; B_0}, Z_3 = \frac{B_1}{A_0 \; or \; B_0},$$

and

$$a_{1p} = -(\lambda + 2\mu Q_p^2)(\frac{v_0}{v_p})^2 - Q_p \frac{S_1}{k_0}(\frac{v_0}{v_p}) - \gamma\frac{\zeta_p}{k_p^2}(\frac{v_0}{v_p})^2, \; (p = 1, 2),$$

$$a_{13} = -2\mu Q_3 \sin\theta_0 (\frac{v_0}{V_3}) - \sin\theta_0 \frac{S_1}{k_0},$$

$$a_{2p} = -2\mu Q_p \sin\theta_0 (\frac{v_0}{v_p}) - \sin\theta_0 \frac{S_2}{k_0},\ (p=1,2),$$

$$a_{23} = \mu[Q_3^2 (\frac{v_0}{V_3})^2 - \sin^2\theta_0] + Q_3 \frac{S_2}{k_0} (\frac{v_0}{V_3}),$$

$$a_{3p} = -\frac{\zeta_p}{k_p^2} (\frac{v_0}{v_p})^3 Q_p,\ (p=1,2),\ \ a_{33} = 0,$$

$$Q_p = \sqrt{1-(\frac{v_p}{v_0})^2 \sin^2\theta_0},\ (p=1,2),\ \ Q_3 = \sqrt{1-(\frac{V_3}{v_0})^2 \sin^2\theta_0}.$$

(a) for incident P wave

$$b_1 = \lambda + 2\mu\cos^2\theta_0 - \cos\theta_0 \frac{S_1}{k_1} + \gamma\frac{\zeta_1}{k_1^2},$$

$$b_2 = -2\mu\sin\theta_0\cos\theta_0 + \sin\theta_0 \frac{S_2}{k_1},$$

$$b_3 = \cos\theta_0 \frac{\zeta_1}{k_1^2},$$

(b) for incident SV wave

$$b_1 = -2\mu\sin\theta_0\cos\theta_0 + \sin\theta_0 \frac{S_1}{k_3},$$

$$b_2 = -\mu(1-2\sin^2\theta_0) + \cos\theta_0 \frac{S_2}{k_3},$$

$$b_3 = 0.$$

For $S_1 = 0$ and $S_2 = 0$, the above theoretical analysis reduces to those for the case of traction free surface.

5. Numerical results and discussion

Following values of the relevant parameters at $T_0 = 300K$ are taken

$\rho = 2.7\times10^3$ Kg.m^{-3}, $\lambda = 5.775\times10^{10}$ N.m^{-2}, $\mu = 2.646\times10^{10}$ N.m^{-2},

$K = 0.492\times10^2$ $W.m^{-1}$.deg^{-1}, $c_E = 2.361\times10^2$ J.Kg^{-1}.deg^{-1}, $\tau_0 = 0.05\times10^{-10}$ s.

Using Fortran program of Gauss elimination method with above physical constants, the non-homogeneous system (19) of three equations in reflection coefficients of reflected P, T and SV waves is solved numerically for incidence of P and SV waves.

For incident P wave, the reflection coefficients of reflected P, T and SV waves are shown graphically against the angle of incidence ($0^o < \theta_0 \le 90^o$) in Figures 2 to 4. The variations shown by solid line, solid line with stars, solid lines with circles and solid lines with triangles as center symbols in Figures 2 to 4 correspond to $S_1 = 0, S_2 = 0$; $S_1 = 0.5, S_2 = 0$; $S_1 = 0, S_2 = 0.5$ and $S_1 = 0.5, S_2 = 0.5$, respectively. In Figure 2, for $S_1 = 0.5, S_2 = 0.5$, the reflection coefficients of reflected *P* wave is 0.9137 at $\theta_0 = 1^o$. It decreases to its minimum value 0.5352 at $\theta_0 = 60^o$ and then increases sharply to its maximum value one at $\theta_0 = 90^o$. For $S_1 = 0.5, S_2 = 0.5$, the variations for reflection coefficients of reflected *T* waves are shown by solid line with triangle in Figure 3. The reflection coefficients of reflected *T* wave is 0.4891e-04 at $\theta_0 = 1^o$. It increases to its maximum value 0.1669e-03 at $\theta_0 = 47^o$ and then decreases sharply to its minimum value zero at $\theta_0 = 90^o$. For $S_1 = 0.5, S_2 = 0.5$, the variations of reflection coefficients of reflected *SV* wave are shown graphically by solid line with triangles in Figure 4. The reflection coefficient of reflected *SV* wave is 0.1296e-01 at $\theta_0 = 1^o$. It increases to its maximum value 0.3663 at $\theta_0 = 46^o$ and then decreases to its minimum value zero $\theta_0 = 90^o$. Comparing the different variations of reflection coefficients for reflected waves in Figures 2 to 4, the effects of normal stiffness S_1 and tangential stiffness S_2 are observed.

