

Evaluation of the cost-benefit of standby retrial systems incorporating switching failure and general repair times

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ABSTRACT

A robust power supply system with high availability and acceptable cost/benefit is essential for many service systems, such as communication networks and product manufacturing processes. This investigation is concerned with the evaluation of the cost-benefit of a standby retrial power supply system incorporating standby switching failure and general repair times, which is the first work on the comparative analysis of retrial availability systems incorporating switching failure and general repair times. Four different standby power supply retrial configurations are included and each configuration consists of a different number of primary and standby generators. The investigated system assumes that the time-to-failure and the time-to-repair of the primary and standby generators obey the exponential and general distributions, respectively. We also take that the switching over standbys may be failed into account. By using the supplementary variable technique, the explicit expressions of the steady-state availability for each configuration are derived. A comparative analysis of the availability and the cost-benefit ratio among four retrial systems is presented. We also rank the configurations based on the steady-state availability and cost-benefit ratio for two repair time distributions, Weibull and lognormal. The calculated numerical results can provide managers with decision reference for stable power supply system and cost reduction.

Keywords: Retrial queue, Availability, Switching failure, Cost-benefit, Supplementary variable.

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1. INTRODUCTION

The high-tech fabrication plants (called fabs) play an important role in modern industries, such as packaging and testing, IC design, and wafer foundry. For Fabs, stable power supply system is essential for maintaining his competition. By analysing the data collected from Hsinchu's semiconductor fabs, Hu and Chuah (2003) indicted that the average power consumption of a fab was 2.18 kW/m². However, the power generator may malfunction and power outages are not only causing significant financial losses but also interrupting the production. For ensuring the power supply stability for a fab, the power system has to consist of primary generators conveyed from both the local power company and the backup generators. Notice that the fabs have to build a monitoring system to monitor the process of switching power sources. In fact, the switching from the standby generator to primary generator may fail. On the other side, retrial is common phenomenon in various applications. In a retrial repairing system, an arriving failed component finding that there is no available repairing server would enter the retrial orbit and attempts to obtain his repairing service after a random time. This paper uses a retrial system with standby switching failure to model the power supply system for a fab. We assume that the area and the power demand of a fab are 12000 m² and 24MW, respectively. Four different retrial configurations are included and each configuration

consists of different number of primary and standby generators.

The existing research works on the retrieval systems, such as Yang and Templeton (1987); Falin (1990); Falin and Templeton (1997); Phung-Duc (2019); Artalejo (1990a, 1990b); Artalejo and Gomez-Corral (2008), have reported the most comprehensive concepts and reviews. Wang et al. (2001) studied a retrieval queue with server breakdowns and repairs, and discussed the reliability issue. They obtained the explicit expressions of availability, reliability function of the server, and failure frequency. Wang (2006) studied an M/G/1 retrieval queues with general retrieval times and server breakdowns. For an M/G/1 retrieval system incorporating warm standby components and a repairable server, Ke et al. (2013) investigated the availability of steady-state and developed an efficient algorithm to compute the steady-state availability. For a retrieval and repairable multi-component system incorporating mixed standby components, Kuo et al. (2014) studied the reliability-based measures and performed a sensitivity analysis on the MTTF and the availability.

Most articles on repairable standbys systems have assumed that the switching from the standby generator to primary generator will not fail. In fact, the process of switching over standbys may not be perfect. In a fuzzy environment, Huang et al. (2006) considered a queueing system subject to switching failure and server breakdown. By using a bootstrap method, Liu et al. (2011) examined the statistical inferences of a repairable system with standby switching failure. Hsu et al. (2014) explored an M/M/R MRP subject to switching failure and reboot delay. By applying the supplementary variable technique, Ke et al. (2016) studied an M/G/1 MRP subject to standbys switching failure. Lee (2016) also applied the supplementary variable technique to determine availability for a redundancy model with switching failure and interrupted repairs. Ke et al. (2018) generalized the model proposed by Ke et al. (2016) through the inclusion of an unreliable repairman and assumed that both the recovery time and the repair time follow general distributions.

These literatures pointed out that retrieval behavior and switching failure are general situations happened in common repairable availability systems. Obviously, a model with general repair times is more generalized and extensive for availability evaluation applications. To the best of our knowledge, there is no works on the standby retrieval system incorporating standby switching failure and general repair times. The remainder of this paper is structured as below. Section 2 describes the standbys retrieval system incorporating standbys switching failure and general repair times. Section 3 introduces four different standby retrieval configurations. For each configuration, we derive the explicit expressions for the availability. In section 4, by taking repair time distributions, Weibull and lognormal, into

account, we compare four configurations for the availability according to the numerical values given to the system parameters. Section 5 summarizes the works of this article. Some derivations are presented in Appendices.

2. SYSTEM DESCRIPTION

For the convenience of analysis, we consider that a system requires 24WM power and assume that the power generating capacity of generator is available in units of 24WM, 12WM and 8WM. Before putting standby generators into full operation, they are assumed to be allowed to fail while inactive. Moreover, to identify whether standby generators fail or not and monitor the power switching process, they are continuously monitored by administrating device. Each primary generator malfunctions independently of the state of the others and obeys an exponential time-to-failure distribution with rate λ . Once a primary generator malfunctions, it is immediately replaced by a standby generator which is any available. During the switching process, there is a significant probability q of failure. The available standby generator malfunctions independently of the state of the others and follows an exponential time-to-failure distribution with rate α ($0 < \alpha < \lambda$). In addition, the failure characteristics of standby generator are those of a primary-generator as a standby generator successfully switches into a primary-generator state. In the repair facility, one server is responsible for repairing failed generators and there is no waiting space in front of the server. Therefore, when a failed generator observing the repair server is busy, it will be delivered to the orbit and retry to get his service after waiting a random time. The times to retrieval are exponentially distributed with rate γ . If the server is free at the end of the retrieval queue waiting time, the failed generator obtains services immediately; otherwise, it will be sent back to the retrieval queue. The times to repair of the generators are independent and identically distributed random variables which have a distribution $R(u)$ ($u \geq 0$), a density function $r(u)$ ($u \geq 0$) and a mean repair time r_1 . Suppose that the system fails when the remaining power generating capacity is less than 24WM.

Four different standby retrieval configurations are considered as below. Configuration 1 includes one 24WM primary generator and one 24WM standby generator; configuration 2 includes of two 12WM primary generators and one 12WM standby generator; configuration 3 is composed of two 12WM primary generator and two 12WM standby generator; configuration 4 contains three 8WM primary generators and two 8WM standby generators. For ease of reference, the used notations and probabilities are listed as follows.

λ	Failure rate of primary generators
α	Failure rate of standby generators
q	Unsuccessful switching probability
γ	Retrial rate of generators in orbit
$R(u)$	Distribution function of the repair time
$r(u)$	Probability density function of the repair time
r_1	Mean repair time
$R^*(u)$	Laplace-Stieltjes transform of $R(u)$
$P_{0,n}(t)$	Probability of n generators in orbit at time t when the server is free
$P_{1,n}(t)$	Probability of n generators in orbit at time t when the server is busy
$P_{0,n}$	Steady-state probability of n generators in orbit when the server is free
$P_{1,n}$	Steady-state probability of n generators in orbit when the server is busy
$P_{1,n}^*(s)$	Laplace-Stieltjes transform of $P_{1,n}(t)$
Av	Steady-state availability

3. PROBLEM SOLUTIONS

For each configuration, we first draw the transition-rate diagram. Based on the diagram, we derive the differential equations of each state at time t . Then, we take the Laplace transform on both sides of the differential equations. Finally, by working on these Laplace transform equations, we can obtain the steady-state availability. In order to govern the general repair times, we adopt the following supplementary variables at time t :

$U(t) \equiv$ remaining repair time for the generator,

$N(t) \equiv$ number of failed generators in orbit,

$I(t) \equiv$ the states of the server.

There are two possible states for the server: $I(t) = 0$ represents the server is free, $I(t) = 1$ denotes the server is busy.

Let

$P_{i,n}(u, t)du = \Pr\{N(t) = n, I(t) = i, u < U(t) \leq u + du\}, u \geq 0,$

$P_{i,n}(t) = \int_0^\infty P_{i,n}(u, t)du, i = 0, 1.$

In steady-state, we define

$P_{i,n} = \lim_{t \rightarrow \infty} P_{i,n}(t), i = 0, 1,$

$P_{i,n}(u) = \lim_{t \rightarrow \infty} P_{i,n}(u, t), i = 0, 1.$

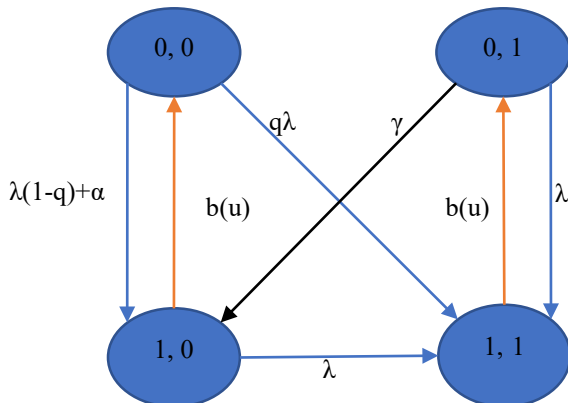


Fig. 1. State-transition-rate diagram of configuration 1

3.1 Configuration 1

Fig. 1 shows the state-transition-rate diagram of configuration 1. From Fig. 1, we have the following steady-state Equations (1)-(4):

$$0 = -(\lambda + \alpha)P_{0,0} + P_{1,0}(0) \quad (1)$$

$$0 = -(\lambda + \gamma)P_{0,1} + P_{1,1}(0) \quad (2)$$

$$-\frac{d}{du}P_{1,0}(u) = (\lambda(1-q) + \alpha)r(u)P_{0,0} - \lambda P_{1,0}(u) + \gamma r(u)P_{0,1} \quad (3)$$

$$-\frac{d}{du}P_{1,1}(u) = \lambda q r(u)P_{0,0} + \lambda P_{1,0}(u) + \lambda r(u)P_{0,1} \quad (4)$$

where we define $P_{0,0}(u) = r(u)P_{0,0}$ and $P_{0,1}(u) = r(u)P_{0,1}$. For detailed derivation that led to the steady-state availability, please refer to Appendix A. The steady-state availability can be expressed as Equation 5.

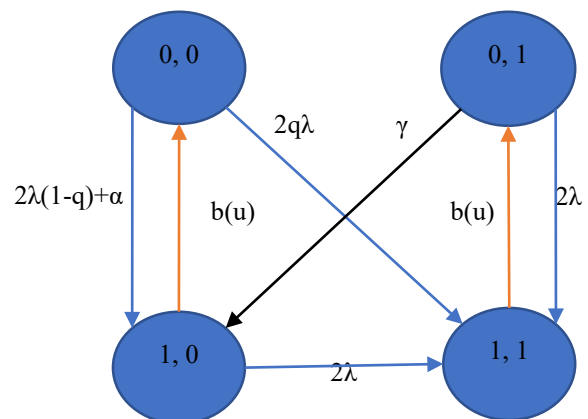


Fig. 2. State-transition-rate diagram of configuration 2

$$Av_1 = 1 - P_{1,1} = P_{0,0} + P_{0,1} + P_{1,0} = \left\{ \frac{\lambda q + \gamma}{\gamma} + \frac{(\lambda + \alpha)(\lambda + \gamma)}{\lambda \gamma} \left[\frac{1 - R^*(\lambda)}{R^*(\lambda)} \right] \right\} P_{0,0} \quad (5)$$

3.2 Configuration 2

For configuration 2, the state-transition-rate diagram is depicted in Fig. 2. Hence, for configuration 2, the following steady-state Equations (6)-(9) can be constructed:

$$0 = -(2\lambda + \alpha)P_{0,0} + P_{1,0}(0) \quad (6)$$

$$0 = -(2\lambda + \gamma)P_{0,1} + P_{1,1}(0) \quad (7)$$

$$-\frac{d}{du}P_{1,0}(u) = (2\lambda(1-q) + \alpha)r(u)P_{0,0} - 2\lambda P_{1,0}(u) + \gamma r(u)P_{0,1} \quad (8)$$

$$-\frac{d}{du}P_{1,1}(u) = 2\lambda q r(u)P_{0,0} + 2\lambda P_{1,0}(u) + 2\lambda r(u)P_{0,1} \quad (9)$$

where we defined $P_{0,0}(u) = r(u)P_{0,0}$ and $P_{0,1}(u) = r(u)P_{0,1}$. Using a similar argument, the explicit expression for the steady-state availability can be obtained as Equation (10). Please see Appendix B for detailed derivation.

3.3 Configuration 3

Fig. 3 presents the state-transition-rate diagram of configuration 3. Hence, for configuration 3, the steady-state Equations (11)-(16) are as below:

$$0 = -(2\lambda + 2\alpha)P_{0,0} + P_{1,0}(0) \quad (11)$$

$$0 = -(2\lambda + \alpha + \gamma)P_{0,1} + P_{1,1}(0) \quad (12)$$

$$0 = -(2\lambda + 2\gamma)P_{0,2} + P_{1,2}(0) \quad (13)$$

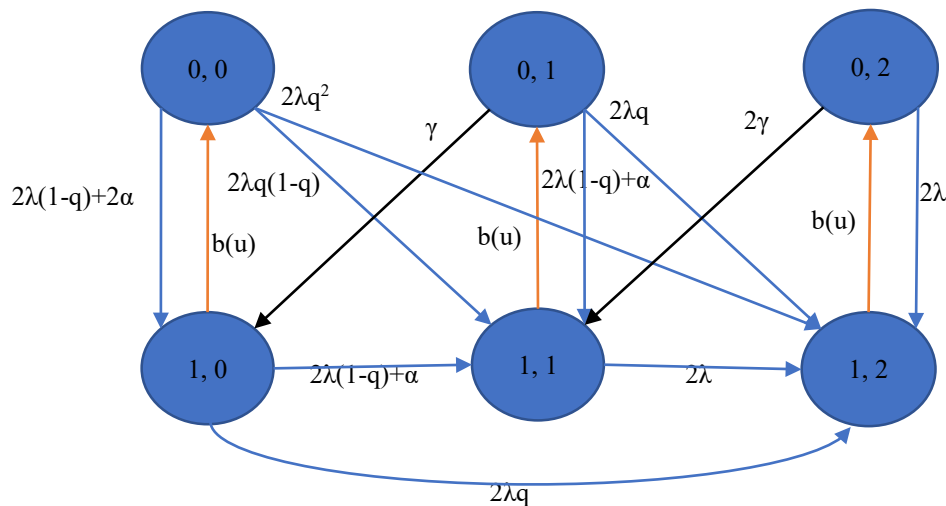


Fig. 3. State-transition-rate diagram of configuration 3

$$Av_2 = 1 - P_{1,1} = P_{0,0} + P_{0,1} + P_{1,0} = \frac{2\lambda q + \gamma}{\gamma} + \frac{(2\lambda + \alpha)(2\lambda + \gamma)}{2\lambda \gamma} \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] P_{0,0} \quad (10)$$

$$-\frac{d}{du}P_{1,0}(u) = (2\lambda(1-q) + 2\alpha)r(u)P_{0,0} - (2\lambda + \alpha)P_{1,0}(u) + \gamma r(u)P_{0,1} \quad (14)$$