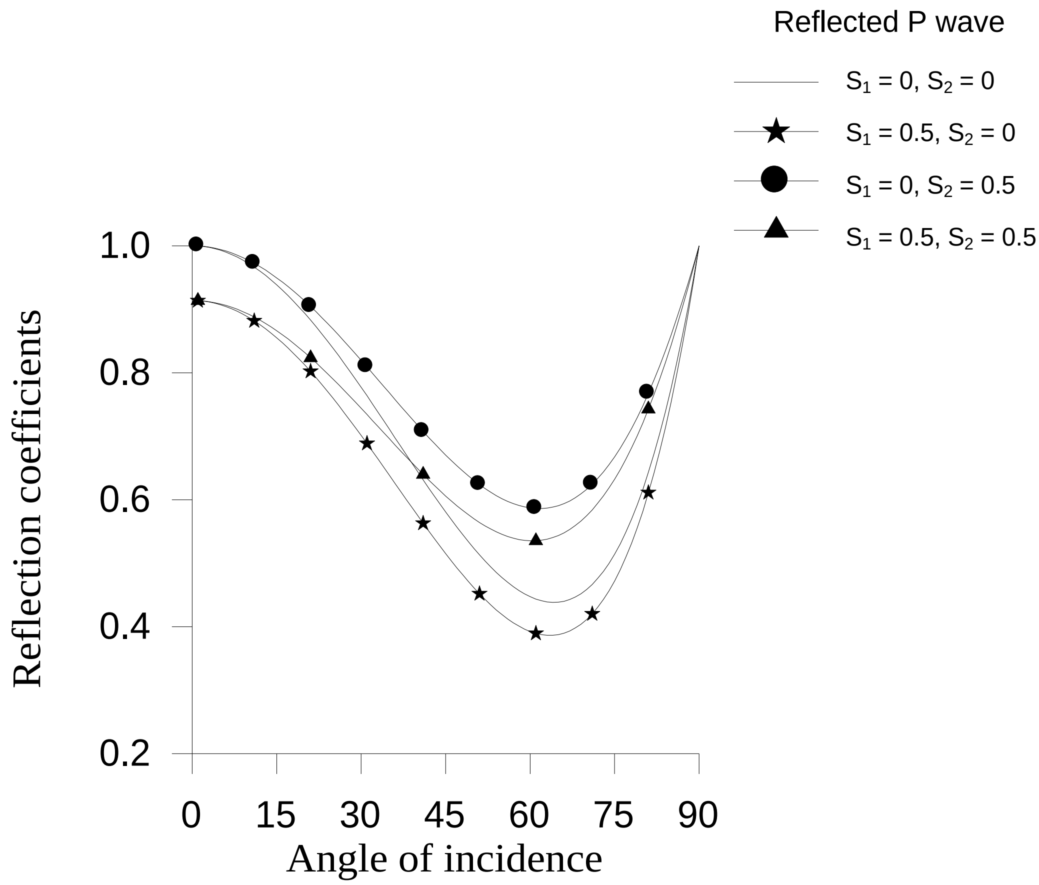

Figure 2. Variations of the reflection coefficients of reflected P wave against the angle of incidence of incident P wave.