$$-\frac{d}{du}P_{1,1}(u) = -2\lambda P_{1,1}(u) + 2\lambda q(1-q)r(u)P_{0,0} + (2\lambda(1-q) + \alpha)P_{1,0}(u) + (2\lambda(1-q) + \alpha)r(u)P_{0,1} + 2\gamma r(u)P_{0,2} \quad (15)$$

$$-\frac{d}{du}P_{1,2}(u) = 2\lambda q P_{1,0}(u) + 2\lambda P_{1,1}(u) + 2\lambda q^2 r(u)P_{0,0} + 2\lambda q r(u)P_{0,1} + 2\lambda r(u)P_{0,2} \quad (16)$$

$$\begin{aligned}
 Av_3 &= P_{0,0} + P_{0,1} + P_{0,2} + P_{1,0} + P_{1,1} \\
 &= \left\{ \frac{(2\lambda q + 2\gamma)(2\lambda q + \gamma)}{2\gamma^2} - \frac{2\lambda q(1-q)}{2\gamma} \right. \\
 &\quad - \frac{(2\lambda(1-q) + 2\alpha)(2\lambda + \alpha + \gamma)}{\gamma} \left(\frac{1}{2\lambda} + \frac{1}{2\gamma} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \\
 &\quad + \frac{(2\lambda + 2\alpha)(2\lambda(1-q) + \alpha)}{\alpha} \left(\frac{1}{2\lambda} + \frac{1}{2\gamma} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \\
 &\quad + \frac{(2\lambda + 2\alpha)(2\lambda q + \gamma)}{2\gamma^2} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \\
 &\quad + \frac{(2\lambda + 2\alpha)(2\lambda + \alpha + \gamma)}{\gamma(2\lambda + \alpha)} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \\
 &\quad - \frac{(2\lambda + 2\alpha)(2\lambda(1-q) + \alpha)(2\lambda + \alpha + 2\gamma)}{2\alpha\gamma(2\lambda + \alpha)} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \\
 &\quad \left. + \frac{(2\lambda + 2\alpha)(2\lambda + \alpha + \gamma)}{\gamma} \left(\frac{1}{2\lambda} + \frac{1}{2\gamma} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \frac{1}{R^*(2\lambda + \alpha)} \right] \right\} P_{0,0}
 \end{aligned} \tag{17}$$

where we defined $P_{0,0}(u) = r(u)P_{0,0}$, $P_{0,1}(u) = r(u)P_{0,1}$ and $P_{0,2}(u) = r(u)P_{0,2}$. Applying a similar argument, we can get the steady-state availability as Equation (17). The detailed derivation is given in Appendix C.

$$0 = -(3\lambda + 2\alpha)P_{0,0} + P_{1,0}(0) \tag{18}$$

$$0 = -(3\lambda + \alpha + \gamma)P_{0,1} + P_{1,1}(0) \tag{19}$$

$$0 = -(3\lambda + 2\gamma)P_{0,2} + P_{1,2}(0) \tag{20}$$

3.4 Configuration 4

For configuration 4, the diagram of state-transition-rate is shown in Fig. 4. Hence, based on the Fig. 4, the steady-state Equations (18)-(23) for configuration 4 are listed as below:

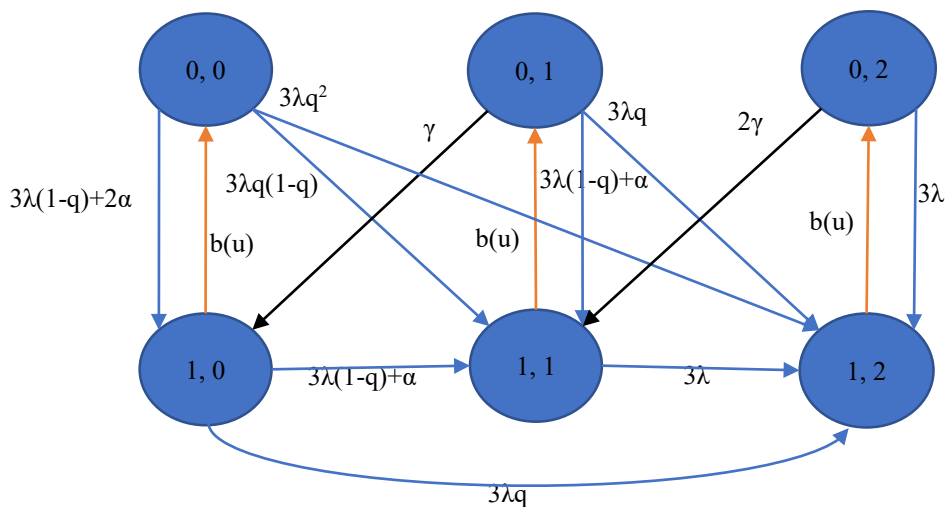


Fig. 4. State-transition-rate diagram of the configuration 4

$$-\frac{d}{du}P_{1,0}(u) = (3\lambda(1-q) + 2\alpha)r(u)P_{0,0} - (3\lambda + \alpha)P_{1,0}(u) + \gamma r(u)P_{0,1} \quad (21)$$

$$-\frac{d}{du}P_{1,1}(u) = (3\lambda(1-q) + \alpha)P_{1,0}(u) - 3\lambda P_{1,1}(u) + 3\lambda q(1-q)r(u)P_{0,0} + (3\lambda(1-q) + \alpha)r(u)P_{0,1} + 2\gamma r(u)P_{0,2} \quad (22)$$

$$-\frac{\partial}{\partial u}P_{1,2}(u) = 3\lambda qP_{1,0}(u) + 3\lambda P_{1,1}(u) + 3\lambda q^2r(u)P_{0,0} + 3\lambda qr(u)P_{0,1} + 3\lambda r(u)P_{0,2} \quad (23)$$

$$\begin{aligned} Av_4 &= P_{0,0} + P_{0,1} + P_{0,2} + P_{1,0} + P_{1,1} \\ &= \left\{ (3\lambda q + \gamma)(3\lambda q + 2\gamma) \frac{1}{2\gamma} \frac{1}{\gamma} - 3\lambda q(1-q) \frac{1}{2\gamma} \right. \\ &\quad - (3\lambda(1-q) + 2\alpha)(3\lambda + \alpha + \gamma) \left(\frac{1}{3\lambda} + \frac{1}{2\gamma} \right) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \\ &\quad + (3\lambda(1-q) + \alpha)(3\lambda + 2\alpha) \left(\frac{1}{3\lambda} + \frac{1}{2\gamma} \right) \frac{1}{\alpha} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \\ &\quad + (3\lambda + 2\alpha)(3\lambda q + \gamma) \frac{1}{2\gamma} \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \\ &\quad + (3\lambda + 2\alpha)(3\lambda + \alpha + \gamma) \frac{1}{(3\lambda + \alpha)} \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \\ &\quad - (3\lambda + 2\alpha)(3\lambda(1-q) + \alpha)(3\lambda + \alpha + 2\gamma) \frac{1}{(3\lambda + \alpha)} \frac{1}{2\gamma} \frac{1}{\alpha} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \\ &\quad \left. + (3\lambda + 2\alpha)(3\lambda + \alpha + \gamma) \left(\frac{1}{3\lambda} + \frac{1}{2\gamma} \right) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)R^*(3\lambda + \alpha)} \right] \right\} P_{0,0} \end{aligned} \quad (24)$$

where we defined $P_{0,0}(u) = r(u)P_{0,0}$, $P_{0,1}(u) = r(u)P_{0,1}$ and $P_{0,2}(u) = r(u)P_{0,2}$. Using a similar argument, the steady-state availability can be obtained as Equation (24). The detailed derivation is given in Appendix D.

4. COMPARISON OF THE FOUR CONFIGURATIONS

In this section, we compare the steady-state availability among four configurations with two different repair time distributions, Weibull and lognormal. We set the parameters $a = \sqrt{2}\mu/2$, $b = 2$ for Weibull distribution and $m = -\ln(\mu) - \frac{1}{2}$, $\sigma = 1$ for lognormal distribution, where μ is the repair rate.

4.1 Comparison of All Configurations Based on their Steady-State Availability

We provide the following cases to investigate the effects of various system parameters on the steady-state availability of four configurations.

- Case 1. Given $\alpha = 0.2\lambda$, $\mu = 1$, $q = 0.1$, $\gamma = 0.5$, varied the values of λ from 0.001 to 0.4.
- Case 2. Given $\lambda = 0.1$, $\alpha = 0.2\lambda$, $q = 0.2$, $\gamma = 0.5$, varied the values of μ from 0.5 to 2.
- Case 3. Given $\lambda = 0.1$, $\alpha = 0.2\lambda$, $\mu = 0.9$, $\gamma = 0.5$, varied the values of q from 0.1 to 0.9.
- Case 4. Given $\lambda = 0.5$, $\alpha = 0.2\lambda$, $\mu = 1$, $q = 0.1$, varied the values of γ from 0.1 to 2.

Tables 1-4 provide the numerical results of the steady-state availability for each configuration for cases 1-4, respectively. From these tables, based on the steady-state availability comparisons, one can find that configuration 1.

Table 1. Comparison of the configurations 1-4 for Av ($\alpha = 0.2\lambda$, $\mu = 1$, $q = 0.1$, $\gamma = 0.5$)

Scope of λ	Results
Weibull repair time	
$0.001 < \lambda < 0.0255$	$Av_1 > Av_3 > Av_2 > Av_4$
$0.0255 < \lambda < 0.4$	$Av_1 > Av_2 > Av_3 > Av_4$
Lognormal repair time	
$0.001 < \lambda < 0.0103$	$Av_1 > Av_3 > Av_4 > Av_2$
$0.0103 < \lambda < 0.0367$	$Av_1 > Av_3 > Av_2 > Av_4$
$0.0367 < \lambda < 0.4$	$Av_1 > Av_2 > Av_3 > Av_4$

Table 2. Comparison of the configurations 1-4 for Av ($\lambda = 0.1$, $\alpha = 0.2\lambda$, $q = 0.2$, $\gamma = 0.5$)

Scope of μ	Results
Weibull repair time	
$0.5 < \mu < 1.783$	$Av_1 > Av_2 > Av_3 > Av_4$
$1.783 < \mu < 2$	$Av_1 > Av_3 > Av_2 > Av_4$
Lognormal repair time	
$0.5 < \mu < 1.383$	$Av_1 > Av_2 > Av_3 > Av_4$
$1.383 < \mu < 2$	$Av_1 > Av_3 > Av_2 > Av_4$

Table 3. Comparison of the configurations 1-4 for Av ($\lambda = 0.1$, $\alpha = 0.2\lambda$, $\mu = 1$, $\gamma = 0.5$)

Scope of q	Results
Weibull repair time	
$0.1 < q < 0.402$	$Av_1 > Av_2 > Av_3 > Av_4$
$0.402 < q < 0.9$	$Av_1 > Av_3 > Av_2 > Av_4$
Lognormal repair time	
$0.1 < q < 0.317$	$Av_1 > Av_2 > Av_3 > Av_4$
$0.317 < q < 0.864$	$Av_1 > Av_3 > Av_2 > Av_4$
$0.864 < q < 0.9$	$Av_1 > Av_2 > Av_3 > Av_4$

Table 4. Comparison of the configurations 1-4 for Av ($\lambda = 0.5$, $\alpha = 0.2\lambda$, $\mu = 1$, $q = 0.1$)

Scope of γ	Results
Weibull repair time	
$0.1 < \gamma < 0.246$	$Av_1 > Av_3 > Av_2 > Av_4$
$0.246 < \gamma < 2$	$Av_1 > Av_2 > Av_3 > Av_4$
Lognormal repair time	
$0.1 < \gamma < 0.217$	$Av_1 > Av_3 > Av_2 > Av_4$
$0.217 < \gamma < 2$	$Av_1 > Av_2 > Av_3 > Av_4$

may be the best configuration. However, because each configuration consumes different costs during the construction process, as a standard for comparing these four configurations, the $cost/Av$ rate may be fairer than Av .

4.2 Comparison of All Configurations Based on their Cost/Benefit Ratios

We consider that the different configurations may have different costs. When different configurations are fairly compared, these costs should be considered. Table 5 lists the size-proportional costs for the primary generators and standby generators. From this table, the cost (C_i) for each configuration i ($i = 1, 2, 3, 4$) can be calculated and are given in Table 6. Next, we compare $cost/Av$ for each configuration by using the four aforementioned cases. Tables 7-10 depicts the results, respectively. We find from

Tables 7 and 8 that the optimal configuration based on $cost/Av$ value depends on the value of λ and μ . We observe that configuration 1 is the optimal configuration as $0.001 < \lambda < 0.0085$; configuration 3 is the optimal configuration as $0.0087 < \lambda < 0.1113$ or $1.683 < \mu < 2$; but when $0.1113 < \lambda < 0.4$ or $0.5 < \mu < 1.683$, configuration 4 is the optimal configuration. In the case of the repair time following the lognormal distribution, we find that configuration 1 is the optimal configuration as $0.001 < \lambda < 0.0085$; configuration 3 is the optimal configuration when $0.0087 < \lambda < 0.2851$ or $1.769 < \mu < 2$; but the optimal configuration 4 when $0.2851 < \lambda < 0.4$ or $0.5 < \mu < 1.769$. We can easily see from Table 9 that the optimal configuration based on $cost/Av$ value is configuration 3, and from Table 10, the optimal configuration is configuration 4.

Table 5. The costs for the primary generators and standby generators

Primary generators	Cost (in \$)	Standby generators	Cost (in \$)
24MW	6×10^6	24MW	5.4×10^6
12MW	3×10^6	12MW	2.7×10^6
8MW	2×10^6	8MW	1.8×10^6

Table 6. The costs for each configuration

Configuration	Cost (in \$)
1	11.4×10^6
2	8.7×10^6
3	11.4×10^6
4	9.6×10^6

Table 7. Comparison of the configurations 1-4 for cost/ Av ($\alpha = 0.2\lambda$, $\mu = 1$, $q = 0.1$, $\gamma = 0.5$)

Scope of λ	Results
Weibull repair time	
$0.001 < \lambda < 0.0085$	$C_1/Av_1 > C_3/Av_3 > C_4/Av_4 > C_2/Av_2$
$0.0085 < \lambda < 0.1113$	$C_3/Av_3 > C_1/Av_1 > C_4/Av_4 > C_2/Av_2$
$0.1113 < \lambda < 0.3135$	$C_4/Av_4 > C_3/Av_3 > C_1/Av_1 > C_2/Av_2$
$0.3135 < \lambda < 0.4$	$C_4/Av_4 > C_3/Av_3 > C_1/Av_1 > C_2/Av_2$
Lognormal repair time	
$0.001 < \lambda < 0.0087$	$C_1/Av_1 > C_3/Av_3 > C_4/Av_4 > C_2/Av_2$
$0.0087 < \lambda < 0.1158$	$C_3/Av_3 > C_1/Av_1 > C_4/Av_4 > C_2/Av_2$
$0.1158 < \lambda < 0.2851$	$C_3/Av_3 > C_4/Av_4 > C_1/Av_1 > C_2/Av_2$
$0.2851 < \lambda < 0.4$	$C_4/Av_4 > C_3/Av_3 > C_1/Av_1 > C_2/Av_2$