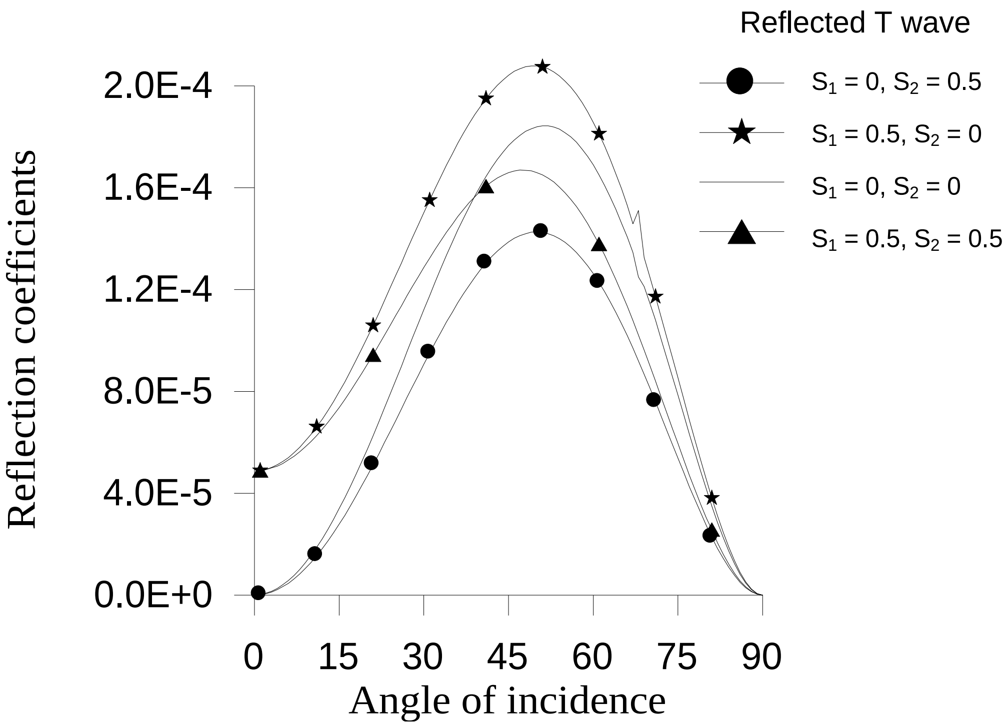

Figure 3. Variations of the reflection coefficients of reflected T wave against the angle of incidence of incident P wave.

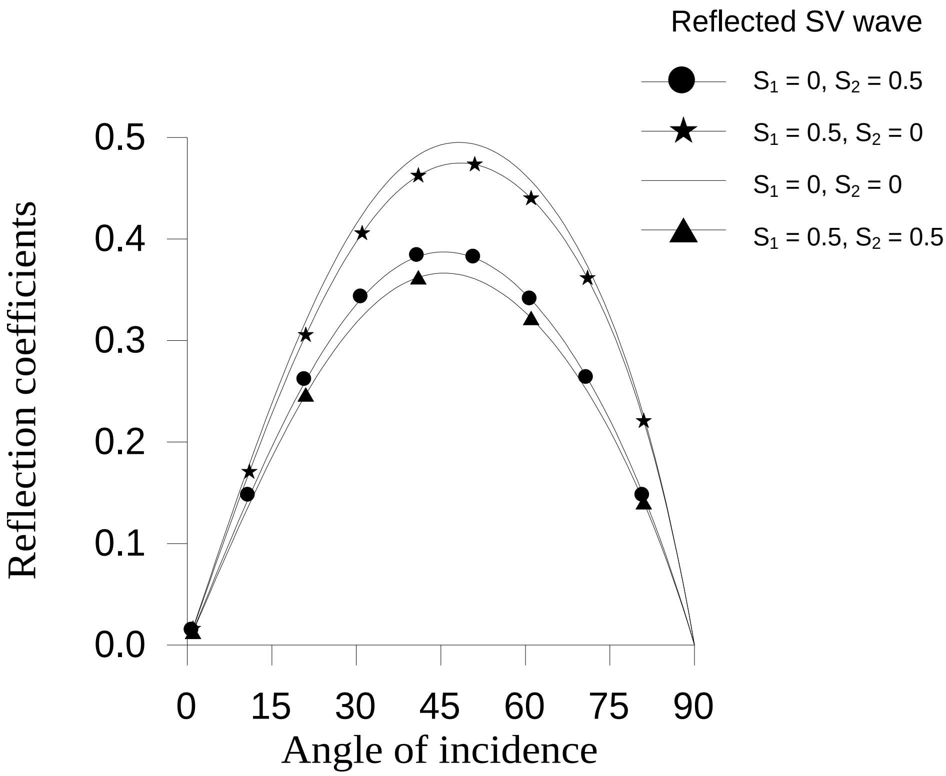

Figure 4. Variations of the reflection coefficients of reflected SV wave against the angle of incidence of incident P wave.