Table 8. Comparison of the configurations 1-4 for cost/ Av ($\lambda = 0.1$, $\alpha = 0.2\lambda$, $q = 0.2$, $\gamma = 0.5$)

Scope of μ	Results
Weibull repair time	
$0.5 < \mu < 1.283$	$C_4/Av_4 > C_3/Av_3 > C_2/Av_2 > C_1/Av_1$
$1.283 < \mu < 1.683$	$C_4/Av_4 > C_3/Av_3 > C_1/Av_1 > C_2/Av_2$
$1.683 < \mu < 2$	$C_3/Av_3 > C_4/Av_4 > C_1/Av_1 > C_2/Av_2$
Lognormal repair time	
$0.5 < \mu < 1.257$	$C_4/Av_4 > C_3/Av_3 > C_2/Av_2 > C_1/Av_1$
$1.257 < \mu < 1.769$	$C_4/Av_4 > C_3/Av_3 > C_1/Av_1 > C_2/Av_2$
$1.769 < \mu < 2$	$C_3/Av_3 > C_4/Av_4 > C_1/Av_1 > C_2/Av_2$

Table 9. Comparison of the configurations 1-4 for cost/ Av ($\lambda = 0.1$, $\alpha = 0.2\lambda$, $\mu = 1$, $\gamma = 0.5$)

Scope of q	Results
Weibull repair time	
$0.1 < q < 0.9$	$C_3/Av_3 > C_1/Av_1 > C_4/Av_4 > C_2/Av_2$
Lognormal repair time	
$0.1 < q < 0.9$	$C_3/Av_3 > C_1/Av_1 > C_4/Av_4 > C_2/Av_2$

Table 10. Comparison of the configurations 1-4 for cost/ Av ($\lambda = 0.5$, $\alpha = 0.2\lambda$, $\mu = 1$, $q = 0.1$)

Scope of γ	Results
Weibull repair time	
$0.1 < \gamma < 2$	$C_4/Av_4 > C_3/Av_3 > C_2/Av_2 > C_1/Av_1$
Lognormal repair time	
$0.1 < \gamma < 2$	$C_4/Av_4 > C_3/Av_3 > C_2/Av_2 > C_1/Av_1$

5. CONCLUSIONS

This research has studied the evaluation of cost-benefit of four standby retrial power supply configurations with standby switching failure and general repair times, which is the first work on the comparative investigation relative to retrial availability systems incorporating switching failure and general repair times. We utilized the supplementary variable method to derive the explicit expressions of the steady-state availability for each configuration and make the comparison. Finally, we ranked four configurations based on the steady-state availability and the cost/benefit ratio for two different repair time distributions, Weibull and lognormal. The numerical results revealed that the optimal configuration based on cost/benefit value depended on the values of λ and μ . The developed results can provide managers with decision reference for stable power supply system and cost reduction. In the future, we can lengthen this work to the fault of primary or standby generators may not be detected. The fellow researchers also can extend this investigation for an unreliable repair server.

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APPENDIX

Appendix A. Derivation of the steady-state availability for configuration 1.

From (1) and (2), we obtain

$$P_{1,0}(0) = (\lambda + \alpha) P_{0,0}, \quad (A.1)$$

$$P_{1,1}(0) = (\lambda + \gamma) P_{0,1}. \quad (A.2)$$

Further define

$$R^*(s) = \int_0^\infty e^{-su} dR(u),$$

$$P_{i,n}^*(s) = \int_0^\infty e^{-su} P_{i,n}(u) du,$$

$$P_{i,n} = P_{i,n}(0) = \int_0^\infty P_{i,n}(u) du,$$

$$\int_0^\infty e^{-su} \frac{d}{du} P_{i,n}(u) du = sP_{i,n}^*(s) - P_{i,n}(0).$$

After taking the Laplace-Stieltjes on both sides of (3) and (4), we have

$$(\lambda - s)P_{1,0}^*(s) = (\lambda(1-q) + \alpha)R^*(s)P_{0,0} + \gamma R^*(s)P_{0,1} - P_{1,0}(0), \quad (A.3)$$

$$-sP_{1,1}^*(s) = \lambda q R^*(s)P_{0,0} + \lambda R^*(s)P_{0,1} + \lambda P_{1,0}^*(s) - P_{1,1}(0). \quad (A.4)$$

Setting $s = \lambda$ and $s = 0$ into (A.3) and using (A.1), we have

$$P_{0,1} = \frac{1}{\gamma} \left\{ \lambda q + (\lambda + \alpha) \left[\frac{1 - R^*(\lambda)}{R^*(\lambda)} \right] \right\} P_{0,0}, \quad (A.5)$$

and

$$P_{1,0} = P_{1,0}^*(0) = \left(\frac{\lambda + \alpha}{\lambda} \right) \left[\frac{1 - R^*(\lambda)}{R^*(\lambda)} \right] P_{0,0}. \quad (A.6)$$

Setting $s = 0$ into (A.4), we have

$$0 = \lambda q P_{0,0} + \lambda P_{0,1} + \lambda P_{1,0} - P_{1,1}(0). \quad (A.7)$$

Differentiating (A.3) with respect to s and setting $s = 0$ in the result,

$$\lambda P_{1,0}^{*(1)}(0) = P_{1,0} - r_1 \left[(\lambda(1-q) + \alpha) P_{0,0} + \gamma P_{0,1} \right], \quad (A.8)$$

where $r_1 = -R^{*(1)}(0)$. Likewise, differentiating (A.4) with respect to s and setting $s = 0$ in the result,

$$P_{1,1} = P_{1,1}^*(0) = r_1 \left[\lambda q P_{0,0} + \lambda P_{0,1} \right] - \lambda P_{1,0}^{*(1)}(0). \quad (A.9)$$

After doing some manipulations, we have

$$P_{1,1} = \left\{ r_1 \left[(\lambda + \alpha) + \frac{\lambda q (\lambda + \gamma)}{\gamma} \right] + (\lambda + \alpha) \left(\frac{(\lambda + \gamma) r_1}{\gamma} - \frac{1}{\lambda} \right) \left[\frac{1 - R^*(\lambda)}{R^*(\lambda)} \right] \right\} P_{0,0}. \quad (A.10)$$

Substituting (A.5), (A.6) and (A.10) into the following normalizing condition $P_{0,0} + P_{0,1} + P_{1,0} + P_{1,1} = 1$, we obtain

$$P_{0,0} = \frac{\gamma R^*(\lambda)}{(\lambda + \alpha)(r_1(\lambda + \gamma) + 1) + [\gamma(r_1 \lambda q + 1) - (\lambda(1-q) + \alpha)(r_1 \lambda + 1)] R^*(\lambda)}. \quad (A.11)$$

Therefore, the steady-state availability can be expressed as

$$Av_1 = 1 - P_{1,1} = P_{0,0} + P_{0,1} + P_{1,0} = \left\{ \frac{\lambda q + \gamma}{\gamma} + \frac{(\lambda + \alpha)(\lambda + \gamma)}{\lambda \gamma} \left[\frac{1 - R^*(\lambda)}{R^*(\lambda)} \right] \right\} P_{0,0}. \quad (A.12)$$

Appendix B. Derivation of the steady-state availability for configuration 2.