For incident *SV* wave, the reflection coefficients of reflected *P*, *T* and *SV* waves are shown graphically against the angle of incidence ($0^o < \theta_0 \leq 29^o$) in Figures 5 to 7. The variations shown by solid line, solid line with stars, solid lines with circles and solid lines with triangles as center symbols in Figures 5 to 7 corresponds to $S_1 = 0, S_2 = 0$; $S_1 = 0.5, S_2 = 0$; $S_1 = 0, S_2 = 0.5$ and $S_1 = 0.5, S_2 = 0.5$, respectively. In Figure 5, for $S_1 = 0.5, S_2 = 0.5$, the reflection coefficient of reflected *P* wave is 0.5458e-01 at $\theta_0 = 1^o$. It increases to its maximum value 1.703 at $\theta_0 = 29^o$. In Figure 6, for $S_1 = 0.5, S_2 = 0.5$, the reflection coefficient of reflected *T* wave is 0.3085e-04 at $\theta_0 = 1^o$. It increases to its maximum value 0.4207e-03 at $\theta_0 = 20^o$ and then decreases sharply to 0.1137e-03 at $\theta_0 = 29^o$. In Figure 7, for $S_1 = 0.5, S_2 = 0.5$, the reflection coefficient of reflected *SV* wave is 0.6472 at $\theta_0 = 1^o$. It decreases to its minimum value 0.56e-01 at $\theta_0 = 28^o$ and then increases to the value 0.1136 at $\theta_0 = 29^o$. Comparing the different variations of reflection coefficients for reflected waves in Figures 5 to 7, the effects of normal stiffness S_1 and tangential stiffness S_2 are observed.

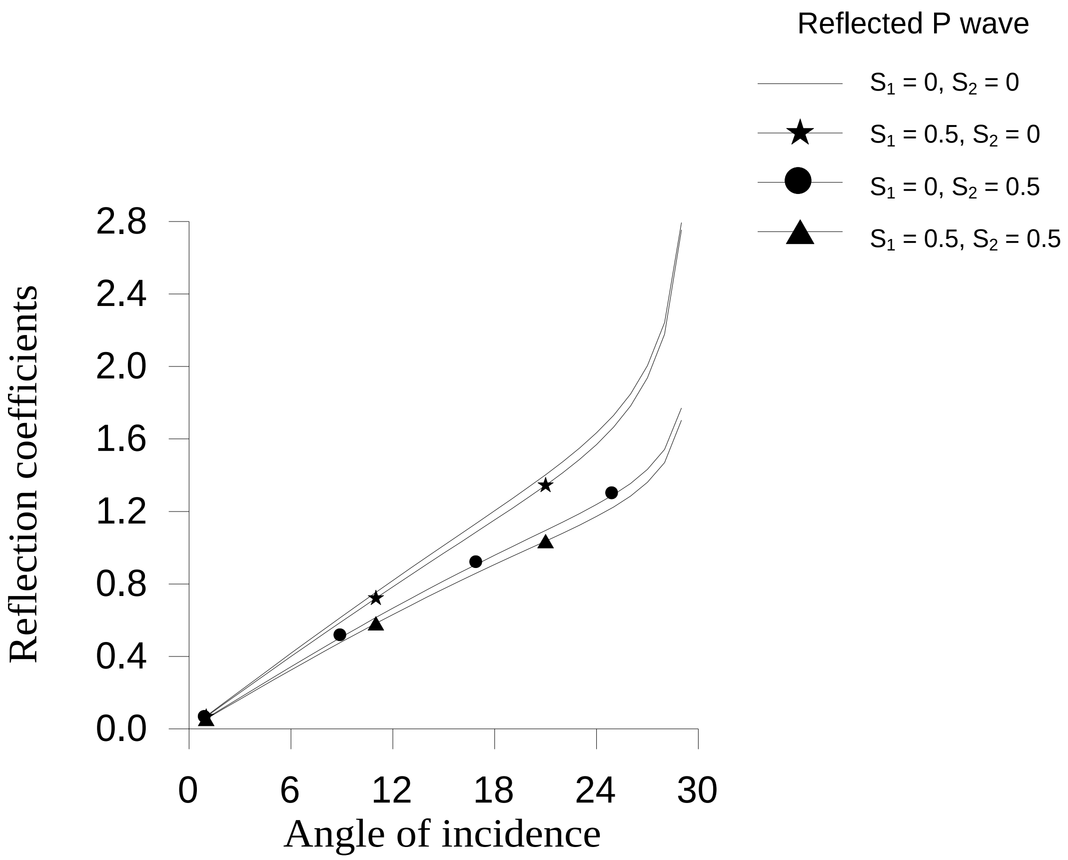

Figure 5. Variations of the reflection coefficients of reflected P wave against the angle of incidence of incident SV wave.

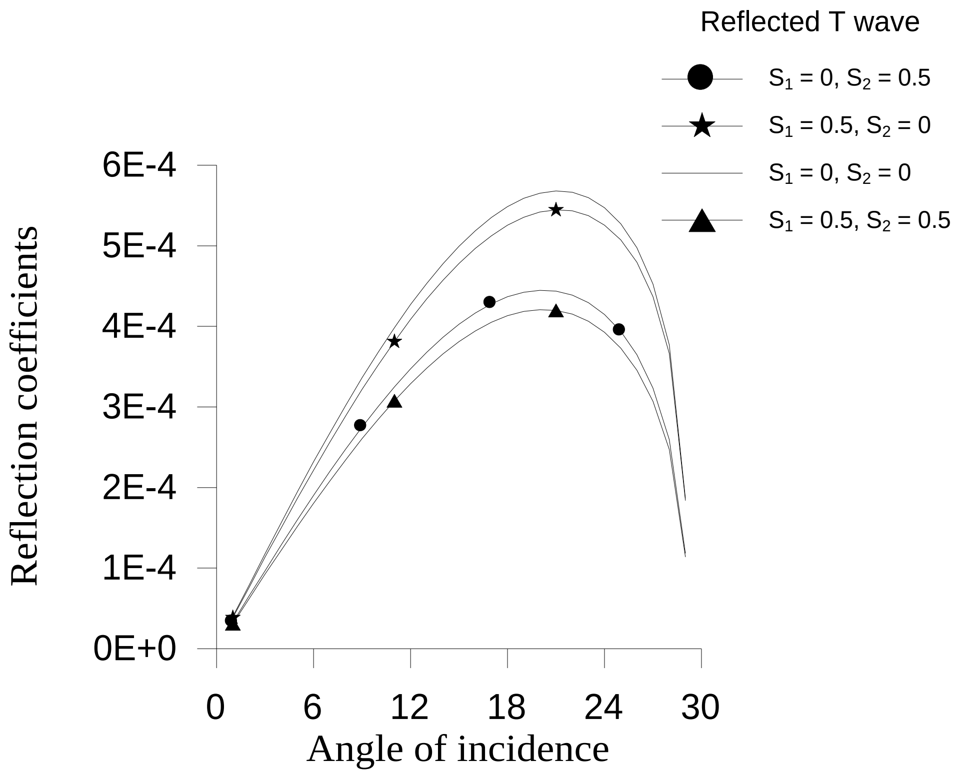

Figure 6. Variations of the reflection coefficients of reflected T wave against the angle of incidence of incident SV wave.