From (6) and (7), we have

$$P_{1,0}(0) = (2\lambda + \alpha)P_{0,0}, \quad (B.1)$$

$$P_{1,1}(0) = (2\lambda + \gamma)P_{0,1}. \quad (B.2)$$

Taking the Laplace-Stieltjes on both sides of (8) and (9) and using (B.1) - (B.2),

$$(2\lambda - s)P_{1,0}^*(s) = (2\lambda(1-q) + \alpha)R^*(s)P_{0,0} + \gamma R^*(s)P_{0,1} - (2\lambda + \alpha)P_{0,0}, \quad (B.3)$$

$$-sP_{1,1}^*(s) = 2\lambda q R^*(s)P_{0,0} + 2\lambda P_{1,0}^*(s) + 2\lambda R^*(s)P_{0,1} - (2\lambda + \gamma)P_{0,1}. \quad (B.4)$$

Setting $s = 2\lambda$ and $s = 0$ into (B.3), we have

$$P_{0,1} = \frac{1}{\gamma} \left\{ 2\lambda q + (2\lambda + \alpha) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \right\} P_{0,0}, \quad (B.5)$$

and

$$P_{1,0} = P_{1,0}^*(0) = \left(\frac{2\lambda + \alpha}{2\lambda} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] P_{0,0}, \quad (B.6)$$

Differentiating (B.3) with respect to s and setting $s = 0$ in the result,

$$2\lambda P_{1,0}^{*(1)}(0) = P_{1,0} - r_1 \left[(2\lambda(1-q) + \alpha) P_{0,0} + \gamma P_{0,1} \right], \quad (B.7)$$

Likewise, differentiating (B.4) with respect to s and setting $s = 0$ in the result,

$$P_{1,1} = P_{1,1}^*(0) = r_1 \left[2\lambda q P_{0,0} + 2\lambda P_{0,1} \right] - 2\lambda P_{1,0}^{*(1)}(0). \quad (B.8)$$

Hence, we have

$$P_{1,1} = \left\{ (2\lambda + \alpha) r_1 + \frac{2\lambda q (2\lambda + \gamma) r_1}{\gamma} + (2\lambda + \alpha) \left[\frac{(2\lambda + \gamma) r_1}{\gamma} - \frac{1}{2\lambda} \right] \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \right\} P_{0,0}. \quad (B.9)$$

To find $P_{0,0}$, we substitute (B.5), (B.6) and (B.9) into the following normalizing condition $P_{0,0} + P_{0,1} + P_{1,0} + P_{1,1} = 1$ and obtain

$$P_{0,0} = \frac{\gamma R^*(2\lambda)}{(2\lambda + \alpha)(1 + r_1(2\lambda + \gamma)) + [\gamma(1 + 2\lambda q r_1) - (2\lambda r_1 + 1)(2\lambda(1-q) + \alpha)] R^*(2\lambda)} \quad (B.10)$$

We assumed that the state (1, 1) is a system down state. For configuration 2, we have

$$Av_2 = P_{0,0} + P_{0,1} + P_{1,0} = \frac{2\lambda q + \gamma}{\gamma} + \frac{(2\lambda + \alpha)(2\lambda + \gamma)}{2\lambda \gamma} \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] P_{0,0}. \quad (B.11)$$

Appendix C. Derivation of the steady-state availability for configuration 3.

From (11)-(13),

$$P_{1,0}(0) = (2\lambda + 2\alpha) P_{0,0}, \quad (C.1)$$

$$P_{1,1}(0) = (2\lambda + \alpha + \gamma) P_{0,1}, \quad (C.2)$$

$$P_{1,2}(0) = (2\lambda + 2\gamma) P_{0,2}. \quad (C.3)$$

Using Laplace transforms as before and using (C.1)-(C.3), (14)-(16) become

$$(2\lambda + \alpha - s) P_{1,0}^*(s) = (2\lambda(1-q) + 2\alpha) R^*(s) P_{0,0} - (2\lambda + 2\alpha) P_{0,0} + \gamma R^*(s) P_{0,1}, \quad (C.4)$$

$$(2\lambda - s) P_{1,1}^*(s) = 2\lambda q (1-q) R^*(s) P_{0,0} + (2\lambda(1-q) + \alpha) P_{1,0}^*(s) + (2\lambda(1-q) + \alpha) R^*(s) P_{0,1} - (2\lambda + \alpha + \gamma) P_{0,1} + 2\gamma R^*(s) P_{0,2}, \quad (C.5)$$

$$-s P_{1,2}^*(s) = 2\lambda q P_{1,0}^*(s) + 2\lambda P_{1,1}^*(s) + 2\lambda q^2 R^*(s) P_{0,0} + 2\lambda q R^*(s) P_{0,1} + 2\lambda R^*(s) P_{0,2} - (2\lambda + 2\gamma) P_{0,2}. \quad (C.6)$$

Setting $s = 2\lambda + \alpha$ and $s = 0$ into (C.4), we have

$$P_{0,1} = \frac{1}{\gamma} \left\{ 2\lambda q + (2\lambda + 2\alpha) \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \right\} P_{0,0}, \quad (C.7)$$

and

$$P_{1,0} = P_{1,0}^*(0) = \left(\frac{2\lambda + 2\alpha}{2\lambda + \alpha} \right) \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] P_{0,0} \quad (C.8)$$

Again, setting $s = 2\lambda$ into (C.4), we have

$$P_{1,0}^*(2\lambda) = \left(\frac{2\lambda + 2\alpha}{\alpha} \right) \left[\frac{R^*(2\lambda) - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] P_{0,0} \quad (C.9)$$

Setting $s = 2\lambda$ into (C.5) yields

$$P_{0,2} = \frac{1}{2\gamma} \left\{ -2\lambda q(1-q)P_{0,0} - (2\lambda(1-q) + \alpha) \left[\frac{1}{R^*(2\lambda)} \right] P_{1,0}^*(2\lambda) - (2\lambda(1-q) + \alpha)P_{0,1} + (2\lambda + \alpha + \gamma) \left[\frac{1}{R^*(2\lambda)} \right] P_{0,1} \right\}$$

It implies that

$$P_{0,2} = \frac{1}{2\gamma} \left\{ 2\lambda q^2(2\lambda + \gamma) \frac{1}{\gamma} + (2\lambda + 2\alpha)(2\lambda q + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] - (2\lambda(1-q) + \alpha)(2\lambda + 2\alpha) \frac{1}{\alpha} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] + (2\lambda(1-q) + \alpha)(2\lambda + 2\alpha) \frac{1}{\alpha} \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] - (2\lambda(1-q) + 2\alpha)(2\lambda + \alpha + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] + (2\lambda + 2\alpha)(2\lambda + \alpha + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)R^*(2\lambda + \alpha)} \right] \right\} P_{0,0} \quad (C.10)$$

Setting $s = 0$ into (C.5) yields

$$P_{1,1} = P_{1,1}^*(0) = \frac{1}{2\lambda} \left\{ (2\lambda(1-q) + \alpha)P_{1,0} + (2\lambda + \alpha + \gamma) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] P_{0,1} - (2\lambda(1-q) + \alpha)(2\lambda + 2\alpha) \frac{1}{\alpha} \left[\frac{R^*(2\lambda) - R^*(2\lambda + \alpha)}{R^*(2\lambda)R^*(2\lambda + \alpha)} \right] P_{0,0} \right\}$$

This implies that

$$P_{1,1} = \frac{1}{2\lambda} \left\{ (2\lambda(1-q) + \alpha)(2\lambda + 2\alpha) \frac{1}{2\lambda + \alpha} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] - (2\lambda(1-q) + \alpha)(2\lambda + 2\alpha) \frac{1}{\alpha} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \right\}$$

$$\begin{aligned}
 & + (2\lambda(1-q) + \alpha)(2\lambda + 2\alpha) \frac{1}{\alpha} \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \\
 & - (2\lambda(1-q) + 2\alpha)(2\lambda + \alpha + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \\
 & + (2\lambda + 2\alpha)(2\lambda + \alpha + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda) R^*(2\lambda + \alpha)} \right] \Big\} P_{0,0}
 \end{aligned} \tag{C.11}$$

Differentiating (C.4) with respect to s and setting $s = 0$ in the result,

$$(2\lambda + \alpha) P_{1,0}^{*(1)}(0) = -r_1 \left[(2\lambda(1-q) + 2\alpha) P_{0,0} + \gamma P_{0,1} \right] + P_{1,0} \tag{C.12}$$

Differentiating (C.5) with respect to s and setting $s = 0$, we have

$$\begin{aligned}
 2\lambda P_{1,1}^{*(1)}(0) &= P_{1,1} - r_1 \left[2\lambda q(1-q) P_{0,0} + (2\lambda(1-q) + \alpha) P_{0,1} + 2\gamma P_{0,2} \right] \\
 &+ (2\lambda(1-q) + \alpha) P_{1,0}^{*(1)}(0),
 \end{aligned} \tag{C.13}$$

Differentiating (C.6) with respect to s and setting $s = 0$ in the result,

$$P_{1,2} = P_{1,2}^*(0) = r_1 \left[(2\lambda + 2\alpha) P_{0,0} + (2\lambda + \alpha + \gamma) P_{0,1} + (2\lambda + 2\gamma) P_{0,2} \right] - P_{1,0} - P_{1,1} \tag{C.14}$$

Hence, we have

$$\begin{aligned}
 P_{1,2} &= \left\{ \frac{(2\lambda + \alpha)(2\lambda q + \gamma) r_1}{\gamma} + (2\lambda q + \alpha) r_1 + \frac{\lambda q^2 (2\lambda + \gamma)(2\lambda + 2\gamma) r_1}{\gamma^2} \right. \\
 &+ \frac{(2\lambda + 2\alpha)(2\lambda(1-q) + \alpha)}{\alpha} \left(\frac{(2\lambda + 2\gamma) r_1}{2\gamma} - \frac{1}{2\lambda} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \\
 &- \frac{(2\lambda(1-q) + 2\alpha)(2\lambda + \alpha + \gamma)}{\gamma} \left(\frac{(2\lambda + 2\gamma) r_1}{2\gamma} - \frac{1}{2\lambda} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \\
 &- \frac{2\lambda + 2\alpha}{2\lambda + \alpha} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] + \frac{(2\lambda + 2\alpha)(2\lambda + \alpha + \gamma) r_1}{\gamma} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \\
 &+ \frac{(2\lambda + 2\alpha)(2\lambda(1-q) + \alpha)}{\alpha} \left(\frac{1}{(2\lambda + \alpha)} - \frac{(2\lambda + 2\gamma) r_1}{2\gamma} \right) \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \\
 &+ \frac{(2\lambda + 2\alpha)(2\lambda q + \gamma)(2\lambda + 2\gamma) r_1}{2\gamma^2} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \\
 &\left. + \frac{(2\lambda + 2\alpha)(2\lambda + \alpha + \gamma)}{\gamma} \left(\frac{(2\lambda + 2\gamma) r_1}{2\gamma} - \frac{1}{2\lambda} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda) R^*(2\lambda + \alpha)} \right] \right\} P_{0,0}
 \end{aligned} \tag{C.15}$$

Using the following normalizing condition $P_{0,0} + P_{0,1} + P_{0,2} + P_{1,0} + P_{1,1} + P_{1,2} = 1$ and doing some algebraic manipulation, we can compute $P_{0,0}$. We assumed that the state (1, 2) is system down state. For configuration 3, the explicit expression for the steady-state availability is given by

$$Av_3 = P_{0,0} + P_{0,1} + P_{0,2} + P_{1,0} + P_{1,1}$$

$$\begin{aligned}
 &= \left\{ \frac{(2\lambda q + 2\gamma)(2\lambda q + \gamma)}{2\gamma^2} - \frac{2\lambda q(1-q)}{2\gamma} + \frac{(2\lambda + 2\alpha)(2\lambda q + \gamma)}{2\gamma^2} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \right. \\
 &+ \frac{(2\lambda + 2\alpha)(2\lambda + \alpha + \gamma)}{\gamma(2\lambda + \alpha)} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \\
 &- \frac{(2\lambda + 2\alpha)(2\lambda(1-q) + \alpha)(2\lambda + \alpha + 2\gamma)}{2\alpha\gamma(2\lambda + \alpha)} \left[\frac{1 - R^*(2\lambda + \alpha)}{R^*(2\lambda + \alpha)} \right] \\
 &- \frac{(2\lambda(1-q) + 2\alpha)(2\lambda + \alpha + \gamma)}{\gamma} \left(\frac{1}{2\lambda} + \frac{1}{2\gamma} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \\
 &+ \frac{(2\lambda + 2\alpha)(2\lambda(1-q) + \alpha)}{\alpha} \left(\frac{1}{2\lambda} + \frac{1}{2\gamma} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \right] \\
 &\left. + \frac{(2\lambda + 2\alpha)(2\lambda + \alpha + \gamma)}{\gamma} \left(\frac{1}{2\lambda} + \frac{1}{2\gamma} \right) \left[\frac{1 - R^*(2\lambda)}{R^*(2\lambda)} \frac{1}{R^*(2\lambda + \alpha)} \right] \right\} P_{0,0}
 \end{aligned} \tag{C.16}$$

Appendix D. Derivation of the steady-state availability for configuration 4.

From (18)-(20), we have

$$P_{1,0}(0) = (3\lambda + 2\alpha)P_{0,0}, \tag{D.1}$$

$$P_{1,1}(0) = (3\lambda + \alpha + \gamma)P_{0,1}, \tag{D.2}$$

$$P_{1,2}(0) = (3\lambda + 2\gamma)P_{0,2}. \tag{D.3}$$

Using Laplace transforms as before and using (D.1) - (D.3), (21) - (23) reduce to

$$(3\lambda + \alpha - s)P_{1,0}^*(s) = (3\lambda(1-q) + 2\alpha)R^*(s)P_{0,0} - (3\lambda + 2\alpha)P_{0,0} + \gamma R^*(s)P_{0,1}, \tag{D.4}$$

$$\begin{aligned}
 (3\lambda - s)P_{1,1}^*(s) &= (3\lambda(1-q) + \alpha)P_{1,0}^*(s) + 3\lambda q(1-q)R^*(s)P_{0,0} + 2\gamma R^*(s)P_{0,2} \\
 &+ (3\lambda(1-q) + \alpha)R^*(s)P_{0,1} - (3\lambda + \alpha + \gamma)P_{0,1},
 \end{aligned} \tag{D.5}$$

$$\begin{aligned}
 -sP_{1,2}^*(s) &= 3\lambda qP_{1,0}^*(s) + 3\lambda P_{1,1}^*(s) + 3\lambda q^2 R^*(s)P_{0,0} + 3\lambda q R^*(s)P_{0,1} \\
 &+ 3\lambda R^*(s)P_{0,2} - (3\lambda + 2\gamma)P_{0,2}.
 \end{aligned} \tag{D.6}$$

Setting $s = 3\lambda + \alpha$ and $s = 0$ into (D.4), we have

$$P_{0,1} = \frac{1}{\gamma} \left\{ 3\lambda q + (3\lambda + 2\alpha) \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \right\} P_{0,0}, \tag{D.7}$$

and

$$P_{1,0} = P_{1,0}^*(0) = \left(\frac{3\lambda + 2\alpha}{3\lambda + \alpha} \right) \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] P_{0,0} \tag{D.8}$$

Again, setting $s = 3\lambda$ into (D.4), it implies that

$$P_{1,0}^*(3\lambda) = \frac{(3\lambda + 2\alpha)}{\alpha} \left[\frac{R^*(3\lambda) - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] P_{0,0} \tag{D.9}$$