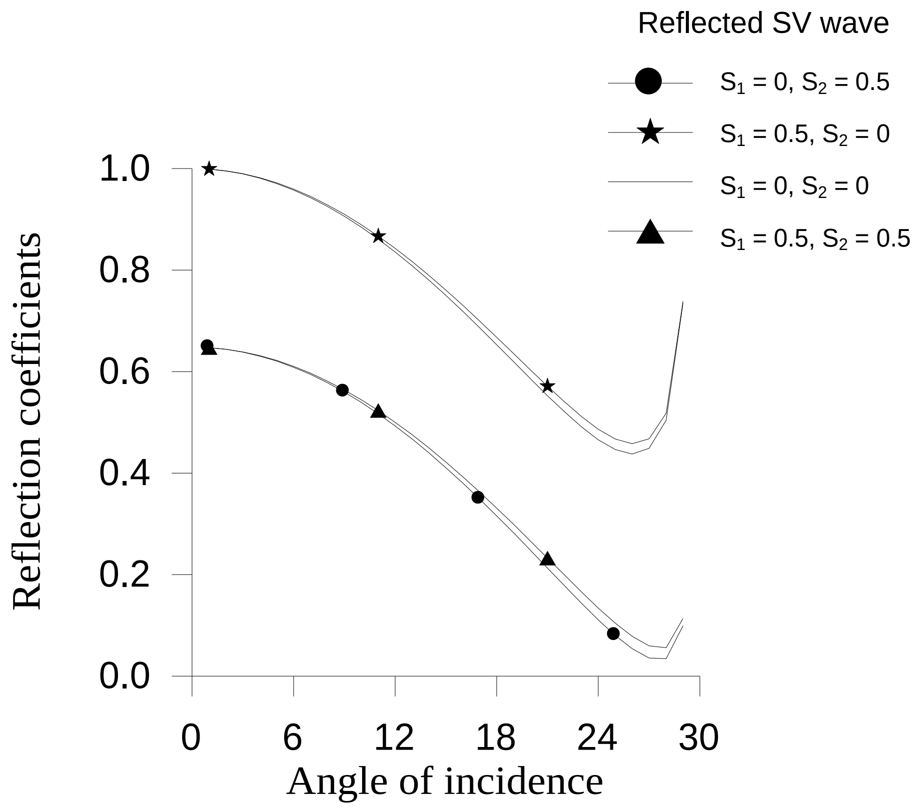

Figure 7. Variations of the reflection coefficients of reflected SV wave against the angle of incidence of incident SV wave.

6. Conclusions

A thermoelastic solid half-space with non-free surface is considered for reflection of P and SV waves. Using appropriate field potentials in required boundary conditions at non-free surface, a non-homogeneous system of three equations in the reflection coefficients of reflected P, T and SV waves is obtained for incidence of both P and SV waves. Using a Fortran program, this system of equations is solved numerically for relevant physical constants of the model. The reflection coefficients are also plotted against the angle of incidence for different sets of the coefficients of normal and tangential stiffness. The reflection coefficients of various reflected waves depend on the coefficients of stiffness at each angle of incidence. Following specific remarks may be concluded from the general discussion on numerical results:

(i) For incident P wave, the reflection coefficients of reflected P have maximum values one at $\theta_0 = 90^o$ (grazing incidence) and different minimum values near angle of incidence $\theta_0 = 60^o$ for different stiffness combinations. Maximum effects of normal and tangential stiffness on reflection coefficients of P wave are observed at angles near the angle of incidence $\theta_0 = 60^o$ and minimum effects of stiffness coefficients are observed at angles near grazing and normal incidences.

(ii) For incident P wave, the reflection coefficients of reflected T and SV wave have minimum values zero at grazing incidence and different maximum values at angles near angle of incidence $\theta_0 = 47^o$ for different stiffness combinations. Maximum effects of normal and tangential stiffness on reflection coefficients of T and SV wave are observed at angles near the angle of incidence $\theta_0 = 47^o$ and minimum effects of stiffness coefficients are observed at angles near grazing and normal incidences.

(iii) For incident SV wave, the maximum values of reflection coefficients of reflected P and T waves are observed at angles near angles of incidence $\theta_0 = 29^o$ and $\theta_0 = 20^o$. The reflection coefficients of these reflected waves are minimum near angle of normal incidence. The effect of stiffness coefficients on the reflection coefficients of P and T waves is minimum at normal and critical incidence. However, it is observed maximum in a range of angle of incidences near critical incidence.

(iv) For incident SV wave, the maximum values of reflection coefficients of reflected SV wave are observed at angle of incidence $\theta_0 = 1^o$. The reflection coefficient of SV wave is minimum at $\theta_0 = 28^o$. The effect of stiffness coefficients on the reflection coefficient of SV wave is minimum at normal and critical incidence. However, it is observed maximum in a range of angle of incidences close to the critical incidence.

The present numerical results may provide useful information for experimental scientists working in the field of wave propagation in solids, mining tremors and drilling into the crust of the earth.

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