Setting $s = 3\lambda$ into (D.5), we get

$$P_{0,2} = \frac{1}{2\gamma} \left\{ -3\lambda q(1-q)P_{0,0} - (3\lambda(1-q) + \alpha) \frac{1}{R^*(3\lambda)} P_{1,0}^* (3\lambda) \right. \\ \left. - (3\lambda(1-q) + \alpha) P_{0,1} + (3\lambda + \alpha + \gamma) \frac{1}{R^*(3\lambda)} P_{0,1} \right\}.$$

After doing some manipulations, we obtain

$$P_{0,2} = \frac{1}{2\gamma} \left\{ 3\lambda q(3\lambda q + \gamma) \frac{1}{\gamma} - 3\lambda q(1-q) \right. \\ \left. - (3\lambda(1-q) + \alpha) \frac{(3\lambda + 2\alpha)}{\alpha} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \right. \\ \left. + (3\lambda + 2\alpha)(3\lambda q + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \right. \\ \left. + (3\lambda(1-q) + \alpha) \frac{(3\lambda + 2\alpha)}{\alpha} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \right. \\ \left. - (3\lambda(1-q) + 2\alpha)(3\lambda + \alpha + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \right. \\ \left. + (3\lambda + 2\alpha)(3\lambda + \alpha + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \frac{1}{R^*(3\lambda + \alpha)} \right] \right\} P_{0,0}. \quad (D.10)$$

Setting $s = 0$ into (D.5), we get

$$P_{1,1} = P_{1,1}^*(0) = \frac{1}{3\lambda} \left\{ (3\lambda(1-q) + \alpha) P_{1,0} + (3\lambda + \alpha + \gamma) \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] P_{0,1} \right. \\ \left. - (3\lambda(1-q) + \alpha) \frac{(3\lambda + 2\alpha)}{\alpha} \left[\frac{R^*(3\lambda) - R^*(3\lambda + \alpha)}{R^*(3\lambda) R^*(3\lambda + \alpha)} \right] P_{0,0} \right\}.$$

It implies that

$$P_{1,1} = \frac{1}{3\lambda} \left\{ (3\lambda(1-q) + \alpha) \frac{(3\lambda + 2\alpha)}{\alpha} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \right. \\ \left. - (3\lambda(1-q) + 2\alpha)(3\lambda + \alpha + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \right. \\ \left. - (3\lambda(1-q) + \alpha)(3\lambda + 2\alpha) \frac{3\lambda}{\alpha(3\lambda + \alpha)} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \right. \\ \left. + (3\lambda + 2\alpha)(3\lambda + \alpha + \gamma) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda) R^*(3\lambda + \alpha)} \right] \right\} P_{0,0}. \quad (D.11)$$

Differentiating (D.4) with respect to s and setting $s = 0$ in the result, we finally get

$$(3\lambda + \alpha)P_{1,0}^{*(1)}(0) = P_{1,0} - r_1 \left[(3\lambda(1-q) + 2\alpha)P_{0,0} + \gamma P_{0,1} \right], \quad (D.12)$$

Differentiating (D.5) with respect to s and setting $s = 0$ in the result, we obtain

$$3\lambda P_{1,1}^{*(1)}(0) = (3\lambda(1-q) + \alpha)P_{1,0}^{*(1)}(0) + P_{1,1} - r_1 \left[3\lambda q(1-q)P_{0,0} + 2\gamma P_{0,2} + (3\lambda(1-q) + \alpha)P_{0,1} \right], \quad (D.13)$$

Similarly, differentiating (D.6) with respect to s and setting $s = 0$ in the result, we find that

$$P_{1,2} = P_{1,2}^*(0) = r_1 \left[(3\lambda + 2\alpha)P_{0,0} + (3\lambda + \alpha + \gamma)P_{0,1} + (3\lambda + 2\gamma)P_{0,2} \right] - P_{1,0} - P_{1,1}. \quad (D.14)$$

Finally, yields

$$\begin{aligned} P_{1,2} = & \left\{ \frac{3\lambda q(3\lambda + \alpha + \gamma)r_1}{\gamma} + \frac{3\lambda q^2(3\lambda + \gamma)(3\lambda + 2\gamma)r_1}{2\gamma^2} + (3\lambda + 2\alpha)r_1 \right. \\ & + \frac{(3\lambda + 2\alpha)(3\lambda(1-q) + \alpha)}{\alpha} \left(\frac{(3\lambda + 2\gamma)r_1}{2\gamma} - \frac{1}{3\lambda} \right) \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \\ & - \frac{(3\lambda(1-q) + 2\alpha)(3\lambda + \alpha + \gamma)}{\gamma} \left(\frac{(3\lambda + 2\gamma)r_1}{2\gamma} - \frac{1}{3\lambda} \right) \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \\ & + \frac{(3\lambda + 2\alpha)(3\lambda q + \gamma)(3\lambda + 2\gamma)r_1}{2\gamma^2} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \\ & + \frac{(3\lambda + 2\alpha)(3\lambda + \alpha + \gamma)r_1}{\gamma} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] - \frac{3\lambda + 2\alpha}{3\lambda + \alpha} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \\ & + \frac{(3\lambda + 2\alpha)(3\lambda(1-q) + \alpha)}{\alpha} \left(\frac{1}{(3\lambda + \alpha)} - \frac{(3\lambda + 2\gamma)r_1}{2\gamma} \right) \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \\ & \left. + \frac{(3\lambda + 2\alpha)(3\lambda + \alpha + \gamma)}{\gamma} \left(\frac{(3\lambda + 2\gamma)r_1}{2\gamma} - \frac{1}{3\lambda} \right) \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)R^*(3\lambda + \alpha)} \right] \right\} P_{0,0}. \end{aligned} \quad (D.15)$$

Using the normalizing condition $P_{0,0} + P_{0,1} + P_{0,2} + P_{1,0} + P_{1,1} + P_{1,2} = 1$, we can compute $P_{0,0}$. We assume that the state (1, 2) is system down state. For configuration 4, the explicit expression for the steady-state availability is given by:

$$\begin{aligned} Av_4 = & P_{0,0} + P_{0,1} + P_{0,2} + P_{1,0} + P_{1,1} \\ = & \frac{3\lambda q + \gamma}{\gamma} + 3\lambda q^2(3\lambda + \gamma) \frac{1}{2\gamma^2} + (3\lambda + 2\alpha)(3\lambda q + \gamma) \frac{1}{2\gamma} \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \\ & + (3\lambda + 2\alpha)(3\lambda + \alpha + \gamma) \frac{1}{(3\lambda + \alpha)} \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \\ & - (3\lambda + 2\alpha)(3\lambda(1-q) + \alpha)(3\lambda + \alpha + 2\gamma) \frac{1}{(3\lambda + \alpha)} \frac{1}{2\gamma} \frac{1}{\alpha} \left[\frac{1 - R^*(3\lambda + \alpha)}{R^*(3\lambda + \alpha)} \right] \end{aligned}$$

$$\begin{aligned}
 & - (3\lambda(1-q) + 2\alpha)(3\lambda + \alpha + \gamma) \left(\frac{1}{3\lambda} + \frac{1}{2\gamma} \right) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \\
 & + (3\lambda(1-q) + \alpha)(3\lambda + 2\alpha) \left(\frac{1}{3\lambda} + \frac{1}{2\gamma} \right) \frac{1}{\alpha} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda)} \right] \\
 & + (3\lambda + 2\alpha)(3\lambda + \alpha + \gamma) \left(\frac{1}{3\lambda} + \frac{1}{2\gamma} \right) \frac{1}{\gamma} \left[\frac{1 - R^*(3\lambda)}{R^*(3\lambda) R^*(3\lambda + \alpha)} \right] \Bigg\} P_{0,0} .
 \end{aligned}
 \tag{D.16}$$